

Operator product expansion coefficient functions in terms of composite operators only: Singlet case

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A method for calculating coefficient functions of the operator product expansion, which was previously derived for the nonsinglet case, is generalized for the singlet coefficient functions. The resulting formula defines coefficient functions entirely in terms of corresponding singlet composite operators without applying to elementary (quark and gluon) fields. Both “diagonal” and “nondiagonal” gluon coefficient functions in the product expansion of two electromagnetic currents are calculated in QCD. Their renormalization properties are studied.

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I. INTRODUCTION

The light-cone (LC) operator product expansion (OPE) [1] (see also [2,3]) permanently receives much attention, since it enables us to separate contributions to cross sections coming from large and small distances in a variety of processes. It was proposed as a generalization of the OPE at short distances [4] in order to describe deep inelastic scattering (DIS) of leptons off nucleons.

In Refs. [5] the T -product of two scalar currents near the LC was defined in term of so-called bi-local light-ray

composite fields. Later on it was shown that the local LC expansion can be obtained by performing a Taylor expansion of the nonlocal one [6]. The nonlocal expansion is more general, but in the present paper we restrict ourselves to considering standard local OPE.

The OPE for the T -product of two electromagnetic currents is of particular importance to practical application. It can be written in the form (see, for instance, [7]):

$$TJ_{\mu}^{\text{em}}(x)J_{\nu}^{\text{em}}(0) = -g_{\mu\nu} \left\{ \sum_{m=2}^{\infty} \sum_{l=1}^m C_{m,l}^{NS}(x^2) \frac{i^m}{m!} x^{\mu_1} \dots x^{\mu_m} \frac{1}{6} \left[O_{NS,\mu_1\dots\mu_m}^{3,m,l}(0) + \frac{1}{\sqrt{3}} O_{NS,\mu_1\dots\mu_m}^{8,m,l}(0) \right] \right. \\ \left. + \frac{2}{9} \left[\sum_{m=2}^{\infty} \sum_{l=1}^m C_{m,l}^F(x^2) \frac{i^m}{m!} x^{\mu_1} \dots x^{\mu_m} O_{F,\mu_1\dots\mu_m}^{m,l}(0) + \sum_{m=2}^{\infty} \sum_{l=1}^{m-1} C_{m,l}^V(x^2) \frac{i^m}{m!} x^{\mu_1} \dots x^{\mu_m} O_{V,\mu_1\dots\mu_m}^{m,l}(0) \right] \right\} + \dots, \quad (1)$$

where the dots denote contributions from other Lorentz structures. This expansion contains both nonsinglet (triplet $O_{NS,\mu_1\dots\mu_m}^{3,m,l}$ and octet $O_{NS,\mu_1\dots\mu_m}^{8,m,l}$) composite operators and singlet (quark $O_{F,\mu_1\dots\mu_m}^{m,l}$ and gluon $O_{V,\mu_1\dots\mu_m}^{m,l}$) composite operators. The quantities $C_{m,l}^{NS}$, $C_{m,l}^F$ and $C_{m,l}^V$ are called coefficient functions (CFs) of the OPE.

As a rule, the internal sums in l are omitted in (1) since neglected terms do not contribute to DIS structure functions (see, for instance, [3]). Below we will discuss this point in more detail.

The standard approach to calculations of the OPE CFs is to apply for perturbation theory by considering the scattering of leptons off *elementary* (quark and gluon) *off-shell* fields. In Ref. [7] a new method for calculating CFs was proposed which does not explicitly depend on elementary fields, but instead defines the CFs entirely in terms of Green functions of the currents and/or composite operators.

In our previous paper [7] the *nonsinglet* case was studied. In the present paper we generalize our results for the *singlet case*. In Sec. II we derive a closed representation for the singlet CFs in terms of vacuum matrix elements of the composite operators. In Sec. III we calculate the singlet CFs in perturbative QCD and demonstrate that our main formula not only reproduces well-known expression for the gluon CF, but enables us to obtain the CFs of all gradient singlet operators in the OPE. The renormalization of singlet quark and gluon composite operators and their CFs is considered in Sec. IV. A number of useful mathematical formulas is collected in Appendix A (integrals) and Appendix B (sums).

II. OPE COEFFICIENT FUNCTIONS AND VACUUM MATRIX ELEMENTS OF COMPOSITE OPERATORS

The cross section of deep inelastic lepton-nucleon scattering (DIS) is related with the hadronic tensor (see, for instance, [3])

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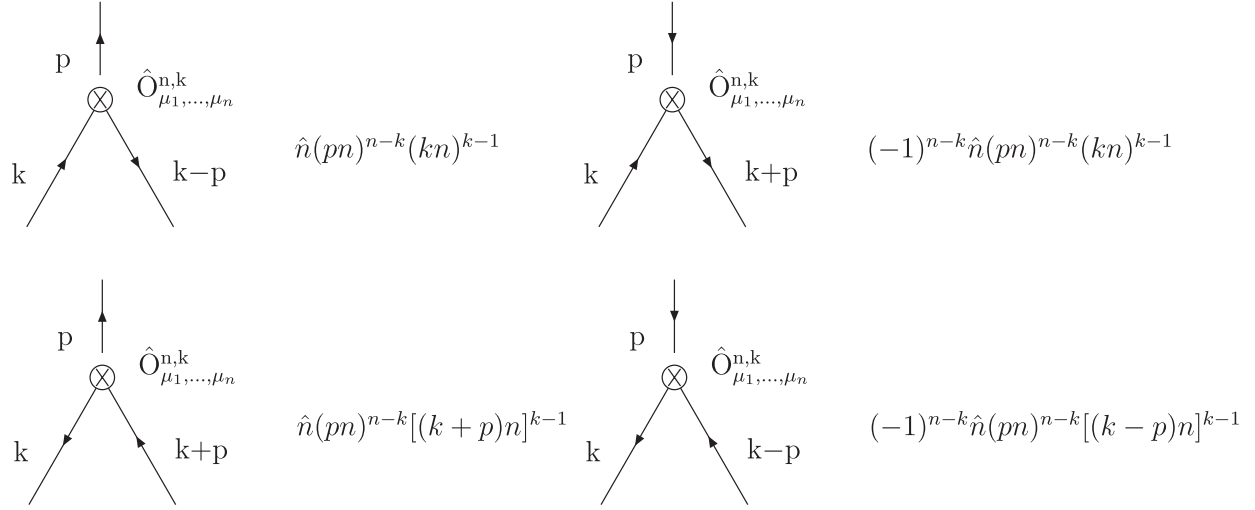


FIG. 1. Feynman rules for the quark composite operators $O_{\mu_1, \dots, \mu_n}^{n,k}$ in the leading (zero) order in strong coupling α_s .

$$W_{\mu\nu}(p, q) = 2\pi^2 \int d^4x e^{iqx} \langle p | T J_\mu^{\text{em}}(x) J_\nu^{\text{em}}(0) | p \rangle. \quad (2)$$

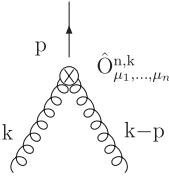
Here $|p\rangle$ means a nucleon state, and $J_\mu^{\text{em}}(x)$ is an electromagnetic current:

$$J_\mu^{\text{em}}(x) = \bar{\Psi}(x) \gamma_\mu \hat{Q} \Psi(x), \quad (3)$$

where $\Psi(x)$ is a quark field. The electric charge operator in (3),

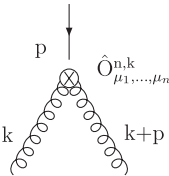
$$\hat{Q} = \frac{1}{2} \left(\lambda^3 + \frac{1}{\sqrt{3}} \lambda^8 \right), \quad (4)$$

obeys the equation



$$(pn)^{n-k-1} (kn)^{k-1}$$

$$\times [-g_{\mu\nu} kn(k-p)n - n_\mu n_\nu k(k-p) + n_\mu k_\nu (k-p)n + (k-p)_\mu n_\nu kn]$$



$$(-1)^{n-k-1} (pn)^{n-k-1} (kn)^{k-1}$$

$$\times [-g_{\mu\nu} kn(k+p)n - n_\mu n_\nu k(k+p) + n_\mu k_\nu (k+p)n + (k+p)_\mu n_\nu kn]$$

FIG. 2. Feynman rules for the gluon composite operators $O_{\mu_1, \dots, \mu_n}^{n,k}$ in the leading (zero) order in strong coupling α_s .

$$\hat{Q}^2 = \frac{1}{6} \left(\lambda^3 + \frac{1}{\sqrt{3}} \lambda^8 \right) + \frac{2}{9} \lambda^0, \quad (5)$$

were λ^a ($a = 1, 2, \dots, 8$) are the Gell-Mann matrices, $\text{Sp}(\lambda^a) = 0$, $\text{Sp}(\lambda^a \lambda^b) = 2\delta_{ab}$, and λ^0 is the identity matrix.

For DIS of a charged lepton, the hadronic tensor (2) has two independent tensor structures [3]:

$$W_{\mu\nu}(p, q) = \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2} \right) \frac{1}{2x_B} F_1(x_B, Q^2) + \left(p_\mu - q_\mu \frac{pq}{q^2} \right) \left(p_\nu - q_\nu \frac{pq}{q^2} \right) \frac{2x_B}{Q^2} F_2(x_B, Q^2), \quad (6)$$

where $Q^2 = -q^2$,

$$x_B = Q^2/2pq \quad (7)$$

is the Bjorken variable, and structure functions F_1 , F_2 depend on these invariant variables. In the Bjorken limit, $Q^2 \rightarrow \infty$, x_B is fixed, the structure functions $F_{1,2}(Q^2, x)$ are defined via one-nucleon matrix elements of the composite operators which enter OPE (1).

Near the light-cone, leading contributions to matrix elements come from twist-2 operators. In QCD, the non-singlet quark twist-2 (traceless) gauge-invariant operators¹ are of the form ($1 \leq l \leq m$):

$$O_{NS, \mu_1, \dots, \mu_m}^{a, m, l}(x) = i^{m-1} \mathbf{S} \partial_{\mu_{l+1}} \dots \partial_{\mu_m} \bar{\Psi}(x) \gamma_{\mu_1} D_{\mu_2} \dots D_{\mu_l} \lambda^a \Psi(x) + (\text{terms proportional to } g_{\mu_i \mu_j}), \quad (8)$$

¹Non-gauge-invariant composite operators in the OPE will be discussed in the end of Sec. IV.

where operator \mathbf{S} means a complete symmetrization in Lorentz indices,

$$D_\mu = \partial_\mu + ig t_a A_\mu^a \quad (9)$$

is a covariant derivative, and $A_\mu^a(x)$ is a gluon field. The singlet quark twist-2 operators ($1 \leq l \leq m$),

$$O_{F,\mu_1 \dots \mu_m}^{m,l}(x) = i^{m-1} \mathbf{S} \partial_{\mu_{l+1}} \dots \partial_{\mu_m} \bar{\Psi}(x) \gamma_{\mu_1} D_{\mu_2} \dots D_{\mu_l} \Psi(x) + (\text{terms proportional to } g_{\mu_i \mu_j}), \quad (10)$$

can mix with the gluon twist-2 operators ($1 \leq l \leq m-1$)

$$O_{V,\mu_1 \dots \mu_m}^{m,l}(x) = i^{m-2} \mathbf{S} \text{Sp} \partial_{\mu_{l+1}} \dots \partial_{\mu_{m-1}} F_{\mu_1 \alpha}(x) D_{\mu_2} \dots D_{\mu_l} F_{\mu_n}^\alpha(x) + (\text{terms proportional to } g_{\mu_i \mu_j}). \quad (11)$$

Feynman rules for these composite operators which will be used for our further calculations are presented in Figs. 1–3. They have to be considered as a generalization of well-known Feynman rules [8] for the case $p \neq 0$.

If the OPE (1) is applied to DIS, only operators of the type

$$O_{NS,\mu_1 \dots \mu_m}^{a,m}(x) = i^{m-1} \mathbf{S} \bar{\Psi}(x) \gamma_{\mu_1} D_{\mu_2} \dots D_{\mu_m} \lambda^a \Psi(x) + (\text{terms proportional to } g_{\mu_i \mu_j}), \quad (12)$$

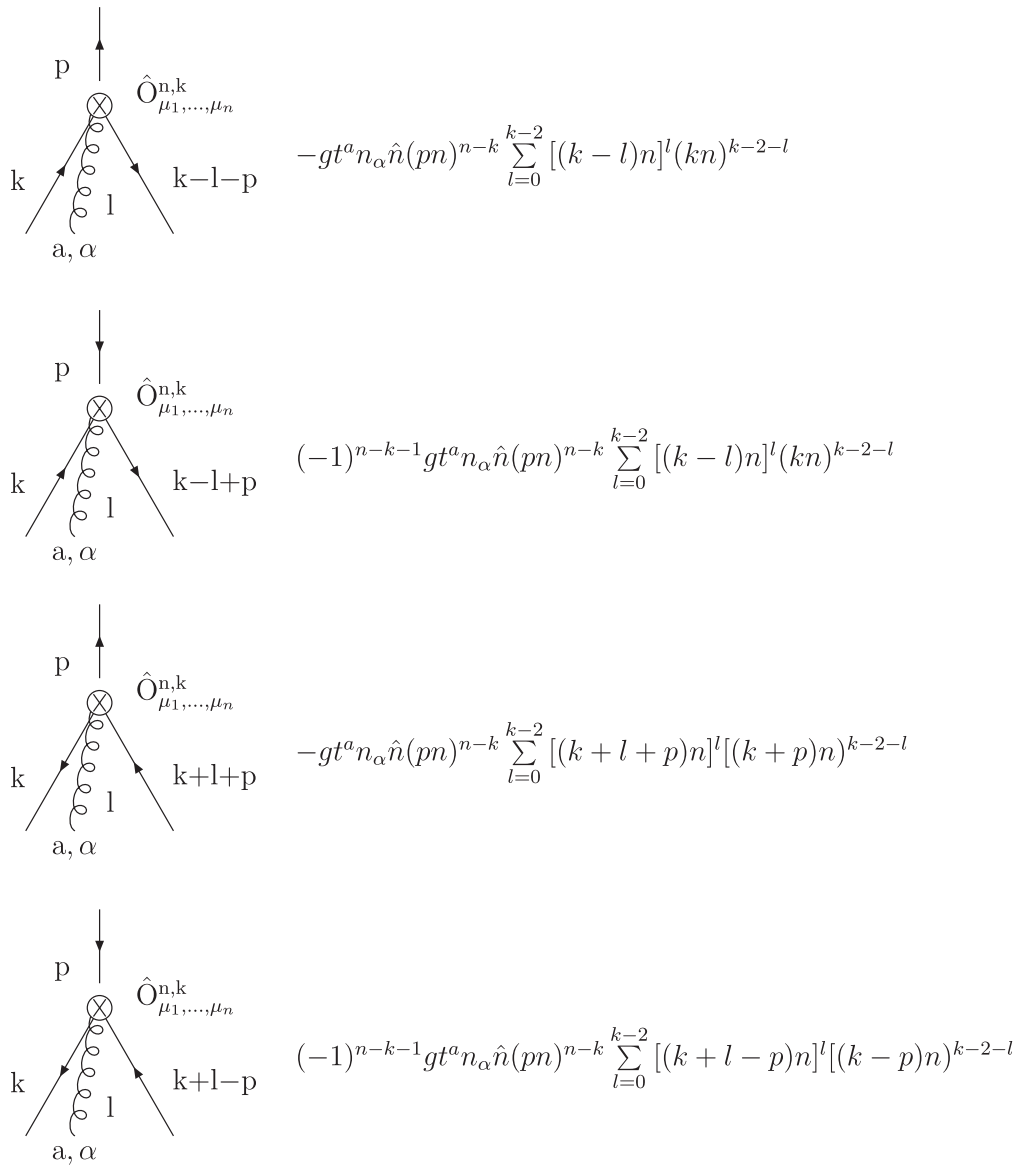


FIG. 3. Feynman rules for the quark composite operators $O_{\mu_1 \dots \mu_n}^{n,k}$ in the first order in strong coupling α_s .

$$O_{F,\mu_1\dots\mu_m}^m(x) = i^{m-1} \mathbf{S} \bar{\Psi}(x) \gamma_{\mu_1} D_{\mu_2} \dots D_{\mu_m} \Psi(x) \\ + (\text{terms proportional to } g_{\mu_i \mu_j}), \quad (13)$$

$$O_{V,\mu_1\dots\mu_m}^m(x) = i^{m-2} \mathbf{S} \text{Sp} F_{\mu_1 \alpha}(x) D_{\mu_2} \dots D_{\mu_{m-1}} F_{\mu_m}^\alpha(x) \\ + (\text{terms proportional to } g_{\mu_i \mu_j}) \quad (14)$$

are relevant. It is due to the fact that any composite operator $O_{A,\mu_1\dots\mu_m}^{m,l}$ ($A = NS, F, V$) with at least one full derivative² gives no contribution to a forward matrix element $\langle p | O_{A,\mu_1\dots\mu_m}^{m,l} | p \rangle$. In our notation,

$$O_{NS,\mu_1\dots\mu_m}^{a,m} = O_{NS,\mu_1\dots\mu_m}^{a,m,m}, \\ O_{F,\mu_1\dots\mu_m}^m = O_{F,\mu_1\dots\mu_m}^{m,m}, \\ O_{V,\mu_1\dots\mu_m}^m = O_{V,\mu_1\dots\mu_m}^{m,m-1}. \quad (15)$$

In what follow, the operators (15) will be called ‘‘major’’ or ‘‘diagonal’’ composite operators, while the quark operators with $1 \leq l \leq m-1$ and gluon operators with $1 \leq l \leq m-2$ will be referred to as ‘‘nondiagonal’’ composite operators. Correspondingly, we define:

$$C_m^{NS} = C_{m,m}^{NS}, \quad C_m^F = C_{m,m}^F, \quad C_m^V = C_{m,m-1}^V. \quad (16)$$

For nonforward matrix elements (for instance, describing deeply virtual Compton scattering), all composite operators contribute. Namely, we have the relations³:

$$\langle p + \Delta | O_{NS,\mu_1\dots\mu_m}^{a,m,l} | p \rangle = \Delta_{\mu_{l+1}} \dots \Delta_{\mu_m} \langle p + \Delta | O_{NS,\mu_1\dots\mu_l}^{a,l} | p \rangle, \\ \langle p + \Delta | O_{F,\mu_1\dots\mu_m}^{m,l} | p \rangle = \Delta_{\mu_{l+1}} \dots \Delta_{\mu_m} \langle p + \Delta | O_{F,\mu_1\dots\mu_l}^l | p \rangle, \\ \langle p + \Delta | O_{V,\mu_1\dots\mu_m}^{m,l} | p \rangle = \Delta_{\mu_{l+1}} \dots \Delta_{\mu_{m-1}} \langle p + \Delta | O_{V,\mu_1\dots\mu_l}^l | p \rangle. \quad (17)$$

The major operators in the RHS of Eq. (17) are defined above in Eq. (15).

As usual, we assume that $C_{m,l}^A(x^2)$ are tempered generalized functions (this is explicit in perturbative calculations), so the symbolic relation

$$x^{\mu_1} \dots x^{\mu_m} = (-2i)^m \frac{q^{\mu_1} \dots q^{\mu_m}}{(-q^2)^m} (-q^2)^m \left(\frac{\partial}{\partial q^2} \right)^m \quad (18)$$

holds in connection with the Fourier transform in (1).

The approach, developed in our previous paper [7] for the nonsinglet CFs, should be generalized for the singlet case. To do this, let us

²As one can see from Eqs. (8), (10), and (11), the total number of full derivatives is equal to $m-l$ or $m-l-1$ for the quark or gluon composite operator, respectively.

³Here $\langle p |$ and $| p + \Delta \rangle$ are one-particle states with 4-momenta p_μ and $(p + \Delta)_\mu$.

- (1) take T -product of both sides of the OPE (1) by a singlet composite operator $O_{A,\nu_1\dots\nu_n}^{n,k}(z)$, with $A = F$ or V ,
- (2) imbed all resulting operator products between vacuum states.

As a result, we obtain from Eq. (1) the following relation between vacuum matrix elements of the operator products and OPE coefficient functions:

$$\int d^4 x e^{iqx} \int d^4 z e^{ipz} \langle T \tilde{J}_\mu^{\text{em}}(x) \tilde{J}_\nu^{\text{em}}(0) O_{A,\nu_1\dots\nu_n}^{n,k}(z) \rangle \\ = -g_{\mu\nu} \left\{ \sum_{m=0}^{\infty} \sum_{l=1}^m 2^m \frac{q^{\mu_1} \dots q^{\mu_m}}{(-q^2)^m} \tilde{C}_{m,l}^F(q^2) \right. \\ \times \int d^4 z e^{ipz} \langle T O_{F,\mu_1\dots\mu_m}^{m,l}(0) O_{A,\nu_1\dots\nu_n}^{n,k}(z) \rangle \\ + \sum_{m=0}^{\infty} \sum_{l=1}^{m-1} 2^m \frac{q^{\mu_1} \dots q^{\mu_m}}{(-q^2)^m} \tilde{C}_{m,l}^V(q^2) \\ \times \int d^4 z e^{ipz} \langle T O_{V,\mu_1\dots\mu_m}^{m,l}(0) O_{A,\nu_1\dots\nu_n}^{n,k}(z) \rangle + \dots, \quad (19)$$

where $\tilde{C}_{m,l}^A(q^2)$ is a Fourier transform of $C_{m,l}^A(x^2)$,

$$\tilde{C}_{m,l}^A(q^2) = \frac{1}{m!} (-q^2)^m \left(\frac{\partial}{\partial q^2} \right)^m \int d^4 x e^{iqx} C_{m,l}^A(x^2), \quad (20)$$

and a new notation,

$$\tilde{J}_\mu^{\text{em}}(x) = \bar{\Psi}(x) \gamma_\mu \lambda^0 \Psi(x), \quad (21)$$

is introduced. In other words, only a *singlet part* of the product of two electromagnetic currents [see Eqs. (3) and (5)] gives a contribution to (19).

Let n_μ be a light-cone 4-vector not orthogonal to 4-momentum p_μ :

$$n_\mu^2 = 0, \quad pn \neq 0. \quad (22)$$

Throughout the paper, we will work in the limit

$$p \rightarrow 0, \quad p^2 < 0. \quad (23)$$

Let us underline that the limit $p^2 \rightarrow 0$ does not assume the limit $p_\mu \rightarrow 0$. On the contrary, given $n_\mu = q_\mu - p_\mu q^2 / [pq + \sqrt{(pq)^2 - p^2 q^2}]$, one gets $pn \simeq pq$ at $p^2 \simeq 0$.

It is useful to convolute vacuum matrix elements with the projector

$$\frac{n^{\nu_1} \dots n^{\nu_n}}{(pn)^n}, \quad (24)$$

and define the following invariant structure,

$$\frac{n^{\nu_1} \dots n^{\nu_n}}{(pn)^n} \int d^4 x e^{iqx} \int d^4 z e^{ipz} \langle T \tilde{J}_\mu^{\text{em}}(x) \tilde{J}_\nu^{\text{em}}(0) O_{A,\nu_1\dots\nu_n}^{n,k}(z) \rangle \\ = -\frac{4}{9} g_{\mu\nu} F_A^{n,k}(\omega, Q^2, p^2) + \dots \quad (25)$$

It depends on invariant variables p^2 , Q^2 and dimensionless variable

$$\omega = 1/x_B = 2pq/Q^2. \quad (26)$$

The vacuum matrix element of the T -product of two composite operators has the following Lorentz structure ($A, B = F, V$):

$$\begin{aligned} & \int d^4z e^{ipz} \langle T O_{B,\mu_1 \dots \mu_m}^{m,l}(0) O_{A,\nu_1 \dots \nu_n}^{n,k}(z) \rangle \\ &= 2p_{\mu_1} \dots p_{\mu_m} p_{\nu_1} \dots p_{\nu_n} \langle O_B^{m,l} O_A^{n,k} \rangle(p^2) \\ &+ (\text{terms proportional to } g_{\mu_i \mu_j} p^2, g_{\nu_i \nu_j} p^2, g_{\mu_i \nu_j} p^2). \end{aligned} \quad (27)$$

Equation (27) means:

$$\begin{aligned} & \frac{n^{\nu_1} \dots n^{\nu_n}}{(pn)^n} \int d^4z e^{ipz} \langle T O_{B,\mu_1 \dots \mu_m}^{m,l}(0) O_{A,\nu_1 \dots \nu_n}^{n,k}(z) \rangle \\ &= 2p_{\mu_1} \dots p_{\mu_m} \langle O_B^{m,l} O_A^{n,k} \rangle(p^2). \end{aligned} \quad (28)$$

Note that both $F_A^{n,k}(\omega, Q^2, p^2)$ and $\langle O_B^{m,l} O_A^{n,k} \rangle(p^2)$ are dimensionless quantities.

Let us note that at $p^2 \rightarrow 0$ vacuum matrix elements of composite operators of higher twists are suppressed by powers of p^2 with respect to the vacuum matrix elements of twist-2 operators (28). Thus, our approach enables us to isolate a contribution from twist-2 operators.

At fixed Q^2 and p^2 , 3-point Green function $\langle T J_\mu^{\text{em}} J_\nu^{\text{em}} O_A^{n,k} \rangle$ has a discontinuity in the variable $(q+p)^2$ for $(q+p)^2 \geq 0$ (that is, for $\omega \geq 1$). By using the dispersion relation for $F_A^{n,k}(\omega, Q^2, p^2)$,

$$\begin{aligned} & F_A^{n,k}(\omega, Q^2, p^2) \\ &= \frac{1}{\pi} \int_1^\infty \frac{d\omega'}{\omega' - \omega} \text{Im} F_A^{n,k}(\omega', q^2, p^2) \\ &= \frac{1}{2\pi i} \sum_{m=0}^\infty \omega^m \int_1^\infty d\omega' \omega'^{-m-1} \text{disc}_\omega F_A^{n,k}(\omega', q^2, p^2), \end{aligned} \quad (29)$$

one can derive from Eqs. (19), (25), and (28):

$$\begin{aligned} & \left[\sum_{l=1}^m \tilde{C}_{m,l}^F(Q^2/\mu^2) \langle O_F^{m,l} O_A^{n,k} \rangle(p^2/\mu^2) \right. \\ & \left. + \sum_{l=1}^{m-1} \tilde{C}_{m,l}^V(Q^2/\mu^2) \langle O_V^{m,l} O_A^{n,k} \rangle(p^2/\mu^2) \right]_{p^2 \rightarrow 0} \\ &= \left[\frac{1}{2\pi i} \int_0^1 dx_B x_B^{m-1} \right. \\ & \left. \times \text{disc}_{(p+q)^2} F_A^{n,k}(x_B, Q^2/p^2, p^2/\mu^2) \right]_{p^2 \rightarrow 0}. \end{aligned} \quad (30)$$

Strictly speaking, possible divergencies must be subtracted from the dispersion relation (29). However, it does not alter

our scheme provided the integrals in the r.h.s. of Eq. (30) converge (remember that $m \geq 2$).

In (30) we took into account that both matrix elements of renormalized composite operators and CFs depend on the *renormalization* scale μ . In what follows, we take μ to be equal to the *regularization* scale $\bar{\mu}$, which arises in dimensional regularization [9], when one changes an integration volume, $d^4k \rightarrow \bar{\mu}^{(4-D)} d^Dk$.

Neither side of Eq. (30) has dependence on n except for the trivial factor $(-1)^n$. By setting $k = 1, 2, \dots, 2m-1$, we thus obtain a set of $2m-1$ algebraic equations for the singlet OPE CFs $\tilde{C}_{m,l}^F$ ($1 \leq l \leq m$) and $\tilde{C}_{m,l}^V$ ($1 \leq l \leq m-1$).⁴

Formula (30) gives an operator definition of the OPE CFs in term of vacuum matrix elements of composite operators.⁵ It is important to stress that our definition of the OPE CFs is unambiguous and it does not lean on a notion of quark and gluon distributions. The latter are defined via nucleon matrix elements of the quark or gluon composite operator, while the coefficient functions are independently defined via vacuum matrix elements of the product of composite operators.

III. CALCULATIONS OF SINGLET COEFFICIENT FUNCTIONS IN PERTURBATIVE QCD

The formula (30) is a generalization of a corresponding formula for a nonsinglet case which was derived in our previous paper [7]:

$$\begin{aligned} & \left[\sum_{l=1}^m \tilde{C}_{m,l}^{NS}(Q^2/\mu^2) \langle O_{NS}^{m,l} O_{NS}^{n,k} \rangle(p^2/\mu^2) \right]_{p^2 \rightarrow 0} \\ &= \left[\frac{1}{2\pi i} \int_0^1 dx_B x_B^{m-1} \right. \\ & \left. \times \text{disc}_{(p+q)^2} F_{NS}^{n,k}(x_B, Q^2/p^2, p^2/\mu^2) \right]_{p^2 \rightarrow 0}. \end{aligned} \quad (31)$$

By using this formula, the following expressions for the nonsinglet CFs were calculated in QCD [7]:

$$[\tilde{C}_{m,m}^{NS}]^{(0)} = \frac{1}{2}[1 + (-1)^m], \quad (32)$$

and

$$[\tilde{C}_{m,l}^{NS}]^{(0)} = \frac{1}{2}(-1)^l \binom{m}{l}, \quad (33)$$

for $l = 0, 1, \dots, m-1$.⁶ Here and in what follows superscript “(0)” means that a corresponding quantity is calculated in zero order in strong coupling.

In the next order in α_s , we obtained the following expressions [7]:

⁴Since $n \geq k$, the index n must be chosen larger than $2m-1$.

⁵The electromagnetic current (3) is a particular case of a quark composite operator with zero anomalous dimension.

⁶Everywhere $\binom{n}{m}$ denotes a binomial coefficient.

$$[\tilde{C}_{m,m}^{NS}(Q^2/\mu^2)]^{(1)} = \frac{\alpha_s}{8\pi} C_F \ln\left(\frac{Q^2}{\mu^2}\right) \left[1 + (-1)^m\right] \times \left[-4 \sum_{j=2}^m \frac{1}{j} - 1 + \frac{2}{m(m+1)}\right], \quad (34)$$

and

$$[\tilde{C}_{m,l}^{NS}(Q^2/\mu^2)]^{(1)} = \frac{\alpha_s}{4\pi} C_F \ln\left(\frac{Q^2}{\mu^2}\right) \left\{ \frac{1}{2} (-1)^l \binom{m-1}{l-1} \times \left[-4 \sum_{j=2}^l \frac{1}{j} - 1 + \frac{2}{l(l+1)}\right] + \left(\frac{1}{m-l} - \frac{1}{m+1}\right) + \sum_{k=l+1}^m (-1)^k \times \left(\frac{m-1}{k-1}\right) \left(\frac{1}{k-l} - \frac{1}{k+1}\right) \right\}, \quad (35)$$

for $l = 0, 1, \dots, m-1$. Here and in what follows superscript “(1)” means that a corresponding quantity is calculated in the first order in strong coupling constant.

Let us stress that we did not demand from the very beginning that the major CF, $\tilde{C}_{m,m}^{NS}$, should be equal to zero for odd m [see Eqs. (32) and (34)]. On the contrary, it is a consequence of the fact that electromagnetic interactions conserve P-parity. Remember that DIS structure function $F_2(x_B, Q^2)$ is an even function of Bjorken variable x_B , and its nonzero moments, $F_2(n, Q^2)$, are defined by quantities $\tilde{C}_{n,n}^A(Q^2/\mu^2) \langle p | O_A^{n,n} | p \rangle(\mu^2)$, with even n . That is why we expect that the gluon major CF should be also proportional to the factor $[1 + (-1)^m]$ [see (57) below].

For a convenience, let us for a moment rewrite our main relation (31) in symbolic form:

$$\langle JJO_A \rangle = C^F \langle O_F O_A \rangle + C^V \langle O_V O_A \rangle. \quad (36)$$

Then we get from (36):

$$[\langle JJO_V \rangle]^{(0)} = [C^F]^{(0)} [\langle O_F O_V \rangle]^{(0)} + [C^V]^{(0)} \langle O_V O_V \rangle^{(0)}. \quad (37)$$

Since $[\langle O_V O_F \rangle]^{(0)} = [\langle JJO_V \rangle]^{(0)} = 0$, while $[\langle O_V O_V \rangle]^{(0)}$ is nonzero, we get

$$\sum_{l=1}^{m-1} [\tilde{C}_{m,l}^V]^{(0)} [\langle O_V^{m,l} O_V^{n,k} \rangle]^{(0)} = 0. \quad (38)$$

Equality (38) is valid for all integer m, n , and $1 \leq k \leq n-1$. Thus, we conclude that

$$[\tilde{C}_{m,l}^V]^{(0)} = 0, \quad (39)$$

for all integer m , and $1 \leq l \leq m-1$.

Analogously, we obtain from (36):

$$[\langle JJO_F \rangle]^{(0)} = [C^F]^{(0)} [\langle O_F O_F \rangle]^{(0)} + [C^V]^{(0)} \langle O_V O_F \rangle^{(0)}, \quad (40)$$

$$[\langle JJO_F \rangle]^{(1)} = [C^F]^{(0)} [\langle O_F O_F \rangle]^{(1)} + [C^F]^{(1)} \langle O_F O_F \rangle^{(0)} + [C^V]^{(0)} [\langle O_V O_F \rangle]^{(1)} + [C^V]^{(1)} \langle O_V O_F \rangle^{(0)}. \quad (41)$$

Taking into account that $[C^V]^{(0)} = 0$ (39) and $[\langle O_V O_F \rangle]^{(0)} = 0$, we find:

$$[\langle JJO_F \rangle]^{(0)} = [C^F]^{(0)} [\langle O_F O_F \rangle]^{(0)}, \quad (42)$$

$$[\langle JJO_F \rangle]^{(1)} = [C^F]^{(0)} [\langle O_F O_F \rangle]^{(1)} + [C^F]^{(1)} \langle O_F O_F \rangle^{(0)}. \quad (43)$$

These equations are identical to those derived for the *non-singlet* quark CF in our paper [7]. As a result, we find that the singlet quark CFs coincide with the corresponding nonsinglet quark CFs in zero and first order in α_s ⁷:

$$[\tilde{C}_{m,l}^F]^{(0)} = [\tilde{C}_{m,l}^{NS}]^{(0)}, \quad [\tilde{C}_{m,l}^F]^{(1)} = [\tilde{C}_{m,l}^{NS}]^{(1)}, \quad (44)$$

with $[\tilde{C}_{m,l}^{NS}]^{(0)}$ and $[\tilde{C}_{m,l}^{NS}]^{(1)}$ given by expressions (32)–(35).

Now let us turn to QCD calculations of the gluon CFs in the first order in strong coupling constant by using our main formula (30). We work in the dimensional regularization [9] and use the $\overline{\text{MS}}$ -scheme to renormalize ultraviolet divergences. All results of our calculations are gauge-invariant since we sum all diagrams in each order of perturbation theory. Let us remember that in order to find the OPE CFs, we have to retain only leading terms in the limit $p^2 \rightarrow 0$. This significantly simplifies the calculations. We will restrict ourselves by considering leading terms in $\ln(Q^2/\mu^2)$, although our main formula (30) enables one to calculate subleading terms as well. In other words, along with the limit $p^2 \rightarrow 0$, we are interested in large values of variable Q^2 .

Starting from (36), we can schematically write:

$$[\langle JJO_V \rangle]^{(1)} = [C^F]^{(0)} [\langle O_F O_V \rangle]^{(1)} + [C^V]^{(1)} \langle O_V O_V \rangle^{(0)}. \quad (45)$$

In full detail, Eq. (45) looks like the following:

$$\left\{ \frac{1}{2\pi i} \int_0^1 dx_B x_B^{m-1} [\text{disc}_{(p+q)^2} F_V^{n,k}(x_B, Q^2/p^2, p^2/\mu^2)]^{(1)} \right\}_{p^2 \rightarrow 0} = \left\{ \sum_{l=1}^m [\tilde{C}_{m,l}^F]^{(0)} [\langle O_F^{m,l} O_V^{n,k} \rangle]^{(1)} + \sum_{l=1}^{m-1} [\tilde{C}_{m,l}^V(Q^2/\mu^2)]^{(1)} [\langle O_V^{m,l} O_V^{n,k} \rangle]^{(0)} \right\}_{p^2 \rightarrow 0}. \quad (46)$$

The quantities $[\tilde{C}_{m,l}^F]^{(0)}$ are already known [see (44), (32), and (33)], while the other terms in (46) should be calculated.

⁷Note, however, that $[\tilde{C}_{m,l}^F]^{(n)} \neq [\tilde{C}_{m,l}^{NS}]^{(n)}$, for $n \geq 2$.

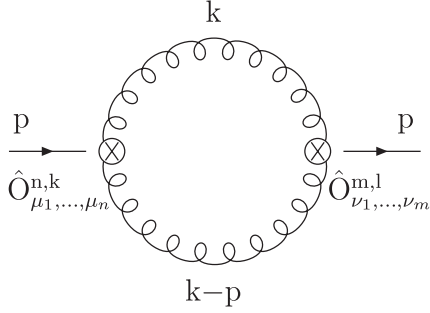


FIG. 4. The diagrams for the propagator of the gluon composite operator $\langle O_V^{n,k} O_V^{m,l} \rangle^{(0)}$ in zero order in strong coupling α_s .

The propagator of the gluon composite operator is shown in Fig. 4, and one gets⁸:

$$\begin{aligned} & \langle O_V^{n,k} O_V^{m,l} \rangle^{(0)}(p^2/\mu^2) \\ &= i(-1)^{n+1} \frac{1}{8\pi^2} \ln\left(\frac{\mu^2}{-p^2}\right) B(k+2, l+2). \end{aligned} \quad (47)$$

The vacuum matrix element of the product of two singlet composite operators is given by the diagram in Fig. 5. The calculations result in the following expression:

$$\begin{aligned} & \langle O_V^{n,k} O_F^{m,l} \rangle^{(1)}(p^2/\mu^2) \\ &= i(-1)^n C_F \frac{\alpha_s}{4\pi^3} \left[\ln\left(\frac{\mu^2}{-p^2}\right) \right]^2 \left\{ \left(\frac{2}{k} - \frac{2}{k+1} + \frac{1}{k+2} \right) \right. \\ & \quad \times \left[\frac{1}{k+l+2} - B(k+3, l) \right] \\ & \quad - \frac{1}{k} \left[\frac{1}{k+l+1} - B(k+2, l) \right] \\ & \quad \left. - \left(\frac{1}{k} - \frac{2}{k+1} + \frac{1}{k+2} \right) \frac{l-1}{l(l+1)} \right\}, \end{aligned} \quad (48)$$

Now we are able to calculate the second term in Eq. (46):

$$\begin{aligned} & \sum_{l=1}^m [\tilde{C}_{m,l}^F]^{(0)} \langle O_V^{n,k} O_F^{m,l} \rangle^{(1)}(p^2/\mu^2) \\ &= i(-1)^n C_F \frac{\alpha_s}{32\pi^3} \left[\ln\left(\frac{\mu^2}{-p^2}\right) \right]^2 \frac{1}{m(m+1)(m+2)} \\ & \quad \times \left\{ \left[\frac{m-2}{k} + \frac{2(m+2)}{(k+1)(k+2)} + \frac{m(m+1)+2}{k+m+2} \right. \right. \\ & \quad \left. \left. - \frac{m(m+2)}{k+m+1} \right] - [(m-2)B(k, m+3) \right. \\ & \quad \left. + m(m+2)B(k+1, m+2)] \right\}, \end{aligned} \quad (49)$$

Summation in l was made with the help of Eqs. (B1)–(B8) from Appendix B. The relation between beta-functions (integer k, m),

⁸Everywhere $B(x, y)$ means beta-function.

$$\begin{aligned} & \left(\frac{2}{k} - \frac{2}{k+1} + \frac{1}{k+2} \right) B(k+3, m) - \frac{1}{k} B(k+2, m) \\ &= - \frac{1}{m(m+1)(m+2)} [(m-2)B(k, m+3) \\ & \quad + m(m+2)B(k+1, m+2)], \end{aligned} \quad (50)$$

was also used.

The diagrams which contribute to vacuum matrix element with two currents are shown in Fig. 6. Omitting details of calculations, let us give the result:

$$\begin{aligned} & \frac{1}{2\pi i} \int_0^1 dx x^{m-1} \text{disc}_{(p+q)^2} \langle J J O_V^{n,k} \rangle^{(1)}(x_B, Q^2/p^2, p^2/\mu^2) \\ &= i(-1)^n C_F \frac{\alpha_s}{32\pi^3} \ln\left(\frac{Q^2}{-p^2}\right) \ln\left(\frac{\mu^2}{-p^2}\right) \\ & \quad \times \frac{1}{m(m+1)(m+2)} \left\{ \left[\frac{m-2}{k} + \frac{2(m+2)}{(k+1)(k+2)} \right. \right. \\ & \quad \left. \left. + \frac{m(m+1)+2}{k+m+2} - \frac{m(m+2)}{k+m+1} \right] \right. \\ & \quad \left. - [(m-2)B(k, m+3) + m(m+2)B(k+1, m+2)] \right\}, \end{aligned} \quad (51)$$

In order to obtain Eqs. (47)–(51), we used integrals (A1)–(A3) from Appendix A.

Equations (46)–(51) result in a set of equations for the gluon CFs. Namely, for any integer $m \geq 2$ we obtain algebraic equations for $[\tilde{C}_{m,l}^V(Q^2/\mu^2)]^{(1)}$, with $1 \leq l \leq m-1$:

$$\begin{aligned} & \sum_{l=1}^{m-1} [\tilde{C}_{m,l}^V(Q^2/\mu^2)]^{(1)} B(k+2, l+2) \\ &= C_F \frac{\alpha_s}{4\pi} \ln\left(\frac{Q^2}{\mu^2}\right) \frac{1}{m(m+1)(m+2)} \left\{ \left[\frac{m-2}{k} \right. \right. \\ & \quad \left. \left. + \frac{2(m+2)}{(k+1)(k+2)} + \frac{m(m+1)+2}{k+m+2} - \frac{m(m+2)}{k+m+1} \right] \right. \\ & \quad \left. - [(m-2)B(k, m+3) + m(m+2)B(k+1, m+2)] \right\}. \end{aligned} \quad (52)$$

Note that these equations hold for all integer $k \geq 1$, but for our purposes it is enough to consider only $m-1$ equations corresponding to $k = 1, 2, \dots, m-1$.⁹

The solution of Eqs. (52) is a sum of two terms one of which is nonzero only for even m , while another is nonzero only for odd m :

⁹The other equations which correspond to $k \geq m$ will be also satisfied, as one could see from an explicit expression for our solution (53)–(55).

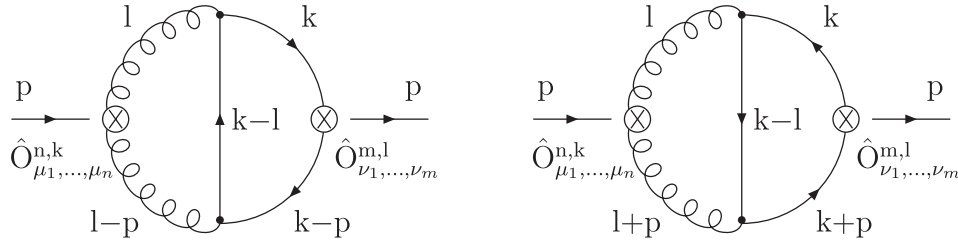


FIG. 5. The diagrams for the mixing of the composite operators $\langle O_V^{n,k} O_F^{m,l} \rangle^{(1)}$ in the first order in strong coupling α_s .

$$[\tilde{C}_{m,l}^V(Q^2/\mu^2)]^{(1)} = \frac{[1 + (-1)^m]}{2} [\tilde{C}_{m,l}^V(Q^2/\mu^2)]_{\text{even}}^{(1)} + \frac{[1 - (-1)^m]}{2} [\tilde{C}_{m,l}^V(Q^2/\mu^2)]_{\text{odd}}^{(1)}. \quad (53)$$

The first term in (53) is defined for $1 \leq l \leq m-1$

$$[\tilde{C}_{m,l}^V(Q^2/\mu^2)]_{\text{even}}^{(1)} = C_F \frac{\alpha_s}{4\pi} \ln\left(\frac{Q^2}{\mu^2}\right) \left\{ \left(\frac{1}{m} - \frac{2}{m+1} + \frac{2}{m+2} \right) \times \left[(-1)^l \binom{m}{l+1} - (m-l) \right] - \frac{1}{m+1} \times \left[(-1)^l \binom{m-1}{l+1} - (m-l-1) \right] \right\}, \quad (54)$$

while the second term in (53) is nonzero for $1 \leq l \leq m-2$:

$$[\tilde{C}_{m,l}^V(Q^2/\mu^2)]_{\text{odd}}^{(1)} = C_F \frac{\alpha_s}{4\pi} \ln\left(\frac{Q^2}{\mu^2}\right) \frac{1}{m(m+1)(m+2)} \times \left\{ (-1)^l [(l-m+1)(m+2) + m(m+1) + 2] \times \left(\binom{m}{l+1} - [(l-m+1)(m-2) + m(m+1) + 2] \right) \right\}. \quad (55)$$

Note that $(\tilde{C}_{m,l}^V)_{\text{odd}}$ (55) gives no contribution to $\tilde{C}_{m,l}^V$ (53) for $l = m-1$ due to relation $[(-1)^{m-1}][(-1)^m + 1] = 0$. It is rather easy to demonstrate that (53) does obey set of

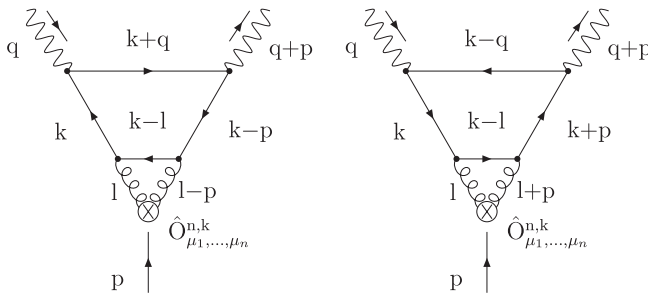


FIG. 6. The diagrams for the matrix element $\langle JJ O_V^{n,k} \rangle^{(1)}$ in the first order in strong coupling α_s .

Eqs. (52) for any $m \geq 2$, $k \geq 1$, if one uses formulas (B9)–(B12) from Appendix B. Indeed, these formulas lead us to the relations:

$$\begin{aligned} & \sum_{l=1}^{m-1} [\tilde{C}_{m,l}^V(Q^2/\mu^2)]_{\text{even}}^{(1)} \mathbf{B}(k+2, l+2) \\ &= \sum_{l=1}^{m-1} [\tilde{C}_{m,l}^V(Q^2/\mu^2)]_{\text{odd}}^{(1)} \mathbf{B}(k+2, l+2) \\ &= C_F \frac{\alpha_s}{4\pi} \ln\left(\frac{Q^2}{\mu^2}\right) \frac{1}{m(m+1)(m+2)} \left\{ \left[\frac{m-2}{k} + \frac{2(m+2)}{(k+1)(k+2)} + \frac{m(m+1)+2}{k+m+2} - \frac{m(m+2)}{k+m+1} \right] \right. \\ & \quad \left. - [(m-2)\mathbf{B}(k, m+3) + m(m+2)\mathbf{B}(k+1, m+2)] \right\}, \end{aligned} \quad (56)$$

In particular, it follows from Eqs. (53) and (54), that

$$[\tilde{C}_{m,m-1}^V(Q^2/\mu^2)]^{(1)} = C_F \frac{\alpha_s}{4\pi} \ln\left(\frac{Q^2}{\mu^2}\right) [1 + (-1)^m] \left(\frac{1}{m} - \frac{2}{m+1} + \frac{2}{m+2} \right). \quad (57)$$

As one can see, major CF (57) is defined by well-known anomalous dimension [10]¹⁰

$$\gamma_{FV}^m = \frac{\alpha_s}{\pi} C_F \frac{(m+1)^2 + (m+1) + 2}{m(m+1)(m+2)}. \quad (58)$$

Thus, we have reproduced the standard expression for the major CF, $\tilde{C}_{m,m-1}^V(Q^2/\mu^2)$, and, which is more important, have calculated “gradient” gluon CFs, $\tilde{C}_{m,l}^V(Q^2/\mu^2)$ ($l = 1, 2, \dots, m-2$), in the first order in strong coupling α_s .

IV. RENORMALIZATION OF SINGLET COMPOSITE OPERATORS AND COEFFICIENT FUNCTIONS

Let us consider in more detail products of renormalized composite operators and corresponding renormalized CFs which enter the OPE of two electromagnetic currents (1).

¹⁰In order to obtain the expression for γ_{FV}^n in standard notations, one has to replace index m by $(n-1)$ in (58).

Both singlet composite operators, $O_F^{m,l}$ and $O_V^{m,l}$, depend on the renormalization scale μ_0 , and mix with each other under rescaling $\mu_0 \rightarrow \mu$:

$$\begin{aligned}
 O_F^{m,k}(\mu_0^2) &= \sum_{l=1}^k (\hat{Z}^{FF})_l^k(\mu_0^2/\mu^2) O_F^{m,l}(\mu^2) \\
 &\quad + \sum_{l=1}^{k-1} (\hat{Z}^{FV})_l^k(\mu_0^2/\mu^2) O_V^{m,l}(\mu^2), \\
 O_V^{m,k}(\mu_0^2) &= \sum_{l=1}^{k+1} (\hat{Z}^{VF})_l^k(\mu_0^2/\mu^2) O_F^{m,l}(\mu^2) \\
 &\quad + \sum_{l=1}^k (\hat{Z}^{VV})_l^k(\mu_0^2/\mu^2) O_V^{m,l}(\mu^2), \quad (59)
 \end{aligned}$$

where \hat{Z} are the matrices of a finite renormalization of the composite operators. In its turn, (59) means that

$$\begin{aligned}
 \tilde{C}_{m,m-1}^V(Q^2/\mu^2) &= \tilde{C}_{m,m}^F(Q^2/\mu_0^2) (\hat{Z}^{FV})_{m-1}^m(\mu_0^2/\mu^2) \\
 &\quad + \tilde{C}_{m,m-1}^V(Q^2/\mu_0^2) (\hat{Z}^{VV})_{m-1}^m(\mu_0^2/\mu^2), \quad (60)
 \end{aligned}$$

and

$$\begin{aligned}
 \tilde{C}_{m,l}^V(Q^2/\mu^2) &= \sum_{k=l+1}^m \tilde{C}_{m,k}^F(Q^2/\mu_0^2) (\hat{Z}^{FV})_l^k(\mu_0^2/\mu^2) \\
 &\quad + \sum_{k=l}^m \tilde{C}_{m,k}^V(Q^2/\mu_0^2) (\hat{Z}^{VV})_l^k(\mu_0^2/\mu^2), \quad (61)
 \end{aligned}$$

for $l = 1, 2, \dots, m-2$.

Since the quantity μ_0 is an arbitrary scale, one can put $\mu_0^2 = Q^2$ in (60) and (61), and obtain:

$$\begin{aligned}
 \tilde{C}_{m,m-1}^V(Q^2/\mu^2) &= \tilde{C}_{m,m}^F(1) (\hat{Z}^{FV})_{m-1}^m(Q^2/\mu^2) \\
 &\quad + \tilde{C}_{m,m-1}^V(1) (\hat{Z}^{VV})_{m-1}^m(Q^2/\mu^2), \quad (62)
 \end{aligned}$$

and

$$\begin{aligned}
 \tilde{C}_{m,l}^V(Q^2/\mu^2) &= \sum_{k=l+1}^m \tilde{C}_{m,k}^F(1) (\hat{Z}^{FV})_l^k(Q^2/\mu^2) \\
 &\quad + \sum_{k=l}^{m-1} \tilde{C}_{m,k}^V(1) (\hat{Z}^{VV})_l^k(Q^2/\mu^2), \quad (63)
 \end{aligned}$$

for $l = 1, 2, \dots, m-2$. As a result, we find equations for the *leading* parts of the gluon CFs in the first order in the strong coupling ($1 \leq l \leq m-1$):

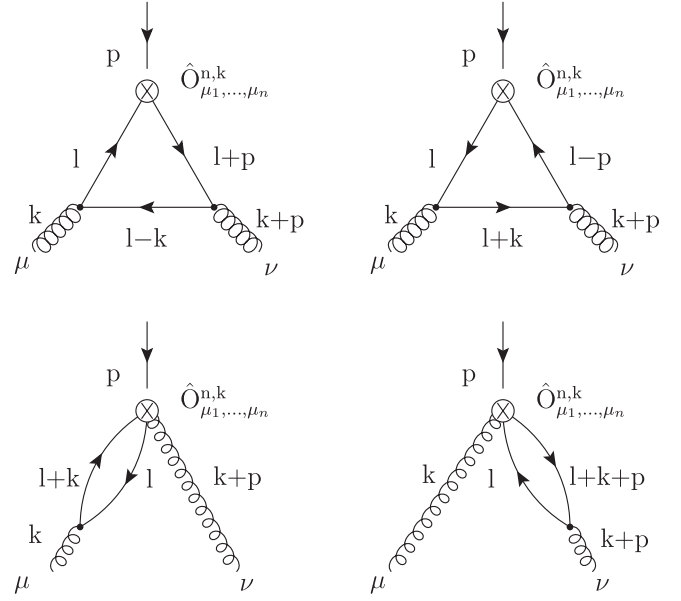


FIG. 7. The diagrams which give contribution to the renormalization of the quark composite operator $O_F^{n,k}$ in the first order in strong coupling α_s .

$$[\tilde{C}_{m,l}^V(Q^2/\mu^2)]^{(1)} = \sum_{k=l+1}^m [\tilde{C}_{m,k}^F]^{(0)} [(\hat{Z}^{FV})_l^k(Q^2/\mu^2)]^{(1)}. \quad (64)$$

In deriving relation (64), we took into account that $[\tilde{C}_{m,l}^V]^{(0)} = 0$ for all m and $1 \leq l \leq m-1$ (39). By using explicit form of $[\tilde{C}_{m,k}^F]^{(0)}$ [see (44) and (32)], these equations can be written as follows ($1 \leq l \leq m-1$):

$$\begin{aligned}
 [\tilde{C}_{m,l}^V(Q^2/\mu^2)]^{(1)} &= [1 + (-1)^m][(\hat{Z}^{FV})_l^m(Q^2/\mu^2)]^{(1)} \\
 &\quad + \sum_{k=l+1}^{m-1} (-1)^k \binom{m-1}{k-1} [(\hat{Z}^{FV})_l^k(Q^2/\mu^2)]^{(1)}. \quad (65)
 \end{aligned}$$

Note that the last term in (65) is identically zero at $l = m-1$.

The mixing of singlet quark operators (10) and gluon operators (11) is defined by the set of diagrams presented in Fig. 7.¹¹ The sum of divergent parts of these diagrams is given by the expression:

$$\begin{aligned}
 &[-g_{\mu\nu}kn(k+p)n - n_\mu n_\nu k(k+p) + n_\mu k_\nu(k+p)n + k_\mu n_\nu kn] C_F \frac{\alpha_s}{8\pi\varepsilon} \frac{1}{(-p^2)} \sum_{l=0}^{k-1} (-1)^{n-l-1} (kn)^{l-1} (pn)^{n-l-1} \\
 &\quad \times \left\{ \left(\frac{1}{k} - \frac{2}{k+1} + \frac{2}{k+2} \right) (k-l) - \frac{1}{k+1} (k-l-1) - (-1)^l \left[\left(\frac{1}{k} - \frac{2}{k+1} + \frac{2}{k+2} \right) \binom{k}{l+1} - \frac{1}{k+1} \binom{k-1}{l+1} \right] \right\}. \quad (66)
 \end{aligned}$$

¹¹The expressions for unrenormalized singlet quark and gluon composite operators are presented in Figs. 1 and 2, respectively.

In deriving (66), basic integrals (A4)–(A7) from Appendix A were used. Let us remember the Feynman rule for the unrenormalized gluon operator $O_V^{n,k}$ (see Fig. 2):

$$(-1)^{n-k-1}(kn)^{k-1}(pn)^{n-k-1}[-g_{\mu\nu}kn(k+p)n - n_\mu n_\nu k(k+p) + n_\mu k_\nu(k+p)n + k_\mu n_\nu kn]. \quad (67)$$

As a result, the matrix of the finite renormalization in Eq. (64) has the following form ($1 \leq l \leq k-1$):

$$\begin{aligned} & [(\hat{Z}^{FV})_l^k(Q^2/\mu^2)]^{(1)} \\ &= C_F \frac{\alpha_s}{8\pi} \ln\left(\frac{Q^2}{\mu^2}\right) \left\{ \left(\frac{1}{k} - \frac{2}{k+1} + \frac{2}{k+2} \right) (k-l) \right. \\ & \quad - \frac{1}{k+1} (k-l-1) - (-1)^l \left[\left(\frac{1}{k} - \frac{2}{k+1} \right. \right. \\ & \quad \left. \left. + \frac{2}{k+2} \right) \binom{k}{l+1} - \frac{1}{k+1} \binom{k-1}{l+1} \right] \left. \right\}. \quad (68) \end{aligned}$$

In particular, it follows from (68):

$$\begin{aligned} & [(\hat{Z}^{FV})_{k-1}^k(Q^2/\mu^2)]^{(1)} \\ &= C_F \frac{\alpha_s}{8\pi} \ln\left(\frac{Q^2}{\mu^2}\right) \left[1 + (-1)^k \left(\frac{1}{k} - \frac{2}{k+1} + \frac{2}{k+2} \right) \right]. \quad (69) \end{aligned}$$

As one can see from (69), $[(\hat{Z}^{FV})_{k-1}^k]^{(1)} = 0$, for $k=1$. It is a consequence of the fact that the singlet operator $\bar{\Psi}(x)\gamma_\mu\Psi(x)$ do not mix with the gluon operators.¹²

From (65) and (69), the expression for the major gluon CF (57) follows which was obtained in the previous section by solving set of Eqs. (52). In order to obtain $[\tilde{C}_{m,l}^V(Q^2/\mu^2)]^{(1)}$ for $1 \leq l \leq m-2$, one should calculate the sum in k in Eq. (65). It can be done with the help of formulas (B13)–(B19) from Appendix B. As a result, we come to expressions (53)–(55), derived above on the basis of our main formula (30).

Finally, by using Eqs. (B9)–(B12) from Appendix B, one can obtain:

$$\begin{aligned} & \sum_{l=1}^{m-1} [\tilde{C}_{m,m}^F]^{(0)} [(\hat{Z}^{FV})_l^m(Q^2/\mu^2)]^{(1)} \mathbf{B}(k+2, l+2) \\ &= C_F \frac{\alpha_s}{4\pi} \ln\left(\frac{Q^2}{\mu^2}\right) \left[1 + (-1)^m \frac{1}{m(m+1)(m+2)} \right. \\ & \quad \times \left[\left[\frac{m-2}{k} + \frac{2(m+2)}{(k+1)(k+2)} + \frac{m(m+1)+2}{k+m+2} \right. \right. \\ & \quad \left. \left. - \frac{m(m+2)}{k+m+1} \right] - [(m-2)\mathbf{B}(k, m+3) \right. \\ & \quad \left. \left. + m(m+2)\mathbf{B}(k+1, m+2) \right] \right]. \quad (70) \end{aligned}$$

Correspondingly, with the help of formulas (B13)–(B19) and (B9)–(B12), we are able to find:

$$\begin{aligned} & \sum_{l=1}^{m-1} \sum_{k=l+1}^{m-1} [\tilde{C}_{m,k}^F]^{(0)} [(\hat{Z}^{FV})_l^k(Q^2/\mu^2)]^{(1)} \mathbf{B}(k+2, l+2) \\ &= C_F \frac{\alpha_s}{4\pi} \ln\left(\frac{Q^2}{\mu^2}\right) \left[1 - (-1)^m \frac{1}{m(m+1)(m+2)} \right. \\ & \quad \times \left[\left[\frac{m-2}{k} + \frac{2(m+2)}{(k+1)(k+2)} + \frac{m(m+1)+2}{k+m+2} \right. \right. \\ & \quad \left. \left. - \frac{m(m+2)}{k+m+1} \right] - [(m-2)\mathbf{B}(k, m+3) \right. \\ & \quad \left. \left. + m(m+2)\mathbf{B}(k+1, m+2) \right] \right]. \quad (71) \end{aligned}$$

Thus, we have successfully reproduced Eqs. (54) and (55).

It is well known that any Green function with an insertion of *one* composite operator is multiplicatively renormalized [11], while Green functions with insertion of *two (or more)* composite operators need additive counterterms [12]. Nevertheless, as was shown in [7], renormalization group equations for the CFs have no additive terms, provided the corresponding composite operators have zero vev.¹³

It was found that some *gauge-invariant* singlet composite operators can mix with *gauge-variant* ones under renormalization [14]. This problem is present for the simplest of these operators, the energy-momentum tensor $\theta_{\mu\nu}$, already in the leading order in strong coupling.

Let O and N represent a set of gauge-independent and non-gauge-independent operators, respectively. It was proven that renormalized and unrenormalized operators of these types are related by a triangular matrix [14]:

$$\begin{pmatrix} O \\ N \end{pmatrix}_R = \begin{pmatrix} \mathbf{Z}_{OO} & \mathbf{Z}_{ON} \\ 0 & \mathbf{Z}_{NN} \end{pmatrix} \begin{pmatrix} O \\ N \end{pmatrix}_U, \quad (72)$$

where \mathbf{Z}_{AB} are matrices. In other words, gauge-variant operators do not mix with gauge-invariant operators under the renormalization. Correspondingly, anomalous dimensions of gauge-independent operators can be determined from matrix \mathbf{Z}_{OO} alone. Moreover, in so-called *physical* (axial) gauge, $n_\mu A^\mu = 0$, n_μ being a constant light-like vector [15], a renormalization procedure does not require gauge-variant counterterms for the gauge-invariant composite operators at all [16].

It follows from (72) that in the OPE renormalized CFs of gauge-independent operators change under rescaling of renormalization mass, $\mu_0 \rightarrow \mu$, as follows:

$$C_O(\mu^2) = C_O(\mu_0^2) \hat{\mathbf{Z}}_{OO}(\mu_0^2/\mu^2), \quad (73)$$

¹²Obviously, it should takes place in *all* orders in α_s .

¹³The renormalization properties of the composite operators with nonzero vev were studied in [13].

where \hat{Z}_{OO} is a matrix of finite renormalization. All physical matrix elements of gauge-variant operators vanish [14].¹⁴ Using a complete set of hadronic states $|n\rangle$, we find that

$$\langle NO \rangle = \sum_n \langle N|n\rangle \langle n|O \rangle = 0. \quad (74)$$

Thus, a presence of gauge-variant composite operators in the OPE (1) have no influence on our method of determining CFs in terms of vacuum matrix elements. Indeed, multiplying elements of the sum $\sum_m [C_O^m O_m + C_N^m N_m]$ by one of the gauge-invariant operators, O_n , and putting them between vacuum states, we exclude a contribution from the gauge-variant operators to our main equation (30).

Taking all said above into account, we did not include gauge-dependent composite operators in the OPE (1).

V. CONCLUSIONS AND DISCUSSIONS

As it was shown in the present paper, the singlet CFs of the OPE of two currents can be explicitly expressed in terms of the Green functions of the corresponding composite operators without explicit use of the elementary (quark and gluon) fields. Our main equation (30) is a generalization of an analogous formula which was previously obtained for the singlet case [7]. It is necessary to stress that our formula holds in *any* renormalization scheme in contrast with other prescriptions (see, for instance, [19]).

As an illustration of a validity of our scheme, the gluon CFs were calculated in QCD in the first order of the strong coupling constant. It is important to note that both diagonal CFs, $\tilde{C}_{m,m-1}^g(Q^2/\mu^2)$ (57), and nondiagonal CFs, $\tilde{C}_{m,l}^g(Q^2/\mu^2)$ ($1 \leq l \leq m-2$) (53)–(55), were simultaneously obtained. The renormalization of these composite operators and their CFs were also considered.

For further discussion, let us rewrite a set of equations for the singlet quark and gluon CFs in the following symbolic form:

$$\begin{aligned} \langle JJO_q \rangle &= C_q \langle O_q O_q \rangle + C_g \langle O_g O_q \rangle, \\ \langle JJO_g \rangle &= C_q \langle O_q O_g \rangle + C_g \langle O_g O_g \rangle. \end{aligned} \quad (75)$$

These equations must be considered as the set of matrix equations (for simplicity, summations in l are omitted). It is also assumed that the procedure described in details in Section II is applied to all matrix elements in (75).¹⁵

Let us define “reduced” matrix elements of the quark and gluon composite operators in the n -th order of perturbation theory ($n \geq 1$):

¹⁴The calculations made in [17] contradict this result. However, it was shown by explicit calculations [18] that the proof in [17] breaks down, and conclusions of paper [14] remain true.

¹⁵See derivation of formula (30).

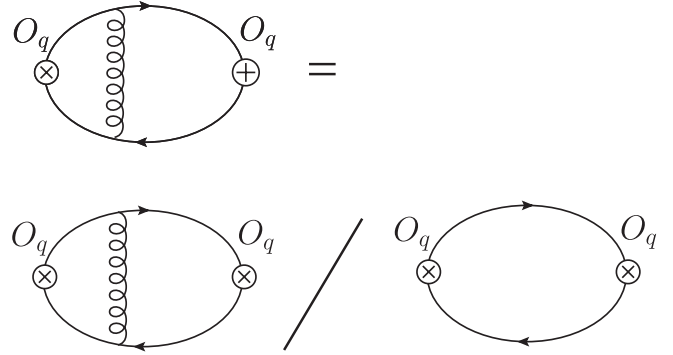


FIG. 8. The redefinition of the matrix element of the quark composite operators in the first order in strong coupling α_s .

$$\begin{aligned} \langle \widehat{O}_A \widehat{O}_q \rangle^{(n)} &= \langle O_A O_q \rangle^{(n)} [\langle O_q O_q \rangle^{(0)}]^{-1}, \\ \langle \widehat{O}_A \widehat{O}_g \rangle^{(n)} &= \langle O_A O_g \rangle^{(n)} [\langle O_g O_g \rangle^{(0)}]^{-1}, \end{aligned} \quad (76)$$

where $A = q, g$ (see Figs. 8 and 9). Analogously, we can define reduced matrix elements which contain both composite operators and electromagnetic currents ($n \geq 0$):

$$\begin{aligned} \langle \widehat{JJO}_q \rangle^{(n)} &= \langle JJO_q \rangle^{(n)} [\langle O_q O_q \rangle^{(0)}]^{-1}, \\ \langle \widehat{JJO}_g \rangle^{(n)} &= \langle JJO_g \rangle^{(n)} [\langle O_g O_g \rangle^{(0)}]^{-1} \end{aligned} \quad (77)$$

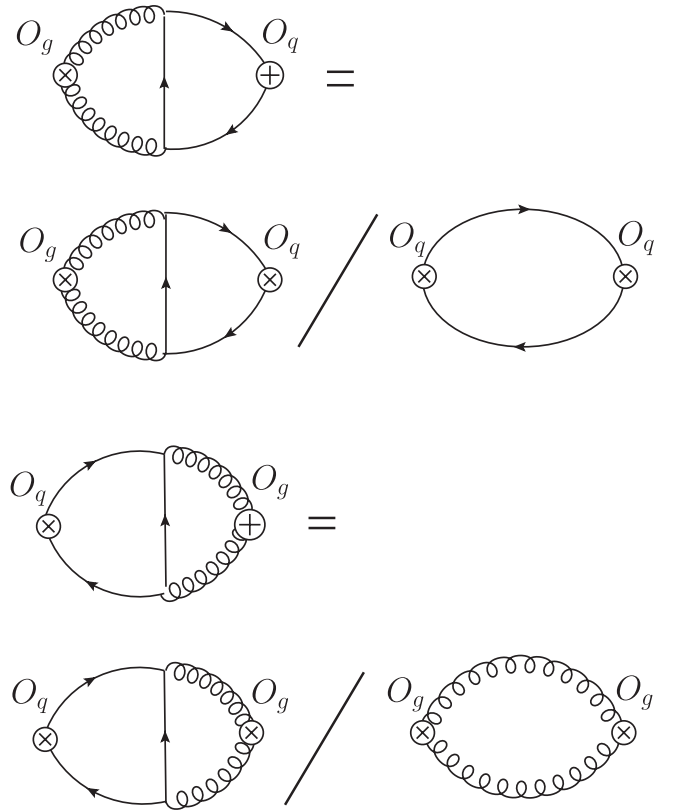


FIG. 9. The redefinition of the matrix elements of the quark and gluon composite operators in the first order in strong coupling α_s .

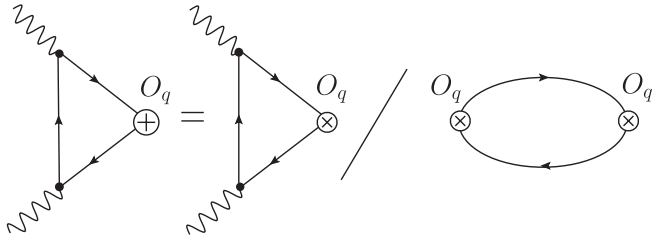


FIG. 10. The redefinition of the matrix element $\langle JJO_q \rangle$ in zero order in strong coupling α_s .

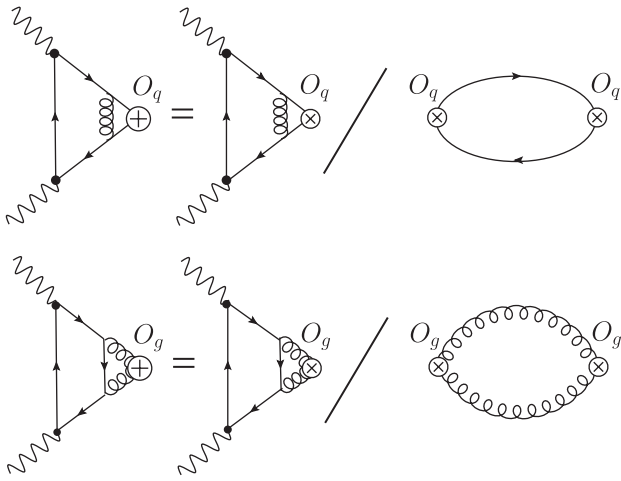


FIG. 11. The redefinition of the matrix elements $\langle JJO_q \rangle$ and $\langle JJO_g \rangle$ in the first order in strong coupling α_s .

(see Figs. 10 and 11). Note that each of the matrix elements $\langle O_A O_B \rangle^{(n)}$ and $\langle JJO_A \rangle^{(n)}$ has a divergency related with a divergency of a corresponding Feynman graphs as a whole.¹⁶ However, these divergences cancel in the *reduced* matrix elements $\langle \widehat{O}_A \widehat{O}_B \rangle^{(n)}$ and $\langle \widehat{JJO}_A \rangle^{(n)}$.

From (75) we get

$$\begin{aligned} \langle JJO_q \rangle^{(0)} &= C_q^{(0)} \langle O_q O_q \rangle^{(0)} + C_g^{(0)} \langle O_g O_q \rangle^{(0)}, \\ \langle JJO_g \rangle^{(0)} &= C_q^{(0)} \langle O_q O_g \rangle^{(0)} + C_g^{(0)} \langle O_g O_g \rangle^{(0)}. \end{aligned} \quad (78)$$

Since $\langle JJO_g \rangle^{(0)} = \langle O_g O_q \rangle^{(0)} = \langle O_q O_g \rangle^{(0)} = 0$, while $\langle O_g O_g \rangle^{(0)} \neq 0$, we immediately obtain in the leading (zero) order in the strong coupling α_s :

$$C_g^{(0)} = 0, \quad (79)$$

$$C_q^{(0)} = \langle \widehat{JJO}_q \rangle^{(0)}. \quad (80)$$

¹⁶Note that zero-order matrix elements $\langle O_q(p)O_q(-p) \rangle^{(0)}$, $\langle O_g(p)O_g(-p) \rangle^{(0)}$, and $\text{disc}_{(p+q)^2} \langle J(q)J(-p+q)O_q(p) \rangle^{(0)}$ are proportional to $\ln(1/p^2)$ at $p^2 \rightarrow 0$.

$$C_q^{(1)} = \left[\text{triangle diagram with } O_q \text{ vertex and gluon loop} + \dots \right]$$

$$- \left[\text{triangle diagram with } O_q \text{ vertex} \times \left[\text{circular diagram with } O_q \text{ and } O_g \text{ vertices} + \dots \right] \right]$$

FIG. 12. The singlet quark coefficient function of the OPE in the first order in strong coupling α_s .

In the first order in α_s , we derive the following expressions for the singlet quark and gluon CFs:

$$C_q^{(1)} = \langle \widehat{JJO}_q \rangle^{(1)} - \langle \widehat{JJO}_q \rangle^{(0)} \langle \widehat{O}_q \widehat{O}_q \rangle^{(1)}, \quad (81)$$

$$C_g^{(1)} = \langle \widehat{JJO}_g \rangle^{(1)} - \langle \widehat{JJO}_g \rangle^{(0)} \langle \widehat{O}_q \widehat{O}_g \rangle^{(1)}. \quad (82)$$

The first-order contributions to the singlet CFs are presented in Figs. 12 and 13, respectively. In the next order the singlet quark CF looks like

$$C_g^{(1)} = \left[\text{triangle diagram with } O_g \text{ vertex and gluon loop} \right]$$

$$- \left[\text{triangle diagram with } O_q \text{ vertex} \times \left[\text{circular diagram with } O_q \text{ and } O_g \text{ vertices} + \dots \right] \right]$$

FIG. 13. The gluon coefficient function of the OPE in the first order in strong coupling α_s .

$$\begin{aligned}
 C_q^{(2)} &= \langle \widehat{JJO}_q \rangle^{(2)} - [\langle \widehat{JJO}_q \rangle^{(1)} - \langle \widehat{JJO}_q \rangle^{(0)} \langle \widehat{O}_q \widehat{O}_q \rangle^{(1)}] \\
 &\quad \times \langle \widehat{O}_q \widehat{O}_q \rangle^{(1)} - [\langle \widehat{JJO}_g \rangle^{(1)} - \langle \widehat{JJO}_g \rangle^{(0)} \langle \widehat{O}_q \widehat{O}_g \rangle^{(1)}] \\
 &\quad \times \langle \widehat{O}_g \widehat{O}_q \rangle^{(1)} - \langle \widehat{JJO}_q \rangle^{(0)} \langle \widehat{O}_q \widehat{O}_q \rangle^{(2)}. \quad (83)
 \end{aligned}$$

Correspondingly, the gluon CF has the form:

$$\begin{aligned}
 C_g^{(2)} &= \langle \widehat{JJO}_g \rangle^{(2)} - [\langle \widehat{JJO}_q \rangle^{(1)} - \langle \widehat{JJO}_q \rangle^{(0)} \langle \widehat{O}_q \widehat{O}_q \rangle^{(1)}] \\
 &\quad \times \langle \widehat{O}_q \widehat{O}_g \rangle^{(1)} - [\langle \widehat{JJO}_g \rangle^{(1)} - \langle \widehat{JJO}_g \rangle^{(0)} \langle \widehat{O}_q \widehat{O}_g \rangle^{(1)}] \\
 &\quad \times \langle \widehat{O}_g \widehat{O}_g \rangle^{(1)} - \langle \widehat{JJO}_q \rangle^{(0)} \langle \widehat{O}_q \widehat{O}_g \rangle^{(2)}. \quad (84)
 \end{aligned}$$

In the same way, singlet quark and gluon CFs can be calculated at any order.

One more advantage of our approach is that it treats uniformly the CFs of light quarks ($q = u, d, s$) and CFs of heavy quarks ($Q = c, b$). For instance, singlet heavy quark CF, C_Q , is given by the same formulas (81) and (83). At the same time, the so-called *double counting* problem does not arise at all.¹⁷

The leading parts of the first terms in Eqs. (81)–(84), $\langle \widehat{JJO}_A \rangle^{(n)}$ ($A = q, g$) are proportional to $\ln^n(Q^2/p^2)$, while the leading parts of the matrix elements $\langle \widehat{O}_A \widehat{O}_B \rangle^{(n)}$ ($A, B = q, g$) are proportional to $\ln^n(\mu^2/p^2)$. Thus, the role of all terms in the r.h.s. of Eqs. (81)–(84), except the first one, is to *cancel* p^2 -dependence in the final expression for the singlet CFs:

$$C_A^{(1)} = N_A^{(1)} \left[\ln\left(\frac{Q^2}{p^2}\right) - \ln\left(\frac{\mu^2}{p^2}\right) \right] = N_A^{(1)} \ln\left(\frac{Q^2}{\mu^2}\right), \quad (85)$$

$$\begin{aligned}
 C_A^{(2)} &= N_A^{(2)} \left[\ln^2\left(\frac{Q^2}{p^2}\right) - 2 \ln\left(\frac{Q^2}{p^2}\right) \ln\left(\frac{\mu^2}{p^2}\right) + \ln^2\left(\frac{\mu^2}{p^2}\right) \right] \\
 &= N_A^{(2)} \ln^2\left(\frac{Q^2}{\mu^2}\right), \quad (86)
 \end{aligned}$$

where constants $N_A^{(n)}$ are known from explicit calculations.

An analogy can be drawn between our formulas (81)–(84) and diagram approach to calculating CFs.¹⁸ Namely, the quantity $\langle \widehat{JJO}_A \rangle$ should be associated with the amplitude shown in Fig. 14, with A being the type of off-shell parton with 4-momentum p . The n -th order contribution to the imaginary part of this amplitude grows as $\ln^n(Q^2/p^2)$ at $p^2 \rightarrow 0$ (or, equivalently, at $Q^2 \rightarrow \infty$). The detailed diagram analysis can be found in Ref. [21], where it was shown that the μ^2 -dependence drops out if a gauge-invariant set of QCD diagrams is taken into account in each order of perturbation theory.

¹⁷One possible solution of double counting problem, which appears in deriving CFs of heavy quarks at the diagram level, was proposed in [20].

¹⁸At the moment, we restrict ourselves by the diagonal CFs which survive in DIS.

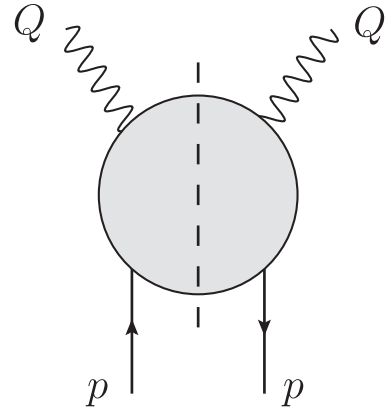


FIG. 14. The hadronic part of the amplitude of deep inelastic lepton scattering off a parton with 4-momentum p ($p^2 < 0$). The dotted line means that the imaginary part of the amplitude should be taken.

The quantity $\langle \widehat{O}_A \widehat{O}_B \rangle$ should be associated with the so-called cut vertex [22] depicted in Fig. 15, where upper (lower) lines in Fig. 15 correspond to partons of type $A(B)$. Being integrated in 4-momentum k of the upper parton, this diagram in the n -th order has a singularity $\ln^n(\mu^2/p^2)$ at $p^2 \rightarrow 0$.

Our results can be applied to studying *generalized* parton distributions (GPDs)¹⁹ [23–26] which permanently attract a great amount of interest.²⁰ They parameterize nonperturbative parton correlation functions in the nucleons and interpolate between the ordinary parton distribution functions (PDFs), which can be measured in DIS, and the elastic form factors. GPDs appear in cross sections of deeply virtual Compton scattering (DVCS), hard lepton production of vector mesons, as well as in diffractive Z^0 -production in ep -collision. They were also introduced in the context of the spin structure of the nucleons [25].

If the OPE (1) is applied to DIS, only diagonal operators of the type $O_{F, \mu_1 \dots \mu_m}^m = O_{F, \mu_1 \dots \mu_m}^{m,m}$ ($O_{V, \mu_1 \dots \mu_m}^m = O_{V, \mu_1 \dots \mu_m}^{m,m-1}$) are important, since the forward matrix elements of these operators with $1 \leq l \leq m-1$ ($1 \leq l \leq m-2$) are zero. However, for DVCS and other processes mentioned above, all operators contribute proportionally to $(p-p')_{\mu_{l+1}} \dots (p-p')_{\mu_m}$. The invariant structures of matrix elements of these operators related to the GPDs. Thus, our scheme of calculating CFs of the nondiagonal composite operators $O_{A, \mu_1 \dots \mu_m}^{m,l}$ ($A = F, V$) becomes quite important.

For the first time, the very notion of nonforward distribution function was introduced in Ref. [28], in which the statement was made that its Fourier transformation “*can be interpreted as the distribution of partons in momentum*”

¹⁹Also called *off-forward* or *nonforward* PDs.

²⁰For the first time, nonforward QCD planar ladder diagrams were studied in [27].

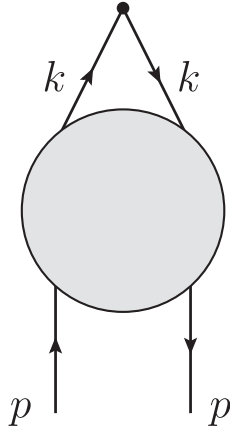


FIG. 15. The partonic cut vertex. The solid lines represent quarks or gluons fields. The “target” parton is off-shell, $p^2 < 0$. Integration in 4-momentum k is made.

fraction x and in impact parameter b_\perp .” Later on, it was shown that GPDs in the limit when the momentum transfer is purely transverse describe the distributions of unpolarized (polarized) partons in the transverse plane [29]. Impact parameter dependent PDFs satisfy positivity constraints which justify their physical interpretation as probability densities [29].

In conclusion, let us stress again that Eq. (30), which defines the singlet CFs in term of the Green functions of the composite operators, does not apply to perturbation theory at all. Therefore, our results can be used for calculating CFs of the OPE by nonperturbative methods.

All Feynman graphs presented in the present paper (Figs. 1–15) were prepared with the use of Axodraw package [30] and JaxoDraw graphical user interface [31].

ACKNOWLEDGMENTS

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APPENDIX A

In this Appendix we have presented several basic D -dimensional integrals ($D = 4 - 2\varepsilon$) which are needed

for our calculations. Light-cone 4-vector n_μ is defined in the main text (22). Only *divergent parts* of the integrals are shown.

$$(\mu^2)^\varepsilon \int \frac{d^D k}{(2\pi)^D} \frac{(kn)^m}{k^2(k+p)^2} = \frac{i}{16\pi^2} \frac{1}{\varepsilon} \left(\frac{\mu^2}{-p^2} \right)^\varepsilon \frac{(-1)^m}{m+1} (pn)^m, \quad (\text{A1})$$

$$\begin{aligned} (\mu^2)^\varepsilon \int \frac{d^D k}{(2\pi)^D} \frac{(kn)^m}{(k+p)^2(k-l)^2} \\ = \frac{i}{16\pi^2} \frac{1}{\varepsilon} \left[\frac{\mu^2}{-(l+p)^2} \right]^\varepsilon \frac{(-1)^m}{m+1} \sum_{p=0}^m (-1)^p (ln)^p (pn)^{m-p}, \end{aligned} \quad (\text{A2})$$

$$\begin{aligned} (\mu^2)^\varepsilon \int \frac{d^D k}{(2\pi)^D} \frac{(kn)^m [(k+p)n]^n}{k^2(k+p)^2} \\ = \frac{i}{16\pi^2} \frac{1}{\varepsilon} \left(\frac{\mu^2}{-p^2} \right)^\varepsilon (-1)^m (pn)^{m+n} \text{B}(m+1, n+1), \end{aligned} \quad (\text{A3})$$

where $\text{B}(x, y)$ is the beta-function, and $m \geq 0$. In the next four integrals $m \geq 1$ is assumed:

$$\begin{aligned} (\mu^2)^\varepsilon \int \frac{d^D k}{(2\pi)^D} \frac{k_\mu (kn)^m}{k^2(k+p)^2} \\ = \frac{i}{16\pi^2} \frac{1}{\varepsilon} \left(\frac{\mu^2}{-p^2} \right)^\varepsilon \frac{(-1)^{m-1}}{m+2} (pn)^{m-1} \\ \times \left[-n_\mu \frac{m}{2(m+1)} p^2 + p_\mu (pn) \right], \end{aligned} \quad (\text{A4})$$

$$\begin{aligned} (\mu^2)^\varepsilon \int \frac{d^D k}{(2\pi)^D} \frac{k_\mu (kn)^m}{k^2(k+p)^2(k-l)^2} \\ = \frac{i}{32\pi^2} \frac{1}{\varepsilon} \left[\frac{\mu^2}{-(l+p)^2} \right]^\varepsilon \frac{(-1)^{m-1}}{m+1} n_\mu \\ \times \sum_{p=0}^{m-1} (-1)^p (ln)^p (pn)^{m-p-1}, \end{aligned} \quad (\text{A5})$$

$$\begin{aligned} (\mu^2)^\varepsilon \int \frac{d^D k}{(2\pi)^D} \frac{k_\mu (kn)^m}{(k+p)^2(k-l)^2} = \frac{i}{16\pi^2} \frac{1}{\varepsilon} \left[\frac{\mu^2}{-(l+p)^2} \right]^\varepsilon \frac{(-1)^m}{m+1} \left\{ \frac{1}{2} n_\mu (l+p)^2 \left[\sum_{p=0}^{m-1} (-1)^p (p+1) (ln)^p (pn)^{m-p-1} \right. \right. \\ \left. \left. - \frac{1}{m+2} \sum_{p=0}^{m-1} (-1)^p (p+1)(p+2) (ln)^p (pn)^{m-p-1} \right] \right. \\ \left. + (l_\mu + p_\mu) \frac{1}{m+2} \sum_{p=0}^m (-1)^p (p+1) (ln)^p (pn)^{m-p} - p_\mu \sum_{p=0}^m (-1)^p (ln)^p (pn)^{m-p} \right\}, \end{aligned} \quad (\text{A6})$$

$$\begin{aligned}
 (\mu^2)^\varepsilon \int \frac{d^D k}{(2\pi)^D} \frac{k_\mu k_\nu (kn)^m}{k^2(k+p)^2(k-l)^2} &= \frac{i}{32\pi^2} \frac{1}{\varepsilon} \left[\frac{\mu^2}{-(l+p)^2} \right]^\varepsilon \frac{(-1)^m}{(m+1)(m+2)} \left\{ g_{\mu\nu} \sum_{p=0}^m (-1)^p (ln)^p (pn)^{m-p} \right. \\
 &- (l_\mu n_\nu + n_\mu l_\nu) \sum_{p=0}^{m-1} (-1)^p (p+1)(ln)^p (pn)^{m-p-1} \\
 &+ (p_\mu n_\nu + n_\mu p_\nu) \sum_{p=0}^{m-1} (-1)^p (m-p)(ln)^p (pn)^{m-p-1} \\
 &- \frac{1}{2} n_\mu n_\nu \left[k^2 \sum_{p=0}^{m-2} (-1)^p (p+1)(ln)^p (pn)^{m-p-2} \right. \\
 &+ p^2 \sum_{p=0}^{m-2} (-1)^p (m-p-1)(ln)^p (pn)^{m-p-2} \\
 &\left. \left. + (l+p)^2 \sum_{p=0}^{m-2} (-1)^p (p+1)(m-p-1)(ln)^p (pn)^{m-p-2} \right] \right\} \quad (A7)
 \end{aligned}$$

APPENDIX B

In this Appendix we collected formulas which are needed for calculating sums presented in the text and getting compact expressions.²¹ Everywhere below $\binom{n}{m}$ denotes a binomial coefficient, and $\mathbf{B}(x, y)$ beta-function. For integer $m, n \geq 0$, one has

$$\mathbf{B}(m+1, n+1) = \left[(m+n+1) \binom{m+n}{n} \right]^{-1}.$$

Let us first consider summation in index l :

$$\begin{aligned}
 \sum_{l=1}^{m-1} (-1)^l \binom{m-1}{l-1} \mathbf{B}(k+3, l) \\
 = -(-1)^m \mathbf{B}(k+3, m) - \frac{1}{k+m+2}, \quad (B1)
 \end{aligned}$$

$$\begin{aligned}
 \sum_{l=1}^{m-1} (-1)^l \binom{m-1}{l-1} \frac{1}{l+1} \mathbf{B}(k+1, l+2) \\
 = -(-1)^m \frac{1}{m+1} \mathbf{B}(k+1, m+2) \\
 - \frac{1}{(k+m+1)(k+m+2)(k+1)}, \quad (B2)
 \end{aligned}$$

²¹All the formulas collected in this section were derived by using several table sums with binomial coefficients [32].

$$\begin{aligned}
 \sum_{l=1}^{m-1} (-1)^l \binom{m-1}{l-1} \frac{1}{(l+1)(l+2)} \mathbf{B}(k, l+3) \\
 = -(-1)^m \frac{1}{(m+1)(m+2)} \mathbf{B}(k, m+3) \\
 - \frac{1}{(k+m+1)(k+m+2)k(k+1)}, \quad (B3)
 \end{aligned}$$

$$\begin{aligned}
 \sum_{l=1}^{m-1} (-1)^l \binom{m-1}{l-1} \frac{1}{l(l+1)(l+2)} \mathbf{B}(k, l+3) \\
 = -(-1)^m \frac{1}{m(m+1)(m+3)} \mathbf{B}(k, m+3) \\
 - \frac{1}{(k+m+2)k(k+1)(k+2)}, \quad (B4)
 \end{aligned}$$

$$\begin{aligned}
 \sum_{l=1}^{m-1} (-1)^l \binom{m-1}{l-1} \frac{1}{l+k} \\
 = -\mathbf{B}(k+1, m) - (-1)^m \frac{1}{m+k}. \quad (B5)
 \end{aligned}$$

In particular, we get from (B5):

$$\sum_{l=1}^{m-1} (-1)^l \binom{m-1}{l-1} \frac{1}{l} = -\frac{1}{m} [1 + (-1)^m], \quad (B6)$$

$$\sum_{l=1}^{m-1} (-1)^l \binom{m-1}{l-1} \frac{1}{l+1} = -\frac{1}{m} + \frac{1}{m+1} [1 - (-1)^m], \quad (B7)$$

$$\begin{aligned}
 \sum_{l=1}^{m-1} (-1)^l \binom{m-1}{l-1} \frac{1}{l+2} \\
 = -\frac{1}{m+2} (-1)^m - \frac{2}{m(m+1)(m+2)}. \quad (B8)
 \end{aligned}$$

Next four sums in l contain the same beta-function $\mathbf{B}(k+2, l+2)$:

$$\begin{aligned} & \sum_{l=1}^{m-1} (-1)^l \binom{m}{l+1} \mathbf{B}(k+2, l+2) \\ &= -\frac{m(m-1)}{(k+m+2)(k+2)(k+3)}, \end{aligned} \quad (\text{B9})$$

$$\begin{aligned} & \sum_{l=1}^{m-1} (-1)^l \binom{m}{l+1} (m-l-1) \mathbf{B}(k+2, l+2) \\ &= -\frac{m(m-1)(m-2)}{(k+m+1)(k+2)(k+3)}, \end{aligned} \quad (\text{B10})$$

$$\begin{aligned} \sum_{l=1}^{m-1} \mathbf{B}(k+2, l+2) &= -\mathbf{B}(k+1, m+2) \\ &+ \frac{2}{(k+1)(k+2)(k+3)}, \end{aligned} \quad (\text{B11})$$

$$\begin{aligned} & \sum_{l=1}^{m-1} (l+2) \mathbf{B}(k+2, l+2) \\ &= -\mathbf{B}(k, m+3) - (m+2) \mathbf{B}(k+1, m+2) \\ &+ (m+1) \mathbf{B}(k+2, m+1) + \frac{6}{(k+1)(k+2)(k+3)}. \end{aligned} \quad (\text{B12})$$

The following four sums in k are also used during calculations. The Kronecker symbols $\delta_{n,m}$ guarantee that all the sums are equal to zero for $l = m-1$:

$$\begin{aligned} & \sum_{k=l+1}^{m-1} (-1)^l \binom{m}{k-1} \frac{1}{k} \binom{k}{l+1} \\ &= -(-1)^m \frac{1}{m} \binom{m}{l+1} [1 - \delta_{0, m-l-1}], \end{aligned} \quad (\text{B13})$$

$$\begin{aligned} & \sum_{k=l+1}^{m-1} (-1)^l \binom{m}{k-1} \frac{1}{k+1} \binom{k}{l+1} \\ &= -(-1)^m \binom{m}{l+1} \left[\frac{1}{m+1} - \frac{1}{m} \delta_{0, m-l-1} \right] \\ &+ (-1)^l \frac{1}{m(m+1)}, \end{aligned} \quad (\text{B14})$$

$$\begin{aligned} & \sum_{k=l+1}^{m-1} (-1)^l \binom{m}{k-1} \frac{1}{k+2} \binom{k}{l+1} \\ &= -(-1)^m \binom{m}{l+1} \left[\frac{1}{m+2} - \frac{1}{m} \delta_{0, m-l-1} \right] \\ &+ 2(-1)^l \frac{l+2}{m(m+1)(m+2)}, \end{aligned} \quad (\text{B15})$$

$$\begin{aligned} & \sum_{k=l+1}^{m-1} (-1)^l \binom{m}{k-1} \frac{1}{k+1} \binom{k-1}{l+1} \\ &= -(-1)^m \frac{1}{m+1} \binom{m-1}{l+1} \\ &+ (-1)^l \frac{1}{m} \left[\frac{l+2}{m+1} - \delta_{0, m-l-1} \right]. \end{aligned} \quad (\text{B16})$$

Note that the latter sum is equal to zero both for $l = m-1$ and $l = m-2$.

$$\begin{aligned} & \sum_{k=l+1}^{m-1} (-1)^l \binom{m}{k-1} \frac{1}{k} \\ &= -\frac{1}{m} \left[(-1)^l \binom{m-1}{l} + (-1)^m \right], \end{aligned} \quad (\text{B17})$$

$$\begin{aligned} & \sum_{k=l+1}^{m-1} (-1)^l \binom{m}{k-1} \frac{1}{k+1} \\ &= (-1)^l \frac{1}{m} \left[\frac{1}{m+1} \binom{m}{l+1} - \binom{m-1}{l} \right] - (-1)^m \frac{1}{m+1}, \end{aligned} \quad (\text{B18})$$

$$\begin{aligned} & \sum_{k=l+1}^{m-1} (-1)^l \binom{m}{k-1} \frac{1}{k+2} \\ &= -(-1)^l \left[\frac{1}{m+2} \binom{m+1}{l+2} - \frac{2}{m+1} \binom{m}{l+2} \right] \\ &+ \frac{1}{m} \binom{m-1}{l+2} - (-1)^m \frac{1}{m+2}. \end{aligned} \quad (\text{B19})$$

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