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Emergent IR dual 2d CFTs in charged AdS_5 black holes

Jan de Boer,[1,2,*] Maria Johnstone,[3,†] M. M. Sheikh-Jabbari,[4,‡] and Joan Simón[3,§]

[1]*Instituut voor Theoretische Fysica, Science Park 904 Postbus 94485, 1090 GL Amsterdam, The Netherlands*
[2]*Gravitation and AstroParticle Physics, Amsterdam*
[3]*School of Mathematics and Maxwell Institute for Mathematical Sciences, King's Buildings, Edinburgh EH9 3JZ, United Kingdom*
[4]*School of Physics, Institute for Research in Fundamental Sciences (IPM), P.O.Box 19395-5531, Tehran, Iran*

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We study the possible dynamical emergence of IR conformal invariance describing the low energy excitations of near-extremal R-charged global AdS_5 black holes. We find interesting behavior especially when we tune parameters in such a way that the relevant extremal black holes have classically vanishing horizon area, i.e. no classical ground-state entropy, and when we combine the low energy limit with a large N limit of the dual gauge theory. We consider both near-BPS and non-BPS regimes and their near horizon limits, emphasize the differences between the local AdS_3 throats emerging in either case, and discuss potential dual IR 2d CFTs for each case. We compare our results with the predictions obtained from the Kerr/CFT correspondence, and obtain a natural quantization for the central charge of the near-BPS emergent IR CFT which we interpret in terms of the open strings stretched between giant gravitons.

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I. INTRODUCTION

The microscopic understanding of nonextremal black holes is an important problem in theoretical physics. Their universal Rindler near horizon geometries and the existence of chiral Virasoro algebras generated by diffeomorphisms preserving this structure raises the possibility of having a conformal field theory (CFT) description for these systems [1],[1] generalizing the structure uncovered in AdS_3 [4,5].

Progress was recently achieved by pursuing these ideas for finite extremal black holes whose near horizon geometry includes an AdS_2 factor.[2] In [7,8], it was shown that one can semiclassically associate a chiral Virasoro algebra to the near-horizon geometry of finite extremal black holes, using asymptotic symmetry group considerations [9]. The existence of a dual chiral CFT accounting for the black hole entropy was also conjectured.

Despite the success in computing the entropy of extremal black holes, either using the Kerr/CFT correspondence [7,8] or the entropy function formalism based on the enhancement of symmetry of their near horizons [10], there are arguments reviewed in Sec. II suggesting these AdS_2 geometries do not generically represent a decoupled conformal field theory. Even if they would, the AdS/CFT machinery would suggest these theories may be dynamically trivial [11–13], in the sense that they only contain degeneracy of the vacuum in their spectrum.

As mentioned the near horizon limit of general extremal black holes does not leave any room for having nontrivial dynamics. As we will discuss in the next section, however, this general picture does have some exceptions. Here, we focus on circumstances under which the near horizon limit of extremal black holes exhibit nontrivial dynamics and analyze the emergence of low energy dynamical conformal symmetry. Our general philosophy is presented in Sec. II and is exemplified in subsequent sections by the analysis of near-extremal static R-charged AdS_5 black holes and their near horizon geometries in order to study their low energy excitations (see Fig. 1 to illustrate our setup). This setup was already considered in [14–16]. Here we build on the scaling limits discussed there and make connection with the more recent Kerr/CFT perspective [7,8].

We focus on black holes whose extremal limit has a vanishing horizon area (in units of AdS_5 radius). They belong to the family of Extremal Vanishing Horizon (EVH) black holes whose near horizon geometry develops a local AdS_3-like throat, signalling the possible existence of an (IR) dual 2d CFT which captures the low energy dynamics around the background of EVH black holes. As we will discuss in the next section, a generic excitation around an EVH black hole, a near-EVH black hole, is specified by two parameters which in the near horizon (IR) limit appear as excitations around the AdS_3 throat, turning it to a BTZ geometry. On the other hand, being asymptotically AdS_5 black holes, the EVH black holes in this class have also a dual ultraviolet (UV) CFT

*J.deBoer@uva.nl
†M.J.F.Johnstone@sms.ed.ac.uk
‡jabbari@theory.ipm.ac.ir
§j.simon@ed.ac.uk

[1]See [2] for a more recent discussion and [3] for a review of these ideas in a more general holographic context.

[2]This is a theorem in $d = 4$, 5 dimensions, and it extends to higher dimensions, under some isometry assumptions [6]. The theorem also allows global AdS_3 geometries for a class of horizons generated by static null Killing vectors. This possibility is different from the one we will discuss in this note.

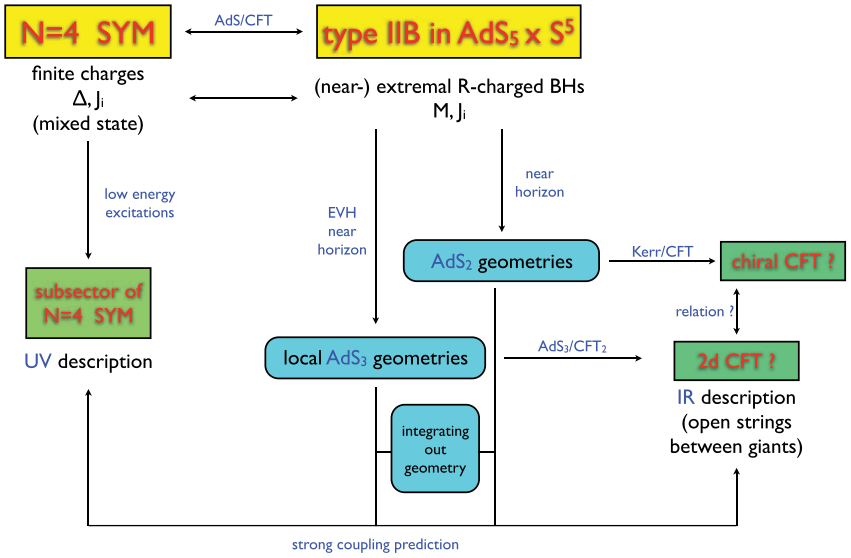

FIG. 1 (color online). Embedding our setup in an UV CFT using an extremal asymptotically AdS black hole, studying its low energy excitations through its near horizon geometry and identifying potential IR 2d CFTs describing them.

description, as a thermal mixed state, using the standard AdS/CFT correspondence [17,18]. In the near-BPS case, they can even be microscopically interpreted as distributions of smeared giant gravitons [19]. If this UV CFT is nonsingular and defined on a compact space, which is true for our case where the UV CFT is $\mathcal{N} = 4$ SYM on $R \times S^3$, its spectrum is gapped and at low enough energies above the EVH black hole, no dynamics should be left.

To circumvent this conclusion we will take *large central charge* limits of this UV CFT, keeping the AdS radius fixed, i.e. large N limits. Besides the well known possibility involving planar black holes, briefly reviewed in Sec. VI, one can also consider vanishing horizon black holes keeping the near-extremal entropy, or rather its density, fixed. We are primarily interested in understanding whether the emergent local AdS_3 geometries appearing in these cases describe the low energy excitations of the original UV CFT in terms of an infra-red (IR) 2d CFT.

In Secs. IV and V, we identify two distinct regimes where to study this phenomena: a near-BPS and a non-BPS regime. In Secs. IV and V, we study these by taking different near horizon limits. We discuss the important geometrical differences between the two, identify the relevant sectors of $\mathcal{N} = 4$ SYM in each case and compute the standard CFT_2 parameters which we compare with Kerr/CFT predictions in section VII. Further advantages of our approach are the natural quantization of the central charges emerging in the near-BPS regime and the potential BMN-like [20] interpretation that our near horizon limits offer. In either case, we attempt to provide an interpretation for our results and comment on the importance/limitations of taking the near horizon limit in our summary and outlook. In the Appendix we discuss near horizon limit of EVH black holes in the family of Myers-Perry black holes [21].

II. GENERAL PHILOSOPHY

A generic asymptotically AdS_{d+1} black hole is described by a thermal mixed state in the dual UV CFT theory. Excitations above it will generically have a gap if the latter is defined on $\mathbb{R} \times S^{d-1}$ and is nonsingular. Thus, probing the system at sufficiently low energies above the black hole, but below the gap, one expects to keep the degeneracy of the ground state of extremal black hole (black hole entropy), but no nontrivial dynamics.

This argument suggests that if there is any emergent CFT in the deep IR, associated with the near-horizon geometry, the latter will contain no nontrivial dynamics. In particular, for generic extremal black holes, whose near horizon geometries include AdS_2 throats, we would conclude that such IR CFTs would only contain the vacuum state and its degeneracy. This seems to be consistent with arguments such as AdS_2-fragmentation [22], AdS_2/CFT_1 considerations [11] or the absence of gravitational perturbations preserving the near horizon of 4d extremal Kerr [23].[3]

This conclusion can be bypassed, as illustrated in Fig. 2, if one violates one of the above assumptions:

(i) if the UV CFT is defined on a *noncompact space*, its spectrum will be continuous and nontrivial physical

[3]This conclusion is expected to be much more subtle in a generic situation, given the existence of multicenter AdS_2 configurations when the cosmological constant vanishes. As already emphasized in [22], these classical configurations survive the low energy limit. Recently, further configurations were found in different supergravity theories sharing this same feature. A proper microscopic understanding of these is not known, though they were already argued to correspond to a physical situation where the Higgs and Coulomb branches of the dual gauge theory coincide [22].

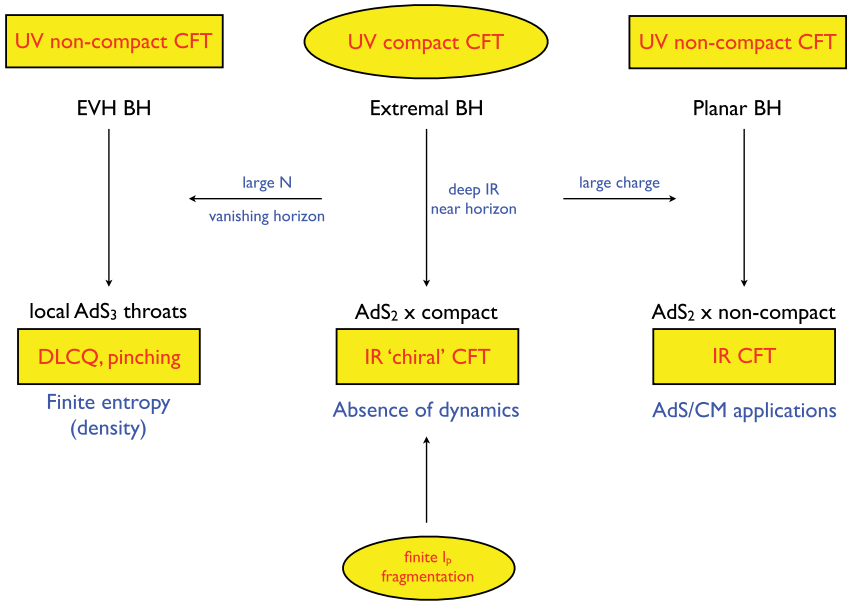

FIG. 2 (color online). Large N limits in nonsingular UV CFTs to get nontrivial physical excitations above extremal black holes at low energies.

excitations may exist at low energies. Noncompactness of the boundary theory also implies noncompactness of the black hole horizon and consequently, the vanishing of the two dimensional Newton's constant obtained from dimensional reduction of the gravity theory over the near horizon geometry of extremal black hole. (The AdS_2 space appearing in the near horizon geometry is a solution to this 2d gravity theory.) The latter also bypasses the fragmentation argument. This setup has prominently appeared in some recent applications of the AdS/CFT correspondence to condensed matter systems [24,25]. Even in these cases, there is no evidence for decoupling of the UV and IR physics (though some nonanalytical features are seemingly captured by the AdS_2 throats).

(ii) if we *decrease the mass gap* $\delta\Delta_{\rm UV}$ of the strongly coupled dual UV CFT. At weak coupling, this implies a large central charge limit, both in a near-BPS and far from BPS situations, in view of the gap $\delta\Delta_{\rm UV} \sim 1/c$ one obtains from the long string picture. At strong coupling and for BPS spectrum, the same conclusion was shown to hold in [26], using the emergence of deep throats. As far as we know, this conclusion has not been extended to strongly coupled far from BPS situations, where the standard lore is that such spectrum will look random, so that $\delta\Delta_{\rm UV} \sim e^{-S}$ [27]. Either way, string theory realizations of these scenarios typically involve a large charge limit. For a given temperature, these would give rise to a divergent entropy. To keep the latter *finite*, one must combine the large charge limit with a *vanishing horizon* limit, which in turn also demands *vanishing temperature* (extremal) limit of the original black hole. In the case of asymptotic AdS_5 EVH black holes, as we will discuss in this paper, this corresponds to a certain large N limit. In both BPS and non-BPS EVH cases taking the large N limit together with near extremal limit will open up the possibility of having nontrivial excitations and dynamics.

In any statistical mechanical system in equilibrium entropy is a positive-definite function of charges and temperature and the entropy can vanish only at zero temperature, i.e. the vanishing entropy limit of any system corresponds to its low temperature (IR) expansion.[4] Therefore, we consider the low temperature IR expansion for the gravitational (Bekenstein-Hawking) entropy of a black hole

$$S(q_i, N; T) = S_0(q_i; N) + S_1(q_i; N)T + S_2(q_i; N)T^2 + \cdots \tag{2.1}$$

where q_i stand for the different black hole charges and N for the rank of the dual gauge group or a quantum number playing a similar role. Generic extremal black holes have nonzero $S_0(q_i; N)$, providing the dominant contribution to the entropy in this IR limit. There may be specific extremal black holes for which the coefficients S_n are zero for $n < k$. In that situation, the leading contribution to the entropy is $S \sim S_k(q_i; N)T^k$ and one may speculate on the existence of

[4]We note that the converse, the usual statement of third law of thermodynamics, does not hold in the cases involving extremal black holes; i.e. extremal black holes generically have a nonzero finite entropy.

a dual IR $k+1$ dimensional CFT, since $S \sim c_k T^k$ follows from conformal invariance with c_k being some effective central charge.[5]

The possibility that such an IR CFT may be supported by a near horizon local AdS_{k+2} throat has already appeared in the literature in the case $k=1$. The fact that the near horizon geometry of an EVH black hole has a local AdS_3 throat was originally pointed out in [28] for extremal 5d Kerr black holes with one vanishing angular momentum, where the near horizon geometry involves a *pinching* AdS_3 orbifold.[6] As discussed in the Appendix, this statement can be easily generalized to higher dimensional Myers-Perry black holes [21]: extremal Myers-Perry black holes with *one* vanishing angular momentum develop such throats. The appearance of an AdS_3 throat for R-charged AdS_d (for $d=4, 5$) black holes was reported in [14–16] when one of the R-charges is parametrically smaller than the rest. It was recently proved that the near horizon geometry of any EVH solution of four dimensional Einstein-Maxwell-Dilaton theory has a pinching AdS_3 orbifold throat [29]. Interestingly, in the Kerr/CFT context, attempts to give a microscopic derivation of the conjecture also ended up exploring regions in parameter space where the horizon became of zero size [30,31]. These ideas were extended, under certain conditions, for extremal vanishing horizons in four and five dimensions in [32–34].

A common feature of all these $k=1$ examples giving rise to local AdS_3 throats is that both horizon area and temperature of the extremal black hole tend to zero keeping their ratio finite

$$A, T \to 0 \quad \text{with} \quad \frac{A}{T} \text{ finite} \tag{2.2}$$

It is this ratio that suggests the potential emergence of a gravitational thermodynamical system in 3 dimensions having a 2d CFT dual with finite central charge. This is indeed the philosophy advocated in [29] giving rise to the so called EVH/CFT correspondence.

To sum up, as illustrated in Fig. 3, one starts with a black hole in AdS whose Bekenstein-Hawking entropy, at low temperatures, satisfies

$$S_{10\mathrm{d}} = \frac{A}{4G_N^{(10)}} \propto T. \tag{2.3}$$

This relation holds in a regime of black hole charges determining a specific set of UV CFT charges Δ_{UV} and J_i. The question is whether there exists an alternative description for the physics of low-lying excitations with quantum numbers $\delta\Delta_{\mathrm{UV}}$ and δJ_i satisfying $\delta\Delta_{\mathrm{UV}} \ll \Delta_{\mathrm{UV}}$. We explore the proposal that this alternative description is in terms of a 2d CFT.

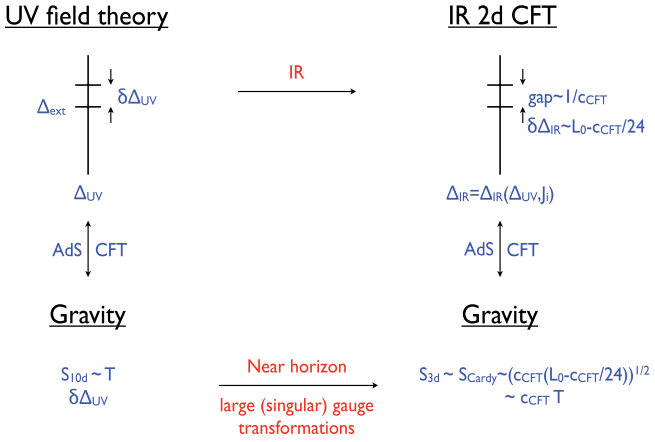

FIG. 3 (color online). Set of scales and IR description of UV excitations.

Geometrically, near-extremal horizon limits typically involve nontrivial (singular) large gauge transformations defining the near horizon geometry (IR description) in terms of the isometry coordinates of the boundary geometry (UV CFT theory)

$$\begin{aligned} &r = r_h + \epsilon\rho, \qquad \varphi_i^{\mathrm{IR}} = \varphi_i^{\mathrm{IR}}(\varphi_i^{\mathrm{UV}}, t^{\mathrm{UV}}, \epsilon), \\ &t^{\mathrm{IR}} = t^{\mathrm{IR}}(\varphi_i^{\mathrm{UV}}, t^{\mathrm{UV}}, \epsilon) \qquad \epsilon \to 0. \end{aligned} \tag{2.4}$$

This suggests, as also expected from a purely field theoretical perspective, the existence of a nontrivial relation between the UV and IR Hamiltonians. If the IR theory is conformal, there will therefore be an interesting relation between quantum numbers of the form $\Delta_{\mathrm{IR}} = \Delta_{\mathrm{IR}}(\Delta_{\mathrm{UV}}, J_i)$.[7] Notice there is no guarantee the emergent 2d CFT would be local in the original UV description. This is expected not to be the case whenever the charges involved correspond to R-charges (internal charges). It would be very interesting to develop a renormalization group perspective (interpreted as integrating out geometry [36]) on these nontrivial relations.

In view of the 10 dimensional thermodynamical relation (2.3), one is looking for a reinterpretation of the UV spectrum in terms of an effective IR 2d CFT, as illustrated in Fig. 3, whose central charge and energy fluctuations satisfy

$$c_{\mathrm{CFT}} \propto \frac{S_{10\mathrm{d}}}{T}, \qquad L_0 - \frac{c_{\mathrm{CFT}}}{24} \sim \delta\Delta_{\mathrm{IR}}. \tag{2.5}$$

[5]Note that a similar low temperature expansion and similar reasoning also applies to the well established (near-BPS) black p-brane solutions, which for $k=2, 3, 5$ lead to the usual (maximally supersymmetric) AdS_{k+2}/CFT_{k+1} examples.

[6]The word pinching AdS_3 orbifold was coined in [13], where the simplest possible EVH black hole, namely, the massless BTZ black hole, and its possible near horizon limits were discussed. The pinching AdS_3 orbifold is a singular geometry which can be thought of as AdS_3/Z_K in the $K \to \infty$ limit.

[7]For example, in a near-BPS situation, one expects $\Delta_{\mathrm{IR}} = \Delta_{\mathrm{UV}} - \sum_i J_i$. We will explicitly see this feature emerging in section IV. The importance of these singular large gauge transformations for extremal black holes has been emphasized in [3,35].

If the IR CFT has a gravitational dual, presumably related to the near horizon geometry of the initial (near-)extremal black hole, one would, in particular, expect the Brown-Henneaux relation [4]

$$c_{\rm CFT} = \frac{3\ell_3}{2G_N^{(3)}} \tag{2.6}$$

to hold, which will provide a useful consistency check of this framework.

Notice that keeping the *entropy finite*, while $A \to 0$, will require us to rescale the 10d Newton's constant. In our setup, which keeps the UV AdS radius fixed, this requires an $N \to \infty$ limit, which we will discuss in more detail in the upcoming sections.

III. VANISHING HORIZON LIMITS FOR *R*-CHARGED AdS_5 BLACK HOLES

In this section, we review the characterization of extremal vanishing horizons among *R*-charged AdS_5 black holes. These are solutions of type IIB supergravity with constant dilaton, and metric and RR 4-form potential given by [37]

$$\begin{aligned} ds_{10}^2 &= \sqrt{\Delta}\left(-\frac{f}{H_1H_2H_3}dt^2 + \frac{dr^2}{f} + r^2 d\Omega_3^2\right) \\ &\quad + \frac{L^2}{\sqrt{\Delta}}\left(\sum_{i=1}^{3} H_i(d\mu_i^2 + \mu_i^2[d\phi_i + a_i dt]^2)\right) \\ C_4 &= -\frac{r^4}{L}\Delta dt \wedge d^3\Omega - L\sum_{i=1}^{3}\tilde{q}_i\mu_i^2\left(Ld\phi_i - \frac{q_i}{\tilde{q}_i}dt\right)\wedge d^3\Omega. \end{aligned} \tag{3.1}$$

The configuration is determined by a set of scalar functions $\{H_i, f, \Delta\}$ and gauge fields $\{a_i\}$

$$H_i = 1 + \frac{q_i}{r^2}, \qquad a_i = \frac{\tilde{q}_i}{q_i}\frac{1}{L}\left(\frac{1}{H_i} - 1\right), \quad i = 1, 2, 3 \tag{3.2a}$$

$$f = 1 - \frac{\mu}{r^2} + \frac{r^2}{L^2}H_1H_2H_3,$$

$$\Delta = H_1H_2H_3\left[\frac{\mu_1^2}{H_1} + \frac{\mu_2^2}{H_2} + \frac{\mu_3^2}{H_3}\right], \tag{3.2b}$$

the unit radius 3-sphere metric $d\Omega_3^2$ and a further 2-sphere $\mu_1^2 + \mu_2^2 + \mu_3^2 = 1$.

These solutions have four independent parameters $\{\mu, q_i\}$ determining the mass and *R*-charges of the black hole

$$\Delta = ML = \frac{\pi L}{4G_N^{(5)}}\left(\frac{3}{2}\mu + q_1 + q_2 + q_3\right), \tag{3.3a}$$

$$J_i = \frac{\pi L}{4G_N^{(5)}}\tilde{q}_i = \frac{\pi L}{4G_N^{(5)}}\sqrt{q_i(\mu + q_i)} \tag{3.3b}$$

in terms of the five dimensional Newton's constant

$$G_N^{(5)} = \frac{G_N^{(10)}}{(\pi^3 L^5)} = \frac{\pi}{2}\frac{L^3}{N^2}. \tag{3.4}$$

The $\mu = 0$ case corresponds to $\tilde{q}_i = q_i$ and $\Delta = J_1 + J_2 + J_3$. This is the BPS limit. Thus μ measures the deviation from BPSness. These singular configurations were interpreted as distributions of smeared giant gravitons in [19], where the flux quantization conditions

$$\frac{N_i}{N} = \frac{2J_i}{N^2} = \frac{\tilde{q}_i}{L^2}, \qquad i = 1, 2, 3 \tag{3.5}$$

were derived for each of the three types of giants supporting these black holes. Here N_i is the number of giant gravitons in each stack. Since each giant type involves a different 3-cycle in the transverse 5-sphere [38], pairs of giants belonging to different types intersect on circles. This observation was used in [14,15,39] to argue that two *R*-charged AdS_5 black holes should allow a dual 2d CFT description defined on the S^1 where giants intersect and with central charge proportional to the total number of such intersections, i.e. $c \sim N_iN_j$ $i \neq j$. This interpretation will play an important role when we discuss the near-BPS regime.

Extremality vs charges: For completeness, we review the conditions under which *finite* extremal *R*-charged black holes appear [37]:

(a) For *single R*-charge configurations characterized by $\{\mu, q_1\}$, the condition for extremality coincides with the condition for the black hole to be BPS, *i.e.* $\mu \to 0$. However, as one may easily check, no local AdS_3 geometry appears as one takes the near-horizon limit. The situation is similar to Myers-Perry black holes with two or more of the angular momenta vanishing (*cf.* the discussions in the Appendix).

(b) For *two R*-charge configurations characterized by $\{\mu, q_2, q_3\}$, horizons exist for $\mu > \mu_c \equiv q_2q_3/L^2$. Extremality is achieved when $\mu = \mu_c$. Thus, the scale $\mu - \mu_c$ measures the amount of nonextremality.

(c) For *three R*-charge black hole, with three generic charges of the same order of magnitude, horizons exist for μ above a certain quantity and below which we have a naked singularity [37]. For our purposes, the is important about black holes in this class is that as soon as the extremal limit is achieved, the horizon size is necessarily *finite*. Thus, in this regime, EVH black holes can not appear.[8]

EVH vs thermodynamics: the energy, entropy and temperature for these black holes are [37]

[8]If one of the charges is parametrically smaller than the other two, the three-charge system can under favorable circumstances be viewed as a perturbation of the EVH configuration identified in b), as we will describe in detail in the following.

$$\Delta = LM = \frac{N^2}{2L^2}\left(\frac{3}{2}\mu + q_1 + q_2 + q_3\right),$$
$$S = \frac{\pi N^2}{L^3}\sqrt{(r_+^2 + q_1)(r_+^2 + q_2)(r_+^2 + q_3)}, \tag{3.6}$$
$$T = \frac{f'(r_+)r_+^3}{4\pi\sqrt{(r_+^2 + q_1)(r_+^2 + q_2)(r_+^2 + q_3)}},$$

where r_+^2 is the (outer) horizon radius, defined as the largest root of $f(r)$ in (3.2).

We are interested in studying the regime of parameters where the thermodynamical conditions (2.2) hold. If N and L are fixed, the vanishing of the area requires $\prod_i(r_+^2 + q_i) \sim \lambda \to 0$. Achieving this while keeping the ratio A/T finite, requires $f'(r_+)r_+^3 \sim \lambda$, which includes the standard extremality condition. To study this limit carefully, we express two of the independent parameters of the solution, such as μ and q_1, in terms of the remaining q_2, q_3 and the outer and inner horizons $r_\pm$

$$\begin{aligned} q_1 &= r_+^2 r_-^2 C(r_\pm, q_2, q_3), \\ \mu - \mu_c &= (r_+^2 + r_-^2)\mu_c C(r_\pm, q_2, q_3) \\ &\quad + r_+^2 r_-^2((q_2 + q_3)C(r_\pm, q_2, q_3) - 1) \end{aligned} \tag{3.7}$$

where the scalar function $C(r_\pm, q_2, q_3)$ is

$$C(r_\pm, q_2, q_3) \equiv \frac{L^2 + q_2 + q_3 + r_+^2 + r_-^2}{\mu_c L^2 - r_+^2 r_-^2}. \tag{3.8}$$

The function $f(r)$ characterizing the existence of horizons becomes

$$f(r) = \frac{(r^2 - r_+^2)(r^2 - r_-^2)}{L^2 r^4}(r^2 + \mu_c L^2 C(r_\pm, q_2, q_3)). \tag{3.9}$$

Whenever the charges q_2 and q_3 are parametrically larger than q_1, it follows that (2.2) requires $q_2, q_3 \gg r_+^2 \sim \epsilon^2$ with $\epsilon \to 0$ and $q_1 \sim \epsilon^\alpha$ with $\alpha \geq 2$. In this regime, entropy and temperature behave like

$$S \sim N^2\sqrt{\frac{\mu_c}{L^2}}\epsilon, \quad TL \sim \frac{\epsilon}{\sqrt{\mu_c/L^2}} \Rightarrow \frac{S}{TL} \sim N^2\frac{\mu_c}{L^2}, \tag{3.10}$$

in agreement with the general setup described in Sec. II.

There are two physically distinct situations compatible with the above regime:

(1) Near-extremal *near-BPS* case in which q_2, $q_3 \sim \epsilon$. This corresponds to a dilute giant graviton approximation in which the black hole temperature remains finite, whereas the ratio

$$\frac{S}{TL} \propto N_2 N_3 \tag{3.11}$$

is proportional to the total number of giant graviton intersections, using the quantization conditions (3.5).

(2) Near-extremal *non-BPS* case in which q_2, q_3 remain finite and the temperature scales to zero.

As argued in Sec. II, any attempt to make the entropy finite will require to take $N \to \infty$. In the non-BPS case, see Sec. V, this will be achieved by requiring $N^2\epsilon =$ finite but large. As we discuss in Sec. IV, the near-BPS limit will turn out to be more subtle. Even though finite entropy is thermodynamically achieved by $N\epsilon \sim 1$, near horizon considerations and reliability of classical supergravity will instead suggest to consider $N\epsilon^2$ to be finite but large.

In the next sections, we will discuss how these physical differences are encoded in the properties of the candidate emergent 2d IR CFTs, i.e. their central charges and energy levels, after explicitly identifying the near horizon geometries of the R-charged AdS_5 black holes (3.1) in the two regimes described above.

IV. NEAR-BPS R-CHARGED ADS_5 EVH BLACK HOLES

In this section, we study the near-extremal near-BPS limit

$$\mu \to 0, \qquad \mu - \mu_c \to 0 \quad \text{with} \quad L \text{ fixed}.$$

This forces the R-charges q_i to scale to zero. Thus, we will be working in some dilute giant graviton approximation. Following [14,15], we consider the near horizon limit[9]

$$\begin{aligned} &r = \epsilon\tilde{\rho}, \qquad \theta_i = \theta_i^0 - \epsilon^{1/2}\hat{\theta}_i, \qquad 0 \leq \theta_i^0 \leq \pi/2, \\ &\mu - \mu_c = \epsilon^2\hat{M}, \qquad q_i = \epsilon\hat{q}_i, \qquad q_1 = \epsilon^2\hat{q}_1, \\ &\psi_i = \frac{1}{\epsilon^{1/2}}\left(\phi_i - \frac{t}{L}\right), \qquad i = 2, 3, \end{aligned} \tag{4.1}$$

while keeping $\tilde{\rho}$, $\hat{q}_i$, $\hat{q}_1$, $\hat{M}$, θ_i^0, L, ψ_i fixed. Choosing $\mu_1^0 = \cos\theta_1^0$, the resulting near horizon metric is [15]

$$\begin{aligned} ds^2 &= \epsilon\left[\mu_1^0 ds_6^2 + \frac{L^2}{R_S^2\mu_1^0}ds_{\mathcal{M}_4}^2\right] \quad \text{with} \\ R_S^4 &= \hat{q}_2\hat{q}_3 \equiv \hat{\mu}_c L^2. \end{aligned} \tag{4.2}$$

Here, $ds_{\mathcal{M}_4}^2$ stands for the metric

$$ds_{\mathcal{M}_4}^2 = \sum_{i=2,3}\hat{q}_i(d\hat{\mu}_i^2 + (\mu_i^0)^2 d\psi_i^2), \tag{4.3}$$

where $d\mu_i = -\epsilon^{1/2}d\hat{\mu}_i$ $i = 2, 3$.[10] The 6d Lorentzian metric describes a local $AdS_3 \times S^3$

[9] We have assumed $\mu_1 \sim \mu_1^0 \neq 1$. For a detailed discussion regarding this possibility, see [14,15].

[10] In terms of an explicit parameterization consistent with the choice $\mu_1^0 = \cos\theta_1^0$, we have $d\hat{\mu}_2 = \cos\theta_1^0\cos\theta_2^0 d\hat{\theta}_1 - \sin\theta_1^0\sin\theta_1^0 d\hat{\theta}_2$ and $d\hat{\mu}_2 = \cos\theta_1^0\sin\theta_2^0 d\hat{\theta}_1 + \sin\theta_1^0\cos\theta_1^0 d\hat{\theta}_2$.

$$ds_6^2 = -\frac{(y^2-y_+^2)(y^2-y_-^2)}{y^2R_S^2}dt^2 + \frac{R_S^2y^2}{(y^2-y_+^2)(y^2-y_-^2)}dy^2 + y^2\left(d\phi_1 - \frac{y_+y_-}{y^2R_S}dt\right)^2 + R_S^2d\Omega_3^2, \tag{4.4}$$

in terms of the new radial coordinate

$$y^2 = \frac{L^2}{R_S^2}(\tilde{\rho}^2 + \hat{q}_1). \tag{4.5}$$

We want to stress that t and ϕ_1 were *not* rescaled in (4.1), which is consistent with the finiteness of the temperature discussed in Sec. III in the near-BPS regime, and ϕ_i were forced to rotate at the speed of light, in L units, matching the rotating velocity of the constituent giant gravitons. The coordinates ψ_i parameterizing deviations from this corotation become effectively noncompact. The near horizon geometry is characterized by two parameters: $\hat{M}$ and $\hat{q}_1$. These describe near-extremality. It is convenient to define $\mu = \epsilon^2\hat{\mu}$ and $\mu_c = \epsilon^2\hat{\mu}_c$ so that $\hat{\mu}_c = \hat{q}_2\hat{q}_3/L^2$ to ease the notation below. Lastly, notice that to achieve an overall scaling in the metric, one is forced to geometrically focus on a strip $\theta \sim \theta_i^0$ of the transverse 5-sphere. We will return to this point when interpreting the entropy of the metric (4.2).

Notice both AdS$_3$ and S^3 spaces have equal radii $\ell_3 = R_S$. The parameters $y_\pm$ are determined in terms of the original black hole parameters by

$$y_+^2 + y_-^2 = \frac{L^2}{R_S^2}\left(\hat{M} + 2\hat{q}_1\right) = R_S^2\left(\frac{\hat{\mu} + 2\hat{q}_1}{\hat{\mu}_c} - 1\right),$$
$$y_+y_- = \frac{L^2}{R_S^2}\sqrt{(\hat{M} + \hat{\mu}_c + \hat{q}_1)\hat{q}_1} = R_S^2\sqrt{\frac{(\hat{\mu} + \hat{q}_1)\hat{q}_1}{\hat{\mu}_c^2}}. \tag{4.6}$$

Depending on the values of $\hat{\mu}$ and $\hat{q}_1$ the locally AdS$_3$ part of geometry (4.4) corresponds to different quotients of AdS$_3$ (for example see [40] or Appendix B of [15]). Note that $\hat{\mu} \geq 0$ and $\hat{q}_1 \geq 0$ in our conventions, which implies that $\hat{M}/\hat{\mu}_c \geq -1$ ($\hat{M}$ can be negative).

- For the special case of $\hat{\mu} = 0 (\hat{M}/\hat{\mu}_c = -1)$, $\hat{q}_1 = 0$ we have global AdS$_3$. This corresponds to the near horizon limit of 1/4 BPS AdS$_5$ black hole.
- When $\hat{M} \leq -2(\sqrt{\hat{q}_1(\hat{\mu} + \hat{q}_1)} + \hat{q}_1)$, $\hat{M}^2 \geq 4\hat{\mu}_c\hat{q}_1$ we have a conic space. This can happen when $\hat{M} + 4\hat{q}_1 \leq 0$ and $4\hat{q}_1 \leq \hat{\mu}_c$. The 1/8 BPS AdS$_5$ black hole with $\mu = 0, 4q_1 \leq \mu_c$ falls in this class.
- For any real value of $y_\pm$ corresponding to $\hat{M}^2 \geq 4\hat{q}_1\hat{\mu}_c$, $\hat{M} \geq 0$ we have BTZ black holes. For the special case of $\hat{q}_1 = 0$, $\hat{\mu} = \hat{\mu}_c$ we have massless BTZ and for $\hat{q}_1 = \hat{M}^2/(4\hat{\mu}_c)$ we have extremal BTZ. The mass and angular momentum of the BTZ black hole is given by

$$M_{\rm BTZ} = \frac{y_+^2 + y_-^2}{8G_N^{(3)}\ell_3^2}, \qquad J_{\rm BTZ} = \frac{2y_+y_-}{8G_N^{(3)}\ell_3}. \tag{4.7}$$

- Finally for $\hat{M}^2 < 4\hat{q}_1\hat{\mu}_c$ and $|\hat{M} + 2\hat{q}_1| < 2\sqrt{\hat{q}_1(\hat{\mu} + \hat{q}_1)}$ the geometry has a naked singularity. The special case of 1/8 BPS R-charged black hole $\mu = 0$, $q_1 > 4\mu_c$ falls into this class. This geometry can correspond to BPS rotating D-string like excitations in the AdS$_3$. This latter case, however, should be explored in more detail, which we postpone to future works.

Black hole vs. near horizon entropies: Using (3.6), the entropy of the full black hole in the limit of charges defined in (4.1) equals

$$S = \pi\frac{R_S^3}{L^4}y_+(N\epsilon)^2, \tag{4.8}$$

where we already used (4.5). It is not surprising to check that the entropy of the near horizon geometry (4.2) does *not* match this result

$$S_{\rm 3d} = \frac{2\pi y_+}{4G_3} = 8S\mu_2^0\mu_3^0\mathcal{V}_{\mathcal{M}_4}, \tag{4.9}$$

where $\mathcal{V}_{\mathcal{M}_4}$, which is kept finite in the limit, is defined in (4.10). This mismatch is physically expected because in taking the near horizon limit (4.1) we were forced to focus on a strip in the transverse 5-sphere. To derive this result, some comments and definitions are in order:

(1) By construction, the volume of the flat noncompact manifold $\mathcal{M}_4$ is infinite. Since the local coordinates describing the latter are ϵ dependent, one can provide a natural regularization by keeping ϵ very small, but finite. This gives rise to

$$\text{vol}\,\mathcal{M}_4 = (2\pi)^2\frac{R_S^4}{\epsilon^2}\mu_2^0\mu_3^0, \tag{4.10}$$

which defines $\mathcal{V}_{\mathcal{M}_4}$.

(2) The 3d Newton's constant is computed in the standard fashion, using the regularization mentioned above, and properly dealing with the factor $\epsilon\mu_1^0$ in front of the 3d metric when comparing

$$\frac{1}{G_{10}}\int d^{10}x\sqrt{-\det g_{10}}\mathcal{R}_{10} = \frac{1}{G_3}\int d^3x\sqrt{-\det g_3}\mathcal{R}_3.$$

Proceeding in this way, one finds

$$\frac{1}{G_3} = 16(N\epsilon)^2\frac{R_S^3}{L^4}\mu_2^0\mu_3^0 = \frac{16N^2\epsilon^4}{\ell_3}\frac{\text{vol}\mathcal{M}_4}{(2\pi)^2L^4}, \tag{4.11}$$

justifying (4.9).

It is interesting to emphasize that focusing on a ''strip'' of a black hole horizon, when taking the near horizon limit, is generic in nonextremal black holes. The difference is that the 2d geometry close to the generic nonextremal horizon is Rindler, whereas the (near)-BPS and (near) EVH case studied here give rise to AdS_3 (BTZ). Technically, this occurs to guarantee analyticity in the ϵ expansion of the black hole metric components when taking its near horizon limit. The latter ensures the limiting metric remains a solution to supergravity equations. Conceptually, if one thinks of the horizon as the location where the black hole degrees of freedom live (at least from the perspective of an observer at infinity), it is clear that such near horizon description will never reproduce the correct entropy because the latter loses the information on the curvature of the original horizon by approximating it with a flat tangent plane.

Gravitationally, and for the reasons just mentioned, it is natural to interpret the so obtained near horizon Bekenstein-Hawking entropy (4.2) as an *entropy density*. This is more even so in the particular example discussed in this subsection, given the microscopic interpretation of the BPS R-charged black holes as a distribution of smeared giant gravitons on the 5-sphere [19] and the arguments provided in [14] identifying the open strings stretched between these giants as responsible for the entropy of their near-BPS limits. The dependence on the point where the strip lies, i.e. $\mu_2^0\mu_3^0$, provides the natural measure where to integrate such density. Not surprisingly, one finds

$$\int_{\mu_2^2+\mu_3^2\leq 1} \mu_2\mu_3 d\mu_2 d\mu_3 = \frac{1}{8}. \tag{4.12}$$

That is, if we suitably sum over the entropies of each BTZ black hole located at different strips (different values of μ_2, μ_3), given in (4.9), we recover the entropy of the original 5d black hole. Unfortunately, it is not clear to us what the process of integrating over this entropy density means in the language of 2d CFTs that naturally would arise as the dual descriptions of the near horizon geometries (4.2).[11]

A. Nontrivial IR dynamics and scaling of N

The entropy of the original black hole (4.8) goes to zero, for finite N, as a consequence of the dilute giant graviton approximation. Given the overall ϵ scaling in the near horizon metric (4.2), it is natural to interpret the latter as a rescaling of the 10d Planck scale, i.e. $\ell_p^4 \to \epsilon^2\ell_p^4$, as we usually do in the decoupling limits leading to the AdS/CFT correspondence [18]. Keeping L finite requires N to scale as $N\epsilon^2 \sim 1$ if g_s remains fixed, i.e. $\alpha' \to \epsilon\alpha'$. (Note that by $N\epsilon^2 \sim 1$ we mean $N\epsilon^2$ is kept finite but large in the near horizon limit.) This is the same scaling considered in [15]. Given the noncompactness of the transverse space in the limit (4.1), it is the entropy density that one should require to keep finite

$$s = \frac{S_{3d}}{\mathrm{vol}\mathcal{M}_4} \propto (N\epsilon^2)^2 \quad \text{finite but large} \Rightarrow N\epsilon^2 \sim 1. \tag{4.13}$$

Thus, both considerations are consistent with the same scaling. We provide two further physical arguments for why $N\epsilon^2 \sim 1$ can be a meaningful limit to study:

(1) One can estimate the *mass density* of open strings stretched between intersecting giants as

$$m_{\mathrm{open}} = \frac{M_{\mathrm{open}}}{\mathrm{length}\mathcal{M}_4} \sim \epsilon^{1/2}\frac{L\delta\hat{\theta}}{l_s^2}\frac{1}{L\epsilon^{-1/2}} \sim \frac{\sqrt{g_s}}{L^2}\delta\hat{\theta}\sqrt{N\epsilon^2}. \tag{4.14}$$

where we used $L^4 = 4\pi g_s l_s^4 N$ and kept g_s fixed. Thus, requiring energy density finiteness of these excitations also dictates the scaling $N\epsilon^2 \sim 1$.

(2) The smallest distance computed in the near horizon metric (4.2) is of order $\ell \sim L\sqrt{\epsilon}$ and the curvature invariants of the near horizon metric (4.2) are of order ℓ^{-2}. In order to have a valid supergravity approximation in which stringy corrections are small we need to require $\ell \gtrsim l_s$. Since $\ell/\ell_s \sim (N\epsilon^2)^{1/4}$, for a fixed g_s, validity of supergravity leads to $N\epsilon^2 \sim 1$. The validity of the supergravity description also demands $sL^4 \gtrsim 1$ where s is the entropy density. This latter, as discussed above, is also satisfied with $N\epsilon^2 \sim 1$ scaling.[12]

B. Comments on $\mathcal{N} = 4$ SYM and 2d CFT descriptions

Our near-BPS black holes have a UV description as thermal states in $\mathcal{N} = 4$ SYM. On the other hand, our AdS_3 near horizon geometry hints at the emergence of an IR 2d CFT dual description. Here we advocate a more direct connection between the two CFTs and will make some further comments in Sec. VII A.

[11] It is possible that requiring to have a consistent on-shell near horizon geometry is responsible for the focusing on a horizon 'strip'. Recently, there have been discussions trying to argue that the low energy physics in (non-)extremal black holes is described by a 2d CFT, without appealing to its near horizon geometry, but to the wave equations satisfied by probe fields on the geometry [41,42]. If one would take a similar attitude in these black holes, one can envision keeping the information about the full black hole geometry. We will come back to this point in Sec. VII when comparing our results with the Kerr/CFT predictions.

[12] One could have considered scaling $N\epsilon \sim 1$ so that the entropy, and not its density, remains finite. However, as pointed out, with such a scaling the overall scale of the near horizon metric will be $L/\sqrt{N}$ which is much smaller than string scale and the gravitational description is no longer valid.

The evaluation of the gravitational charges $\{M, J_i\}$ in the near-BPS limit (4.1) identifies the sector of the Hilbert space in $\mathcal{N}=4$ SYM that we are focusing on:

$$\Delta = \frac{N^2\epsilon}{2L^2}\left(\hat{q}_2+\hat{q}_3+\frac{\epsilon}{2}\left(3\hat{\mu}_c+2\hat{q}_1+3\hat{M}\right)\right), \tag{4.15a}$$

$$J_1 = \frac{(N\epsilon)^2}{2L^2}\sqrt{\hat{q}_1(\hat{\mu}_c+\hat{M}+\hat{q}_1)}, \tag{4.15b}$$

$$J_i = \frac{N^2\epsilon}{2L^2}\left(\hat{q}_i+\frac{\epsilon}{2}\left(\hat{\mu}_c+\hat{M}\right)+\mathcal{O}(\epsilon^2)\right), \quad i=2,3. \tag{4.15c}$$

In the regime $N\epsilon^2 \sim 1$ where the entropy density remains finite, two remarks are in order:

(1) The dominant divergent contributions to energy Δ and R-charges J_i scale like $N^{3/2}$ [15].

(2) All quantities measuring the magnitude of the deviation from the BPS EVH solution, J_1 and $\Delta - J_2 - J_3$, diverge like $(N\epsilon)^2$, while their densities $J_1/\mathrm{vol}\mathcal{M}_4$ and $(\Delta - J_2 - J_3)/\mathrm{vol}\mathcal{M}_4$ remain finite. The latter are related to the mass and angular momentum of the BTZ black holes geometries obtained in the near horizon limit.

Here, we propose an interpretation along the lines of BMN [20]. In that case, the geometrical limit is a Penrose limit [43] corresponding to focusing the dynamics onto a sector of $\mathcal{N}=4$ SYM where Δ and J scaled like $N^{1/2}$ keeping $\Delta - J$ finite. Similarly, our limit (4.1) identifies the set of "almost-quarter-BPS" operators characterized by [15]

$$\begin{aligned} &\Delta, J_2, J_3 \sim N^{3/2}, \qquad \lambda_{t'\mathrm{Hooft}} = g_{YM}^2 N \sim N \to \infty \\ &\Delta - J_2 - J_3, J_1 \sim N, \qquad \lambda_{\mathrm{eff}} = \frac{g_{YM}^2 N}{N_2 N_3}, \end{aligned} \tag{4.16}$$

where $N_2, N_3 \sim N^{1/2}$ are the numbers of giants. As explained earlier, these degrees of freedom are expected to be associated with open strings stretched between smeared giant gravitons rather than with closed strings in the bulk [14]. It is not clear to us whether simplifications similar to those that appear in the standard BMN setup will occur here, but it is interesting to point out that preliminary steps in this direction have been taken in [44].

If we take the appearance of AdS_3 and BTZ geometries as serious evidence of the existence of a dual IR CFT, we can use the standard AdS_3/CFT_2 dictionary to connect the BTZ mass and angular momentum to L_0 and $\bar{L}_0$ of this conjectured dual CFT [40]

$$\begin{aligned} L_0 - \frac{c}{24} &= \frac{M_{BTZ}\ell_3 + J_{BTZ}}{2} = \frac{c}{24}\left(\frac{y_+ + y_-}{\ell_3}\right)^2, \\ \bar{L}_0 - \frac{c}{24} &= \frac{M_{BTZ}\ell_3 - J_{BTZ}}{2} = \frac{c}{24}\left(\frac{y_+ - y_-}{\ell_3}\right)^2, \end{aligned} \tag{4.17}$$

where we have used (4.7). It is straightforward to check that (4.15) yields

$$\Delta - J_2 - J_3 \pm J_1 = \frac{N^2\epsilon^2}{4}\left(\frac{\ell_3}{L}\right)^4\left(\frac{y_+ \pm y_-}{\ell_3}\right)^2. \tag{4.18}$$

We may then identify

$$\begin{aligned} \frac{L^4}{\mathrm{vol}\mathcal{M}_4}(\Delta - J_2 - J_3 + J_1) &= L_0 - \frac{c}{24}, \\ \frac{L^4}{\mathrm{vol}\mathcal{M}_4}(\Delta - J_2 - J_3 - J_1) &= \bar{L}_0 - \frac{c}{24}, \end{aligned} \tag{4.19}$$

and

$$c = 6N_2N_3\frac{L^4}{\mathrm{vol}\mathcal{M}_4}, \tag{4.20}$$

where N_2, N_3 are the numbers of giants. The central charge c has essentially the same form as in usual D1–D5 system, but now the central charge is proportional to density of giant gravitons; $L_0 \pm \bar{L}_0$ are examples of the quantities Δ_{IR} and J_{IR} discussed in Sec. II. We also note that for all vales of $\hat{\mu}, \hat{q}_1 \geq 0$, L_0, $\bar{L}_0$ have a non-negative spectrum and hence the proposed 2d CFT is unitary. The vacuum of the 2d CFT, $L_0 = \bar{L}_0 = 0$ corresponds to the 1/4 BPS black hole with $\mu = q_1 = 0$.

V. NON-BPS R-CHARGED AdS_5 NEAR-EVH BLACK HOLES

In this section, we analyze the non-BPS version of the near-EVH limit described in Sec. IV. This requires studying $\mu - \mu_c \to 0$ keeping μ_c finite. Thus, the charges q_i $i=2, 3$ will be kept finite. Since the deep interior of these non-BPS extremal black holes resembles the one of massless BTZ black holes, the discussion in [13] will apply. Thus, there will exist two different near horizon limits: one giving rise to a pinching AdS_3 orbifold, where the periodicity of the compact dimension goes to zero and a second giving rise to the null self-dual orbifold [45]. We want to stress that *a priori*, one may have not expected any decoupled geometry in this regime given the unbounded nature of the Hamiltonian in this sector. We leave this point to the discussion section.

Emergence of pinching orbifolds: Since pinching orbifolds allow to explore the physics near extremality,[13] we will parameterize the outer (r_+) and inner (r_-) horizons in (3.9) in terms of

$$r_\pm^2 = r_*^2 \pm \delta r_*^2, \tag{5.1}$$

in the limit $r_*, \delta r_* \to 0$. We define the non-BPS near-EVH near-horizon limit as

[13]This was shown in some detail in the Appendix of [13].

$$r = \epsilon \frac{(q_2 q_3)^{1/4}}{L} y, \qquad r_* = \epsilon \frac{(q_2 q_3)^{1/4}}{L} \rho_*,$$
$$\delta r_* = \epsilon \frac{(q_2 q_3)^{1/4}}{L} \delta\rho_*, \qquad t = -\frac{L}{(q_2 q_3)^{1/4}} \frac{\tau}{\epsilon},$$
$$\phi_1 = \frac{\varphi}{\epsilon}, \qquad \phi_i = \psi_i - \frac{\tilde{q}_i}{q_i} \frac{1}{(q_2 q_3)^{1/4}} \frac{\tau}{\epsilon}, \qquad i = 2, 3$$
$$q_1 = \epsilon^4 \hat{q}_1, \qquad \mu - \mu_c = \epsilon^2 \hat{M}, \qquad \epsilon \to 0, \tag{5.2}$$

keeping all parameters and coordinates in the right hand sides fixed. The resulting metric is [15]

$$ds^2 = \mu_1 ds^2(\ell_3) + \frac{L^2}{\mu_1} ds^2_{\mathcal{M}_7}. \tag{5.3}$$

This involves a 7d Euclidean compact metric

$$ds^2_{\mathcal{M}_7} = \frac{\sqrt{q_2 q_3}}{L^2} \mu_1^2 d\Omega_3^2 + \sqrt{q_2/q_3}(d\mu_2^2 + \mu_2^2 d\psi_2^2) + \sqrt{q_3/q_2}(d\mu_3^2 + \mu_3^2 d\psi_3^2), \tag{5.4}$$

and a 3d lorentzian locally AdS_3 metric

$$ds^2(\ell_3) = -\frac{(y^2 - y_+^2)(y^2 - y_-^2)}{y^2 \ell_3^2} d\tau^2 + \frac{\ell_3^2 y^2}{(y^2 - y_+^2)(y^2 - y_-^2)} dy^2 + y^2 \left(d\varphi + \frac{y_+ y_-}{y^2 \ell_3} d\tau \right)^2, \tag{5.5}$$

having radius ℓ_3

$$\ell_3^2 = \sqrt{q_2 q_3} \frac{L^2}{h^2}, \qquad h^2 \equiv L^2 + q_2 + q_3, \tag{5.6}$$

and a periodic φ satisfying $\varphi \sim \varphi + 2\pi\epsilon$.

The near-horizon geometry depends on two parameters $\hat{M}$ and $\hat{q}_1$. Depending on their values, the 3d geometry describes an extremal, nonextremal, massive, or massless BTZ geometry with outer and inner horizons $y_\pm^2 = \rho_*^2 \pm \delta\rho_*^2$ given by[14]

$$(y_+ \pm y_-)^2 = \frac{L^4}{h^2 \sqrt{q_2 q_3}} \left(\hat{M} \pm 2 \frac{h}{L^2} \sqrt{\hat{q}_1 q_2 q_3} \right). \tag{5.7}$$

It was shown in [15] that there exists a nontrivial consistent truncation of type IIB with a constant dilaton, metric and self-dual 5-form to six dimensions of the form

$$ds^2 = \mu_1 ds_6^2 + \frac{L^2}{\mu_1} (\sqrt{q_2/q_3}(d\mu_2^2 + \mu_2^2 d\psi_2^2) + \sqrt{q_3/q_2}(d\mu_3^2 + \mu_3^2 d\psi_3^2)). \tag{5.8}$$

Notice this is indeed of the form found above with $ds_6^2 = ds^2(\ell_3) + \sqrt{q_2 q_3}/L^2 d\Omega_3^2$, where the 6d part is a space of negative constant scalar curvature $R_6 = -\frac{6(q_2+q_3)}{L^2\sqrt{q_2 q_3}}$.

The linear ϵ periodicity in φ affects the ADM mass and angular momentum of the pinching BTZ geometries described above [46]

$$M_{\rm BTZ} = \frac{y_+^2 + y_-^2}{8\ell_3^2 G_N^{(3)}} = \frac{N^2\epsilon}{4} \frac{\hat{M}}{L^2 (q_2 q_3)^{1/4}},$$
$$J_{\rm BTZ} = \frac{y_+ y_-}{4\ell_3 G_N^{(3)}} \epsilon = \frac{N^2 \epsilon}{2} \sqrt{\frac{\hat{q}_1 q_2 q_3}{L^6}}, \tag{5.9}$$

where we used the value for the 3d Newton's constant $G_N^{(3)}$ [15]

$$\frac{1}{G_N^{(3)}} = \frac{L^3}{G_N^{(10)}} (q_2 q_3)^{3/4} (2\pi)^2 (\mathrm{vol} S^3) \int \mu_2 \mu_3 d\mu_2 d\mu_3 = 2N^2 \frac{(q_2 q_3)^{3/4}}{L^4}. \tag{5.10}$$

The Hawking temperature is finite and ϵ independent

$$T_{\rm BTZ} = \frac{(y_+^2 - y_-^2)}{2\pi\ell_3^2 y_+} = \frac{h^2 \delta\rho_*^2}{\pi L^2 \sqrt{q_2 q_3} \sqrt{\rho_*^2 + \delta\rho_*^2}}, \tag{5.11}$$

since it can be computed requiring the absence of conical singularities in its Euclidean continuation. Notice $T_{\rm BTZ}$ differs from the 5d black hole temperature T (3.6) by

$$T = \frac{(q_2 q_3)^{1/4}}{L} T_{\rm BTZ} \epsilon. \tag{5.12}$$

This relation is consistent with the scaling of the time coordinate t in the near horizon limit (5.2).

The null orbifold appearance: Following the discussion in [13], there should exist a second inequivalent near horizon limit when we restrict ourselves to deformations preserving extremality

$$y_+ = y_- \Longleftrightarrow \hat{M} = 2 \frac{h}{L^2} \sqrt{\hat{q}_1 q_2 q_3}. \tag{5.13}$$

Indeed, for this subset of excitations, we can modify the singular large gauge transformations appearing in (5.2) to

$$t = -\frac{L}{(q_2 q_3)^{1/4}} \frac{\tau}{\epsilon^2}, \qquad \phi_1 = \varphi - \frac{\tau}{\ell_3 \epsilon^2}, \tag{5.14}$$

$$\phi_i = \chi_i + \frac{\tilde{q}_i}{q_i^2 L^2} (q_2 q_3)^{1/4} y_+^2 \tau - \frac{\tilde{q}_i}{q_i} \frac{1}{(q_2 q_3)^{1/4}} \frac{\tau}{\epsilon^2}, \qquad i = 2, 3.$$

[14]The geometry is BTZ black hole if $(y_+ \pm y_-)^2 \geq 0$. If $(y_+ \pm y_-)^2 < 0$ we have a conic space and if $(y_+ + y_-)^2 > 0$, $(y_+ - y_-)^2 < 0$ we have a timelike naked singularity. The latter two are only possible when the original R-charged black hole violates the extremality bound.

Note that in the above expression $\tilde{q}_i = \sqrt{q_i(\mu + q_i)}$ has an expansion in powers of ϵ because $\mu = \mu_c + \epsilon^2 \hat{M}$. The resulting metric is as in (5.3), but there are some important differences:

(a) The 3d metric corresponds to the null self-dual orbifold

$$ds^2(\ell_3) = \ell_3^2 \frac{d\sigma^2}{4\sigma^2} + 2\sigma d\varphi \frac{d\tau}{\ell_3}, \tag{5.15}$$

where $\sigma = y^2 - y_+^2$.

(b) The null AdS_3 orbifold is nontrivially fibered over the 7d transverse space by replacing the $d\psi_i$ in (5.4) with $d\chi_i - A_i \sigma d\tau$, with $A_i = \frac{\sqrt{\mu_c q_i}}{q_i^2 L^2}(q_2 q_3)^{1/4}$. This turns on some constant electric fields in the transverse space.

(c) The periodicity in φ remains ϵ independent. Thus, this limit involves no pinching.

A. Nontrivial IR dynamics and scaling of N

In this section, we want to reexamine the existence of nontrivial dynamics in the IR limits taken in (5.2) and (5.14). Since the near horizon geometries so obtained are equivalent to the ones studied for (near) extremal BTZ black holes [12,13], the viewpoint we adopt here is to assume the existence of a 2d CFT, that we shall refer to as "parent" CFT and which will capture some of the IR dynamics. The different large gauge transformations involved in (5.2) and (5.14) correspond to focusing on different sectors in the same theory. Thus, different subsectors of $\mathcal{N} = 4$ SYM share some features with certain subsectors of these parent CFTs. Whenever the pinching AdS_3 emerges, both chiral sectors of this CFT are decoupled [13], whereas when φ remains 2π periodic, the time coordinate scaling (5.14) is appropriately interpreted as an infinite boost [47], allowing us to interpret the near horizon as describing the DLCQ limit of the original nonchiral 2d CFT [12,47]. The null self-dual orbifold corresponds to the $p^+ = 0$ sector of the latter [13,48].

Consider the non-BPS IR limit (5.2) first. From a gravitational point of view, since (5.2) does not involve any focusing in the transverse dimensions, the entropy evaluated in the near horizon pinching geometry, as expected, equals the one of the original black hole

$$S = \pi \frac{\sqrt{q_2 q_3}}{L^2} \frac{r_h}{L} N^2 = S_{3d} = \frac{2\pi \epsilon y_+}{4G_3}$$
$$= \pi \frac{(q_2 q_3)^{3/4}}{L^3} \frac{y_+}{L} N^2 \epsilon. \tag{5.16}$$

In the original black hole geometry, the linear ϵ scaling is due to the smallness of the horizon, i.e. $r_h \sim \epsilon$. In the 3d near horizon geometry, it is due to the pinching. Thus, the entropy vanishes in the limit $\epsilon \to 0$ while keeping N fixed.

It is standard to reproduce the gravitational entropy of an AdS_3 throat using Cardy's formula [5]. In the presence of a nontrivial pinching, this may be a bit more subtle. When computing the mass and angular momentum of the 3d pinching geometry (5.9), we took into account the nontrivial periodicity of the S^1 circle. Using the same spacetime perspective, the standard Brown-Henneaux central charge [4] will also acquire such a linear ϵ dependence

$$c_{AdS_3} = \frac{3\ell_3}{2G_3}\epsilon = \frac{3\mu_c}{Lh} N^2 \epsilon. \tag{5.17}$$

This agrees with the *spacetime* CFT approach described in [49], when describing AdS_3 orbifolds. In this approach, both the central charge and the excitations $L_0 - c/24$ and $\bar{L}_0 - c/24$ scale like $N^2\epsilon$

$$L_0 + \bar{L}_0 - \frac{c}{12} = \frac{N^2\epsilon}{4}\frac{\hat{M}}{Lh}, \qquad L_0 - \bar{L}_0 = \frac{N^2\epsilon}{2}\sqrt{\frac{\hat{q}_1 q_2 q_3}{L^6}}. \tag{5.18}$$

Having assumed the existence of a dual 2d parent CFT, the near horizon limit (5.2) corresponds to an IR limit of such theory in which both chiral sectors are decoupled and we are left with no dynamics [13]. This is a direct consequence of the pinching appearing in the near horizon geometry (5.3). This conclusion is true while holding Newton's constant fixed, but if we allow the latter to scale to zero as the near horizon (IR) limit is taken, one can keep the central charge and the entropy finite [13]. This is explicit in our setup since the rank of the SU(N) gauge group in the original $\mathcal{N} = 4$ SYM controls the 3d Newton's constant (5.10). In other words, we can scale $N \to \infty$, keeping L *fixed* so that $N^2\epsilon$ remains *finite*

$$N \to \infty \quad \text{with} \quad L, N^2\epsilon \text{ fixed} \tag{5.19}$$

It is manifest that in such a double scaling limit all relevant thermodynamical and conformal field theory quantities will remain *finite*. (Note that since the volume of the $\mathcal{M}_7$ manifold remains finite, both 3d and 10d Newton constants scale to zero in the same way, as N^{-2}.)

We can now consider the same EVH triple scaling limit as in [29]:

$$A, T, G_N \to 0, \qquad \frac{A}{G_N}, \frac{A}{T} \text{ finite}, \tag{5.20}$$

where A/G_N is the entropy of the EVH black hole (reproduced by Cardy formula of the dual 2d CFT) and the ratio A/T (up to numerical coefficients) is equal to the central charge of the dual CFT.

The spacetime CFT perspective used in (5.17) and (5.18) can now be understood as the orbifold projection of the dual parent 2d CFT [13,49]. The pinching orbifold can then be understood as an *ensemble* at temperature $T \sim \epsilon$ describing excitations $L_0 - c/24$, $\bar{L}_0 - c/24 \sim N^2\epsilon^2$ in this CFT theory with central charge

$$c_{CFT} = \frac{3\ell_3}{2G_3} = \frac{3\mu_c}{Lh} N^2. \tag{5.21}$$

We will comment on the relation between this parent CFT and the one emerging in Kerr/CFT in Sec. VII where we also remark on the dual 2d (parent) CFT description of the null orbifold obtained through (5.14).

B. $\mathcal{N}=4$ SYM interpretation

Given the dictionary between bulk charges and $\mathcal{N}=4$ SYM quantum numbers, we can identify the sector of the $\mathcal{N}=4$ Hilbert space being explored in the near horizon limit (5.2)

$$\Delta = \frac{N^2}{2L^2}\left(\frac{3}{2}\mu_c + q_2 + q_3\right) + \frac{3\hat{M}}{4L^2}(N\epsilon)^2, \tag{5.22a}$$

$$J_i = \frac{N^2}{2L^2}\sqrt{q_i(\mu_c + q_i)} + \frac{\hat{M}}{4L^2}\sqrt{\frac{q_i}{\mu_c + q_i}}(N\epsilon)^2, \qquad i = 2, 3 \tag{5.22b}$$

$$J_1 = \frac{\sqrt{\mu_c \hat{q}_1}}{2L^2}(N\epsilon)^2. \tag{5.22c}$$

Any bulk probe field has a Fourier expansion in terms of both the UV and IR isometries. In the UV theory, the conformal dimension $\Delta_{\rm UV}$ and R-charges J_a $a=1,2,3$ will naturally be related to the eigenvalues of the vector fields

$$\Delta_{\rm UV} = iL\frac{\partial}{\partial t}, \qquad J_a = -i\frac{\partial}{\partial \phi_a}, \qquad a = 1, 2, 3. \tag{5.23}$$

Similarly, in the IR theory, there are natural vector fields to use, related to the UV ones through the nontrivial singular large gauge transformations included in (5.2), giving rise to the IR eigenvalues $\Delta_{\rm IR}$ and $\mathcal{J}$

$$\Delta_{\rm IR} \equiv -i\ell_3\frac{\partial}{\partial\tau} = -\frac{\ell_3}{(q_2q_3)^{1/4}\epsilon}\left(\Delta - \frac{2L^2}{N^2}\sum_{i=2,3}\frac{J_i^2}{q_i}\right) \tag{5.24a}$$

$$\mathcal{J} \equiv -i\frac{\partial}{\partial\varphi} = \frac{J_1}{\epsilon}. \tag{5.24b}$$

$\mathcal{E}$ and $\mathcal{J}$ may directly be related to the 2d CFT charges.[15] Explicitly, one can see that

$$\Delta_{\rm IR} = \mathcal{E} - \mathcal{E}_c = L_0 + \bar{L}_0 - \frac{c}{12}, \qquad \mathcal{J} = L_0 - \bar{L}_0, \tag{5.25}$$

where $\mathcal{E}_c$ is computed for $\Delta = \Delta_c$ and $c = c_{{\rm AdS}_3}$ is given in (5.17) and L_0 and $\bar{L}_0$ are given in (5.18). This precise matching is intriguing, as we are working with a sector in the $\mathcal{N}=4$ SYM with large scaling dimensions which is far from BPS and one would not expect a protection due to supersymmetry. As another intriguing fact, although it is not clear that the extremality bound in gravity has a specific meaning in the $\mathcal{N}=4$ SYM theory, it seems that "extremality" also brings some sort of protected-ness and that $\Delta = \Delta(\mu=\mu_c)$, $J_i = J_i(\mu=\mu_c)$ provides a well-defined ground state for the BMN-type sector and possibly for a decoupled theory. This BMN-type sector, will hence contain operators with Δ, J_2, J_3 of order N^2, while certain combination of these (given in (5.24)) remain finite [15].

Even though it would be tempting to interpret the central charge (5.17) in terms of intersecting giant gravitons, the microscopic understanding of the non-BPS regime is not established. In particular the $1/h$ factor does not have a clear origin in that framework.

The null orbifold case: We can repeat the procedure for the second limit (5.14) giving rise to the null orbifold. The condition $y_+ = y_-$ forces

$$\hat{M} = 2\frac{h}{L^2}\sqrt{\hat{q}_1 q_2 q_3}.$$

Using (5.22a) and (5.14) one can show that

$$i\ell_3\frac{\partial}{\partial\tau} = \frac{N^2 q_2 q_3}{4L^3 h\epsilon^2} + \frac{N^2\sqrt{\hat{q}_1 q_2 q_3}}{Lh^2}\left(\frac{q_2+q_3}{2L^2}\right) + \mathcal{O}(N^2\epsilon^2).$$

The first term in the above is basically what was called $\mathcal{E}_c$ and the term proportional to N^2 is measuring being out of EVH point. One would have naively expected the N^2 piece not to be present. These terms may be related to the energy of electric fields discussed in item 2. below (5.15).

VI. PLANAR BLACK HOLES AS INFINITE CHARGE BLACK HOLES

As discussed in Sec. II, a second possibility to avoid the conclusion that extremal black holes in AdS have a dual CFT with no dynamics is to consider black holes with noncompact horizons. In the spirit of this note, we want to remind the readers of the well known result that such black holes, the so called planar AdS black holes, can be viewed as a large N limit, i.e. an infinite charge limit, of global R-charged AdS black holes.

To illustrate this relation, consider an R-charged black hole in global ${\rm AdS}_{d+1}$ characterized by the radius R of the $(d-1)$-sphere, its mass M and its R-charge J. Conformal invariance implies the equivalence between systems with parameters

$$(M, J, R) \sim (\lambda^{-1}M, J, \lambda R) \qquad \text{(conformal invariance)}$$

Since planar AdS black holes have infinite M and J but their mass and charge densities are finite, one way to derive them from their global versions is to combine the rescaling

[15]The second equality in (5.24a) may be understood in an intuitive way. The BPS giants are spherical branes moving with speed of light on a circle in the S^5. Similarly the near extremal (far from BPS case) could be interpreted as massive (topologically) spherical 3-branes which are behaving like *non-relativistic* objects which are rotating with angular momentum J_i over circles with radii R_i, $R_i^2 = \frac{L^2}{(q_2q_3)^{1/2}}q_i$ (5.4). Therefore, the kinetic energy of this rotating branes is proportional to $\sum J_i^2/q_i$.

$$M \to \lambda^d M_0, \qquad J \to \lambda^{d-1} J_0 \qquad \text{(charge rescaling)}$$

with a conformal transformation in the limit $\lambda \to \infty$

$$(M, J, R) \Rightarrow (\lambda^{d-1} M_0, \lambda^{d-1} J_0, \lambda R) \qquad \lambda \to \infty. \quad (6.1)$$

In this way, charges go to infinity as the volume of the boundary theory, keeping their densities (M_0, J_0) finite, without modifying the AdS curvature radius L.

One consequence of this procedure is to relate correlation functions of operators $\mathcal{O}$ in a planar AdS black hole background to correlation functions in the original global AdS black hole

$$\langle \mathcal{O}(x)\mathcal{O}(y)\rangle^{\text{planar}}_{M_0,Q_0} \equiv \lim_{\lambda\to\infty} \lambda^{-2\Delta} \langle \mathcal{O}(\lambda^{-1}x)\mathcal{O}(\lambda^{-1}y)\rangle_{\lambda^d M_0, \lambda^{d-1} Q_0} \quad (6.2)$$

where Δ is the conformal weight of $\mathcal{O}$. Notice how the boundary points on the $(d-1)$-sphere where operators $\mathcal{O}$ are inserted get rescaled, due to conformal invariance, as one increases the mass and R-charge of the black hole. There are similar expressions for higher n-point functions.

Planar RN AdS black holes: The procedure reviewed above was already applied for the R-charge AdS_5 black holes (3.1) in [37,50]. Focusing on the equal R-charge black hole, i.e. $q_i = q$ $i = 1, 2, 3$, the combined charge rescaling and conformal transformation amount to

$$\mu \to \lambda^4 \mu_0, \qquad q \to \lambda^2 q_0, \qquad r \to \lambda r, \qquad t \to \lambda^{-1} t \quad (6.3)$$

For large λ, this will rescale the ADM mass by λ^4 and the R-charge $\tilde{q}$ by λ^3, as desired. The net result of the transformation is to:

(1) replace $f(r)$ in (3.2) with

$$f = -\frac{\mu_0}{r^2} + \frac{r^2}{L^2}\left(1 + \frac{q_0}{r^2}\right)^3, \quad (6.4)$$

keeping H_i and Δ unmodified.

(2) replace the physical R-charge $\tilde{q} = \sqrt{\mu_0 q_0}$.

(3) rescale the 3-sphere metric as $d\Omega_3^2 \to \lambda^2 d\Omega_3^2$, so that in the $\lambda \to \infty$, it gets replaced by $\mathbb{R}^3$, making explicit the noncompactness of both the boundary theory and the black hole horizon.

The resulting metric corresponds to the planar RN AdS black hole. Its low temperature regime and its $AdS_2 \times \mathbb{R}^3 \times S^5$ near horizon were studied in [25] in connection with quantum criticality and the emergence of IR CFTs. Their work gives explicit evidence for the existence of nontrivial dynamics in these setups.

Identifying the AdS_2 *isometries in the UV theory?* Planar black holes arise as in the large charge limit of ordinary black holes. In addition, extremal planar black holes are presumably unstable against backreaction as mentioned above. It would therefore be sufficient to find approximate $SL(2)$ generators in the UV theory. In fact, it would be sufficient to find UV generators L_k, which obey

$$P_{\text{near-extremal}}([L_k, L_l] - (k-l)L_{k+l}) = \mathcal{O}(N^{-1}, Q^{-1}) \quad (6.5)$$

where Q represents the charges of the extremal black hole, and the projection on the left-hand side is a projection onto states in Hilbert space which are very close to extremality. The near-horizon limit is then implemented by this projection operator, in the planar limit the terms of order $1/Q$ can be neglected, and in the supergravity limit the terms of order $1/N$ can be neglected. Thus, if (6.5) is satisfied, then they are candidate near-horizon isometries of planar black holes in the supergravity description. We leave a precise description of these isometries to future work.

VII. COMPARISON WITH EXTREMAL BH/CFT

In this section, we investigate the behavior of the CFT data, i.e. central charges and Frolov-Thorne temperatures, provided by the Kerr/CFT correspondence [7], and its extremal black hole extension [8], when the extremal horizon size vanishes. When the latter occurs, the AdS_2 near horizon responsible for the boundary conditions proposed in [7,8] may disappear, giving rise to the local AdS_3 throats discussed in this note.

The extremal black hole/CFT correspondence [8] applies to near horizon geometries of the form

$$ds^2 = A\left(-\rho^2 dt^2 + \frac{d\rho^2}{\rho^2}\right) + \sum_{\alpha=1}^{n-1} F_\alpha dy_\alpha^2 + \sum_{i,j=1}^{n-1+\epsilon} \tilde{g}_{ij}\tilde{e}_i\tilde{e}_j,$$
$$\tilde{e}_i = d\phi_i + k_i \rho dt. \quad (7.1)$$

When certain boundary conditions are applied to these near horizon geometries, one discovers that the asymptotic symmetry group includes a single Virasoro algebra extension for each of the compact U(1) isometries ∂_{ϕ_i} [51]. Their central charges equal [51]

$$c_i = \frac{6 k_i S_{\text{bh}}}{\pi}, \quad (7.2)$$

where S_{bh} stands for the entropy of the original *finite* extremal black hole. The application of Cardy's formula

$$S = 2\pi\sqrt{\frac{c_i}{6}\left(L_0^i - \frac{c_i}{24}\right)} = \frac{\pi^2}{3} c_i T_i = S_{\text{bh}}, \quad (7.3)$$

always reproduces S_{bh} since the CFT temperature T_i equals

$$k_i = \frac{1}{2\pi T_i}, \qquad T_i = -\frac{T_H'^0}{\Omega_i'^0}, \quad \text{with}$$
$$T_H'^0 \equiv \frac{\partial T_{\text{H}}}{\partial r_+}\bigg|_{r_+=r_0}, \qquad \Omega_i'^0 \equiv \frac{\partial \Omega_i}{\partial r_+}\bigg|_{r_+=r_0}, \quad (7.4)$$

where $T_H(r_+)$ and $\Omega_i(r_+)$ are the temperature and angular velocities on the outer horizon r_+ and r_0 stands for its extremal value.

Finite extremal R-charged AdS_5 black holes fit this discussion. Since these have three U(1) isometries describing independent rotations in S^5, there should exist three inequivalent chiral CFTs reproducing the black hole entropy. This system was analyzed in [52], where the following Frolov-Thorne temperatures were computed

$$T_i = \frac{(r_0^2 + q_i)^2(q_1q_2q_3 + r_0^6)}{\pi\tilde{q}_i L r_0^7\sqrt{H_1H_2H_3(r_0)}}. \tag{7.5}$$

These fix the central charges through (7.2).

Because of the analysis performed in previous sections, we will assume the bulk entropy scales like ϵ^γ with $\epsilon \to 0$. Given the emergence of local AdS_3 throats in these situations, we are interested in matching the AdS_3/CFT_2 dictionary to the limiting values of the Kerr/CFT predictions above and, whenever possible, interpret and justify the latter results in terms of the former. One can distinguish three different physical cases consistent with this entropy behavior and (7.3):

(1) *Finite* central charge c_i, but vanishing level $L_0^i - \frac{c_i}{24} \sim \epsilon^{2\gamma}$. From an AdS_3 perspective, this would correspond to keeping a *finite* gap in the CFT, but sending the level to zero (vacuum). Notice the CFT temperature scales like the entropy $T_i \sim \epsilon^\gamma$.

(2) *Finite* level $L_0 - \frac{c_i}{24}$, but central charge scaling like $c_i \sim \epsilon^{2\gamma}$. From an AdS_3 perspective, this generates an infinite gap in the CFT, keeping the level finite. Thus, the CFT temperature scales inversely to the entropy, $T_i \sim \epsilon^{-\gamma}$.

(3) *Vanishing* central charge and level according to

$$c_i \sim \epsilon^\alpha, \qquad L_0^i - \frac{c_i}{24} \sim \epsilon^\beta \qquad \alpha + \beta = \gamma.$$

If $\alpha, \beta > 0$, the system is pushed to its vacuum while generating an infinite gap. The CFT temperature scales like $T_i \sim \epsilon^{\gamma/2-\alpha}$ and remains finite when $\alpha = \beta$.

Here we identify each of the regimes described above with the different extremal vanishing horizon limits studied in previous sections.

A. Near-BPS discussion

The purpose of this section is to describe the limiting behavior of the three CFTs that reproduce the entropy for extremal finite R-charged AdS_5 black holes in the near-BPS regime (4.1) and to identify which one matches, if any, with the 2d CFT that one may associate with the near horizon AdS_3 geometry (4.2). Geometrically, it is expected the latter should correspond to the chiral CFT based on the Virasoro extension of the isometry direction along which giant gravitons intersect. Thus, one expects c_1 and T_1 to have different scaling behavior from the two remaining Kerr/CFT descriptions.

Let us analyze the CFT temperatures first. In the near-BPS limit (4.1), these behave like

$$T_1 \to \frac{\sqrt{\hat{q}_2\hat{q}_3}}{\pi L}\frac{(\hat{r}_+^2 + \hat{q}_1)^{3/2}}{\hat{r}_+^4}\sqrt{\frac{\hat{q}_1}{\hat{\mu} + \hat{q}_1}},$$
$$T_i \sim \frac{1}{\epsilon} \qquad i = 2, 3 \tag{7.6}$$

Since the black hole entropy scales like $S \sim (N\epsilon)^2$, the central charges also have two distinctive behaviors

$$c_1 \sim (N\epsilon)^2, \qquad c_i \sim \frac{(N\epsilon^2)^2}{\epsilon} \qquad i = 2, 3 \tag{7.7}$$

For finite N, all central charges scale to zero, consistent with the vanishing of the bulk entropy and the dilute giant graviton approximation in which the number of physical degrees of freedom is being scaled to zero.

Consider the regime $N\epsilon^2 \sim 1$ when the bulk *entropy density* is finite. The analogue notion in the CFT is carried by a central charge density. Using the volume of the transverse space computed in Sec. IV, $c_1/\epsilon^{-2} \sim (N\epsilon^2)^2$ remains *finite*, while the other two central charge densities $c_i/\epsilon^{-2} \sim \epsilon \to 0$ would still indicate a breaking down of this effective description.[16] This confirms our expectation that the surviving CFT is the one living on the intersection of giant gravitons.

Let us explore more thoroughly the Kerr/CFT description with CFT data (c_1, T_1). To compare with the 2d CFT dual to the near horizon AdS_3 geometry (4.2), we must satisfy the extremality condition, i.e. $y_+ = y_-$ or equivalently, $\hat{M} = 2\sqrt{\hat{\mu}_c\hat{q}_1}$. Standard AdS_3/CFT_2 tells us

$$T_L = \frac{y_+ + y_-}{2\pi\ell_3} = \frac{y_+}{\pi\ell_3}, \qquad T_R = \frac{y_+ - y_-}{2\pi\ell_3} = 0. \tag{7.8}$$

Thus, we want to compare T_1 with the chiral temperature T_L. Using (4.5) and (4.6), we learn

$$y_+^2 = \frac{L^2}{\sqrt{\hat{q}_2\hat{q}_3}}\sqrt{\hat{q}_1(\hat{M} + \hat{\mu}_c + \hat{q}_1)} \Rightarrow \hat{r}_+^2 = \sqrt{\hat{\mu}_c\hat{q}_1}.$$

Thus, both temperatures are equal

$$T_L = \frac{y_+}{\pi(\hat{q}_2\hat{q}_3)^{1/4}} = T_1.$$

Furthermore, using (7.2), the corresponding Kerr/CFT central charge equals

$$c_1 = \frac{3}{\pi^2}\frac{S}{T_1} = 3N^2\frac{q_2}{L^2}\frac{q_3}{L^2} = 3N_2N_3, \tag{7.9}$$

where we used the quantization conditions (3.5). The latter agrees with the total number of giant intersections, independently of whether N is scaled or not. Notice how a proper microscopic understanding of the system quantizes this specific CFT central charge reported in [52].

[16]Notice the $N\epsilon \sim 1$ scaling keeping the entropy finite keeps c_1 finite, whereas $c_i \sim \epsilon \to 0$. Thus the breaking down of these two CFTs also remains for this scaling limit.

Comparison with the near horizon analysis: As stressed in Sec. IV, our near horizon limit (4.1) focused on a strip of the transverse 5-sphere. Consequently, its Bekenstein-Hawking entropy did not match the full black hole entropy (4.9). This suggests that if one uses the standard dictionary between AdS_3 bulk data and 2d dual CFT quantities, the central charge so obtained will only capture the number of intersecting giant gravitons present in the focused strip. Indeed, using the Brown-Henneaux central charge [4] and the 3d Newton's constant (4.11), we obtain

$$c = \bar{c} = \frac{3\ell_3}{2G_N^{(3)}} = 3N_2N_3(8\mu_2^0\mu_3^0 g_{\mathcal{M}_4}). \tag{7.10}$$

As usual, Cardy's formula is consistent with Bekenstein-Hawking (4.9),

$$S_{\text{Cardy}} = 2\pi\sqrt{\frac{c(L_0 - c/24)}{6}} + 2\pi\sqrt{\frac{\bar{c}(\bar{L}_0 - \bar{c}/24)}{6}} = S_{3\text{d}}, \tag{7.11}$$

using $L_0 - \frac{c}{24} = (M_{\text{BTZ}}\ell_3 + J_{\text{BTZ}})/2$ and $\bar{L}_0 - \frac{\bar{c}}{24} = (M_{\text{BTZ}}\ell_3 - J_{\text{BTZ}})/2$, with M_{BTZ}, J_{BTZ} given in (4.7).

We do not understand either in field theory or in gravitational terms, what the proper connection is between the description above and the one emerging in Kerr/CFT. We do want to emphasize that our approach did insist on taking a near horizon limit to study the IR properties of the system and keeping the latter on-shell. In this near-BPS regime, this forced us to lose part of the degrees of freedom responsible for the full black hole entropy. Both the Kerr/CFT central charge (7.9) and our near horizon analysis do suggest the potential existence of a 2d CFT which is *not* intrinsically localized in the horizon. Similar observations have been made using low energy probes in (non-) extremal black holes [41,42], without explicitly relying on the near horizon geometry.[17]

It would be very interesting to provide *any* technical evidence confirming the structures uncover in gravity, along the lines of [44], by which the sector (4.16) in $\mathcal{N} = 4$ SYM may be equivalently described by a (non-) chiral CFT whose degrees of freedom should be the open strings stretched between giant gravitons.

B. Non-BPS extremal discussion

The purpose of this section is to extend the previous discussion to the near extremal non-BPS limit (7.5). Consider first the limiting behavior of the three Frolov-Thorne temperatures (5.2)

$$T_1 = \frac{\epsilon}{\pi}\frac{\sqrt{\hat{q}_1}}{\hat{r}_+}, \qquad T_i \sim \frac{1}{\epsilon}, \qquad i = 2, 3 \tag{7.12}$$

Since the black hole entropy scales like $S \sim N^2\epsilon$, central charges also have two distinctive behaviors:

$$c_1 \sim N^2, \qquad c_i \sim (N\epsilon)^2 \qquad i = 2, 3. \tag{7.13}$$

For finite N, c_i $i = 2, 3$ scale to zero, indicating the physical degrees of freedom scale to zero, whereas c_1 remains finite.

When $N^2\epsilon \sim 1$, to keep the entropy finite, the same conclusion holds for c_i: $c_i \sim \epsilon \to 0$, whereas c_1 diverges. This suggests two of the Kerr/CFT descriptions are breaking down, as expected, whereas the third one corresponds to a double scaling limit in which the CFT gap is scaled to zero, while one scales the vacuum energy $L_0 - c_1/24 \sim \epsilon$ to zero. This is precisely the setup described in [13], but now embedded in the AdS/CFT correspondence and explicitly implemented by an $N \to \infty$ limit.

Let us explore the relevant Kerr/CFT description more closely. Notice T_1 is related to the 2d chiral CFT temperature T_L in (7.8) by

$$T_1 = \frac{\epsilon}{\pi}\frac{h\hat{r}_+}{\sqrt{q_2q_3}} = \epsilon T_L = \epsilon\frac{y_+}{\pi\ell_3}, \tag{7.14}$$

where we used $r_+ = \epsilon\hat{r}_+$ and the identity

$$\hat{r}_+ = \frac{(q_2q_3)^{1/4}}{L}y_+ \quad \text{with} \quad y_+^2 = L^2\frac{\sqrt{\hat{q}_1}}{h}, \tag{7.15}$$

encoding the extremality condition $y_+ = y_-$, equivalent from (5.7), to $\hat{M} = 2\frac{h}{L^2}\sqrt{\hat{q}_1q_2q_3}$.

Comparing the central charge c_1 with c_{AdS_3} computed in the spacetime superconformal algebra associated to the near horizon pinching geometry (5.3), one finds[18]

$$c_1 = \frac{3\pi}{2G_5}\frac{q_2q_3}{h} = \frac{c_{\text{AdS}_3}}{\epsilon} = c_{\text{CFT}}. \tag{7.16}$$

Thus, the Kerr/CFT central charge equals the central charge of the parent 2d CFT in (5.21), that is the Brown-Henneaux central charge if we did not have the pinching.

Interpretation: The Kerr/CFT central charge c_1 is *finite* before scaling Newton's constant. Because of the regime of charges tested in the near-EVH limit, there is an AdS_3 throat emerging in the near horizon gravitational description that allows us to identify this CFT as the parent 2d CFT alluded to in Sec. VA. The pinching orbifold is a *thermal* state in this parent CFT exploring very low energies, $L_0 - c/24 \to N^2\epsilon^2$ at low temperatures $T_1 \sim \epsilon$. The spacetime conformal algebra perspective discussed in [49] and mentioned in section VA corresponds to the "long string sector" of this parent CFT.

[17] If we start with a generic BPS or extremal black hole, take the near horizon limit first (as is done in [8,51]) and then take the nearly vanishing horizon limit, as we have done in this section, we obtain a different geometry than when we first take the near-EVH black hole limit and then take the near horizon limit, as we did in Secs. IV and VI. Nonetheless, despite of having different geometries, as we have shown the entropies obtained in these two cases are equal.

[18] One must reinsert $G_5 = \pi L^3/(2N^2)$ into the expressions written in [52], where it was set to one, to reproduce this result.

This picture is basically the same as the one advocated in [13] for a similar situation in a much simpler setting of EVH BTZ black hole (i.e. massless BTZ): Taking the near horizon limit first and then going to near-EVH region, if we did not scale the central charge (or N), one would have ended up with the null self-dual orbifold. However, if we took the near-EVH limit first and then focused on the near horizon we end up with a pinching AdS_3 orbifold. One may start with some (parent) CFT with finite central charge and explores excitations with very low energy. Taking the $N \to \infty$ limit enables us to keep some nontrivial dynamics by considering the double scaling limit $L_0 - c/24 \to 0$ and $c \to \infty$, keeping the entropy *finite*.

Comment on the null self-dual orbifold: In (5.14), we identified a different near horizon limit giving rise to a different local AdS_3 throat, corresponding to the null self-dual orbifold. It is easy to see the central charge describing this throat is the same as in the previous discussion. The difference in scaling in the time coordinate and the absence of pinching are consistent with the chiral nature of the dual CFT description,[19] which is identified with the one emerging in the limiting Kerr/CFT description. In this case, though, the absence of pinching suggests the surviving chiral sector is *not* decoupled. It is worth stressing the appearance of nontrivial gauge fields in the transverse dimensions. Note also the discussion in footnote [17]. These are reminiscent of the deformations recently discussed in [53]. It would be interesting to understand the physics of these.

VIII. SUMMARY AND OUTLOOK

In this work we continued studying aspects Extremal Vanishing Horizon (EVH) black holes in the family of static R-charged AdS_5 black holes, envisioning that a generic black hole in this family can be understood as excitations above the EVH black hole. Generic (non-BPS) EVH black holes are defined as black holes with A, T, $G_N \to 0$ while the ratios A/T and A/G_N remain finite, where A is the horizon area, T is the Hawking temperature and G_N is the Newton constant (BPS EVH black holes have some other subtleties we discussed in detail, see also below). We showed that in the near horizon limit of a generic EVH black hole one obtains an approximate AdS_3 throat. This AdS_3 throat for a near-EVH black hole turns to a BTZ black hole. In the case of EVH KK black hole it was shown that the near horizon limit is indeed a decoupling limit [29]. Unfortunately, we have not shown that the near horizon limit for the case of R-charged EVH black holes is a decoupling limit.

We discussed that R-charged AdS_5 EVH black holes can be BPS or non-BPS and that the near horizon limit is only well-defined in the 10d uplift of the black hole. In the case of BPS EVH black holes taking the near horizon limit we were forced to also blow up a four dimensional part $\mathcal{M}_4$ of the 10d geometry, and hence it is appropriate to consider "density" of physical quantities over $\mathcal{M}_4$ and demand these densities to remain finite in the near horizon limit. In the non-BPS case the 4d part of the geometry remains of finite volume while the AdS_3 throat becomes a pinching orbifold of AdS_3, a feature seemingly generic to all non-BPS EVH black holes. We discussed the possibility to resolve the pinching orbifold using large N and large central charge limits.

The appearance of an approximate near horizon AdS_3 throat motivated the EVH/CFT proposal [29]: near-EVH black holes or low energy excitations around an EVH black hole is described by a subsector of a 2d CFT. Moreover, when dealing with asymptotic AdS_5 black holes, there is also a UV dual CFT ($\mathcal{N} = 4$ SYM in this case). Based on the gravity picture we then proposed a relation between the IR and UV dual CFTs, in which the IR CFT is related to a BMN-like sector of the UV CFT. Exploring and establishing this proposal further is an open interesting question. In particular, in the non-BPS case, the IR physics is not governed by ground states, and it is therefore difficult to see how one could obtain an IR Hamiltonian which is not unbounded from below.

All emergent IR CFTs, whether related to AdS_3 or AdS_2, share various common features. One such feature is that the diffeomorphisms and associated Virasoro algebras that emerge involve in some nontrivial way an S^1, which can be either an S^1 in space on which the field theory lives (for rotating black holes) or an internal S^1 (for R-charged black holes). Understanding the relevant Virasoro algebras in the latter case requires one to construct "diffeomorphisms in R-charge space" directly in the field theory. Though a difficult problem, finding these could provide a key towards unraveling the way in which conformal symmetry emerges in field theory. To give some idea of the sort of structure we might be looking for, consider a 2d CFT with $N = (2, 2)$ supersymmetry. Such theories have a spectral flow symmetry, and let us denote the operator which spectral flows by n units (so it maps the NS and R sector into themselves) by S_n. If we bosonize the U(1) current in the $\mathcal{N} = (2, 2)$ algebra and call the resulting boson χ, then S_n is crudely speaking something like multiplication by $e^{in\chi}$. Since the zero modes of the current J_0 is approximately $J_0 \sim \partial_\chi$, the operators $d_n \equiv S_n J_0$ resemble standard diff S^1 generators in the χ-direction. Indeed, one finds the Virasoro algebra $[d_n, d_m] = \frac{c}{3}(m - n)d_{m+n}$. We leave a further examination of these operators, and possible generalizations to other field theories to future work.

We also discussed how the EVH/CFT and Kerr/CFT proposals are related to each other along the lines of discussions in [12,13,32–34]: The chiral CFT appearing in Kerr/CFT in the near-EVH locus in the parameter space corresponds to the DLCQ of the 2d CFT whose pinching

[19]The latter may be understood as the DLCQ limit of a non-chiral 2d CFT [12,47].

orbifold limit appears in the EVH/CFT. This, together with our discussions here, can potentially be used to identify the microscopic degrees of freedom of the dual 2d CFT and it would be interesting to pursue this further.

The EVH black holes are not limited to static ones and can be stationary, e.g. the one considered in [29]. Within the class of asymptotic AdS_5 black holes we have a more general family of EVH black holes which involve rotation as well as R-charge. This class of charged-rotating AdS_5 EVH black holes will be studied in a future publication [54].

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APPENDIX: NEAR-HORIZON LIMITS OF SINGULAR MYERS-PERRY BLACK HOLES

As mentioned the appearance of AdS_3 factors are not limited to asymptotic AdS_5 R-charged black holes and seems to be generic to any EVH black hole. One such example was 4d KK black hole discussed in [29]. Here we present another class which generalizes the Bardeen-Horowitz 5d Kerr EVH black hole [28]. Let us start with revisiting the case of [28]. Their starting point is the 5d Myers-Perry black hole [21] which generically come with angular momenta. These black holes are specified by three parameters, ADM mass m and two angular momentum parameters a, b. We take the extremal limit for which $m = m(a, b)$, and then send one of the two angular momenta, say the b parameter, to zero in order to obtain a vanishing horizon area, singular black hole. (In general the EVH black holes are naked singularities.) The resulting metric reads

$$ds^2 = -d\tilde{t}^2 + \frac{a^2}{\rho^2}(d\tilde{t} - a\sin^2\theta d\tilde{\phi})^2 + (\tilde{r}^2 + a^2)\sin^2\theta d\tilde{\phi}^2 + \tilde{r}^2\cos^2\theta d\tilde{\psi}^2 + \rho^2 d\theta^2 + \frac{\rho^2}{\tilde{r}^2} d\tilde{r}^2, \tag{A1}$$

where a parameterizes the remaining angular momentum, and $\rho^2 = \tilde{r}^2 + a^2\cos^2\theta$. The near-horizon limit is obtained by

$$\tilde{r} = \epsilon r, \qquad \tilde{t} = t/\epsilon, \qquad \tilde{\phi} = \phi + \tilde{t}/a, \qquad \tilde{\psi} = \psi/\epsilon, \tag{A2}$$

and taking $\epsilon \to 0$. The result is [28]

$$ds^2 = \cos^2\theta\left[-\frac{r^2}{a^2}dt^2 + \frac{a^2}{r^2}dr^2 + r^2 d\psi^2\right] + a^2\left[\cos^2\theta d\theta^2 + \frac{\sin^2\theta}{\cos^2\theta}d\phi^2\right]. \tag{A3}$$

This has the structure of a warped AdS_3, but the AdS_3 collapses to zero size at $\theta = \pi/2$. In addition, the rescaling of $\tilde{\psi}$ implies that ψ has periodicity $2\pi\epsilon$, in other words the ψ circle shrinks to zero size in the $\epsilon \to 0$ limit, which is the "vanishing periodicity" pathology or pinching AdS_3 orbifold problem which seems to be generic behavior for near-horizon limit of all *non-BPS EVH black holes*.

One may extend the above limit to near-EVH 5d Kerr by "perturbative addition" of b, *i.e.* by allowing nonzero, but small b parameter and small deviation of m parameter from extremality. It is straightforward to show that with appropriate scaling of b and out-of-extremality (when they scale like ϵ^2) we obtain a geometry as (A3) but with AdS_3 metric replaced with a BTZ geometry. The angular momentum of the BTZ metric is measured by b/ϵ^2 parameter while its mass by the out-of-extremality as $(m - m_{\text{ext}})/\epsilon^2$, such that if the original near-EVH geometry is extremal in the near-horizon limit we obtain an extremal BTZ. Note, however, that this BTZ is a "pinching BTZ orbifold", *i.e.* it is a BTZ black hole built upon the pinching AdS_3 orbifold.

Inspired by the above near horizon limits and despite the fact that the geometry (A3) has a (naked) singularity at $\theta = \pi/2$, one can show that upon the reduction of 5d Einstein theory with the reduction ansatz

$$ds^2 = \cos^2\theta g_{\mu\nu}dx^\mu dx^\nu + R^2\left[\cos^2\theta d\theta^2 + \frac{\sin^2\theta}{\cos^2\theta}d\phi^2\right], \tag{A4}$$

we obtain a 3d Einstein gravity with cosmological constant $-\frac{1}{R^2}$. The 3d and 5d Newton constants are then related as $G_3 = \frac{G_5}{\pi R^2}$. The vacuum solutions to the 3d gravity theory obtained from this reduction are hence of the form of BTZ black holes, or other quotients of AdS_3 and the corresponding Brown-Henneaux central charge is $c = \frac{3\pi}{2}\frac{R^3}{G_5}$.

Higher dimensional Myers-Perry black holes: The above limit can be generalized to Myers-Perry black holes in $d = 2n + 1$ dimensions. There one has n angular momentum parameters a_i. For the extremal black holes in this class, if one of a_i's, say a_n, is zero we have an EVH black hole. In the near horizon limit of this EVH black hole we obtain the metric

$$ds^2 = \mu_n^2\left[-\alpha r^2 dt^2 + \frac{1}{\alpha r^2}dr^2 + r^2 d\psi^2\right] + \sum_{i=1}^{n-1}\left[a_i^2 d\mu_i^2 + a_i^2\mu_i^2\left(1+\frac{\mu_i^2}{\mu_n^2}\right)d\phi_i^2\right], \quad \text{(A5)}$$

where ψ was the angle corresponding to the vanishing angular momentum a_n, the ϕ_i are the angles corresponding to the other angular momenta, and μ_i are coordinates on S^{n-1} that obey $\sum_{i=1}^{n}\mu_i^2 = 1$. The ψ direction in the AdS_3 part has a vanishing periodicity and the parameter α is equal to $\alpha = \sum_{i=1}^{n-1}\frac{1}{a_i^2}$. One may also consider near-EVH near-horizon limit by a "perturbative addition" of a_n and moving slightly away from extremality, as we described above for the 5d example. In this case one will get a (pinching) BTZ geometry instead of (pinching) AdS_3.

The angle ψ in (A5) once more has vanishing periodicity (the 3d part is a pinching AdS_3). One can see from the explicit form of the solution that taking a second angular momentum to zero does not lead to any well-defined metric. In particular, there does not seem to be an obvious generalization that leads to AdS_d metrics with $d > 3$, but it would be interesting to explore this in more detail.

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