Constraining the infrared behavior of the soft-wall AdS/QCD model

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By requiring the correct Regge behavior in both meson and nucleon sectors, we determine the infrared asymptotic behavior of various background fields in the soft-wall anti-de Sitter/QCD model, including the dilaton, the warp factor, and the scalar vacuum expectation value. We then use a simple parametrization which smoothly connects these IR limits and their usual UV limits. The resulting spectrum is compared with experimental data, and the agreement between them is good.

I. INTRODUCTION

Quantum chromodynamics (QCD) has been established as the genuine theory of strong interaction for nearly 40 years. Quarks and gluons are identified as the fundamental degrees of freedom. QCD is asymptotically free in the UV limit, so people can use standard techniques of perturbation theory to study the processes with large momentum transfer, like deep inelastic scatterings, etc. In the IR region, however, the coupling constant becomes strong. Now, the effective participants of strong interactions are various hadrons, like π, ρ, N, etc., while quarks and gluons are confined inside these particles. Perturbation theory cannot be directly used here. People need to develop various effective models to describe the low-energy hadron physics.

Anti-de Sitter (AdS)/QCD is one of them and has been densely researched for the last several years. This methodology stems from the idea of the large N expansion due to 't Hooft [1] and is directly motivated by the AdS/conformal field theory correspondence [2–4] in string theory. AdS/QCD is a bottom-up approach. It associates QCD operators, like chiral currents and the quark condensate, to bulk fields propagating in a five-dimensional (5D) space, which tends to AdS$_5$ as the fifth coordinate z goes to zero. There are mainly two versions: the hard-wall model [5–7] and the soft-wall model [8]. The former can correctly describe the chiral symmetry breaking (χSB) and low-lying hadron states. The latter is developed for the purpose of realizing the meson Regge behavior due to the linear confinement in QCD. People find that it is necessary to introduce the dilaton background which is quadratic growth in the deep IR region $z \to \infty$. It is further studied in Ref. [9] in order to correctly incorporate the χSB. AdS/QCD also has interesting relations with the light-front dynamics [10]. The UV limits of various background fields can be easily fixed. For instance, the warp factor should tend to that of the AdS space in order to reflect the conformal invariance of the high-energy fixed point of QCD, and the UV behavior of the vacuum expectation value (VEV) of the bulk scalar is determined by the pattern of χSB. Therefore, works on

soft-wall models mainly focus on various improvements in the IR region. However, it still seems arbitrary to some extent.

The main result of this paper is a way to fix the IR asymptotic behavior of various background fields: the dilaton $\Phi(z)$, the warp factor $a(z)$, and the scalar VEV $v(z)$. We achieve this just by requiring the model has the correct Regge-type spectrum in both meson and nucleon sectors. Nucleons can also be realized [11] in the framework of AdS/QCD by introducing 5D Dirac spinors which correspond to the baryon operators. In Ref. [12], nucleons are extended to the soft-wall model with an asymptotically linear spectrum in both meson and nucleon sectors. Some other works considering mesons and baryons at the same time can be found in Refs. [13–15]. The main drawback of the model in Ref. [12] is that, although both mesons and nucleons have linear spectra, the spectral slopes of vectors and that of axial vectors are different, which is inconsistent with experimental data. Actually, we can argue that it is impossible to improve this if only adjusting the form of the potential and the scalar VEV. The way around this is to allow—actually we *have to* allow, for being consist with data—the mass of a bulk field being z-dependent. This idea has also been suggested in the literature, e.g. Refs. [16,17]. Except for conserved currents, a generic operator will have nonzero anomalous dimension, which is scale-dependent due to the running coupling constant of QCD. According to the well-known dimension-mass relation, the mass of the corresponding bulk field should be z-dependent, since the fifth coordinate z can be interpreted as the inverse of the four-dimensional (4D) energy scale. What we find is that, by requiring the Regge-type spectrum is properly realized in both meson and nucleon sectors, the IR asymptotic behaviors of various background fields are totally fixed. Then, using a simple parametrization which smoothly connects these IR limits and their usual UV limits, we can make predictions and compare them with the observed data. Our philosophy is to reduce the uncertainty of the model as much as possible by the use of known facts.

II. THE MODEL AND CONSTRAINTS

The soft-wall AdS/QCD model is defined in a five-dimensional bulk with the metric

$$ds^2 = G_{MN}dx^M dx^N = a^2(z)(\eta_{\mu\nu}dx^\mu dx^\nu - dz^2),$$
$$0 < z < \infty. \qquad (2.1)$$

The factor $a(z)$ is called a warp factor, which tends to z^{-1} as $z \to 0$. There is also a background dilaton $\Phi(z)$ which is assumed to be $O(z^2)$ as $z \to \infty$, in order to have a Regge-type spectrum in the meson sector [8]. According to the general rules of the gauge/gravity duality, there are two 5D gauge fields, L_M^a and R_M^a, which correspond to 4D chiral currents $J_L^{a\mu} = \bar{q}_L \gamma^\mu t^a q_L$ and $J_R^{a\mu} = \bar{q}_R \gamma^\mu t^a q_R$. The quark bilinear operator $\bar{q}_L^i q_R^j$ is also an important 4D operator for χSB. Its holographic dual is a 5D 2×2 matrix-valued complex scalar field $X = (X^{ij})$, which is in the bifundamental representation of the 5D gauge group $SU(N_f)_L \times SU(N_f)_R$, with N_f being the number of quark flavors. The bulk action for the meson sector is

$$S_M = \int d^4x\, dz\sqrt{G}e^{-\Phi}\,\mathrm{Tr}\left\{-\frac{1}{4g_5^2}(F_L^2 + F_R^2)\right.$$
$$\left. + |DX|^2 - m_X^2|X|^2\right\}. \qquad (2.2)$$

By matching with QCD, $g_5^2 = 12\pi^2/N_c = 4\pi^2$. The covariant derivative of X is $D_M X = \partial_M X - iL_M X + iX R_M$. F_L and F_R are the field strengths of the gauge potentials L and R, respectively. The generator t^a is normalized by $\mathrm{Tr}(t^a t^b) = \frac{1}{2}\delta^{ab}$.

The bulk scalar X is assumed to have a z-dependent VEV: $\langle X \rangle = \frac{1}{2}v(z)$. The function $v(z)$ satisfies the equation of motion

$$\partial_z(a^3 e^{-\Phi}\partial_z v) - a^5 e^{-\Phi}m_X^2 v = 0. \qquad (2.3)$$

The mass-square m_X^2 may be z-dependent due to a possible anomalous dimension of $\bar{q}_L q_R$. From Eq. (2.3), we can express m_X^2 as

$$m_X^2 = \frac{v'' + (-\Phi' + 3a'/a)v'}{a^2 v}. \qquad (2.4)$$

To describe vector mesons, define $V_M = (L_M + R_M)/2$, and use the axial gauge $V_5 = 0$. Expand the field V_μ in terms of its Kaluza-Klein modes $V_\mu(x, z) = \sum_n \rho_\mu^{(n)}(x)f_V^{(n)}(z)$, with $f_V^{(n)}(z)$ being eigenfunctions of $-\partial_5(ae^{-\Phi}\partial_5 f_V^{(n)}) = ae^{-\Phi}M_V^{(n)2}f_V^{(n)}$. After integrating out the z-coordinate, we get an effective 4D action for a tower of massive vector fields $\rho_\mu^{(n)}(x)$, which can be identified as the fields of ρ mesons with $M_V^{(n)}$ being their masses. By setting $f_V^{(n)} = e^{\omega/2}\psi_V^{(n)}$, the equation of eigenfunctions can be transformed to a Schrödinger form $-\psi_V^{(n)\prime\prime} + V_V\psi_V^{(n)} = M_V^{(n)2}\psi_V^{(n)}$ with the potential

$$V_V = \frac{1}{4}\omega'^2 - \frac{1}{2}\omega''. \qquad (2.5)$$

where $\omega = \Phi - \log a$. Similarly, for axial-vectors, define $A_M = (L_M - R_M)/2$. Also, use the axial gauge $A_5 = 0$, expand $A_\mu(x, z) = \sum_n a_\mu^{(n)}(x)f_A^{(n)}(z)$, and transform the eigenvalue problem for $f_A^{(n)}(z)$ into the Schrödinger form. The resulting potential V_A for axial-vector mesons is

$$V_A = \frac{1}{4}\omega'^2 - \frac{1}{2}\omega'' + g_5^2 a^2 v^2. \qquad (2.6)$$

The corresponding eigenvalue is the mass-square $M_A^{(n)2}$ of the a_1 mesons. Note that there is an additional term $g_5^2 a^2 v^2$, which guarantees the axial-vector resonance is heavier than the vector one with the same radial quantum number.

The spin-1/2 nucleon can also be realized in the AdS/QCD framework by introducing two 5D Dirac spinors $\Psi_{1,2}$, which are charged under the gauge fields L_M and R_M, respectively. The nucleon sector action is [11,12]

$$S_N = \int d^4x\, dz\sqrt{G}(\mathcal{L}_K + \mathcal{L}_I),$$
$$\mathcal{L}_K = i\bar{\Psi}_1\Gamma^M\nabla_M\Psi_1 + i\bar{\Psi}_2\Gamma^M\nabla_M\Psi_2 - m_\Psi\bar{\Psi}_1\Psi_1 \quad (2.7)$$
$$+ m_\Psi\bar{\Psi}_2\Psi_2,$$
$$\mathcal{L}_I = -g_Y\bar{\Psi}_1 X\Psi_2 - g_Y\bar{\Psi}_2 X^\dagger\Psi_1.$$

Here, $\Gamma^M = e_A^M\Gamma^A = z\delta_A^M\Gamma^A$, and $\{\Gamma^A, \Gamma^B\} = 2\eta^{AB}$ with $A = (a, 5)$. We choose the representation as $\Gamma^A = (\gamma^a, -i\gamma^5)$ with $\gamma^5 = \mathrm{diag}(I, -I)$. The covariant derivatives for spinors are $\nabla_M\Psi_1 = \partial_M\Psi_1 + \frac{1}{2}\omega_M^{AB}\Sigma_{AB}\Psi_1 - iL_M\Psi_1$ and $\nabla_M\Psi_2 = \partial_M\Psi_2 + \frac{1}{2}\omega_M^{AB}\Sigma_{AB}\Psi_2 - iR_M\Psi_2$. The only nonzero components of the spin connection ω_M^{AB} is $\omega_\mu^{a5} = -\omega_\mu^{5a} = \frac{1}{z}\delta_\mu^a$. The $\mathcal{L}_I$ part introduces the effects of χSB into the nucleon sector. In Eq. (2.7), we also allow m_Ψ being z-dependent due to a possible anomalous dimension of the baryon operator. Similar to the meson sector, we expand two spinors $\Psi_{a=1,2}$ in terms of its Kaluza-Klein modes

$$\Psi_a(x, z) = \begin{pmatrix} \sum_n N_L^{(n)}(x)f_{aL}^{(n)}(z) \\ \sum_n N_R^{(n)}(x)f_{aR}^{(n)}(z) \end{pmatrix}. \qquad (2.8)$$

The 4D spinors $N^{(n)} = (N_L^{(n)}, N_R^{(n)})^{\mathrm{T}}$ are interpreted as nucleon fields. The internal wave functions f's satisfy four coupled 1st-order differential equations. By acting one more derivative and eliminating two right-handed f's, we get a coupled Sterm-Liouville eigenvalue problem for $f_L^{(n)} \equiv (f_{1L}^{(n)}, f_{2L}^{(n)})^{\mathrm{T}}$. Define $\chi_L^{(n)} = a^2 f_L^{(n)}$; the coupled Schrödinger equation for $\chi_L^{(n)}$ is $-\chi_L^{(n)\prime\prime} + V_N\chi_L^{(n)} = M_N^{(n)2}\chi_L^{(n)}$. The potential matrix V_N is

$$V_N = \begin{pmatrix} m_\Psi^2 a^2 + (m_\Psi a)' + u^2 & u' \\ u' & m_\Psi^2 a^2 - (m_\Psi a)' + u^2 \end{pmatrix}, \qquad (2.9)$$

with $u(z) = \frac{1}{2} g_Y a v$. The eigenvalue $M_N^{(n)2}$ is the mass-square of nucleon and its radial excitations.

Now, we start to analyze the asymptotic behavior of various background fields in the model, i.e. the dilaton $\Phi(z)$, the warp factor $a(z)$, and the scalar VEV $v(z)$. The UV limit is relatively simple to argue. For the warp factor

$$a(z) \sim \frac{L}{z}, \qquad z \rightarrow 0. \qquad (2.10)$$

This is because of the conformal invariance of the UV fixed point. So, the bulk space should be asymptotic 5D AdS. The value of the characteristic length L does not affect the resulting spectrum. For the scalar VEV

$$v(z) \sim Az + Bz^3, \qquad z \rightarrow 0. \qquad (2.11)$$

The linear term corresponds to the explicit χSB due to the quark mass, while the cubic term describes the spontaneous breaking by the nonzero quark condensate. Unlike the warp factor and the scalar VEV, the UV limit of the dilaton, however, cannot be uniquely fixed. The reason is as follows. Since QCD is asymptotically free, the conformal dimension of any operator, at the UV fixed point, is just its classical value, which is 3 for $\bar{q}_L q_R$. Therefore, by the mass-dimension relation $m_X^2 = \Delta(\Delta - 4)$, we have

$$m_X^2(z) \sim -3, \qquad z \rightarrow 0. \qquad (2.12)$$

From the expression (2.4), we can see that the above equation (2.12) holds if and only if $\Phi(z) \sim z^\alpha$ as $z \rightarrow 0$ with $\alpha > 0$. Actually, the UV limit of the dilaton could also depend on the form of the scalar potential in the bulk action [9,12]. So, it is, generally speaking, model-dependent.

Having studied the UV behavior, we now turn to the IR. For the dilaton, it must be

$$\Phi(z) \sim O(z^2), \qquad z \rightarrow \infty, \qquad (2.13)$$

which guarantees $M_V^{(n)2} \sim O(n)$ as $n \rightarrow \infty$ for vector mesons [8]. Suppose $a(z) \sim O(z^\gamma)$ as $z \rightarrow \infty$,[1] we always have $(\log a)' \sim O(z^{-1})$ for any power γ. Therefore, from the expression (2.5) of the potential, only considering vector mesons cannot give any constraint on the IR behavior of the warp factor $a(z)$. One of the key observations of this paper is that we can fix that by the nucleon sector. Look at the nucleon potential matrix (2.9), please. At the IR fixed point, QCD becomes a strongly coupled, but well-defined, conformal field theory. So, the dimension of the baryon operator should be finite, which means that m_Ψ, although it may be z-dependent, must tend to a finite constant as $z \rightarrow \infty$. Consider the diagonal terms of Eq. (2.9) first. $m_\Psi^2 a^2$ must dominate $\pm(m_\Psi a)'$ when z is large. However, which one is dominant between the 1st term $m_\Psi^2 a^2$ and the 3rd

term u^2 is a crucial issue. We determine this by reduction to the absurd. Suppose $u^2 \propto a^2 v^2$ dominates; then, the asymptotic linearity of nucleon spectrum forces $a^2 v^2 \sim O(z^2)$ in the IR. However, note that $u^2 \propto a^2 v^2$ also appears in the axial potential (2.6), there will be another $O(z^2)$ term in addition to the 1st term in $V_A(z)$. Then, this implies the axial-vector spectrum, although still linear, has a different slope with vector mesons, which is inconsistent with experimental data. Therefore, the conclusion is that $m_\Psi^2 a^2$ dominates u^2. Again, by the spectral linearity of nucleons, we obtain the desired IR limit of the warp factor as

$$a(z) \sim O(z), \qquad z \rightarrow \infty. \qquad (2.14)$$

By requiring vectors and axial-vectors have the same spectral slopes, we only know $v(z) \sim O(z^{1-\varepsilon})$ at IR for some positive ε. To further constrain it, we suppose the chiral symmetry is not asymptotically restored [18].[2] This means that $V_A - V_V \propto a^2 v^2$ should tend to some nonzero constant as $z \rightarrow \infty$. Therefore, the IR behavior of the scalar VEV is

$$v(z) \sim O(z^{-1}), \qquad z \rightarrow \infty. \qquad (2.15)$$

As a cross check, we find that $u' \propto (av)'$ tends to zero in the deep IR region. Two Schrödinger equations for nucleons decouple with each other, which means the mass difference between a nucleon state and its parity partner becomes smaller and smaller.[3] This is consistent with the observed data.

Additionally, it can be shown that, with these IR limits, the scalar and pseudoscalar mesons also have asymptotically linear spectral trajectories parallel to those of vectors and axial-vectors. By directly applying the method of Ref. [8], it can be further shown that the relation between the mass-square and the total angular-momentum quantum number J for higher spin mesons is indeed Regge-type. These facts exhibit the consistency of the AdS/QCD model and our asymptotic relations (2.14) and (2.15), which are our main results in the present work.

III. A SIMPLE PARAMETRIZATION

Having determined various asymptotic behaviors of background fields, we will use simple parametrizations which smoothly connect these asymptotes from UV to IR. First, we simply choose

$$\Phi = \kappa^2 z^2. \qquad (3.1)$$

It is shown in Ref. [22] that the sign of the dilaton should be positive to avoid a spurious massless state in the vector

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[1]If we assume a nonpower function, e.g. $a \sim e^z$, it will destroy the linear spectrum in the nucleon sector.

[2]There are some controversies about this issue among experts, see, e.g., Ref. [19] for the opposite opinion.

[3]This asymptotic degeneracy of nucleon states in a parity doublet does not necessarily imply, at least theoretically, the chiral symmetry restoration; see, e.g., Refs. [20,21].

TABLE I.   The theoretical and experimental values of vector meson masses.

ρ	0	1	2	3	4	5	6
m_{th} (MeV)	1003	1306	1550	1759	1947	2118	2276
m_{ex} (MeV)	775.5	1465	1570	1720	1909	2149	2265
error	29.3%	10.8%	1.3%	2.3%	2.0%	1.4%	0.5%

TABLE II.   The theoretical and experimental values of axial-vector meson masses.

a_1	0	1	2	3	4	5
m_{th} (MeV)	1452	1646	1842	2022	2187	2340
m_{ex} (MeV)	1230	1647	1930	2096	2270	2340
error	18.1%	0.0%	4.6%	3.6%	3.7%	0.0%

TABLE III.   The theoretical and experimental values of the spin-1/2 nucleon masses.

N	0	1	2	3	4	5	6
m_{th}(MeV)	937	1434	1583	1783	1842	2029	2065
m_{ex}(MeV)	939	1440	1535	1650	1710	2090	2100
error	0.2%	0.5%	3.2%	8.0%	7.7%	2.9%	1.7%

sector. Since we allow m_X^2 to be z-dependent, the choice (3.1) does not raise difficulties for the correct realization of χSB [17]. We parametrize the warp factor and the scalar VEV as

$$a(z) = \frac{1 + \mu z^2}{z}, \tag{3.2}$$

$$v(z) = \frac{Az + Bz^3}{1 + Cz^4}. \tag{3.3}$$

We have chosen the characteristic length L of AdS$_5$ to be 1. All of these three parametrizations have correct UV and IR behaviors determined in the previous section. By use of Eq. (2.4), the z dependence of m_X^2 has been fixed. Since the dilaton Φ has positive power, m_X^2 has the correct UV limit, i.e. $m_X^2 \sim -3$. In the deep IR, it can be shown that m_X^2 tends to zero. With these parametrizations, we can numerically solve the Schrödinger equation with the potentials $V_V(z)$ in Eq. (2.5) and $V_A(z)$ in Eq. (2.6). By fitting the vector meson masses and those of the axial-vectors, we choose the values of the five parameters as follows

$$\kappa = 415.9 \text{ MeV}, \qquad \mu = 860.4 \text{ MeV};$$
$$A = 2.1 \text{ MeV}, \qquad B = (411.9 \text{ MeV})^3, \tag{3.4}$$
$$C = (733.6 \text{ MeV})^4.$$

Vector mesons and axial-vector mesons both have asymptotically linear mass-squares, with the same slope $4\kappa^2$. The resulting spectra together with the observed values are listed in Table I and II, respectively. The agreement between them is good, especially for higher-resonance states.

For nucleons, we parametrize the bulk spinor mass m_Ψ as

$$m_\Psi = \frac{\frac{5}{2} + \mu_1 z}{1 + \mu_2 z}. \tag{3.5}$$

This parametrization gives the correct UV limit 5/2 which corresponds to the classical dimension 9/2 of the baryon operator by the mass-dimension relation for spinors $m_\Psi = \Delta - 2$. At IR, m_Ψ tends to a constant μ_1/μ_2 which, together with the parameter μ in the warp factor, determines the mass-square slope for nucleons. It is needed to numerically solve the coupled Schrödinger equation with proper boundary conditions [12]. We simply fix $\mu_1 =$

1 GeV, while μ_2 and the Yukawa coupling constant g_Y are chosen as

$$g_Y = 9.2, \qquad \mu_2 = 4573 \text{ MeV}. \tag{3.6}$$

The calculated nucleon masses and the corresponding data are listed in Table III. The agreement between them is also reasonable.

IV. CONCLUSIONS

By requiring the model has correct Regge-type spectrum in both meson and nucleon sectors, we determine the IR behavior of various background fields in the soft-wall AdS/QCD model, including the dilaton $\Phi(z)$, the warp factor $a(z)$, and the scalar VEV $v(z)$. More precisely, our arguments are mainly based on:

(i) $M_n^2 \sim O(n)$ as $n \to \infty$ for both mesons and nucleons.

(ii) The meson spectral slopes are asymptotically equal.

(iii) The distance between the mass-squares of a vector resonance and the corresponding axial-vector resonance tends to a finite nonzero constant.

We use simple parametrizations which smoothly connect these IR limits with their usual UV limits. The agreement between the predictive value and experimental data is good. In the present work, we restrict ourself to the lowest-order effective bulk action. Whether our arguments could supply some constraints on higher-dimensional terms, e.g. scalar potentials, is an interesting further issue. There is another way [23], which is more economic theoretically, to realize baryon states in the framework of holographic QCD by a five-dimensional Skyrmion. It is interesting to see how to realize the linear spectra for baryons in this model and which constraints imposed on various background fields.

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