

**Model to localize gauge and tensor fields on thick branes**A. E. R. Chumbes,<sup>\*</sup> J. M. Hoff da Silva,<sup>†</sup> and M. B. Hott<sup>‡</sup>*DFQ, UNESP Universidade Estadual Paulista, Campus de Guaratinguetá, Av. Dr. Ariberto Pereira da Cunha, 333 CEP 12516-410, Guaratinguetá-SP, Brazil*

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It is shown that the introduction of a suitable function in the higher-dimensional gauge field action may be used in order to achieve gauge bosons localization on a thick brane. The model is constructed upon analogies to the effective coupling of neutral scalar field to electromagnetic field and to the Friedberg-Lee model for hadrons. After that we move forward studying the localization of the Kalb-Ramond field via this procedure.

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**I. INTRODUCTION**

It is well known that in the braneworld paradigm four-dimensional gravity may be localized on a singular brane [1], i.e., a normalizable zero mode arising from the gravitational field fluctuation exists on the brane. In Ref. [2], a nonsingular brane performed by a thick domain wall is considered. In this more realistic case, gravity is also localized on the brane. In general, braneworld models are inspired in string theory and it is expected that a considered model makes contact with some string theory limit in the consideration, for instance, of  $D$ -branes solutions. In this vein, it is important to make effort in order to eliminate some of the differences between the domain-wall approach for braneworld models and the  $D$ -branes solutions.

As already noted, for example, in Ref. [3], an important difference of  $D$ -branes when compared to the domain wall is that while the former supports gauge fields living on it (basically arising from the open strings ending on the  $D$ -branes), it is not always possible to achieve gauge field localization on the domain walls by means of only the space-time curvature acting, i.e., the gauge field effective action term is blind with respect to the warp factor. In other words, as well known [4], the five-dimensional gauge field action

$$-\frac{1}{4} \int d^5x \sqrt{g} F^{MN} F_{MN}, \quad (1)$$

simply blows up after the dimensional reduction. The simplest approach to reach zero modes of gauge fields on the brane is by assuming the existence of bulk gauge fields which could, in principle, to give rise to the four-dimensional gauge sector on the domain wall. Unfortunately such an approach indicates that the gauge fields cannot be localized [4–6].

In order to circumvent this difficulty, some models have appeared in the literature. In the absence of gravity, gauge field localization was extensively considered in, for

instance, Ref. [6] (see also references therein). In the context of curved spaces this issue was also analyzed. In Ref. [3], an additional scalar field—the dilaton—introduced in the five-dimensional action is responsible to drive the gauge field localization, by means of the coupling between the dilaton and the kinetic term of gauge fields. Similar procedure was also adopted in [7]. In Ref. [8] gauge field localization obtained via kinetic terms induced by localized fermions. After all, however, it is relevant to note that there is not a complete mechanism concerning gauge field localization on the brane.

In this paper we shall add one more possibility in order to localize gauge fields in thick branes. From the pragmatic point of view the idea is quite simple and it is based on the same mechanism which provides the localization of spin 1/2 fermion fields in a brane in five-dimensional flat [9] and warped [10] space-time. We just introduce a suitable function in the five-dimensional gauge field Lagrangian, which leads to a normalizable zero mode after the dimensional reduction, namely:

$$-\frac{1}{4} \int d^5x \sqrt{g} G(\phi) F^{MN} F_{MN}. \quad (2)$$

The  $G(\phi)$  is a functional of the scalar field from which the brane originates. To fix ideas one should think in the model obtained in [2]. Therefore two questions are immediately raised: first, since the inserted function depends on the scalar field, it should enter in its field equations contributing to the background constituted by the metric and the scalar field. Second, how to set up the form of such a functional, since any normalizable function could, in principle, act in the same way in the gauge localization scheme. To the first point we should assume that in this effective model  $G(\phi)$  is a function of the minimum energy solution,  $\tilde{\phi}(r)$ , which represents the brane (the domain-wall solution), such that there is no contribution of the gauge field zero mode to the energy of the system, as it happens in the localization of fermion zero mode in the brane. The second question is a little more subtle. While it is true that the procedure explained in the next Section may be successfully repeated with any normalizable function  $G(\tilde{\phi}(r))$ , we

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shall give a physical motivation based upon analogies to the Schwinger's neutral scalar-gauge field coupling [11], to the color dielectric model for the confinement of gluons and quarks [12] and to the quantum mechanics associated to the matter fields localization on branes. It must be remarked that the analogy we shall explore here has many limitations and the  $G(\phi)$  function form shall be regarded more appropriately as inspired on that models.

Apart of that, it is also known that string theory presents plenty of higher spin fields on its spectrum. Therefore it is quite conceivable the study of such fields in the braneworld context. In this vein it is important to analyze the possibility of localize the Kalb-Ramond (K-R) field [13] on the brane. In the context of infinitely thin branes the localized zero mode of the K-R field (interpreted as torsion) is highly suppressed by the size of the extra dimension [14]. Within the framework of thick brane worlds the K-R field was also investigated in [15] and in [16]. In fact, in [15] it was demonstrated that there is no localized tensorial zero mode with the usual thick brane background. It was shown that, in order to localize the zero mode it is necessary a background composed by a membrane described by two real scalar fields with internal structures, or a dilatonic gravitation. Part of this paper is devoted to the use of appropriated smearing out functions in order to localize the K-R zero mode field on the brane. The aforesaid functions, as mentioned, are suitable constructed based upon the same mechanism for the localization of spin 1/2 fermion fields on a 3-brane embedded in flat [9] and warped [10] background.

The paper is structured as follows: in the next section we show, in very simple grounds, that the introduction of the  $G(\bar{\phi})$  function do localize normalizable zero mode gauge fields on the brane, physically motivating the functional form of  $G(\bar{\phi})$ . In Sec. III we take forward our analogy applying, then, a similar procedure to localize the zero mode of the K-R field and stressing an important point concerning the integrability of the smearing out functions. In the last section we conclude.

## II. LOCALIZING GAUGE FIELDS

Before starting our analysis properly, let us briefly set the background by recalling the standard model developed in Ref. [2] for five-dimensional gravity coupled to a real scalar field:

$$S = \int d^5x \sqrt{g} \left( -\frac{1}{4}R + \frac{1}{2}(\partial\phi)^2 - V(\phi) \right), \quad (3)$$

where the Poincare invariant line element is given by

$$ds^2 = e^{2A(r)} \left( dt^2 - \sum_{i=1}^3 dx_i^2 \right) - dr^2. \quad (4)$$

By admitting that the scalar field is dependent on the extra dimension only, the Einstein-Hilbert and scalar field

equations admit minimum energy solutions which are also solutions of the first-order differential equations [2]

$$\frac{d\phi}{dr} = \frac{\partial W(\phi)}{\partial \phi} = W_\phi \quad (5)$$

and

$$\frac{dA}{dr} = -\frac{2}{3}W(\phi), \quad (6)$$

whenever the potential  $V(\phi)$  is written in terms of the superpotential  $W(\phi)$  as [17,18]

$$V(\phi) = \frac{1}{2}W_\phi^2 - \frac{4}{3}W(\phi)^2. \quad (7)$$

In [2] the superpotential is chosen to be given as

$$W(\phi) = 3bc \sin\left(\sqrt{\frac{2}{3b}}\phi\right), \quad (8)$$

which leads to

$$A(r) = -b \ln(2 \cosh(2cr)) \quad (9)$$

and

$$\bar{\phi}(r) = \sqrt{6b} \arctan(\tanh(cr)). \quad (10)$$

The free parameters  $b$  and  $c$  in this model are related to the thickness of the brane ( $c$ ) and the anti-de Sitter curvature ( $bc$ ).

Having fixed the background, let us study the standard protocol for gauge field localization, this time armed with the smearing out  $G(\bar{\phi})$  function. The gauge field Lagrangian is given by Eq. (2). As remarked before, we shall neglect the  $G(\bar{\phi})$  contribution to the background in this effective model. The field equation reads

$$\partial_C(e^{4A}G(\bar{\phi})g^{CE}g^{DB}F_{EB}) = 0. \quad (11)$$

In the gauge  $\partial_\rho A^\rho = 0$  and  $A_4 = 0$ , decomposing the field as  $A_\mu = \sum_n \mathcal{A}_\mu(x)\alpha_n(r)$  one arrives at

$$m_n^2 \alpha_n(r) + e^{2A} \left[ \alpha_n''(r) + \left( \frac{G'(\bar{\phi})}{G(\bar{\phi})} + 2A' \right) \alpha_n'(r) \right] = 0, \quad (12)$$

where prime means derivative with respect to  $r$ . In order to set a typical quantum mechanical problem, let us make the following transformation:

$$\alpha_n(r) = e^{-\gamma(r)} g_n(r). \quad (13)$$

By means of the identification  $2\gamma' = 2A' + G'/G$  the first derivative term disappear and the result is the Schrödinger equation:

$$-g_n''(r) + [\gamma'' + (\gamma')^2 - m_n^2 e^{-2A}]g_n(r) = 0. \quad (14)$$

For the massless zero mode ( $g_0 \equiv g$ ) we have simply

$$-g''(r) + [\gamma'' + (\gamma')^2]g(r) = 0, \quad (15)$$

which may be recast in the following operatorial form:

$$\left(\frac{d}{dr} + \gamma'\right)\left(-\frac{d}{dr} + \gamma'\right)g(r) = 0. \quad (16)$$

Hence we have  $g(r) \sim e^{\gamma(r)}$  and  $\alpha_0$  ends up as a constant, say  $\tilde{\alpha}$ , by Eq. (13). Therefore, the dimensional reduction of (2) leads straightforwardly to

$$S = -\frac{1}{4} \int_{-\infty}^{+\infty} \tilde{\alpha}^2 G(\tilde{\phi}) dr \int d^4 x f^{\mu\nu} f_{\mu\nu}. \quad (17)$$

Obviously, if the  $G(\tilde{\phi})$  function is constant the coupling constant multiplying the usual four-dimensional Maxwell Lagrangian blows up, rendering a delocalized gauge field, that is, the gauge field zero mode permeates the whole bulk.

Besides, it is quite easy to see that there is no negative mass modes in the spectrum for the gauge field. This can be seen by writing Eq. (12) in terms of a conformal-like coordinate  $z = \int^r d\xi e^{-A(\xi)}$ , such that one finds

$$-\frac{d^2 \tilde{g}_n(z)}{dz^2} + \tilde{V}(z) \tilde{g}_n(z) = m_n^2 \tilde{g}_n(z),$$

where  $\tilde{V}(z) = ((d\tilde{\gamma}/dz)^2 - (d^2\tilde{\gamma}/dz^2))$  and we have used the redefinition  $\alpha_n(z) = e^{-\tilde{\gamma}(z)} \tilde{g}_n(z)$  with  $(d\tilde{\gamma}/dz) = (1/2)[(1/G)(dG/dz)] + (dA/dz)$ . The differential equation for  $\tilde{g}_n(z)$  can be factorized as  $DD^\dagger \tilde{g}_n(z) = m_n^2 \tilde{g}_n(z)$  with  $D = (d/dz) - (d\tilde{\gamma}/dz)$ . Then for each one of the normalized eigenstates one has  $0 \leq \int dz |D^\dagger \tilde{g}_n(z)|^2 = m_n^2$ . Therefore the background with the  $G$  function is indeed stable.

### A. Setting up the $G(\tilde{\phi})$ function

The idea that a neutral scalar field might be effectively coupled to a gauge field dates back from the observations of the anomalous decay  $\pi^0 \rightarrow 2\gamma$  mediated by virtual fermions. Such an effective coupling was found by Schwinger [11] together with an effective coupling term of a scalar neutral field to the electromagnetic field. The latter effective coupling would describe the decay of a stationary meson into two parallel polarized photons mediated by a virtual proton-antiproton pair, namely,  $\mathcal{L} = (e^2/12\pi)(g/M)\phi F^{\mu\nu} F_{\mu\nu}$  (see Eq. (5.6) of [11]). In trying to follow this clue it is important to stress that the simple replacement  $G(\tilde{\phi}) \propto \tilde{\phi}(r)$  is not satisfactory to our problem, since  $\tilde{\phi}(r)$  is not normalizable in the entire domain of the extra dimension. This is a peculiar feature of domain-wall solutions, as the one given by Eq. (10), whatever the nonlinear model one uses to describe thick branes.

Friedberg and Lee proposed a phenomenological model [12] to explain nonperturbative effects of QCD at low energies. In that model, hadrons are nontopological solitons of a nonlinear field theory potential involving a phenomenological scalar field,  $\sigma$ , which couples to the quarks by means of a Yukawa coupling and to the gluons by means of a dielectric function, namely  $\mathcal{L} = (-1/4)\kappa(\sigma)F_c^{\mu\nu} F_{\mu\nu}^c$ .

Without going into detail, we just recall that in the Friedberg-Lee model the functional dependence of  $\kappa(\sigma)$  on  $\sigma$  is not crucial, but it has to satisfy some conditions such that the QCD vacuum works as dielectric medium for the chromo electric field and an antidiagnetic medium for the chromo magnetic field, in close analogy to the Meissner effect in superconductors. Those conditions are  $\kappa(0) = 1$ ,  $\kappa(\bar{\sigma}) = 0$ , and  $d\kappa(\bar{\sigma})/d\sigma = 0$ , where  $\bar{\sigma}$  is the expectation value of the scalar field on the QCD vacuum. Such conditions might be suited to the  $G(\tilde{\phi})$  function. Here we set  $G(\tilde{\phi}) = 1$  on the core of the brane, and  $\tilde{\alpha}$  can be conveniently chosen such that  $\int_{-\infty}^{+\infty} \tilde{\alpha}^2 G(\tilde{\phi}) dr = 1$ . The other condition over  $G(\tilde{\phi})$  is  $G(\tilde{\phi}) \rightarrow 0$  asymptotically ( $r \rightarrow \pm\infty$ ), that is, when  $\tilde{\phi}(r \rightarrow \pm\infty)$  goes to the two respective neighbors minima of the potential  $V(\phi)$ .

We have found that some functionals satisfy those conditions. At this point we would like to recall that the warp factor itself, which keeps a connection to  $\tilde{\phi}(r)$ , plays the role of a smearing out weight function to localize gravitons on branes [10], and it would also satisfies the above conditions imposed over  $G(\tilde{\phi})$ . Nevertheless, since we want to localize gauge field on branes embedded in flat space-time too, as in the Rubakov-Shaposhnikov scenario [9], we keep looking for a functional of  $\tilde{\phi}(r)$ . Particularly, in flat space-time Eq. (14) reduces to

$$-g_n''(r) + [\gamma'' + (\gamma')^2]g_n(r) = m_n^2 g_n(r), \quad (18)$$

where  $\gamma' = G'/2G$  is the quantum mechanics superpotential. Such an equation is very similar to the equation for the excitations of the brane (branons) around the domain-wall solution. In this last case the quantum mechanics superpotential is given by [19]

$$\gamma' = W_{\phi\phi}(\tilde{\phi}(r)). \quad (19)$$

Furthermore, Eqs. (18) and (19) also appear in the case of fermion fields localization on branes in flat space-time when the coupling of fermions to the scalar field is inspired on supersymmetry, that is,  $W_{\phi\phi} \bar{\Psi}\Psi$ . As mentioned in the Sec. I, we shall introduce the  $G$  function functional form—as far as possible—resembling the quantum mechanics associated to the matter fields localization on branes. Hence, by keeping the above recurrence also in the case of localization of gauge fields on branes, we set

$$\gamma' = G'/2G = \kappa W_{\phi\phi}(\tilde{\phi}(r)), \quad (20)$$

$\kappa$  being a positive constant, which leads to

$$G(\tilde{\phi}(r)) \propto W_{\phi\phi}^2(\tilde{\phi}(r)). \quad (21)$$

It is important to have in mind at least two central aspects of differences regarding the analogy to be proposed: in the QCD case, the color electric flux is sustained to one dimension exactly by the dielectric vacuum quality. In our model, by means of the  $G(\phi)$  function with suitable boundary conditions (as previously discussed), the flux is concentrated on the brane leading to the localization.

Besides, as it is well known, the aforementioned confinement is accomplished for non-Abelian gauge fields. This is an important difference which highlights the fact that the analogy should not be taken literally. Instead, as we will see, if the conditions on the smearing out function are similar to those for the dielectric function, the localization (in the braneworld sense) of the gauge field is accomplished.

Now we are in position to set up the smearing out functions for both flat and warped space-time. In Ref. [9] one has  $V(\phi) = W_\phi^2/2 = (\lambda/4)(\phi^2 - m^2/\lambda)^2$  and  $\bar{\phi}(r) = (m/\sqrt{\lambda}) \tanh(mr/\sqrt{2})$ ; therefore, one obtains

$$G(\bar{\phi}(r)) = \text{sech}^{4\kappa}(mr/\sqrt{2}). \quad (22)$$

Equation (22) is the appropriated  $G(\bar{\phi})$  function to the flat space, according to our analogy and to the conditions imposed over  $G(\bar{\phi}(r))$ . Such a superpotential, however, is not adequate for the brane worlds scenario in warped space-time, because it implies into a unbound from below potential  $V(\phi)$  as given by Eq. (7). Nevertheless, by taking  $W(\phi)$  as in Eq. (8) together with the domain-wall solution (10), one finds

$$G(\bar{\phi}(r)) = \text{sech}^{2\kappa}(2cr). \quad (23)$$

Both these smearing out functions, (22) and (23), are sharp on the core of the brane and exhibit a narrow bell-shape profile, in such a way that they are normalizable in the entire domain of the extra coordinate.

### III. LOCALIZING THE KALB-RAMOND FIELD

We start with the K-R Lagrangian suitable modified by the multiplication of the smearing out function

$$S = -\frac{1}{12} \int d^5x \sqrt{g} G(\bar{\phi}) H_{MNL} H^{MNL}, \quad (24)$$

where

$$H_{MNL} = \partial_M B_{NL} + \partial_N B_{LM} + \partial_L B_{MN}, \quad (25)$$

is the field strength for the K-R field.

The equation of motion for the field  $B_{MN}$  is given by

$$\partial_Q (\sqrt{g} G(\bar{\phi}) g^{MQ} g^{NR} g^{LS} H_{MNL}) = 0, \quad (26)$$

which with the aid of Eq. (4) can be expressed as

$$e^{2A} G(\bar{\phi}) \partial_\mu H^{\mu\gamma\theta} - \partial_\gamma (G(\bar{\phi}) H^{\gamma\theta}) = 0. \quad (27)$$

With the gauge choice  $B^{\mu r} = 0$ ,  $\partial_\mu B^{\mu\nu} = 0$ , and decomposing the field as  $B^{\gamma\theta} = \sum_{i=1}^n h^{\gamma\theta}(x) U_n(r)$  we have

$$m_n^2 U_n(r) + e^{2A} \left[ U_n''(r) + \frac{G'(\bar{\phi})}{G(\bar{\phi})} U_n'(r) \right] = 0. \quad (28)$$

Just as in the gauge field case, in order to set a typical quantum mechanical problem, it is convenient to perform the following transformation:

$$U_n(r) = e^{-\omega(r)} h_n(r). \quad (29)$$

Now, by means of the identification

$$\omega' = G(\bar{\phi})'/2G(\bar{\phi}), \quad (30)$$

we obtain a Schrödinger-like equation

$$-h_n''(r) + (\omega'' + \omega'^2) h_n(r) = m_n^2 e^{-2A} h_n(r). \quad (31)$$

For the massless zero mode ( $h_0 \equiv h$ ) we simply have

$$-h''(r) + (\omega'' + \omega'^2) h(r) = 0, \quad (32)$$

which may be rewritten in the operatorial form

$$\left( \frac{d}{dr} + \omega' \right) \left( -\frac{d}{dr} + \omega' \right) h(r) = 0. \quad (33)$$

Hence we have  $h(r) \sim e^{\omega(r)}$  and by means of Eq. (29),  $U_0(r)$  ends up as a constant, say  $\alpha$ . Therefore, the dimensional reduction of (24) leads directly to

$$S = -\frac{1}{12} \int_{-\infty}^{+\infty} dr \alpha^2 e^{-2A} G(\bar{\phi}) \int d^4x h_{\alpha\beta\gamma} h^{\alpha\beta\gamma}. \quad (34)$$

In order to reproduce an asymptotic anti-de Sitter bulk, the warp factor  $e^{2A}$  have a Gaussian-like shape peaked at the core of the brane, then  $e^{-2A(r)} \rightarrow \infty$  as  $r \rightarrow \pm\infty$ , for all models used to describe thick branes, and that is the reason for not having a localized zero mode. Hence, if  $G(\bar{\phi})$  is again a convenient smearing out function of  $r$ , it would be possible to localize the K-R zero mode on the brane. Such a smearing out function would also work for flat space ( $e^{-2A(r)} = 1$ ), rendering a localized tensorial field.

#### A. Identifying the smearing out function

The first clue we shall follow in order to set a suitable  $G$  function is the fact that by means of Eqs. (30) and (32); a given  $G$  modify the quantum mechanics potential acting on the modes for the K-R field.

Following this reasoning and the recurrence mentioned in the previous section, we shall identify the  $G$  as in Eq. (21) and check what would be the constraints over  $\kappa$  which make  $\int_{-\infty}^{+\infty} dr \alpha^2 e^{-2A(r)} G(\bar{\phi})$  convergent. We have noted that the conditions over  $\kappa$  are very dependent on the model we are using to describe thick branes. We illustrate that by resorting to the same models we have used in the previous section.

For the case  $W_\phi^2 = (\lambda/2)(\phi^2 - m^2/\lambda)^2$  we have  $\bar{\phi}(r) = (m/\sqrt{\lambda}) \tanh(mr/\sqrt{2})$  and

$$e^{-2A} W_\phi^{2\kappa} \propto \text{sech}^{4\kappa + (8/9)(m^2/\lambda)} \left( \frac{mr}{\sqrt{2}} \right) \times e^{(2m^2/9\lambda) \tanh^2((mr)/(\sqrt{2}))}. \quad (35)$$

Hence, upon integration over the extra dimension the Eq. (35) is convergent for  $\kappa \geq (-2/9)m^2/\lambda$ . Since  $\kappa$  is positive, it is always convergent in this case and the localization of the zero mode for the K-R field is accomplished without any restriction.

Now, keeping in mind the background given in Ref. [2] it is easy to see that the integration along the extra dimension

$$\int_{-\infty}^{+\infty} dr e^{-2A} W_{\phi}^{2\kappa} \propto \int_{-\infty}^{+\infty} dr \operatorname{sech}^{2(\kappa-b)}(2cr) \quad (36)$$

is convergent whenever  $\kappa > b$ . Therefore, differently from the (35) case, here we have a nontrivial constraint over  $\kappa$  which must be fulfilled in order to localize the zero mode for the K-R field. One can note that there will be no restriction over  $\kappa$  if one works in flat geometry, since there is no warp factor.

#### IV. FINAL REMARKS AND OUTLOOK

We have proposed a mechanism that leads to gauge field zero mode localization on thick branes by means of an effective model obtained via the introduction of a smearing out  $G(\phi)$  function in the gauge field Lagrangian.  $G(\phi)$  is a functional of the classical scalar-gravitational field equations solution which originates the brane in a warped space-time, but the procedure can be applied to the case of flat space-time as well.

In order to set up a physically motivated  $G(\phi)$  function, we rely on the Friedberg-Lee phenomenological model proposed to explain nonperturbative effects of QCD at low energies. This model involves a scalar field coupling (via a Yukawa term) to the quarks and also coupling to the gluons by means of a dielectric function. Translating to our problem, the analog  $G(\phi)$  function plays the role of a smearing out dielectric function.

The case of flat geometry is more manageable in the determination of  $G(\phi)$  and we are guided by the problem

$$\int d^4x \mathcal{Q} \bar{\psi}(x) \gamma^\mu A_\mu(r, x) \psi(x) \underbrace{\left\{ \int_{-\infty}^{+\infty} dr e^{3A(r)} [N e^{-2A(r)} f(r)]^2 \right\}}_{=1} \quad (37)$$

Then, one can see that the fermion zero mode does not affect the charge. The four-dimensional coupling  $q$  is related to  $\mathcal{Q}$  as  $q = \tilde{\alpha} \mathcal{Q}$ , where  $\tilde{\alpha}$  is the constant extra-dimensional zero mode for the gauge field, and we obtain the following interaction action involving only the localized modes on the brane  $\int d^4x q \bar{\psi}(x) \gamma^\mu \mathcal{A}_\mu(x) \psi(x)$ .

It is worthwhile to mention that this simple effective model, based upon phenomenological quantum field theory scenarios, might also be applied to the localization of Yang-Mills fields on branes. In fact, we believe that, giving the root of the Friedberg-Lee model itself, the extension of this analogy to the non-Abelian gauge fields localization follows straightforwardly.

The very same procedure is adopted to get the localization of Kalb-Ramond fields on a thick brane. In that case we have found that more restrictive conditions over  $\kappa$  are necessary in order to accomplish the localization and also

of matter fields localization on branes. By leading this recurrence a little further we were able to identify the smearing out function as  $G(\bar{\phi}(r)) \propto W_{\bar{\phi}}^{2\kappa}(\bar{\phi}(r))$ , where  $\kappa$  is a positive constant. Such functional form to the  $G$  function is suitable for gauge field localization for both, flat and warped geometries.

One crucial aspect of gauge field localization is that of a universal coupling to matter. For example, introducing charged five-dimensional fermions, one requires that the zero modes of all independent fermion fields couple with equal strength to the zero mode gauge field. In particular, the extra-dimensional profile of the zero mode fermion should not affect its coupling to the gauge field. To show that in the setting we have been working with, we start with the five-dimensional interaction action for the fermions with the gauge field, namely

$$\int d^5x \sqrt{-g} \mathcal{Q} \bar{\Psi} \Gamma^a A_a \Psi,$$

where  $\mathcal{Q}$  is the coupling constant and  $\Gamma^a = e^{-A(r)} \gamma^\mu \delta_\mu^a$ . We have been using the gauge choice  $A_5 = 0$  and by following the 3rd section of Ref. [19] one sees that the normalized fermion zero mode is given by  $\alpha_f(r) = N e^{-2A(r)} f(r)$ , with  $N$  the normalization constant to be found under the normalization condition  $N^2 \int_{-\infty}^{+\infty} dr, e^{-A} f^2 = 1$ , and  $f(r)$  is a function that depends on the functional form of the Yukawa-like interaction of the scalar field  $\phi(r)$  with the fermions. Then, if one considers only the fermion zero mode on the above action one has

that such restrictions depend on the model one has in hands to describe thick branes.

We also have found that there is a mapping from the quantum mechanics resulting from our approach, namely, Eqs. (14) and (28), into the quantum mechanics for the localized and resonant modes for the vector and tensor gauge fields in dilatonic branes, which were carried out in Refs. [7,15,16]. In the latter, the quantum mechanics potentials for the excitations associated to the vector and tensor gauge fields depend on the warp factor and on  $A'(r)$ ,  $\pi'(r) \propto A'(r)$ , and  $B'(r) \propto A'(r)$ , where  $\pi(r)$  is the dilaton field and  $e^{2B(r)}$  is an extra warp factor from the metric used in the models for dilatonic thick branes. We have noted that the dependence on those terms is such that their resulting quantum mechanics potential is proportional to the quantum mechanics potential found in our approach, provided that the same nonlinear field theory model is used to

describe the thick branes in both cases. Such a mathematical mapping is much clearer when one deals with the model defined by the superpotential (8), because in this case the term  $G(\bar{\phi})'/G(\bar{\phi})$  is proportional to  $A'(r)$ . Such a relation can be used to develop a straightforward analysis of the resonant modes for the vector and tensor gauge fields in our case by resorting to the results found in [7,15,16]. We think

that our results concerning resonant modes will not differ appreciably from theirs.

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