

Sivers effect and transverse single spin asymmetry in $e + p^\uparrow \rightarrow e + J/\psi + X$

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We discuss the possibility of using electroproduction of J/ψ as a probe of gluon Sivers function by measuring single spin asymmetry (SSA) in experiments with transversely polarized protons and electron beams. We estimate SSA for JLab, HERMES, COMPASS, and eRHIC energies using the color evaporation model of charmonium production and find asymmetry up to 25% for certain choices of model parameters which have been used earlier for estimating SSA in the SIDIS and Drell-Yan processes.

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I. INTRODUCTION

In recent years, there has been a lot of interest in investigations of transverse single spin asymmetries (SSA's) in high energy QCD processes as they provide information about spin structure of proton. These asymmetries arise in scattering of a transversely polarized proton off an unpolarized proton if the scattering cross section depends on the direction of polarization.

The single spin asymmetry (SSA) for the inclusive process $A^\uparrow + B \rightarrow C + X$ is defined as

$$A_N = \frac{d\sigma^\uparrow - d\sigma^\downarrow}{d\sigma^\uparrow + d\sigma^\downarrow}, \quad (1)$$

where $d\sigma^{(\uparrow/\downarrow)}$ denotes the cross section for scattering of a transversely polarized hadron A off an unpolarized hadron B , with A upwards (downwards) transversely polarized with respect to the production plane. Large SSA's have been measured in pion production at Fermilab [1] as well as at BNL-RHIC in pp^\uparrow collisions [2]. SSA's have also been observed by the HERMES [3] and COMPASS [4] collaborations, in polarized semi-inclusive deep inelastic scattering. The magnitude of the observed asymmetries have been found to be larger than what is predicted by perturbative quantum chromodynamics (pQCD) [5].

It was first proposed by Sivers [6] that it may be possible to explain this asymmetry by allowing a correlation between the transverse momentum of the quark and the polarization of parent hadron. This approach requires a

pQCD factorization scheme that includes the spin and intrinsic transverse momentum effects. With the inclusion of \mathbf{k}_\perp dependence in parton distribution functions (pdf's) and fragmentation functions (ff's) [7], one is led to a generalized factorization formula called transverse momentum dependent (TMD) factorization [6,8]. TMD factorization in some processes has been proved at leading twist and leading order [9] and has been argued to hold at all orders.

The inclusion of the effect of transverse momentum of partons in pdf's and ff's leads to a new class of parton distributions which are obtained as extensions of usual collinear pdf's and include the effects of spin and transverse momentum of the partons. One of these functions is the Sivers function, which describes the probability of finding an unpolarized parton inside a transversely polarized hadron. The coupling of the transverse momentum of the unpolarized quarks and gluons to the nucleon spin is in fact related to their orbital angular momentum. Thus the Sivers asymmetry gives access to the orbital angular momentum of the partons. The number density of partons inside a proton with transverse polarization \mathbf{S} and momentum \mathbf{P} is parameterized as [10]

$$f_{a/p^\uparrow}(x_a, \mathbf{k}_{\perp a}, \mathbf{S}) \equiv f_{a/p}(x_a, k_{\perp a}) + \frac{1}{2} \Delta^N f_{a/p^\uparrow}(x_a, k_{\perp a}) \hat{\mathbf{S}} \cdot (\hat{\mathbf{P}} \times \hat{\mathbf{k}}_{\perp a}), \quad (2)$$

where $\mathbf{k}_{\perp a}$ is the transverse momentum of the parton, x_a is the longitudinal momentum fraction of parton, $f_{a/p}(x, k_{\perp a})$ is the unpolarized parton distribution and $\Delta^N f_{a/p^\uparrow}(x, k_{\perp a})$ is the Sivers function. In this work, we propose charmonium production as a probe to investigate the Sivers function and as a first step, estimate SSA in

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photoproduction of charmonium in scattering of electrons off transversely polarized protons.

One of the difficulties in getting information about the spin and transverse momentum dependent pdf's and fragmentation functions is that very often two or more of these functions contribute to the same physical observable making it difficult to estimate each single one separately. It has been shown how properly defined SSA's in the Drell-Yan process depend only on the quark Sivers function and the unpolarized quark distributions [10]. The studies of Anselmino *et al.* show that the magnitude of the Sivers asymmetry in the Drell-Yan (DY) process for forthcoming experiments at RHIC, COMPASS, J-PARC, PAX, PANDA, NICA, and SPASCHARM is large [11]. A study of the Sivers effect for pion and kaon production in semi-inclusive deep inelastic scattering (SIDIS) processes has been performed and estimates have been given for experiments at COMPASS and JLab [12–14]. It has been proposed to probe the gluon Sivers function by looking at back-to-back correlations in $p^\dagger p$ interactions at RHIC [15]. Another process that has been suggested as a probe to access gluon Sivers function is $p^\dagger p \rightarrow DX$ as the SSA in this case arises due to the gluon Sivers function alone mainly in the intermediate rapidity region [16].

In this paper, we investigate feasibility of using charmonium production to obtain information about the Sivers function. Charmonium production has been known to be a sensitive tool to study QCD for bound states of heavy quark-antiquark systems. Recently, there has been some discussion about the possibility to use fixed target experiments at LHC for charmonium production with the aim of investigating the quarkonium production mechanism [17]. Photoproduction and electroproduction of charmonium near threshold are expected to throw some light on this mechanism as well as on hadron structure [18]. In fact the connection between charm, charmonium production and the gluon densities has been explored since long for protons-polarized and unpolarized-, nuclei and photons [19]. Here, we study asymmetry in photoproduction (i.e. low virtuality electroproduction) of charmonium in scattering off polarized protons. At LO, this receives contribution only from a single partonic subprocess $\gamma g \rightarrow c\bar{c}$. Hence, SSA in $e + p^\dagger \rightarrow e + J/\psi + X$, if observed, can be used as a clean probe of gluon Sivers function. In addition, charmonium production mechanism can also have implications for this SSA and therefore, its study can help probe the production mechanism for charmonium.

There are three models for charmonium production. In the color singlet model [20] the cross section for charmonium production is factorized into a short distance part for $c\bar{c}$ pair production calculable in perturbation theory and a nonperturbative matrix element for the formation of a bound state, which is produced in a color singlet state. In the color evaporation model, first proposed by Halzen and Matsuda [21] and Fritsch [22] a statistical treatment of color is made and the probability of finding a specific

quarkonium state is assumed to be independent of the color of heavy quark pair. In later versions of this model it has been found that the data are better fitted if a phenomenological factor is included in the differential cross section formula, which depends on a Gaussian distribution of the transverse momentum of the charmonium [23]. A more recent model of charmonium production is the color octet model [24]. This is based on a factorization approach in nonrelativistic QCD, and it allows $c\bar{c}$ pairs to be produced in color octet states. Here again, one requires knowledge of the nonperturbative color octet matrix elements, which are determined through fits to the data on charmonium production. Inclusion of nonzero intrinsic transverse momentum for the colliding partons can help us to understand the discrepancy between these matrix elements determined from the hadroproduction (Tevatron) data and the lepto-production (HERA) data [25,26]. Since the nonrelativistic QCD calculations are done in collinear approximation, it is not surprising that the effects of nonzero transverse momentum of the colliding partons can be large. In this work, we have chosen to work with the color evaporation model (CEM) as its simplicity makes it suitable for an initial study of SSA in the charmonium production.

It has been proposed that the SSA's at leading twist in pQCD, arise from the final state interactions between the outgoing quark and the target spectator system [27]. In case of the Sivers asymmetry, the initial and/or final state interactions can generate a nonzero phase in the amplitude, which through the naive time reversal odd (T -odd) Sivers function then gives rise to the SSA. This interaction is part of the gauge link present in the TMD functions and depends on the specific process under consideration. Thus it introduces a process dependence, in particular, in the T -odd distribution functions [28]. In fact, the direction of the link is opposite in the Drell-Yan process as compared to SIDIS. For a generic hadronic process, it can be completely different from the above two processes. The generation of SSA in J/ψ production in ep^\dagger and pp^\dagger was studied in [29]. It was found therein that existence of nonzero SSA depends on the production mechanism of J/ψ , namely, in ep^\dagger processes nonzero SSA is expected only if the charmonium is produced in the color octet state and for pp^\dagger it should be produced in the color singlet state. However no numerical estimate of the asymmetry was given.

We provide a first estimate of the SSA in the low Q^2 electroproduction (photoproduction) of J/ψ at leading order, using a factorized formula along with models of Sivers function used in the literature. As a first step in our investigation of SSA's in charmonium production, we have used the color evaporation model for charmonium production. According to CEM, the cross section for charmonium production is proportional to the rate of production of $c\bar{c}$ pair integrated over the mass range $2m_c$ to $2m_D$ [30]

$$\sigma = \frac{1}{9} \int_{2m_c}^{2m_D} dM_{c\bar{c}} \frac{d\sigma_{c\bar{c}}}{dM_{c\bar{c}}}, \quad (3)$$

where m_c is the charm quark mass and $2m_D$ is the $D\bar{D}$ threshold.

At leading order (LO) there are no ff's involved and the only contribution to asymmetry comes from the Sivers function. Therefore, we can use this observable to extract information about it.

Here, we have used Weizsacker-Williams (WW) equivalent photon approximation to calculate cross section for the process $e + p^\dagger \rightarrow e + J/\psi + X$. The underlying partonic process at LO is $\gamma g \rightarrow c\bar{c}$ and therefore, the only k_\perp dependent pdf appearing is the gluon Sivers function. For a complete calculation of photoproduction of J/ψ one has to consider higher order contributions and also the resolved photon contributions [23].

To assess numerical estimates, we have used and compared two models of the quark Sivers function obtained from SIDIS data and have estimated the magnitude of asymmetry at JLab, HERMES, COMPASS, and eRHIC center of mass energies. We predict nonzero asymmetry in both rapidity and q_T distribution for the parameter set fitted from experimental data [31].

II. FORMALISM FOR ASYMMETRY IN J/ψ PRODUCTION

A. Color evaporation model

We consider the LO parton model cross section for low virtuality electroproduction (photoproduction) of J/ψ within color evaporation model. According to CEM, the cross section for charmonium production is given by Eq. (3), where $\frac{d\hat{\sigma}_{c\bar{c}}}{dM_{c\bar{c}}^2}$ is calculable perturbatively, $M_{c\bar{c}}$ being the invariant mass of the $c\bar{c}$ pair. The differential cross section for the $\gamma + p \rightarrow J/\psi + X$ is given by

$$\frac{d\sigma^{\gamma p \rightarrow c\bar{c}}}{dM_{c\bar{c}}^2} = \int dx f_{g/p}(x) \frac{d\hat{\sigma}^{\gamma g \rightarrow c\bar{c}}}{dM_{c\bar{c}}^2}, \quad (4)$$

where $f_{g/p}(x)$ is the gluon distribution in the proton.

Using the Weizsacker-Williams approximation [32,33], one can convolute the cross section given by Eq. (4) with a photon flux factor to obtain the electroproduction cross section for $e + p \rightarrow e + J/\psi + X$

$$\frac{d\sigma^{e p \rightarrow e + J/\psi + X}}{dM_{c\bar{c}}^2} = \int dy f_{\gamma/e}(y) \frac{d\hat{\sigma}^{\gamma p \rightarrow c\bar{c}}}{dM_{c\bar{c}}^2}, \quad (5)$$

where y is the energy fraction of electron carried by the photon and $f_{\gamma/e}(y)$ is the distribution function of the photon in the electron given by [34],

$$f_{\gamma/e}(y, E) = \frac{\alpha}{\pi} \left\{ \frac{1 + (1-y)^2}{y} \left(\ln \frac{E}{m} - \frac{1}{2} \right) + \frac{y}{2} \left[\ln \left(\frac{2}{y} - 2 \right) + 1 \right] + \frac{(2-y)^2}{2y} \ln \left(\frac{2-2y}{2-y} \right) \right\}. \quad (6)$$

Thus, the cross section for electroproduction of J/ψ using WW approximation is given by,

$$\sigma^{ep \rightarrow e + J/\psi + X} = \int_{4m_c^2}^{4m_D^2} dM_{c\bar{c}}^2 \int dy dx f_{\gamma/e}(y) f_{g/p}(x) \times \frac{d\hat{\sigma}^{\gamma g \rightarrow c\bar{c}}}{dM_{c\bar{c}}^2}. \quad (7)$$

B. Single spin asymmetry in J/ψ production

To calculate SSA in scattering of electrons off a polarized proton target, we assume a generalization of CEM expression by taking into account the transverse momentum dependence of the Weizsacker-Williams function and gluon distribution function

$$\frac{d\sigma^{e + p^\dagger \rightarrow e + J/\psi + X}}{dM^2} = \int dx_\gamma dx_g [d^2\mathbf{k}_{\perp\gamma} d^2\mathbf{k}_{\perp g}] \times f_{g/p^\dagger}(x_g, \mathbf{k}_{\perp g}) f_{\gamma/e}(x_\gamma, \mathbf{k}_{\perp\gamma}) \times \frac{d\hat{\sigma}^{\gamma g \rightarrow c\bar{c}}}{dM^2}, \quad (8)$$

where $M^2 \equiv M_{c\bar{c}}^2$. We have not written the scale dependence of the quantities on the *right-hand side* explicitly. As mentioned earlier, a generalization of factorization formula involving TMD pdf's and ff's leads to nonzero SSA in DY, SIDIS, and other processes. Therefore, we expect that inclusion of transverse momentum dependence in WW function and generalization of the CEM expression might be a valid approach to estimate SSA in charmonium production.

SSA is defined as in Eq. (1), where in our case $d\sigma^{\uparrow(\downarrow)}$ are single transverse spin-dependent cross sections for $e + p^\dagger \rightarrow e + J/\psi + X$ and $e + p^\dagger \rightarrow e + J/\psi + X$, respectively.

The difference in $d\sigma^\dagger$ and $d\sigma^\downarrow$ is parameterized in terms of the gluon Sivers function

$$d\sigma^\dagger - d\sigma^\downarrow = \int dx_\gamma dx_g d^2\mathbf{k}_{\perp\gamma} d^2\mathbf{k}_{\perp g} \Delta^N f_{g/p^\dagger}(x_g, \mathbf{k}_{\perp g}) \times f_{\gamma/e}(x_\gamma, \mathbf{k}_{\perp\gamma}) d\hat{\sigma}^{\gamma g \rightarrow c\bar{c}}, \quad (9)$$

where $d\hat{\sigma}$ is the elementary cross section for the process $\gamma g \rightarrow c\bar{c}$ given by

$$d\hat{\sigma} = \frac{1}{2\hat{s}} \frac{d^3p_c}{2E_c} \frac{d^3p_{\bar{c}}}{2E_{\bar{c}}} \frac{1}{(2\pi)^2} \times \delta^4(p_\gamma + p_g - p_c - p_{\bar{c}}) \overline{|M_{\gamma g \rightarrow c\bar{c}}|^2}. \quad (10)$$

We rewrite $\frac{d^3p_{\bar{c}}}{2E_{\bar{c}}} = d^4p_{\bar{c}} \delta(p_{\bar{c}}^2 - m_c^2)$ and change the variable to $q = p_c + p_{\bar{c}}$ [10] so that

$$\frac{d^3p_{\bar{c}}}{2E_{\bar{c}}} = d^4q \delta((q - p_c)^2 - m_c^2). \quad (11)$$

Now using the expression for total partonic cross section

$$\hat{\sigma}_0^{\gamma g \rightarrow c\bar{c}}(M^2) = \frac{1}{2\hat{s}} \int \frac{d^3 p_c}{2E_c} \frac{1}{(2\pi)^2} \times \delta((q - p_c)^2 - m_c^2) |\overline{M}_{\gamma g \rightarrow c\bar{c}}|^2 \quad (12)$$

and changing the variables from q_0 and q_L to M^2 and rapidity y so that

$$dM^2 dy = 2dq_0 dq_L. \quad (13)$$

We finally obtain

$$\begin{aligned} \frac{d^4 \sigma^\uparrow}{dy dM^2 d^2 \mathbf{q}_T} - \frac{d^4 \sigma^\downarrow}{dy dM^2 d^2 \mathbf{q}_T} &= \frac{1}{2} \int [dx_\gamma d^2 \mathbf{k}_{\perp \gamma} dx_g d^2 \mathbf{k}_{\perp g}] \\ &\times \Delta^N f_{g/p^i}(x_g, \mathbf{k}_{\perp g}) \\ &\times f_{\gamma/e}(x_\gamma, \mathbf{k}_{\perp \gamma}) \\ &\times \delta^4(p_g + p_\gamma - q) \\ &\times \hat{\sigma}_0^{\gamma g \rightarrow c\bar{c}}(M^2), \end{aligned} \quad (14)$$

where the partonic cross section is given by [35]

$$\begin{aligned} \hat{\sigma}_0^{\gamma g \rightarrow c\bar{c}}(M^2) &= \frac{1}{2} e_c^2 \frac{4\pi\alpha_s}{M^2} \left[\left(1 + \gamma - \frac{1}{2}\gamma^2 \right) \right. \\ &\times \left. \ln \frac{1 + \sqrt{1 - \gamma}}{1 - \sqrt{1 - \gamma}} - (1 + \gamma)\sqrt{1 - \gamma} \right]. \end{aligned} \quad (15)$$

Here, $\gamma = \frac{4m_c^2}{M^2}$ and $M^2 \equiv \hat{s}$.

$\Delta^N f_{g/p^i}(x_g, \mathbf{k}_{\perp g})$ is related to the gluon Sivvers function $\Delta^N f_{g/p^i}(x, k_{\perp g})$ by

$$\Delta^N f_{g/p^i}(x_g, \mathbf{k}_{\perp g}) = \Delta^N f_{g/p^i}(x_g, k_{\perp g}) \hat{\mathbf{S}} \cdot (\hat{\mathbf{P}} \times \hat{\mathbf{k}}_{\perp g}). \quad (16)$$

Similarly the total cross section is given by

$$\begin{aligned} \frac{d^4 \sigma^\uparrow}{dy dM^2 d^2 \mathbf{q}_T} + \frac{d^4 \sigma^\downarrow}{dy dM^2 d^2 \mathbf{q}_T} &= \int [dx_\gamma d^2 \mathbf{k}_{\perp \gamma} dx_g d^2 \mathbf{k}_{\perp g}] \\ &\times f_{g/p}(x_g, \mathbf{k}_{\perp g}) \\ &\times f_{\gamma/e}(x_\gamma, \mathbf{k}_{\perp \gamma}) \\ &\times \delta^4(p_g + p_\gamma - q) \\ &\times \hat{\sigma}_0^{\gamma g \rightarrow c\bar{c}}(M^2). \end{aligned} \quad (17)$$

It is worth mentioning at this point that following the procedure used by Anselmino *et al.* in case of Drell-Yan process, we have been able to write the distribution in M^2 , q_T and y of the produced J/ψ in terms of unpolarized total partonic cross section.

Rewriting the four momentum conservation δ function as

$$\begin{aligned} \delta^4(p_g + p_\gamma - q) &= \delta(E_g + E_\gamma - q_0) \delta(p_{z_g} + p_{z_\gamma} - q_L) \\ &\times \delta^2(\mathbf{k}_{\perp \gamma} + \mathbf{k}_{\perp g} - \mathbf{q}_T) \\ &= \frac{2}{s} \delta\left(x_g - \frac{M}{\sqrt{s}} e^y\right) \delta\left(x_\gamma - \frac{M}{\sqrt{s}} e^{-y}\right) \\ &\times \delta^2(\mathbf{k}_{\perp \gamma} + \mathbf{k}_{\perp g} - \mathbf{q}_T). \end{aligned} \quad (18)$$

one can perform the x_γ and x_g integrations to obtain

$$\begin{aligned} \frac{d^4 \sigma^\uparrow}{dy dM^2 d^2 \mathbf{q}_T} - \frac{d^4 \sigma^\downarrow}{dy dM^2 d^2 \mathbf{q}_T} &= \frac{1}{s} \int [d^2 \mathbf{k}_{\perp \gamma} d^2 \mathbf{k}_{\perp g}] \\ &\times \Delta^N f_{g/p^i}(x_g, \mathbf{k}_{\perp g}) \\ &\times f_{\gamma/e}(x_\gamma, \mathbf{k}_{\perp \gamma}) \\ &\times \delta^2(\mathbf{k}_{\perp \gamma} + \mathbf{k}_{\perp g} - \mathbf{q}_T) \\ &\times \hat{\sigma}_0^{\gamma g \rightarrow c\bar{c}}(M^2) \end{aligned} \quad (19)$$

and

$$\begin{aligned} \frac{d^4 \sigma^\uparrow}{dy dM^2 d^2 \mathbf{q}_T} + \frac{d^4 \sigma^\downarrow}{dy dM^2 d^2 \mathbf{q}_T} &= \frac{2}{s} \int [d^2 \mathbf{k}_{\perp \gamma} d^2 \mathbf{k}_{\perp g}] \\ &\times f_{g/p}(x_g, \mathbf{k}_{\perp g}) \\ &\times f_{\gamma/e}(x_\gamma, \mathbf{k}_{\perp \gamma}) \\ &\times \delta^2(\mathbf{k}_{\perp \gamma} + \mathbf{k}_{\perp g} - \mathbf{q}_T) \\ &\times \hat{\sigma}_0^{\gamma g \rightarrow c\bar{c}}(M^2) \end{aligned} \quad (20)$$

with

$$x_{g,\gamma} = \frac{M}{\sqrt{s}} e^{\pm y}. \quad (21)$$

Integrating Eqs. (19) and (20) over M^2 as prescribed by CEM, we obtain the difference and sum of $\frac{d^3 \sigma^\uparrow}{dy d^2 \mathbf{q}_T}$ and $\frac{d^3 \sigma^\downarrow}{dy d^2 \mathbf{q}_T}$ for J/ψ production.

We follow the convention in Ref. [36] and define the Sivvers asymmetry integrated over the azimuthal angle of J/ψ with a weight factor $\sin(\phi_{q_T} - \phi_S)$:

$$A_N^{\sin(\phi_{q_T} - \phi_S)} = \frac{\int d\phi_{q_T} [d\sigma^\uparrow - d\sigma^\downarrow] \sin(\phi_{q_T} - \phi_S)}{\int d\phi_{q_T} [d\sigma^\uparrow + d\sigma^\downarrow]}, \quad (22)$$

where $d\sigma^\uparrow$ is differential cross section in q_T or y variable and ϕ_{q_T} and ϕ_S are the azimuthal angles of the J/ψ and proton spin, respectively. To evaluate asymmetry in y distribution, we will substitute

$$\begin{aligned} d\sigma^\uparrow - d\sigma^\downarrow &= \int d\phi_{q_T} \int q_T dq_T \int_{4m_c^2}^{4m_D^2} [dM^2] \int [d^2 \mathbf{k}_{\perp g}] \\ &\times \Delta^N f_{g/p^i}(x_g, \mathbf{k}_{\perp g}) f_{\gamma/e}(x_\gamma, \mathbf{q}_T - \mathbf{k}_{\perp g}) \\ &\times \hat{\sigma}_0(M^2) \sin(\phi_{q_T} - \phi_S) \end{aligned} \quad (23)$$

and

$$\begin{aligned} d\sigma^\uparrow + d\sigma^\downarrow &= 2 \int d\phi_{q_T} \int q_T dq_T \int_{4m_c^2}^{4m_D^2} [dM^2] \int [d^2 \mathbf{k}_{\perp g}] \\ &\times f_{g/p}(x_g, \mathbf{k}_{\perp g}) f_{\gamma/e}(x_\gamma, \mathbf{q}_T - \mathbf{k}_{\perp g}) \\ &\times \hat{\sigma}_0(M^2). \end{aligned} \quad (24)$$

Thus at LO, the SSA depends on Weizsacker-Williams function, gluon distribution function and gluon Sivvers

function. We discuss our choice of WW function and Sivers function in the following subsection.

C. Sivers function and Weizsacker-Williams function

In our analysis, we have chosen a kinematical configuration in which proton with momentum \mathbf{P} is moving along z axis and is transversely polarized in y direction so that

$$\hat{S} \cdot (\hat{\mathbf{P}} \times \hat{\mathbf{k}}_{\perp g}) = \hat{k}_{\perp gz} = \cos\phi_{k_{\perp}}, \quad (25)$$

where $\mathbf{k}_{\perp g} = k_{\perp}(\cos\phi_{k_{\perp}}, \sin\phi_{k_{\perp}}, 0)$.

For $k_{\perp g}$ dependence of the unpolarized pdf's, we use a simple factorized and Gaussian form [12]

$$f_{g/p}(x_g, k_{\perp}) = f_{g/p}(x_g) \frac{1}{\pi \langle k_{\perp g}^2 \rangle} e^{-k_{\perp g}^2 / \langle k_{\perp g}^2 \rangle}. \quad (26)$$

We also need a transverse momentum dependent WW function. In our analysis, we have used two choices for it:

(1) A simple Gaussian form as above:

$$f_{\gamma/e}(x_{\gamma}, k_{\perp \gamma}) = f_{\gamma/e}(x_{\gamma}) \frac{1}{\pi \langle k_{\perp \gamma}^2 \rangle} e^{-k_{\perp \gamma}^2 / \langle k_{\perp \gamma}^2 \rangle}. \quad (27)$$

(2) A dipole form:

$$f_{\gamma/e}(x_{\gamma}, k_{\perp \gamma}) = f_{\gamma/e}(x_{\gamma}) \frac{1}{2\pi} \frac{N}{k_{\perp \gamma}^2 + k_0^2}, \quad (28)$$

where N is a normalization constant, which gets cancelled in the asymmetry.

For Sivers function we have used two models in our analysis

(i) In most part of the analysis we use model I [11] which is given by,

$$\Delta^N f_{g/p^1}(x_g, \mathbf{k}_{\perp g}) = \Delta^N f_{g/p^1}(x_g) \frac{1}{\pi \langle k_{\perp g}^2 \rangle} h(k_{\perp g}) \times e^{-k_{\perp g}^2 / \langle k_{\perp g}^2 \rangle} \cos(\phi_{k_{\perp}}), \quad (29)$$

where the gluon Sivers function, $\Delta^N f_{g/p^1}(x_g)$ is defined as

$$\Delta^N f_{g/p^1}(x_g) = 2\mathcal{N}_g(x_g) f_{g/p}(x_g), \quad (30)$$

where $\mathcal{N}_g(x_g)$ is an x -dependent normalization for gluon to be chosen so that the gluon Sivers function obeys the positivity bound

$$\frac{|\Delta^N f_{g/p^1}(x_g, \mathbf{k}_{\perp g})|}{2\hat{f}_{g/p}(x_g, k_{\perp g})} \leq 1, \quad \forall x_g, k_{\perp g} \quad (31)$$

and

$$h(k_{\perp g}) = \sqrt{2} e^{\frac{k_{\perp g}}{M_1}} e^{-k_{\perp g}^2 / M_1^2}, \quad (32)$$

where the gluon Sivers function, $\Delta^N f_{g/p^1}(x_g)$ is given as in model I and M_1 is parameter obtained by fitting

the recent experimental data corresponding to pion and kaon production at HERMES and COMPASS.

The corresponding parameterizations for quark Sivers function $\mathcal{N}_u(x)$ and $\mathcal{N}_d(x)$ have been fitted from SIDIS data and are given by [15],

$$\mathcal{N}_f(x) = N_f x^{a_f} (1-x)^{b_f} \frac{(a_f + b_f)^{(a_f + b_f)}}{a_f^{a_f} b_f^{b_f}}, \quad (33)$$

where a_f, b_f, N_f for u and d quarks are free parameters obtained by fitting the data. However, there is no information available on $\mathcal{N}_g(x)$. In our analysis we have used two choices [15]

(a) $\mathcal{N}_g(x) = (\mathcal{N}_u(x) + \mathcal{N}_d(x))/2$,

(b) $\mathcal{N}_g(x) = \mathcal{N}_d(x)$.

(ii) We also compared the results with model II [14,16] given by

$$\Delta^N f_{g/p^1}(x_g, \mathbf{k}_{\perp g}) = \Delta^N f_{g/p^1}(x_g) \frac{1}{\pi \langle k_{\perp g}^2 \rangle} e^{-k_{\perp g}^2 / \langle k_{\perp g}^2 \rangle} \times \frac{2k_{\perp g} M_0}{k_{\perp g}^2 + M_0^2} \cos(\phi_{k_{\perp}}), \quad (34)$$

and $M_0 = \sqrt{\langle k_{\perp g}^2 \rangle}$, where the gluon Sivers function is given as in model I.

Model I has been used in analysis of SSA in the SIDIS [12] and DY processes [11] and model II has been used for the quark Sivers function to estimate SSA in D meson production at RHIC [16]. A comparison of k_{\perp} dependence of Sivers function in the two models is given by Fig. 7 of Ref. [14]. We will give numerical estimates of asymmetry in photoproduction of charmonium using both of these models and will also compare both the parameterizations for $\mathcal{N}_g(x_g)$ in our analysis in the next section.

III. NUMERICAL ESTIMATES FOR THE ASYMMETRY IN J/ψ PRODUCTION

We will now estimate the magnitude of asymmetry using models I and II for both the parameterizations (a) and (b) and Gaussian form for WW function k_{\perp} dependence.

The values of best fit parameters of Sivers functions we have used are [31]

$$\begin{aligned} N_u &= 0.40, & a_u &= 0.35, & b_u &= 2.6, \\ N_d &= -0.97, & a_d &= 0.44, & b_d &= 0.90, \\ M_1^2 &= 0.19 \text{ GeV}^2. \end{aligned} \quad (35)$$

These parameters are from new HERMES and COMPASS data [37,38] fitted at $\langle Q^2 \rangle = 2.4 \text{ GeV}^2$.

The value of $\langle k_{\perp g}^2 \rangle$ is chosen to be the same as $\langle k_{\perp}^2 \rangle$ for quarks obtained in Ref. [14] by analysis of Cahn effect in unpolarized SIDIS from data collected in different energy and Q^2 ranges assuming a constant Gaussian width. The value of $\langle k_{\perp \gamma}^2 \rangle$ has been chosen to be comparable to $\langle k_{\perp g}^2 \rangle$.

As pointed out in [39], $\langle k_{\perp}^2 \rangle$ in fact depends on energy. However, in this paper, we keep this value fixed as above as we do not expect a large variation in this parameter related to charmonium production in the model concerned (see below).

A. Asymmetry using model I

In model I, the exponential nature of the function $h(k_{\perp})$ enables one to perform the k_{\perp} integration analytically and we obtain the expressions for numerator and denominator in asymmetry as

$$\begin{aligned} \frac{d^3 \sigma^{\uparrow}}{dy d^2 \mathbf{q}_T} - \frac{d^3 \sigma^{\downarrow}}{dy d^2 \mathbf{q}_T} &= \frac{1}{s} \int_{4m_c^2}^{4m_D^2} dM^2 \Delta^N f_{g/p}(x_g) f_{\gamma/e}(x_{\gamma}) \\ &\times \sqrt{2e} \frac{q_T \langle k_S^2 \rangle^2 \exp[-q_T^2 / (\langle k_S^2 \rangle + \langle k_{\perp \gamma}^2 \rangle)]}{M_1 \pi [\langle k_S^2 \rangle + \langle k_{\perp \gamma}^2 \rangle]^2 \langle k_{\perp g}^2 \rangle} \\ &\times \cos(\phi_{q_T}) \hat{\sigma}_0^{\gamma g \rightarrow c\bar{c}}(M^2) \end{aligned} \quad (36)$$

$$\begin{aligned} \frac{d^3 \sigma^{\uparrow}}{dy d^2 \mathbf{q}_T} - \frac{d^3 \sigma^{\downarrow}}{dy d^2 \mathbf{q}_T} &= \frac{1}{s} \int_{4m_c^2}^{4m_D^2} dM^2 \int_0^{k_{\perp g \max}} dk_{\perp g} \int_0^{2\pi} d\phi_{k_{\perp}} k_{\perp g} \Delta^N f_{g/p}(x_g) f_{\gamma/e}(x_{\gamma}) \exp[-k_{\perp g}^2 / \alpha] \exp[-k_{\perp g}^2 / \beta] \\ &\times \exp[-q_T^2 / \alpha] \exp[(2q_T k_{\perp g} \cos(\phi_{k_{\perp}} - \phi_{q_T})) / \alpha] \frac{1}{\pi^2 \alpha \beta} \frac{2k_{\perp g} M_0}{k_{\perp g}^2 + M_0^2} \cos(\phi_{k_{\perp}}) \hat{\sigma}_0^{\gamma g \rightarrow c\bar{c}}(M^2) \end{aligned} \quad (39)$$

and the denominator becomes,

$$\begin{aligned} \frac{d^3 \sigma^{\uparrow}}{dy d^2 \mathbf{q}_T} + \frac{d^3 \sigma^{\downarrow}}{dy d^2 \mathbf{q}_T} &= \frac{2}{s} \int_{4m_c^2}^{4m_D^2} dM^2 \int_0^{k_{\perp g \max}} dk_{\perp g} \int_0^{2\pi} d\phi_{k_{\perp}} k_{\perp g} f_{g/p}(x_g) f_{\gamma/e}(x_{\gamma}) \exp[-k_{\perp g}^2 / \alpha] \exp[-k_{\perp g}^2 / \beta] \\ &\times \exp[-q_T^2 / \alpha] \exp[(2q_T k_{\perp g} \cos(\phi_{k_{\perp}} - \phi_{q_T})) / \alpha] \frac{1}{\pi^2 \alpha \beta} \hat{\sigma}_0^{\gamma g \rightarrow c\bar{c}}(M^2), \end{aligned} \quad (40)$$

where $\alpha = \langle (\mathbf{q}_T - \mathbf{k}_{\perp g})^2 \rangle$, $\beta = \langle k_{\perp g}^2 \rangle$ and $M_0 = \sqrt{\langle k_{\perp g}^2 \rangle} = \sqrt{\beta}$.

We have used $k_{\perp g \max} = 0.5$ GeV, observing that the effective intrinsic motion is limited to $k_{\perp} \leq \sqrt{\langle k_{\perp}^2 \rangle}$ for Gaussian distribution [13].

We have estimated the asymmetry with both kinds of parameterizations [labeled (a) and (b)]. The estimates are obtained using GRV98LO for gluon distribution functions and Weizsaker-Williams function for photon distribution [34]. The scale of the gluon distribution has been taken to be $\hat{s}(= M^2)$ [35]. It is worth pointing out that the scale evolution of the TMD's including the Siverson function has been worked out in [40,41] and recently it has been noted in [42,43] that in SIDIS the evolution indeed affects the Siverson asymmetry. In fact the evolution effectively produces a change in the Gaussian width of the TMDs depending on the scale. The possible effect of such scale dependence is not included in our this, exploratory, study of the asymmetry in J/ψ production and is deferred to a later publication. However, since we are using CEM at LO, the hard scale is between $4m_c^2$ and $4m_D^2$ independent of the

and,

$$\begin{aligned} \frac{d^3 \sigma^{\uparrow}}{dy d^2 \mathbf{q}_T} + \frac{d^3 \sigma^{\downarrow}}{dy d^2 \mathbf{q}_T} &= \frac{2}{s} \int_{4m_c^2}^{4m_D^2} dM^2 f_{g/p}(x_g) f_{\gamma/e}(x_{\gamma}) \\ &\times \frac{\exp[-q_T^2 / (\langle k_{\perp g}^2 \rangle + \langle k_{\perp \gamma}^2 \rangle)]}{\pi [\langle k_{\perp g}^2 \rangle + \langle k_{\perp \gamma}^2 \rangle]} \\ &\times \hat{\sigma}_0^{\gamma g \rightarrow c\bar{c}}(M^2), \end{aligned} \quad (37)$$

where

$$\frac{1}{\langle k_S^2 \rangle} = \frac{1}{M_1^2} + \frac{1}{\langle k_{\perp g}^2 \rangle}. \quad (38)$$

B. Asymmetry using model II

With the k_{\perp} dependent unpolarized pdf's and gluon Siverson function in Eq. (26) and (34) numerator becomes

beam energies and the TMD evolution effects within this rather narrow range will be similar for all experiments considered. Our results are presented in Figs. 1–9. The main features of these plots are summarized below.

In Figs. 1–5 we have shown the asymmetry ($A_N^{\sin(\phi_{q_T} - \phi_S)}$) as a function of rapidity y and q_T respectively for JLab ($\sqrt{s} = 4.7$ GeV), HERMES ($\sqrt{s} = 7.2$ GeV), COMPASS ($\sqrt{s} = 17.33$ GeV) and eRHIC ($\sqrt{s} = 31.6$ GeV and $\sqrt{s} = 158.1$ GeV) energies. The plots are for models I(a) and I(b) for two choices of the gluon Siverson function. We obtain sizable asymmetry in the kinematical regions of all the experiments for model I(b). The asymmetry is smaller in model I(a). The asymmetry decreases with increase of y for JLab. At HERMES it remains almost independent of y for negative rapidity and decreases with increasing y for positive rapidity. For COMPASS as well as for eRHIC lower energy the asymmetry increases with y , reaches a maximum and then decreases. This maximum is reached at $y \approx 0.6$ for COMPASS and at $y \approx 1.2$ for eRHIC for model I(b). For the higher energy at eRHIC no maximum is seen in the rapidity region plotted. The asymmetry increases with the

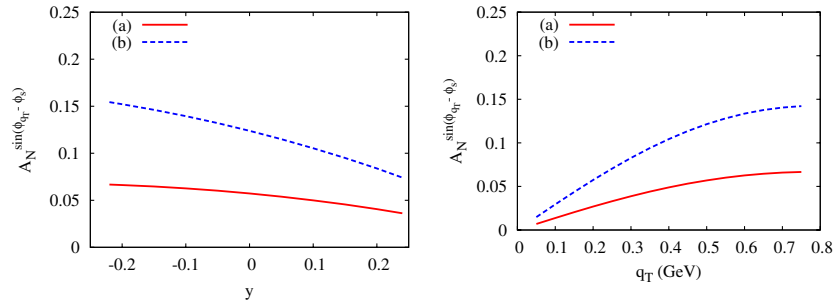


FIG. 1 (color online). The single spin asymmetry $A_N^{\sin(\phi_{q_T} - \phi_S)}$ for the $e + p^1 \rightarrow e + J/\psi + X$ at JLab as a function of y (left panel) and q_T (right panel). The plots are for model I with two parameterizations (a) [solid red line] and (b) [dashed blue line]. The integration ranges are $(0 \leq q_T \leq 1)$ GeV and $(0 \leq y \leq 0.25)$. The results are given at $\sqrt{s} = 4.7$ GeV.

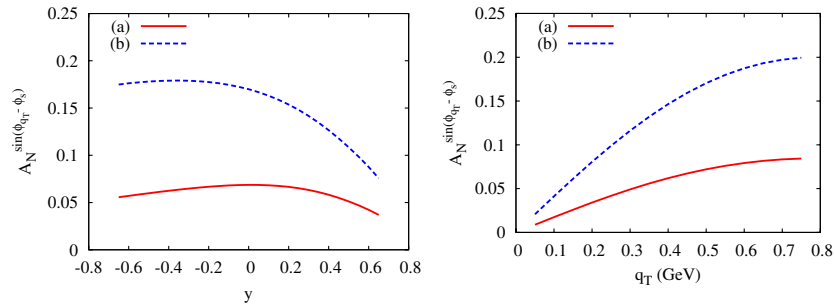


FIG. 2 (color online). The single spin asymmetry $A_N^{\sin(\phi_{q_T} - \phi_S)}$ for the $e + p^1 \rightarrow e + J/\psi + X$ at HERMES as a function of y (left panel) and q_T (right panel). The plots are for model I with two parameterizations (a) [solid red line] and (b) [dashed blue line]. The integration ranges are $(0 \leq q_T \leq 1)$ GeV and $(0 \leq y \leq 0.6)$. The results are given at $\sqrt{s} = 7.2$ GeV.

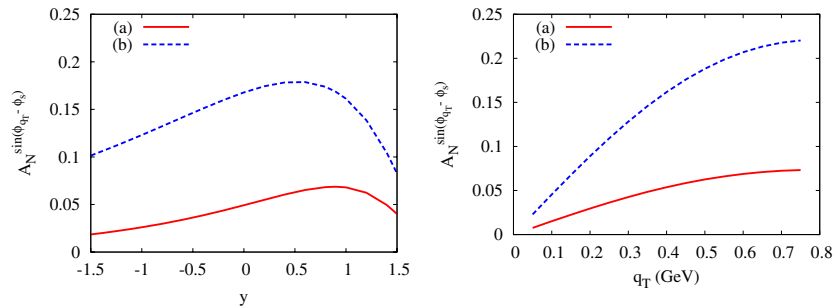


FIG. 3 (color online). The single spin asymmetry $A_N^{\sin(\phi_{q_T} - \phi_S)}$ for the $e + p^1 \rightarrow e + J/\psi + X$ at COMPASS as a function of y (left panel) and q_T (right panel). The plots are for model I with two parameterizations (a) [solid red line] and (b) [dashed blue line]. The integration ranges are $(0 \leq q_T \leq 1)$ GeV and $(0 \leq y \leq 1)$. The results are given at $\sqrt{s} = 17.33$ GeV.

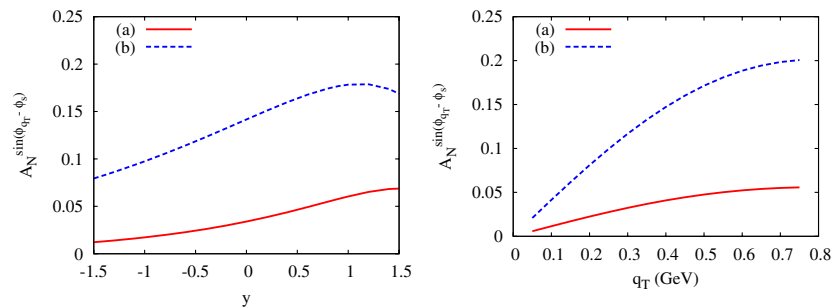


FIG. 4 (color online). The single spin asymmetry $A_N^{\sin(\phi_{q_T} - \phi_S)}$ for the $e + p^1 \rightarrow e + J/\psi + X$ at eRHIC as a function of y (left panel) and q_T (right panel). The plots are for model I with two parameterizations (a) [solid red line] and (b) [dashed blue line]. The integration ranges are $(0 \leq q_T \leq 1)$ GeV and $(0 \leq y \leq 1)$. The results are given at $\sqrt{s} = 31.6$ GeV.

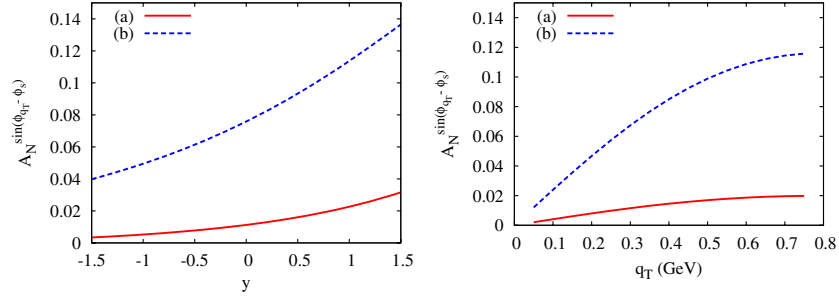


FIG. 5 (color online). The single spin asymmetry $A_N^{\sin(\phi_{q_T} - \phi_S)}$ for the $e + p^1 \rightarrow e + J/\psi + X$ at eRHIC as a function of y (left panel) and q_T (right panel). The plots are for model I with two parameterizations (a) [solid red line] and (b) [dashed blue line]. The integration ranges are $(0 \leq q_T \leq 1)$ GeV and $(0 \leq y \leq 1)$. The results are given at $\sqrt{s} = 158.1$ GeV.

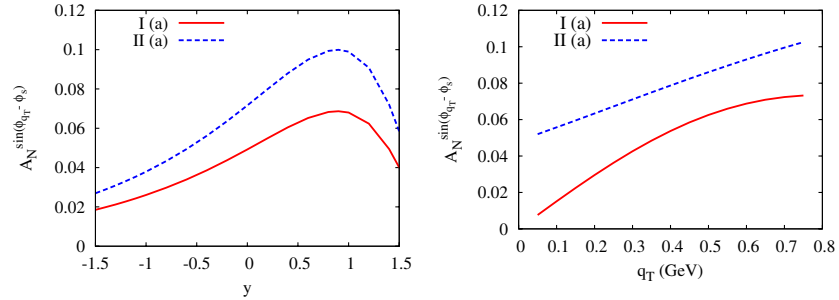


FIG. 6 (color online). The single spin asymmetry $A_N^{\sin(\phi_{q_T} - \phi_S)}$ for the $e + p^1 \rightarrow e + J/\psi + X$ at COMPASS as a function of y (left panel) and q_T (right panel). The plots are for two models I (solid red line) and II (dashed blue line) with parameterization (a). The integration ranges are $(0 \leq q_T \leq 1)$ GeV and $(0 \leq y \leq 1)$. The results are given at $\sqrt{s} = 17.33$ GeV.

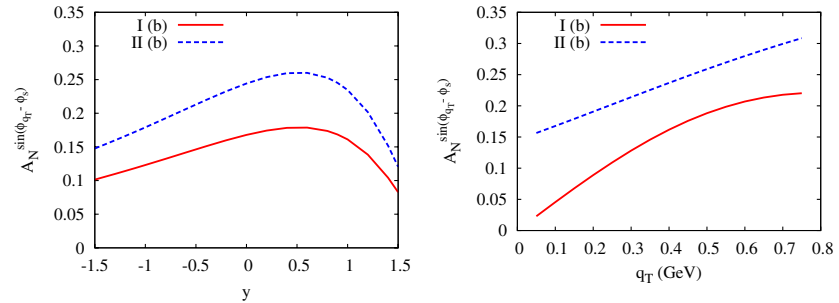


FIG. 7 (color online). The single spin asymmetry $A_N^{\sin(\phi_{q_T} - \phi_S)}$ for the $e + p^1 \rightarrow e + J/\psi + X$ at COMPASS as a function of y (left panel) and q_T (right panel). The plots are for two models I (solid red line) and II (dashed blue line) with parameterization (b). The integration ranges are $(0 \leq q_T \leq 1)$ GeV and $(0 \leq y \leq 1)$. The results are given at $\sqrt{s} = 17.33$ GeV.

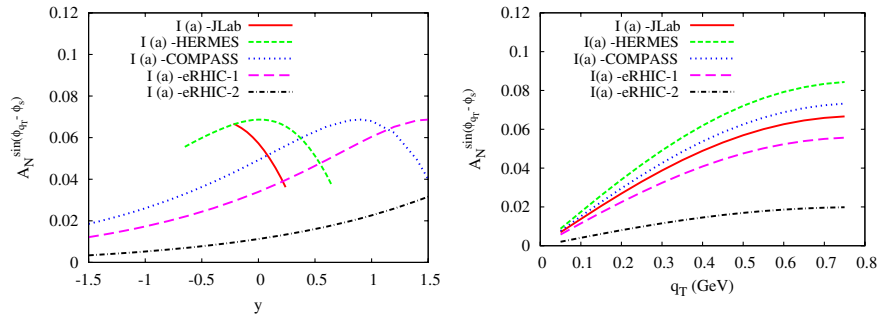


FIG. 8 (color online). The single spin asymmetry $A_N^{\sin(\phi_{q_T} - \phi_S)}$ for the $e + p^1 \rightarrow e + J/\psi + X$ as a function of y (left panel) and q_T (right panel). The plots are for model I with parameterization (a) compared for JLab ($\sqrt{s} = 4.7$ GeV) [solid red line], HERMES ($\sqrt{s} = 7.2$ GeV) [dashed green line], COMPASS ($\sqrt{s} = 17.33$ GeV) [dotted blue line], eRHIC-1 ($\sqrt{s} = 31.6$ GeV) [long dashed pink line] and eRHIC-2 ($\sqrt{s} = 158.1$ GeV) [dot-dashed black line].

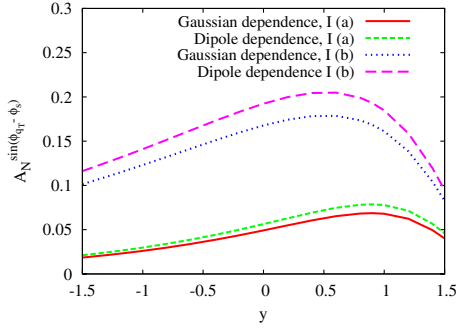


FIG. 9 (color online). The single spin asymmetry $A_N^{\sin(\phi_{q_T} - \phi_S)}$ for the $e + p^\uparrow \rightarrow e + J/\psi + X$ at COMPASS as a function of y . The plots are for model I with two parameterizations (a) [solid red line and dashed green line] and (b) [dotted blue line and long dashed pink line] compared for Gaussian and dipole k_\perp dependence of WW function. The results are given at $\sqrt{s} = 17.33$ GeV.

increase of y . Both model I(a) and model I(b) show similar qualitative behavior. The asymmetry increases with q_T for both models, and for higher values of $q_T \approx 0.6$ – 0.7 GeV, it becomes relatively steady.

In Figs. 6 and 7 we have compared the results for the asymmetry for models I and II at COMPASS energy ($\sqrt{s} = 17.33$ GeV). Figure 6 shows the asymmetry for models I(a) and II(a) and Fig. 7 shows the same for models I(b) and II (b). In model II the asymmetry is larger in magnitude. It increases steadily with q_T in model II unlike in model I where it first increases rapidly then becomes relatively stable with increase of q_T . The difference in the q_T dependence in model I and model II arises due to the different k_\perp dependence of the Siverson function in the two models. Figure 8 shows a comparison of the y and q_T dependence of the asymmetry at JLab, HERMES, COMPASS and eRHIC. We have used model I(a) for this comparison.

So far we calculated the asymmetry using a Gaussian ansatz for the transverse momentum dependence of the WW function given by Eq. (27). We also do a comparative

study taking a different ansatz, namely, the dipole form with $k_0 = 0.7$ GeV.

We have shown the asymmetry as a function of rapidity or COMPASS energy in Fig. 9 for both Ansätze and using models I(a) and I(b). It is seen that the asymmetry does not depend much on the choice of the WW function.

IV. SUMMARY AND CONCLUSION

We have estimated the magnitude of SSA in electroproduction of J/ψ using Weizsacker-Williams equivalent photon approximation and models of Siverson function proposed earlier to estimate the asymmetry in SIDIS and Drell-Yan processes. We used the color evaporation model for charmonium production for the numerical estimate of the asymmetry. Corresponding to each of the two models of the Siverson function, results have been given for two parameterizations of the gluon Siverson function. We have found sizable asymmetry in the energies of COMPASS, HERMES, JLab and eRHIC. We have used two different ansatz for the transverse momentum dependence of the WW function and found that the asymmetry does not depend much on the choice. Our results based on CEM indicate that it may be worthwhile to look at SSA's in charmonium production both from the point of view of comparing different models of charmonium production as well as comparing the different models of gluon Siverson function used for estimating SSA in other processes.

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