Diffraction cone shrinkage speed up with the collision energy

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The multiperipheral ladder structure of the Pomeron leads to the quite natural conclusion that the elastic slope, $B_{\rm el}$, is not a simple linear function of the logarithm of the collision energy. The existing experimental data on the diffraction cone shrinkage provide evidence in favor of such "complicated" energy dependence. An increase of the diffraction cone shrinkage with the beam energy is directly connected with the extreme rise of the total cross section (Froissart limit).

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I. INTRODUCTION

The high-energy behavior of the hadron-hadron scattering is usually described by the Pomeron exchange. A popular parametrization of the elastic scattering amplitude at small momentum transfer takes into account only the Reggeon and Pomeron poles exchanges. The ab elastic scattering amplitude reads

$$T_{ab}(t) = F_a(t)F_b(t)C_P s^{\alpha_P(t)} + F_R(t)C_R s^{\alpha_R(t)}, \qquad (1)$$

where the form factors F_a, F_b, F_R describe the hadronic matter distribution in the incoming hadrons a, b. C_P and C_R are the normalization constants. The contribution of the secondary Reggeon poles [last term in (1)] becomes negligible at $\sqrt{s} \sim 100$ GeV.

From the microscopic point of view the Pomeron is described by the ladder-type diagrams where the energy (longitudinal momentum fraction) in each next cell is a few times smaller than that in the previous cell.[1] To reach the largest cross section we have to consider the chain (sequence) of strong interactions with relatively low partial subenergies. Such a sequence of interactions allows a large cross section which does not decrease with energy $\sigma \propto s^{\alpha_P(0)-1}$.

On another hand, at each step the interaction radius, ρ, changes by the value $\delta\rho \sim 1/k_t$ leading to the "diffusion" in the impact parameter plane. At each step the energy of incoming particle diminishes a few times. Thus, the number of steps is $n \sim \ln s$, and the final radius is $R^2 = R_0^2 + n \cdot (\delta\rho)^2$. Therefore, the Pomeron trajectory $\alpha_P(t)$ depends on the transverse momentum $t = -q_t^2$, and for a not large $|t|$ it can be written as $\alpha_P(t) = 1 + \epsilon + \alpha'_P t$.

Correspondingly, the elastic ab cross section takes the form

$$\frac{d\sigma_{ab}}{dt} = \frac{\sigma_0^2}{16\pi} F_a^2(t) F_b^2(t) \left(\frac{s}{s_0}\right)^{2\epsilon + 2\alpha'_P t}. \qquad (2)$$

The power growth of the "single Pomeron exchange" cross section generated by the ladder diagram reflects the rise of the parton multiplicity, N. Since at each (ladder) step the longitudinal momentum decreases by a few times the mean number of steps $\langle n\rangle \sim c * \ln s$. At each splitting (step) the parton multiplicity increases by a factor 2. Thus, the final parton multiplicity $N \sim 2^{c\ln s} = s^{c\ln 2}$.

The slope of a Pomeron trajectory α'_P accounts for the growth of interaction radius. This is caused by a long chain of intermediate (relatively low energy) interactions whose length increases with $\ln s$. In the case of Gaussian form factors $F_a^2 F_b^2 = \exp(B_0 t)$, we arrive at the slope of elastic cross section $d\sigma/dt = |T(t)|^2/16\pi s^2 \propto \exp(B_{\rm el} t)$

$$B_{\rm el} = B_0 + 2\alpha'^{\rm eff}_P \ln(s/s_0). \qquad (3)$$

While the first term B_0 in (3) depends on the type of the incoming hadrons a and b, the second term $2\alpha'^{\rm eff}_P \ln(s/s_0)$ is universal. In the case of the one-Pomeron exchange it should be the same at any energy and for any type of incoming hadron. This universality was confirmed in the fixed target experiments [2] ($\sqrt{s} < 25$ GeV), where the value of $\alpha'_P = 0.14$ GeV^{-2} was measured.[2]

Donnachie and Landshoff [3] [see Eq. (7)] from the analysis of the $d\sigma_{\rm el}/dt$ distribution, measured at $\sqrt{s} = 52.8$ GeV [4], have obtained much larger value of $\alpha'_P = 0.25$ GeV^{-2}.

II. MORE COMPLICATED BEHAVIOR OF THE SLOPE $B_{\rm el}(s)$

The growth of $\alpha'^{\rm eff}_P$ was by no means unexpected. Recall that in the impact parameter, ρ, representation the amplitude[3]

$$T(\rho) = \frac{1}{8\pi^2 s} \int T(q_t) \exp(i\vec{q}_t\vec{\rho}) d^2 q_t \qquad (4)$$

should satisfy the unitarity equation

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[1] Such a multiperipheral ladder structure of the Reggeon was considered first in [1].

[2] At not too large fixed target energies it was important to account for the secondary Reggeon contribution in the fit [2].

[3] Here we use the normalization $\mathrm{Im}T(t=0) = s\sigma_{\rm tot}$.

$$2\,\mathrm{Im}T(\rho) = |T(\rho)|^2 + G_{\mathrm{inel}}(\rho), \tag{5}$$

with $-t = q_t^2$, while G_{inel} denotes the contribution of all inelastic channels. The solution of (5) reads

$$T(\rho) = i(1 - \exp(-\Omega(\rho, s)/2)). \tag{6}$$

In terms of $T(\rho)$ the total and elastic cross sections and the elastic slope can be written as

$$\sigma_{\mathrm{tot}} = 2\int \mathrm{Im}T(\rho)d^2\rho, \tag{7}$$

$$\sigma_{\mathrm{el}} = \int |T(\rho)|^2 d^2\rho, \tag{8}$$

$$B_{\mathrm{el}}(t=0) = \frac{\int \rho^2 T(\rho)d^2\rho}{2\int T(\rho)d^2\rho} = \langle\rho^2\rangle/2. \tag{9}$$

When the optical (parton) density, i.e. the opacity $\Omega(\rho, s)$, becomes too large we have to account for the multiple interactions which are described by the multi-Pomeron diagrams. Similar to the case of nuclear-nuclear AA collisions, where few nucleon-nucleon pairs can interact simultaneously and screen each other, the corresponding absorptive corrections terminate the growth of elastic amplitude near the black disk limit, when in the impact parameter representation the imaginary part of the elastic scattering amplitude $\mathrm{Im}T(\rho) \to 1$.[4]

Note that while at the center of the disk (small ρ) the amplitude saturates at $\mathrm{Im}T = 1$, it still continues to rise with energy at the periphery (large ρ), leading to the increase of the mean interaction radius and, thus, to the growth of the elastic slope B_{el}. Another way to see the energy variation of B_{el} is to consider the two first diagrams—the one-Pomeron exchange and the two-Pomeron cut. As compared to the one-Pomeron exchange, the two-Pomeron contribution slowly falls down with $-t$. This is because when using the two (few) Pomerons we can distribute the whole transferred momentum more homogeneously between the components (partons) of the initial hadron. However the two-Pomeron amplitude describes the absorptive correction which has the sign opposite to that of the one-Pomeron exchange. Therefore the t dependence of the whole amplitude becomes steeper, and the slope B_{el} increases in the case of $\alpha_P(0) > 1$, when at larger energies the relative contribution of the two-Pomeron cut increases.

Therefore, the effective shrinkage of the diffractive cone is described by the value of $\alpha_P'^{\mathrm{eff}}$ which accounts for both the growth of the radius of individual Pomeron (α_P' of the "bare" Pomeron trajectory) and the decrease (slower increase in comparison with the periphery) of the optical density in the center of the disk due to the absorptive effects which lead to the radius growth with energy. So $\alpha_P'^{\mathrm{eff}} > \alpha_P'$.

[4]These multi-Pomeron diagrams are generated just by the s-channel two particle unitarity; the value of Ω is described by the one-Pomeron (ladder) exchange.

III. THE INCREASE $\alpha_P'^{\mathrm{eff}}$ WITH THE COLLISION ENERGY

Let us emphasize that since the absolute value of the amplitude $T(\rho)$ is limited ($|T(\rho)| < 2$) by the unitarity condition (5), at asymptotically high energies the elastic slope B_{el} cannot grow more slowly than the total cross section. In particular, in the case of a step function $T(\rho) = i\Theta(R-\rho)$ from (7) and (9) we obtain $\sigma_{\mathrm{tot}} = 2\pi R^2$ and $B_{\mathrm{el}} = R^2/4$, that is $B_{\mathrm{el}} = \sigma_{\mathrm{tot}}/8\pi$. At present energies we are not reaching the black disk limit yet, and thus there should be the inequality $B_{\mathrm{el}} > \sigma_{\mathrm{tot}}/8\pi$.

It is evident that the conventional formula for the slope (3) with a constant $\alpha_P'^{\mathrm{eff}}$ is inconsistent with the high energy behavior of the pp cross section, which is described either by the power, $\sigma_{\mathrm{tot}} = \sigma_0(s/s_0)^{.08}$ [5], or by the logarithmic,

$$\sigma_{\mathrm{tot}} = \sigma_0 + c_1\ln(s/s_0) + c_2\ln^2(s/s) \tag{10}$$

(see e.g. [6]), s dependence. Therefore, we have to expect that the value of $\alpha_P'^{\mathrm{eff}}$ increases with energy.

Figure 1 shows the measured values of the elastic t slope $B_{\mathrm{el}}(s)$ (NA8-Gatchina-Cern [2], ISR [4], UA4 [7], CDF [8]), including the new TOTEM result [9].[5]

One can clearly see that the value of $\alpha_P'^{\mathrm{eff}}$ does grow with energy. Such a feature was already mentioned in the CDF [8] publication.

In spite of the fact that the elastic proton-proton amplitude does not reach the black disk limit, the cross section is well described by the logarithmic formula (10) which, contrary to the power behavior [5], satisfies the Froissart

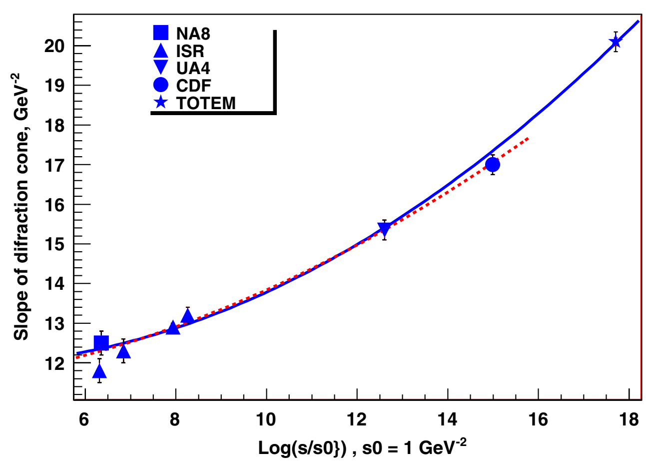

FIG. 1 (color online). The existing measurements of the diffractive cone slope B_{el}. Results of the data fit with the formulae $B_{\mathrm{el}} = B_0 + b_2\ln^2(s/s_0)$ are also shown: the full line reflects the fit with *all* available data, and broken line corresponds to the fit without TOTEM point.

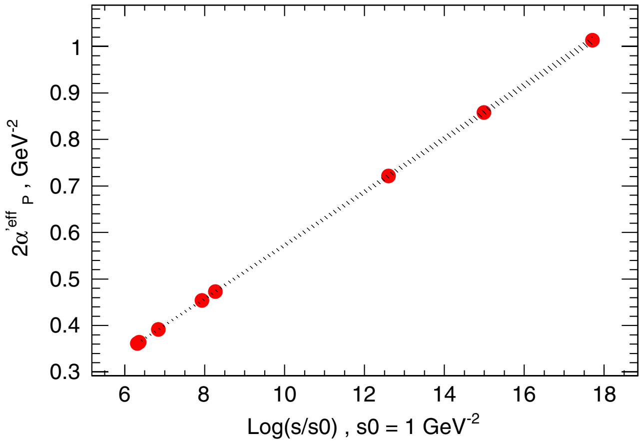

FIG. 2 (color online). The energy dependence of $2\alpha_P'^{\text{eff}}$.

(unitarity) limit. Therefore, we are fitting the energy dependence of the elastic slope, $B_{\text{el}}(s)$ by the second order polynomial

$$B_{\text{el}} = B_0 + b_1 \ln(s/s_0) + b_2 \ln^2(s/s_0). \qquad (11)$$

Asymptotically, the leading $\ln^2(s/s_0)$ term reflects the growth of the interaction radius, $R = R_0 + c_R \ln(s/s_0)$, of the step function $\text{Im}T(\rho) = \Theta(R - \rho)$, while the linear, $b_1 \ln(s/s_0)$ and the constant pre-asymptotic terms account for the starting value, R_0, and the "diffusion" form of edge of our "step function." Recall that the coefficient b_2 (and the analogous coefficient c_2 in the expression for the total cross section in Sec. IV) does not depend on the value of s_0. Changing s_0 we only redefine the coefficients B_0 and b_1. Moreover, at a given beam energy the value of $2\alpha_P'^{\text{eff}} = dB_{\text{el}}/d(\ln(s/s_0))$ is also independent of the scale s_0.

Fitting the $\ln s$ dependence of B_{el} by the second order polynomial (11) we arrive at $b_1 = (-.22 \pm .17)$ GeV^{-2} and $b_2 = (.037 \pm .006)$ GeV^{-2} with a good $\chi^2/\text{NoF} = 7.5/5$, while the fit with the linear function is unacceptable $\chi^2/\text{NoF} = 37.8/6$. In all fits we use $s_0 = 1$ GeV^2.

Note that in the case of $s_0 = 1$ GeV^2 the value of b_1 is consistent with zero. The exclusion of this parameter and the fit with the function

$$B_{\text{el}} = B_0 + b_2 \ln^2(s/s_0) \qquad (12)$$

gives

$$b_2 = (0.028\,60 \pm 0.000\,50)\ \text{GeV}^{-2} \qquad (13)$$

and does not change the statistical significance: $\chi^2/\text{NoF} = 9.2/6$ against of $\chi^2/\text{NoF} = 7.5/5$.

The energy dependence of the $2\alpha_P'^{\text{eff}} = dB_{\text{el}}/d(\ln(s/s_0))$ is shown in Fig. 2.

[5]The NA8 and ISR experiments performed measurements at very small $|t|$ (less than .05 GeV^2). The t range of UA4 and TOTEM was wider ($|t|_{\min} \sim .02$ GeV^2). However, it was proven that the t dependence of the elastic cross section is well described by the simple exponent $\propto \exp(B_{\text{el}} t)$.

IV. FROISSART LIMIT FOR THE DIFFRACTIVE CONE SHRINKAGE

Let us compare the behavior of the slope B_{el} and that of the total pp cross section in the asymptotic black disk (Froissart) limit, when $\sigma_{\text{tot}} = 2\pi R^2$ and $B_{\text{el}} = R^2/4$ (here R is the black disk radius).

The recent fit $\sigma_{\text{tot}} = \sigma_0 + c_1 \ln(s/s_0) + c_2 \ln^2(s/s_0)$ gives $c_2 = (0.2817 \pm 0.0064)$ mb (see Table 1 of [10]), while from $b_2 = 0.037$ GeV^2 we get $c_2(B_{\text{el}}) = 0.375$ mb and from $b_2 = (0.0286 \pm 0.0005)$ GeV^2, obtained in the two-parameter fit, we get a very close value—$c_2(B_{\text{el}}) = (0.294 \pm 0.005)$ mb. This demonstrates the current uncertainty in the coefficient c_2 extracted from the elastic slope behavior. Of course, even at the LHC we are rather far from the complete black disk limit. The proton is still relatively transparent, and the cross section σ_{tot} is less than its geometric value $2\pi R^2$.

However, it is interesting that both the elastic t slope and the total cross section reveal the same $\ln^2 s$ high energy behavior. Starting from the elastic slope we find from the coefficient b_2 the value of c_2, which is close to that obtained from the total cross sections.

The nontrivial fact is that at $\sqrt{s} = 24$ GeV the values of $2\alpha_P'^{\text{eff}} = (0.26 \pm 0.17)$ GeV^{-2} for the 3-parameters fit or $2\alpha_P'^{\text{eff}} = (0.364 \pm 0.003)$ GeV^{-2} for the two-parameter fit are similar to $2\alpha_P' = (0.28 \pm 0.03)$ GeV^{-2} found in the Regge Poles analysis of the "low energy" elastic scattering [2].

One could argue, that our conclusion about the nonlinear $\ln(s)$ behavior of the elastic slope, B_{el}, is based (besides the Regge *theory*) on the only *one measurement* TOTEM [9]. However, the 2-parameter (constant and $\ln^2(s)$) fit of the data *without* the TOTEM point describes the data very well ($\chi^2/\text{NoF} = 4.3/5$) with $b_2 = (0.0257 \pm 0.0013)$ GeV^{-2}. The 2-parameter fit with the linear $\ln(s)$ describes the data with similar statistical confidence ($\chi^2/\text{NoF} = 3.9/5$) with $b_1 = (0.556 \pm 0.030)$ GeV^{-2}. It is just the new TOTEM (LHC) data which justify the presence of the $\ln^2 s$ term and exclude the linear $\ln s$ dependence of the slope B_{el}. This indicates that in the energy region $\sqrt{s} = 2$–7 TeV the role of multi-Pomeron contributions strongly increases.

The multi-Pomeron effects should reveal themselves not only in the elastic scattering but also in the multiparticle production (see the discussion in [11]).

Recall that the recent Donnachie-Landshoff fit [12] includes *two* Pomeron poles. The pole with high intercept $\epsilon = 0.362$ and the pole with $\epsilon = 0.093$. Each of these "effective" poles should produce its own secondaries, and it would be important to observe the two different power of s in the behavior of the inclusive cross sections, $d\sigma/d^3p$, and in the two-particle correlations, including the Bose-Einstein correlations where these two poles will act as two different sources of secondary mesons. Since the slope of the trajectory with a higher intercept is smaller than that for the pole with $\epsilon = 0.093$, we expect that the emission size corresponding to the pole with $\epsilon = 0.362$ should be smaller as well. The authors [12], however, have mentioned that their description of "the TOTEM elastic scattering data is not perfect."

Only the LHC can investigate this energy region performing the energy scan in the way previously realized at the S$p\bar{p}$S collider. We have to measure the total, total-inelastic and elastic cross sections, together with the real part of the elastic scattering amplitude at $|t| = 0$ and the slope of the diffractive cone. Such measurements are being planned, but one should stress that such high precision experiments have to be done at several values of $\sqrt{s}$, starting from 900 GeV up to the highest energy possible.

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