

Revisiting the pure annihilation decays $B_s \rightarrow \pi^+ \pi^-$ and $B^0 \rightarrow K^+ K^-$: The data and the perturbative QCD predictions

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In this work, we recalculate the charmless pure annihilation decays $B_s \rightarrow \pi^+ \pi^-$ and $B^0 \rightarrow K^+ K^-$ by using the perturbative QCD (pQCD) factorization approach, and compare the pQCD predictions with currently available experimental measurements. By numerical calculations and phenomenological analysis, we found the following results: (a) one can provide a consistent pQCD interpretation for both the measured branching ratio $\text{Br}(B_s^0 \rightarrow \pi^+ \pi^-)$ and $\text{Br}(B_d^0 \rightarrow K^+ K^-)$ simultaneously; (b) the pQCD predictions for $\text{Br}(B_s^0 \rightarrow \pi^+ \pi^-)$ obtained by different authors are well consistent with each other; (c) our new pQCD prediction for $\text{Br}(B_d^0 \rightarrow K^+ K^-)$ agrees well with the measured values from CDF and LHCb Collaborations; and (d) the CP -violating asymmetry $\mathcal{A}_{CP}(B_d^0 \rightarrow K^+ K^-) \approx 19\%$, which is large and may be detected at the LHCb and future Super-B factory experiments.

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I. INTRODUCTION

Among the two-body hadronic B meson decays, the pure annihilation decay modes, such as $B_s^0 \rightarrow \pi^+ \pi^-$ and $B^0 \rightarrow K^+ K^-$ decays, are specific in several respects. They can occur only through the annihilation diagrams in the standard model because none of the quarks(anti-quarks) in the final states are the same as those of the initial B meson. And consequently, they are rare decay modes with a branching ratio (Br) at the level of 10^{-7} or less as generally expected. Such decays play a very important role in understanding the annihilation mechanism and determining the strength of the annihilation contribution in B meson charmless hadronic decays, and therefore have been studied intensively by many authors [1–10] in spite of the great difficulties in both the theoretical calculation and the experimental measurements.

In the experiment side, both $\text{Br}(B_s^0 \rightarrow \pi^+ \pi^-)$ and $\text{Br}(B^0 \rightarrow K^+ K^-)$ are measured very recently due to their rareness. At the spring and summer conference of 2011, CDF [11] and LHCb [12] Collaborations reported their first measurement of the decay rates

$$\begin{aligned} \text{Br}(B_s^0 \rightarrow \pi^+ \pi^-) \\ = \begin{cases} (5.7 \pm 1.5(\text{stat}) \pm 1.0(\text{syst})) \times 10^{-7} & \text{CDF} \\ (9.8_{-1.9}^{+2.3}(\text{stat}) \pm 1.1(\text{syst})) \times 10^{-7} & \text{LHCb,} \end{cases} \quad (1) \end{aligned}$$

$$\begin{aligned} \text{Br}(B^0 \rightarrow K^+ K^-) \\ = \begin{cases} (2.3 \pm 1.0(\text{stat}) \pm 1.0(\text{syst})) \times 10^{-7} & \text{CDF} \\ (1.3_{-0.5}^{+0.6}(\text{stat}) \pm 0.7(\text{syst})) \times 10^{-7} & \text{LHCb.} \end{cases} \quad (2) \end{aligned}$$

The statistical significance of LHCb measurement reaches 5.3σ for $B_s \rightarrow \pi^+ \pi^-$ decay, which means an observation for the first time.

In the theory side, we know that it is very hard to make a reliable calculation for pure annihilation decays of B mesons. In the QCD factorization (QCDF) approach [13], for example, one cannot perform a real calculation for the annihilation diagrams due to the end-point singularity, but has to make a rough estimation by parametrizing the annihilation contribution through the treatment $\int_0^1 dx/x \rightarrow X_A = (1 + \rho_A e^{i\phi}) \ln \frac{m_B}{\Lambda_h}$ [5,6], or by using an effective gluon propagator $1/k^2 \rightarrow 1/(k^2 + M_g^2(k^2))$ to avoid enhancements in the soft end-point region [7]. Of course, such parametrization will produce large theoretical uncertainties. For $B_s^0 \rightarrow \pi^+ \pi^-$ and $B^0 \rightarrow K^+ K^-$ decays, the theoretical predictions based on the QCDF approach as given, for example, in Refs.[5–9] are the following:

$$\text{Br}(B_s^0 \rightarrow \pi^+ \pi^-) = \begin{cases} 0.24 \times 10^{-7} \\ (1.24 \pm 0.28) \times 10^{-7} \\ (2.6 \pm 1.0) \times 10^{-7}, \end{cases} \quad (3)$$

$$\text{Br}(B^0 \rightarrow K^+ K^-) = \begin{cases} 0.13 \times 10^{-7} \\ (1.0_{-0.2}^{+0.3} \pm 0.3) \times 10^{-7}. \end{cases} \quad (4)$$

Obviously, the QCDF predictions in Refs. [5–8] are much smaller than the measured results for $\text{Br}(B_s^0 \rightarrow \pi^+ \pi^-)$, while smaller or close to the measured ones for $\text{Br}(B^0 \rightarrow K^+ K^-)$ in Refs. [5,8], respectively.

After CDF's report of the evidence of $B_s \rightarrow \pi^+ \pi^-$ decay, the author of Ref. [10] reinvestigated the role of annihilation topology in the QCDF approach and found that (1) the CDF measurement of $\text{Br}(B_s \rightarrow \pi^+ \pi^-)$ implies a large annihilation scenario with ρ_A around 2 instead of $\rho_A \approx 1$ preferred by all previous studies in QCDF

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approach [5,6,13]; (2) if one assumes universal annihilation parameters ρ_A and ϕ_A for all $B_{d,s} \rightarrow PP$ decay modes, one cannot provide predictions being consistent with all well-measured decays¹; (3) one possible way to solve this problem is to use different (ρ_A, ϕ_A) for different decays, which however means that the predictive power of QCDF approach becomes rather limited. In short, the studies in Ref. [10] tell us that it is very hard to give a consistent QCDF interpretation for $\text{Br}(B_s \rightarrow \pi^+ \pi^-)$ and other well-measured $B_{d,s} \rightarrow PP$ decay modes simultaneously.

In the perturbative QCD (pQCD) factorization approach [14–17], however, the situation becomes rather different. Here, the pure annihilation decays of the B/B_s meson can be calculated perturbatively by employing the Sudakov factors to smear and then to strongly suppress the end-point singularity. In the pQCD factorization approach, for example, the end-point divergence of the factorizable emission diagram Figs. 1(a) and 1(b) in Ref. [18] are regulated by introducing the transverse momentum $k_{i\perp}^2$, i.e.,

$$\frac{1}{k^2} \cdot \frac{1}{p_b^2 - M_B^2} = \frac{1}{M_B^4 x_1 x_3 (1-x_3)} \rightarrow \frac{1}{(1-x_3)M_B^2 + \mathbf{k}_{3\perp}^2} \cdot \frac{1}{x_1 x_3 M_B^2 + (\mathbf{k}_{1\perp}^2 - \mathbf{k}_{3\perp}^2)^2}, \quad (5)$$

where $k^2 = (k_1 - k_3)^2$ and $p_b^2 = (P_1 - k_3)^2$ is the momentum of the gluon propagator and b -quark propagator, respectively. It is easy to see that the end-point divergences for $x_1 = 0$ and $x_3 = (0, 1)$ are removed effectively by introducing small but nonzero $\mathbf{k}_{i\perp}^2$.

For $B_s^0 \rightarrow \pi^+ \pi^-$ decay, it was calculated by employing the pQCD factorization approach in 2004 [2] and 2007 [3], respectively. In Ref. [2], we obtained the first pQCD prediction for the decay rate:

$$\text{Br}(B_s^0 \rightarrow \pi^+ \pi^-) = (4.2 \pm 0.6) \times 10^{-7}. \quad (6)$$

In 2007, Ali *et al.* [3] made a systematic calculation for all $B_s \rightarrow PP, PV, VV$ decays in the pQCD factorization approach and found that

$$\text{Br}(B_s^0 \rightarrow \pi^+ \pi^-) = (5.7_{-1.6}^{+1.8}) \times 10^{-7}. \quad (7)$$

These two pQCD predictions at leading order are well consistent within 1σ error and confirmed by CDF and LHCb measurements as shown in Eqs. (1) and (2). The small difference for the predicted decay rates between Refs. [2,3] comes from the fact that a little different input parameters and distribution amplitudes of π and B_s mesons were used in two studies.

¹The corresponding QCDF predictions for $\text{Br}(B_s \rightarrow K^+ K^-)$ and $\text{Br}(B_d \rightarrow K^0 \bar{K}^0)$ are two times larger than the experimental measurements [10].

In Ref. [4], by employing the pQCD factorization approach, we studied the $B_s \rightarrow PP$ decays with the inclusion of partial next-to-leading order (NLO) contributions, coming from the QCD vertex corrections, the quark loops, the chromomagnetic penguins, and the usage of the NLO Wilson coefficients instead of the leading-order ones. For the pure annihilation decay $B_s \rightarrow \pi^+ \pi^-$, it does not receive the NLO contributions from the QCD vertex corrections, the quark loops, and the chromomagnetic penguins. The leading-order pQCD prediction is $\text{Br}(B_s^0 \rightarrow \pi^+ \pi^-) = (7 \pm 2.5) \times 10^{-7}$, while it becomes $(5.7_{-2.2}^{+2.4}) \times 10^{-7}$ when the NLO Wilson coefficients $C_i(M_W)$, the NLO renormalization group evolution matrix $U(t, m, \alpha)$ [19], and the $\alpha_s(t)$ at two-loop level were employed in the numerical calculation [4].

For $B^0 \rightarrow K^+ K^-$ decay, the known pQCD prediction for its branching ratio was given in 2001 [1],

$$\begin{aligned} \text{Br}(B_d^0 \rightarrow K^+ K^-) &= 3.27 \times 10^{-8}; \\ \text{Br}(\bar{B}_d^0 \rightarrow K^+ K^-) &= 5.90 \times 10^{-8}, \end{aligned} \quad (8)$$

which is much smaller than the measured value as given in Eq. (2) by roughly a factor of 3, in other words, a large discrepancy between the data and the theoretical prediction based on the pQCD factorization approach for $B^0 \rightarrow K^+ K^-$ decay.

It is necessary and interesting to check if one can provide a consistent pQCD interpretation for both the measured $\text{Br}(B_s^0 \rightarrow \pi^+ \pi^-)$ and $\text{Br}(B_d^0 \rightarrow K^+ K^-)$ simultaneously. In this paper, by employing the pQCD factorization approach, we recalculate the pure annihilation decays $B_s^0 \rightarrow \pi^+ \pi^-$ and $B^0 \rightarrow K^+ K^-$ with the usage of the same set of input parameters and wave functions for the mesons involved, in order to check if the new data from CDF and LHCb can be understood in the pQCD approach. Our studies will be helpful to determine the strength of penguin-annihilation amplitudes [20].

The paper is organized as follows. In Sec. II, we give a brief review about the theoretical framework of the pQCD factorization approach and the wave functions for B^0/B_s^0 and π, K mesons involved. We perform the perturbative calculations for considered decay channels in Sec. III, while the numerical results and phenomenological analysis are given in Sec. IV. A short summary also be given in Sec. IV.

II. THEORETICAL FRAMEWORK

In the pQCD approach, the decay amplitude $\mathcal{A}(B_q \rightarrow M_2 M_3)$ with $q = (d, s)$ can be written conceptually as the convolution,

$$\begin{aligned} \mathcal{A}(B_q \rightarrow M_2 M_3) &\sim \int d^4 k_1 d^4 k_2 d^4 k_3 \text{Tr}[C(t) \Phi_B(k_1) \Phi_{M_2}(k_2) \\ &\quad \times \Phi_{M_3}(k_3) H(k_1, k_2, k_3, t)], \end{aligned} \quad (9)$$

where k_i 's are momenta of light quarks included in each meson, and ‘‘Tr’’ denotes the trace over Dirac and color

indices. In the above convolution, $C(t)$ is the Wilson coefficient evaluated at scale t , the function $H(k_1, k_2, k_3, t)$ describes the four-quark operator and the spectator quark connected by a hard gluon. The wave functions $\Phi_B(k_1)$ and Φ_{M_i} describe the hadronization of the quark and antiquark in the B_q meson and the final-state light meson M_i .

We treat the B_q meson as a heavy-light system, and consider the B_q meson at rest for simplicity. By using the light-cone coordinates, the B_q meson momentum P_B and the two final-state mesons' momenta P_2 and P_3 (for M_2 and M_3 , respectively) can be written as

$$\begin{aligned} P_B &= \frac{M_B}{\sqrt{2}}(1, 1, \mathbf{0}_T), & P_2 &= \frac{M_B}{\sqrt{2}}(1 - r_3^2, r_2^2, \mathbf{0}_T), \\ P_3 &= \frac{M_B}{\sqrt{2}}(r_3^2, 1 - r_2^2, \mathbf{0}_T), \end{aligned} \quad (10)$$

where $r_i = m_i/M_B$. For the final-state light mesons made up with (u, d, s) and the corresponding antiquarks, the ratios r_2 and r_3 are small and will be neglected safely. Putting the quark momenta in B_q , M_2 , and M_3 mesons as k_1 , k_2 , and k_3 , respectively, we can choose

$$\begin{aligned} k_1 &= (x_1 P_1^+, 0, \mathbf{k}_{1T}), & k_2 &= (x_2 P_2^+, 0, \mathbf{k}_{2T}), \\ k_3 &= (0, x_3 P_3^-, \mathbf{k}_{3T}). \end{aligned} \quad (11)$$

Then, the integration over k_1^- , k_2^- , and k_3^+ in Eq. (9) will lead to

$$\begin{aligned} \mathcal{A}(B_q \rightarrow M_2 M_3) &\sim \int dx_1 dx_2 dx_3 b_1 db_1 b_2 db_2 b_3 \\ &\times db_3 \cdot \text{Tr}[C(t)\Phi_B(x_1, b_1)\Phi_{M_2}(x_2, b_2) \\ &\times \Phi_{M_3}(x_3, b_3)H(x_i, b_i, t)S_i(x_i) \\ &\times e^{-S(t)}], \end{aligned} \quad (12)$$

where b_i is the conjugate space coordinate of k_{iT} . The large double logarithms ($\ln^2 x_i$) on the longitudinal direction are summed by the threshold resummation, and they lead to the first Sudakov factor $S_i(x_i)$ which smears the end-point singularities on x_i [17]. The Sudakov resummations of large logarithmic corrections, such as the terms proportional to $\alpha_s \log^2[Q/\mathbf{k}_{iT}]$ ($Q \sim m_B$), to the B_q and two final-state-meson wave functions will lead to the second Sudakov factor $e^{-S(t)} = e^{-S_B(t)} \cdot e^{-S_{M_2}(t)} \cdot e^{-S_{M_3}(t)}$. These two kinds of Sudakov factors can together suppress the soft dynamics effectively [17].

In the momentum space, the light-cone wave function of the B_q meson can be defined as [14–16]

$$\Phi_{B_q}(k) = \frac{i}{\sqrt{2N_c}} [(\not{P} + m_{B_q})\gamma_5 \phi_{B_q}(k)]_{\alpha\beta}, \quad (13)$$

where P is the momentum of the B_q meson, k is the momentum carried by the light quark in the B_q meson, and ϕ_{B_q} is the corresponding distribution amplitude.

For the B/B_s mesons, the distribution amplitudes $\phi_B(x, b)$ in the b space can be written as [14–16]

$$\phi_B(x, b) = N_B x^2 (1-x)^2 \exp\left[-\frac{1}{2}\left(\frac{xm_B}{\omega_b}\right)^2 - \frac{\omega_b^2 b^2}{2}\right], \quad (14)$$

and

$$\phi_{B_s}(x, b) = N_{B_s} x^2 (1-x)^2 \exp\left[-\frac{1}{2}\left(\frac{xm_{B_s}}{\omega_{B_s}}\right)^2 - \frac{\omega_{B_s}^2 b^2}{2}\right], \quad (15)$$

where the normalization factors $N_{B_{(s)}}$ are related to the decay constants $f_{B_{(s)}}$ through

$$\int_0^1 dx \phi_{B_{(s)}}(x, b=0) = \frac{f_{B_{(s)}}}{2\sqrt{6}}. \quad (16)$$

Here, the shape parameter ω_b has been fixed at 0.40 GeV by using the rich experimental data on the B mesons with $f_B = 0.19$ GeV. Correspondingly, the normalization constant N_B is 91.745. For B_s meson, considering a small SU(3) symmetry breaking, since s quark is heavier than the u or d quark, the momentum fraction of s quark should be a little larger than that of the u or d quark in the B mesons, we therefore adopt the shape parameter $\omega_{B_s} = 0.50$ GeV [3] with $f_{B_s} = 0.23$ GeV, then the corresponding normalization constant is $N_{B_s} = 63.67$. In order to analyze the uncertainties of theoretical predictions induced by the inputs, we can vary the shape parameters ω_b and ω_{B_s} by 10%, i.e., $\omega_b = 0.40 \pm 0.04$ GeV and $\omega_{B_s} = 0.50 \pm 0.05$ GeV, respectively.

For the π^\pm and K^\pm mesons, we adopt the same set of distribution amplitudes $\phi_{\pi,K}^A(x_i)$ and $\phi_{\pi,K}^{P,T}(x_i)$ as defined in Refs. [21,22]:

$$\begin{aligned} \phi_{\pi(K)}^A(x) &= \frac{3f_{\pi(K)}}{\sqrt{6}} x(1-x) [1 + a_1^{\pi(K)} C_1^{3/2}(t) \\ &+ a_2^{\pi(K)} C_2^{3/2}(t) + a_4^{\pi(K)} C_4^{3/2}(t)], \end{aligned} \quad (17)$$

$$\begin{aligned} \phi_{\pi(K)}^P(x) &= \frac{f_{\pi(K)}}{2\sqrt{2N_c}} \left[1 + \left(30\eta_3 - \frac{5}{2}\rho_{\pi(K)}^2 \right) C_2^{1/2}(t) \right. \\ &\left. - 3 \left[\eta_3 \omega_3 + \frac{9}{20}\rho_{\pi(K)}^2 (1 + 6a_2^{\pi(K)}) \right] C_4^{1/2}(t) \right], \end{aligned} \quad (18)$$

$$\begin{aligned} \phi_{\pi(K)}^T(x) &= -\frac{f_{\pi(K)}}{2\sqrt{6}} t \left[1 + 3 \left(5\eta_3 - \frac{1}{2}\eta_3 \omega_3 - \frac{7}{20}\rho_{\pi(K)}^2 \right) \right. \\ &\left. - \frac{3}{5}\rho_{\pi(K)}^2 a_2^{\pi(K)} \right] (5t^2 - 3), \end{aligned} \quad (19)$$

where $t = 2x - 1$, $\rho_{\pi(K)} = m_{\pi(K)}/m_0^{\pi(K)}$ are the mass ratios [here, $m_0^\pi = m_\pi^2/(m_u + m_d)$ and $m_0^K = m_K^2/(m_s + m_d)$ are the chiral mass of pion and kaon], $a_i^{\pi,K}$ are the Gegenbauer moments, while $C_n^C(t)$ are the Gegenbauer polynomials

$$\begin{aligned}
 C_1^{3/2}(t) &= 3t, & C_2^{1/2}(t) &= \frac{1}{2}(3t^2 - 1), & C_2^{3/2}(t) &= \frac{3}{2}(5t^2 - 1), \\
 C_4^{1/2}(t) &= \frac{1}{8}(3 - 30t^2 + 35t^4), & C_4^{3/2}(t) &= \frac{15}{8}(1 - 14t^2 + 21t^4).
 \end{aligned}
 \tag{20}$$

Under the replacement of $x \rightarrow 1 - x$, only $C_1^{3/2}(t)$ will change its sign, others remain unchanged.

III. PERTURBATIVE CALCULATION IN THE PQCD APPROACH

In the pQCD factorization approach, the four annihilation Feynman diagrams for $B_s \rightarrow \pi^+ \pi^-$ and $B^0 \rightarrow K^+ K^-$ decays are shown in Fig. 1, where (a) and (b) are factorizable diagrams, while (c) and (d) are the nonfactorizable ones. The initial \bar{b} and $s(d)$ quarks annihilate into u and \bar{u} pair, and then form a pair of light mesons by hadronizing with another pair of $d\bar{d}$ ($s\bar{s}$) produced perturbatively through the one-gluon exchange mechanism. Besides the short-distance contributions based on one-gluon exchange, the $q\bar{q}$ pair can also be produced through strong interaction in nonperturbative regime (final state interaction, for example).

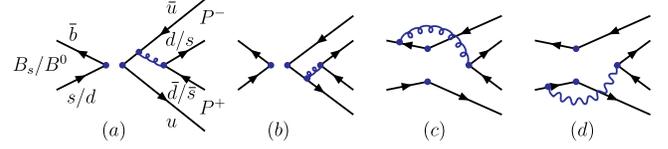


FIG. 1 (color online). The typical annihilation Feynman diagrams for $B_s^0 \rightarrow \pi^+ \pi^-$ and $B^0 \rightarrow K^+ K^-$ decays. (a) and (b) are factorizable diagrams; while (c) and (d) are the nonfactorizable ones.

Final-state-interaction effects in considered decays have been assumed rather small; we do not consider them here.

We will adopt (F^{LL}, F^{LR}, F^{SP}) and (M^{LL}, M^{LR}, M^{SP}) to stand for the contributions of the factorizable [Figs. 1(a) and 1(b)] and nonfactorizable [Figs. 1(c) and 1(d)] annihilation diagrams from the $(V - A)(V - A)$, $(V - A)(V + A)$, and $(S - P)(S + P)$ operators, respectively. By making the analytic calculations, we obtain the following decay amplitudes for both $B_s^0 \rightarrow \pi^+ \pi^-$ and $B^0 \rightarrow K^+ K^-$ decays:

From the factorizable annihilation diagrams Figs. 1(a) and 1(b), we have

(i) $(V - A)(V - A)$ operators:

$$\begin{aligned}
 F^{LL} &= 16\pi C_F M_{B_q}^2 \int_0^1 dx_2 dx_3 \int_0^\infty b_2 db_2 b_3 db_3 \times \{ [x_2 \phi_2^A(x_2) \phi_3^A(x_3) + 2r_2 r_3 (\phi_2^P(x_2) - \phi_2^T(x_2)) \phi_3^P(x_3) \\
 &\quad + 2r_2 r_3 x_2 (\phi_2^P(x_2) + \phi_2^T(x_2)) \phi_3^P(x_3)] \cdot h_a(x_2, x_3, b_2, b_3) E_a(t_a) + [(x_3 - 1) \phi_2^A(x_2) \phi_3^A(x_3) - 4r_2 r_3 \phi_2^P(x_2) \phi_3^P(x_3) \\
 &\quad + 2r_2 r_3 x_3 \phi_2^P(x_2) (\phi_3^P(x_3) - \phi_3^T(x_3))] \cdot h_b(x_2, x_3, b_2, b_3) E_a(t_b) \},
 \end{aligned}
 \tag{21}$$

(ii) $(V - A)(V + A)$ operators:

$$F^{LR} = F^{LL},
 \tag{22}$$

(iii) $(S - P)(S + P)$ operators:

$$\begin{aligned}
 F^{SP} &= 32\pi C_F M_{B_q}^2 \int_0^1 dx_2 dx_3 \int_0^\infty b_2 db_2 b_3 db_3 \times \{ [2r_3 \phi_2^A(x_2) \phi_3^P(x_3) + r_2 x_2 (\phi_2^P(x_2) \\
 &\quad - \phi_2^T(x_2)) \phi_3^A(x_3)] \cdot h_a(x_2, x_3, b_2, b_3) \cdot E_a(t_a) [2r_2 \phi_2^P(x_2) \phi_3^A(x_3) \\
 &\quad + (1 - x_3) r_3 \phi_2^A(x_2) (\phi_3^P(x_3) + \phi_3^T(x_3))] \cdot h_b(x_2, x_3, b_2, b_3) \cdot E_a(t_b) \},
 \end{aligned}
 \tag{23}$$

where $r_2 = m_2/m_{B_q}$, $r_3 = m_3/m_{B_q}$ with $q = (d, s)$ for B_d^0 or B_s^0 decays, and $C_F = 4/3$ is a color factor. The explicit expressions for the convolution functions $E_a(t_{a,b})$, the hard scales $t_{a,b}$, and the hard functions $h_{a,b}(x_i, b_i)$ can be found, for example, in Refs. [18,23].

From the nonfactorizable annihilation diagrams Figs. 1(c) and 1(d), we have

$$\begin{aligned}
 M^{LL} &= \frac{64}{\sqrt{6}} \pi C_F M_{B_q}^2 \int_0^1 dx_1 dx_2 dx_3 \int_0^\infty b_1 db_1 b_2 db_2 \phi_{B_q}(x_1, b_1) \times \{ [(1 - x_3) \phi_2^A(x_2) \phi_3^A(x_3) + (1 - x_3) r_2 r_3 (\phi_2^P(x_2) \\
 &\quad + \phi_2^T(x_2)) (\phi_3^P(x_3) - \phi_3^T(x_3)) + x_2 r_2 r_3 (\phi_2^P(x_2) - \phi_2^T(x_2)) (\phi_3^P(x_3) + \phi_3^T(x_3))] \cdot h_c(x_1, x_2, x_3, b_1, b_2) E_c(t_c) \\
 &\quad + [-x_2 \phi_2^A(x_2) \phi_3^A(x_3) - 4r_2 r_3 \phi_2^P(x_2) \phi_3^P(x_3) + (1 - x_2) r_2 r_3 (\phi_2^P(x_2) + \phi_2^T(x_2)) (\phi_3^P(x_3) - \phi_3^T(x_3)) \\
 &\quad + x_3 r_2 r_3 (\phi_2^P(x_2) - \phi_2^T(x_2)) (\phi_3^P(x_3) + \phi_3^T(x_3))] \cdot h_d(x_1, x_2, x_3, b_1, b_2) E_c(t_d) \},
 \end{aligned}
 \tag{24}$$

$$\begin{aligned}
 M^{LR} = & \frac{64}{\sqrt{6}} \pi C_F M_{B_q}^2 \int_0^1 dx_1 dx_2 dx_3 \int_0^\infty b_1 db_1 b_2 db_2 \phi_{B_q}(x_1, b_1) \times \{ [x_2 r_2 (\phi_2^P(x_2) + \phi_2^T(x_2)) \phi_3^A(x_3) \\
 & - (1 - x_3) r_3 \phi_2^A(x_2) (\phi_3^P(x_3) - \phi_3^T(x_3))] \cdot h_c(x_1, x_2, x_3, b_1, b_2) E_c(t_c) + [(2 - x_2) r_2 (\phi_2^P(x_2) + \phi_2^T(x_2)) \phi_3^A(x_3) \\
 & - (1 + x_3) r_3 \phi_2^A(x_2) (\phi_3^P(x_3) - \phi_3^T(x_3))] \cdot h_d(x_1, x_2, x_3, b_1, b_2) E_c(t_d) \}, \quad (25)
 \end{aligned}$$

$$\begin{aligned}
 M^{SP} = & \frac{64}{\sqrt{6}} \pi C_F M_{B_q}^2 \int_0^1 dx_1 dx_2 dx_3 \int_0^\infty b_1 db_1 b_2 db_2 \phi_{B_q}(x_1, b_1) \times \{ [x_2 \phi_2^A(x_2) \phi_3^A(x_3) + x_2 r_2 r_3 (\phi_2^P(x_2) + \phi_2^T(x_2)) (\phi_3^P(x_3) \\
 & - \phi_3^T(x_3)) + (1 - x_3) r_2 r_3 (\phi_2^P(x_2) - \phi_2^T(x_2)) (\phi_3^P(x_3) + \phi_3^T(x_3))] \cdot h_c(x_1, x_2, x_3, b_1, b_2) E_c(t_c) \\
 & + [-(1 - x_3) \phi_2^A(x_2) \phi_3^A(x_3) - 4 r_2 r_3 \phi_2^P(x_2) \phi_3^P(x_3) + x_3 r_2 r_3 (\phi_2^P(x_2) + \phi_2^T(x_2)) (\phi_3^P(x_3) - \phi_3^T(x_3)) \\
 & + (1 - x_2) r_2 r_3 (\phi_2^P(x_2) - \phi_2^T(x_2)) (\phi_3^P(x_3) + \phi_3^T(x_3))] \cdot h_d(x_1, x_2, x_3, b_1, b_2) E_c(t_d) \}, \quad (26)
 \end{aligned}$$

where $r_{2,3}$ and C_F are defined in the same way as in Eqs. (21)–(23). Again, the explicit expressions of the functions $E_c(t_{c,d})$ and $h_{c,d}$, and the hard scales $t_{c,d}$ can be found in Refs. [18,23].

Because of the isospin symmetry, the contributions to both $B_s^0 \rightarrow \pi^+ \pi^-$ and $B^0 \rightarrow K^+ K^-$ decays from the factorizable annihilation diagrams Figs. 1(a) and 1(b) cancel each other. The total decay amplitudes for the considered decays are therefore written as

$$\mathcal{A}(B_s^0 \rightarrow \pi^+ \pi^-) = V_{ub}^* V_{us} C_2 M^{LL} - V_{tb}^* V_{ts} \{ [C_4 + C_6 - \frac{1}{2} C_8 + C_{10}] M^{LL} + [C_4 + C_6 + C_8 - \frac{1}{2} C_{10}] M^{SP} \}, \quad (27)$$

$$\mathcal{A}(B_d^0 \rightarrow K^+ K^-) = V_{ub}^* V_{ud} C_2 M^{LL} - V_{tb}^* V_{td} \{ [C_4 + C_6 - \frac{1}{2} C_8 + C_{10}] M^{LL} + [C_4 + C_6 + C_8 - \frac{1}{2} C_{10}] M^{SP} \}. \quad (28)$$

The expression of decay amplitude in Eq. (27) is equivalent with those as given in Refs. [2,3] by a proper transformation between M^{LL} and M^{SP} .

IV. NUMERICAL RESULTS AND DISCUSSIONS

Now, it is straightforward to calculate the CP -averaged branching ratios and CP -violating asymmetries for the two considered decays. In numerical calculations, central values of the input parameters will be used implicitly unless otherwise stated. The QCD scale (GeV), masses (GeV), decay constants (GeV), and B_q meson lifetime (ps) being used are the following [24]:

$$\begin{aligned}
 \Lambda_{\text{QCD}} = & 0.25, \quad m_W = 80.41, \quad m_{B^0} = 5.2795, \quad M_{B_s} = 5.3663; \\
 m_\pi = & 0.14, \quad m_K = 0.494, \quad f_\pi = 0.13, \quad f_K = 0.16, \quad (29) \\
 \tau_{B^0} = & 1.525 \text{ ps}, \quad \tau_{B_s} = 1.472 \text{ ps}.
 \end{aligned}$$

As for the CKM matrix elements, we use [24]

$$\begin{aligned}
 \lambda = & 0.2253 \pm 0.0007, \quad A = 0.808_{-0.015}^{+0.022}, \quad (30) \\
 \bar{\rho} = & 0.132_{-0.014}^{+0.022}, \quad \bar{\eta} = 0.341 \pm 0.013.
 \end{aligned}$$

For the Gegenbauer moments and other relevant input parameters, based on the works of [21,22], we use

$$\begin{aligned}
 a_1^\pi = & 0, \quad a_1^K = 0.06 \pm 0.03, \quad a_2^\pi = 0.35 \pm 0.15, \\
 a_2^K = & 0.25 \pm 0.10, \quad a_4^\pi = -0.015, \quad a_4^K = 0, \quad \rho_\pi = m_\pi / m_0^\pi, \\
 \rho_K = & m_K / m_0^K, \quad \eta_3 = 0.015 \pm 0.005, \quad \omega_3 = -3.0 \pm 1.0 \quad (31)
 \end{aligned}$$

with the chiral mass $m_0^\pi = 1.4 \pm 0.1$ GeV, and $m_0^K = 1.9 \pm 0.2$ GeV. In order to check the theoretical errors induced by the uncertainty of the Gegenbauer moments, we vary a_1^K , $a_2^{\pi,K}$, η_3 , and ω_3 in the range of $a_1^K = 0.06 \pm 0.03$, $a_2^\pi = 0.35 \pm 0.15$, $a_2^K = 0.25 \pm 0.10$, $\eta_3 = 0.015 \pm 0.005$, and $\omega_3 = -3.0 \pm 1.0$.

From the decay amplitudes, it is easy to write down the corresponding branching ratio:

$$\text{Br}(B \rightarrow PP) = \frac{G_F^2 m_B^3}{128 \pi^2} \tau_B |\mathcal{A}(B \rightarrow PP)|^2, \quad (32)$$

where $\mathcal{A}(B \rightarrow PP)$ is the decay amplitude as defined in Eqs. (27) and (28).

By using the analytic expressions for the complete decay amplitudes and the input parameters, we calculate the branching ratios and CP -violating asymmetries for both considered decay modes. The numerical results are the following:

$$\text{Br}(B_s^0 \rightarrow \pi^+ \pi^-) = (5.10_{-1.68}^{+1.96} (a_2^\pi)_{-0.19-0.83-0.20}^{+0.25+1.05+0.29}) \times 10^{-7}, \quad (33)$$

$$\mathcal{A}_{CP}(B_s^0 \rightarrow \pi^+ \pi^-) = (-2.3_{-0.3}^{+0.0} (a_2^\pi)_{-0.2-0.2-0.1}^{+0.3+0.1+0.1}) \%, \quad (34)$$

$$\text{Br}(B_d^0 \rightarrow K^+ K^-) = (1.56_{-0.42}^{+0.44} (a_2^K)_{-0.22-0.19-0.09}^{+0.23+0.22+0.13}) \times 10^{-7}, \quad (35)$$

$$\mathcal{A}_{CP}(B_d^0 \rightarrow K^+ K^-) = (18.9_{-1.9}^{+0.2} (a_2^K)_{-2.2-1.4-1.1}^{+1.4+0.1+0.8}) \%, \quad (36)$$

where the first error comes from the theoretical uncertainty of the Gegenbauer moments $a_2^\pi = 0.35 \pm 0.15$ and $a_2^K = 0.25 \pm 0.10$, the small theoretical errors due to the variations of $a_1^K = 0.06 \pm 0.03$, $\eta_3 = 0.015 \pm 0.005$ and $\omega_3 = -3.0 \pm 1.0$ is shown as the second error, the third error includes the uncertainties induced by the parameter $\omega_b = 0.40 \pm 0.04$ GeV and $\omega_{B_s} = 0.50 \pm 0.05$ GeV, as well as the uncertainties of $m_0^\pi = 1.4 \pm 0.1$ GeV and $m_0^K = 1.9 \pm 0.2$ GeV, and the last error comes from the uncertainties of the relevant CKM elements. It is easy to see that the uncertainties from $a_2^{\pi,K}$, ω_b , and ω_{B_s} dominate the theoretical error.

For $B_s^0 \rightarrow \pi^+ \pi^-$ decay, the pQCD prediction for its branching ratio in Eq. (33) agrees very well with the measured results from CDF and LHCb Collaborations [11,12] as shown in Eq. (1). This pQCD prediction also agrees very well with the previous pQCD predictions as given in Refs. [2–4]. The analytical results for the decay amplitudes obtained in this paper are consistent with those as given in Refs. [2–4]. The small difference in numerical pQCD predictions comes from the difference of the input parameters being used in different works.

For $B_d^0 \rightarrow K^+ K^-$ decay, fortunately, the pQCD prediction for its branching ratio in Eq. (35) agrees well with the measured results from CDF and LHCb Collaborations [11,12] as shown in Eq. (2).

It is easy to see that the new pQCD prediction in Eq. (35) is much larger than the previous pQCD prediction as given in Ref. [1]. In order to find the reason for the large difference, we checked the relevant analytical expressions as given in Ref. [1] and found that those analytical results are consistent with our results after proper transformation: $x \rightarrow 1 - x$. The large numerical difference between two pQCD predictions comes from the fact that (a) the distribution amplitudes of the kaon meson used by Chen and Li [1] are very different from those used in this paper; and (b) some improved Gegenbauer moments as given in Ref. [22] are used in this paper.

In Ref. [1], only the axial-vector and pseudoscalar kaon wave functions $\phi_K(x)$ and $\phi'_K(x)$ were considered:

$$\phi_K(x) = \frac{3f_K}{\sqrt{6}} x(1-x) \{1 + 0.51(1-2x) + 0.3[5(1-2x)^2 - 1]\}, \quad (37)$$

$$\phi'_K(x) = \frac{3f_K}{\sqrt{6}} x(1-x). \quad (38)$$

In this paper, however, besides the leading twist-2 $\phi_K^A(x)$ [i.e., the axial-vector $\phi_K(x)$ in Ref. [1]], we also take into account the twist-3 contributions from both ϕ_K^P and ϕ_K^T simultaneously. Based on the analytical expressions as given in Eqs. (17)–(19), one can obtain the numerical expressions for $\phi_K^A(x)$, $\phi_K^P(x)$, and $\phi_K^T(x)$:

$$\phi_K^A(x) = \frac{3f_K}{\sqrt{6}} x(1-x) \{1 - 0.18(1-2x) + 0.375[5(1-2x)^2 - 1]\}, \quad (39)$$

$$\phi_K^P(x) = \frac{f_K}{2\sqrt{6}} \{1 + 0.282(1 - 6x + 6x^2) - 0.012[3 - 30(2x - 1)^2 + 35(2x - 1)^4]\}, \quad (40)$$

$$\phi_K^T(x) = -\frac{f_K}{2\sqrt{6}} (2x-1)[1 + 0.55(1-10x + 10x^2)], \quad (41)$$

by using the central values of the relevant input parameters $a_{1,2,4}^K$, ρ_k , η_3 , and ω_3 , etc., as given in Eqs. (29) and (31).

For the leading twist-2 axial-vector wave function, the $\phi_K^A(x)$ we used is in the same form as $\phi_K(x)$ being used in Ref. [1]. The difference of the coefficients of the second and third term comes from the variation of the values of the corresponding Gegenbauer moments (a_1^K, a_2^K): (a_1^K, a_2^K) = (0.17, 0.20) in Ref. [1], while (a_1^K, a_2^K) = (0.06, 0.25) in this paper, based on recent improvements made in Ref. [22]. The difference of the sign of the second term in $\phi_K^A(x)$ is resulted from the different assignment for the momentum fraction x in Ref. [1] and in this paper: We here use x to denote the momentum fraction of s/\bar{s} quark in the K^\pm meson, instead of the u/\bar{u} quark as assigned in Ref. [1]. The Gegenbauer polynomial $C_1^{3/2}(t) = 3t$ in Eq. (38) will change its sign under the transformation $x \rightarrow 1 - x$.

In Ref. [1], the authors took $\phi'_K(x) = \frac{3}{\sqrt{6}} f_K x(1-x)$ as the pseudoscalar kaon wave function, which was “determined from the data of the $B \rightarrow K\pi$ decays” by Chen and Li, instead of the ordinary $\phi_K^P(x)$ as derived from the QCD sum rule [21,22] and shown in Eq. (39). (For more details of the derivation of $\phi'_K(x)$, see Sec. IV of Ref. [15].) The $\phi'_K(x)$ in Ref. [1] is just the first and leading term of the twist-2 part $\phi_K^A(x)$ and is very different from commonly used ϕ_K^P .

In Ref. [1], the term $\phi_K^T(x)$ was absent. All differences in the relevant wave functions being used in Ref. [1] and in this paper lead to the large difference between the pQCD predictions for the branching ratio $\text{Br}(B^0 \rightarrow K^+ K^-)$ as presented in Ref. [1] and in this paper.

Explicit numerical examinations also show that the leading twist-2 ϕ_K^A provide the dominant contribution to the magnitude of the decay amplitudes and consequently branching ratio $\text{Br}(B^0 \rightarrow K^+ K^-)$:

- (1) When all three terms $\phi_K^{A,P,T}$, or only the leading twist-2 term $\phi_K^A(x)$, are taken into account, we find numerically

$$\begin{aligned} \mathcal{A}(B^0 \rightarrow K^+ K^-) &= \begin{cases} (-0.31 - 2.2\text{Im}) \times 10^{-5} & (\phi_K^A(x) \text{ only}) \\ (-0.82 - 3.6\text{Im}) \times 10^{-5} & (\text{All three terms}); \end{cases} \end{aligned} \quad (42)$$

$$\begin{aligned} \text{Br}(B^0 \rightarrow K^+ K^-) &= \begin{cases} 0.55 \times 10^{-7} & (\phi_K^A(x) \text{ only}) \\ 1.56 \times 10^{-7} & (\text{All three terms}). \end{cases} \end{aligned} \quad (43)$$

- (2) If only the twist-3 term $\phi_K^P(x)$, ϕ_K^T , or both of them are taken into account, we find numerically

$$\begin{aligned} \mathcal{A}(B^0 \rightarrow K^+ K^-) &= \begin{cases} (-0.61 - 0.55 \text{Im}) \times 10^{-5} & (\phi_K^P(x) \text{ only}) \\ (0.06 - 0.27 \text{Im}) \times 10^{-5} & (\phi_K^T(x) \text{ only}); \end{cases} \end{aligned} \quad (44)$$

$$\text{Br}(B^0 \rightarrow K^+ K^-) = \begin{cases} 0.08 \times 10^{-7} & (\phi_K^P(x) \text{ only}) \\ 0.01 \times 10^{-7} & (\phi_K^T(x) \text{ only}). \end{cases} \quad (45)$$

It is straightforward to see from the above numerical results that

- (1) The leading twist-2 term $\phi_K^A(x)$ provides the dominant contribution to the decay amplitude: $\mathcal{A} = (-0.31 - 2.2 \text{Im}) \times 10^{-5}$ if only $\phi_K^A(x)$ is taken into account, while $\mathcal{A} = (-0.61 - 0.55 \text{Im}) \times 10^{-5}$ ($\mathcal{A} = (-0.06 - 0.27 \text{Im}) \times 10^{-5}$) if only $\phi_K^P(x)$ (ϕ_K^T) is taken into account. For the branching ratio, its size would be 10^{-7} , 10^{-8} , or 10^{-9} if only the terms $\phi_K^A(x)$, $\phi_K^P(x)$, or $\phi_K^T(x)$ contribute.
- (2) The enhancements due to the constructive interference between the three parts also play an important role in producing a large branching ratio $\text{Br}(B^0 \rightarrow K^+ K^-)$. One can see that the contributions to the decay amplitude \mathcal{A} from the three terms interfere constructively, which finally leads to a

large branching ratio $\text{Br}(B^0 \rightarrow K^+ K^-) = 1.56 \times 10^{-6}$, partially due to the further magnifying effects since the branching ratio is proportional to the module square of the decay amplitude \mathcal{A} .

As for the CP -violating asymmetry for the considered decays, $\mathcal{A}_{CP}(B_s^0 \rightarrow \pi^+ \pi^-)$ is very small, only about 2% and therefore hardly to be detected even at the LHCb. For $B_d^0 \rightarrow K^+ K^-$ decay, however, its \mathcal{A}_{CP} is relatively large, around 19%, and may be detected at the LHCb experiment or future Super-B factory experiments.

In summary, by employing the pQCD factorization approach, we here recalculated the branching ratios and CP -violating asymmetries of the pure annihilation decays $B_s^0 \rightarrow \pi^+ \pi^-$ and $B^0 \rightarrow K^+ K^-$ with the usage of the wave functions based on the QCD sum rule [21,22] and the improved Gegenbauer moments [22]. By numerical calculations and phenomenological analysis, we found the following results: (a) one can provide a consistent pQCD interpretation for both the measured $\text{Br}(B_s^0 \rightarrow \pi^+ \pi^-)$ and $\text{Br}(B_d^0 \rightarrow K^+ K^-)$ simultaneously; (b) the pQCD predictions for $\text{Br}(B_s^0 \rightarrow \pi^+ \pi^-)$ obtained by different authors are well consistent with each other within 1 standard deviation; (c) our new pQCD prediction for $\text{Br}(B_d^0 \rightarrow K^+ K^-)$ agrees well with the measured values from CDF and LHCb Collaborations; and (d) the CP -violating asymmetry $\mathcal{A}_{CP}(B_s^0 \rightarrow \pi^+ \pi^-) \approx -2.3\%$, may be too small to be detected even at LHCb experiment; (e) $\mathcal{A}_{CP}(B_d^0 \rightarrow K^+ K^-) \approx 19\%$, which is large and may be detected at the LHCb and future super-B factory experiments.

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