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# Chameleon effect and the Pioneer anomaly

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The possibility that the apparent anomalous acceleration of the Pioneer 10 and 11 spacecrafts may be due, at least in part, to a chameleon field effect is examined. A small spacecraft, with no thin shell, can have a more pronounced anomalous acceleration than a large compact body, such as a planet, having a thin shell. The chameleon effect seems to present a natural way to explain the differences seen in deviations from pure Newtonian gravity for a spacecraft and for a planet, and it appears to be compatible with the basic features of the Pioneer anomaly, including the appearance of a jerk term. However, estimates of the size of the chameleon effect indicate that its contribution to the anomalous acceleration is negligible. We conclude that any inverse square component in the anomalous acceleration is more likely caused by an unmodeled reaction force from solar radiation pressure rather than a chameleon field effect.

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## I. INTRODUCTION

The Pioneer anomaly refers to an anomalous acceleration of the Pioneer 10 and 11 spacecrafts that has been inferred for large heliocentric distances of ~20–70 AU, resulting from the presence of an anomalous Doppler shift [1–3]. This small anomalous acceleration $\vec{a}_P$, which is a deviation from the prediction of the Newtonian acceleration $\vec{a}_N$, had previously been taken to have been an essentially constant acceleration with a magnitude of $a_P = 8.74 \pm 1.33 \times 10^{-10}$ m/s$^2$. Recently, however, an analysis of more complete data sets has supported the conclusion that the anomalous Pioneer acceleration $\vec{a}_P$ actually decreases with time with a temporal decay rate of magnitude $\dot{a}_P \approx 1.7 \times 10^{-11}$ m/s$^2$/yr [4]. This anomalous acceleration is seen to act on both the Pioneer 10 and 11 spacecrafts and is directed sunward. In contrast, there appear to be no such anomalous accelerations exhibited by planetary motions, disfavoring a gravitational explanation, unless there is a modified theory of gravity where small objects such as spacecraft are affected differently than planets.

There have been many attempts to explain the anomaly, either due to mundane causes or on the basis of new physics (see, e.g., Ref. [3] and references therein). However, it is possible that some combination of both of these gives rise to the anomaly. Attention here is focused on the possibility that the previously mentioned features of the Pioneer anomaly may be explained, in a rather natural way, by the chameleon effect [5,6]. For an outward-bound spacecraft trajectory, the chameleonic acceleration decreases with distance from the Sun and is therefore expected to give rise to a nonzero jerk term.

The basic aspects of the original Khoury-Weltman chameleon model are briefly reviewed, along with the expression for the chameleonic acceleration of a thick-shelled spacecraft, due to the thin-shelled sun. The basic features of the Pioneer anomaly are presented and compared with those of the chameleon model. Numerical estimates are made, including an estimate of the thin-shell factor for the Sun, allowing a rough determination of the chameleonic acceleration. It is concluded that for a chameleon-matter coupling constant of order unity, $\beta \sim O(1)$, the chameleon acceleration is negligible in comparison to the anomalous Pioneer acceleration and the chameleonic jerk term is negligible in comparison to that reported recently in Ref. [4]. In addition, we simply apply Solar System constraints on the parameterized post-Newtonian (PPN) parameter $\gamma$ obtained from the Cassini mission [7,8], ignoring assumptions concerning the chameleon coupling to matter and estimates of the Sun's thin-shell factor, and again find that the chameleonic acceleration, along with the chameleonic jerk term, are negligible in comparison to those reported for the Pioneer anomaly. We conclude that an explanation of the Pioneer anomaly must likely lie elsewhere.

## II. AN INVERSE SQUARE COMPONENT IN RECENT MEASUREMENTS OF THE PIONEER ANOMALY

In a recent paper, Turyshev *et al.* [4] analyze archived radio Doppler data extending back to February 14, 1979 for Pioneer 10 and January 12, 1980 for Pioneer 11. They produce a record of unmodeled radial acceleration $a_r$ at a two-year sample interval for both spacecraft. We plot these

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accelerations as a function of radial distance from the Sun in Fig. 1. The plotted points can be fit with a simple inverse square curve for each spacecraft, as shown by the two solid lines. When this inverse square component is removed, the resulting accelerations are consistent with the constant value reported previously [2]. In addition, the longer observation interval reveals a residual linear decrease in the acceleration of $(-0.024 \pm 0.005) \times 10^{-10}\ \mathrm{m\,s^{-2}}$ per astronomical unit (AU), much smaller than what was inferred by Turyshev *et al.* from the $a_r$ data without the removal of the inverse square curves. This inverse square component is most likely a result of a mismodeling of solar radiation pressure acting on the spacecraft. It is unlikely that it results from nonisotropic thermal emission from the spacecraft. Based on a model of the spacecraft, including its power subsystem, Anderson *et al.* [2] conclude that the thermal contribution is $(0.55 \pm 0.55) \times 10^{-10}\ \mathrm{m\,s^{-2}}$ directed toward the Sun, and they account for it as a measurement bias in their determination of the magnitude of the anomalous acceleration. It is difficult to argue for anything more than a three-sigma thermal effect, or a maximum contribution of $2.2 \times 10^{-10}\ \mathrm{m\,s^{-2}}$, 25% of the total anomaly. The model used by Anderson *et al.* [2] is given some credence by its successful application to the Cassini spacecraft, where the observed decrease in orbital energy is consistent with the model [9]. Even so, based on their own spacecraft model, Francisco *et al.* [10] claim that the anomaly is 100% thermal. In the following, we address the possibility that the observed inverse square decrease in

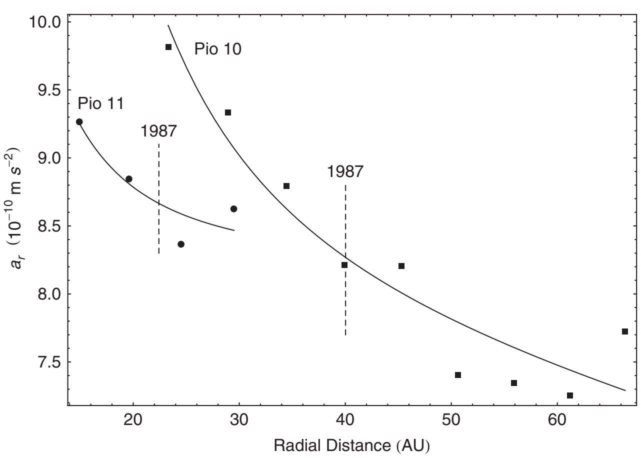


FIG. 1. Measured values of unmodeled radial acceleration $a_r$ (in units of $10^{-10}\ \mathrm{m\,s^{-2}}$) according to Turyshev *et al.* [4], but plotted as a function of radial distance r in astronomical units (AUs) rather than time. The two fitting curves are given by the function $k_0 + k_1 r + k_2/r^2$, where $k_1$ is set to zero for Pioneer 11. The two dashed lines indicate the radii at the beginning of 1987. No data prior to 1987 were used in obtaining the anomalous acceleration of $(8.74 \pm 1.33) \times 10^{-10}\ \mathrm{m\,s^{-2}}$ reported by Anderson *et al.* [2], although after subtraction of the inverse square component $k_2/r^2$ and with a reported measurement bias of $0.90 \times 10^{-10}\ \mathrm{m\,s^{-2}}$ added in [2], the resulting accelerations are well within the standard error of the 2002 result.

the measured acceleration could indeed be a part of the anomaly. By means of calculations based on the so-called chameleon effect, we conclude that this is unlikely.

## III. CHAMELEON EFFECT

### A. Equations of Motion

Basic features of the original chameleon model proposed by Khoury and Weltman in Refs. [5,6] are summarized here, beginning with the Einstein frame (EF) action

$$S = \int d^4x\sqrt{g}\left\{\frac{1}{2\kappa^2}\mathcal{R}[g_{\mu\nu}] + \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right\} + S_m[A^2(\phi)g_{\mu\nu}, \psi], \tag{3.1}$$

where $S_m$ is the matter portion of the action containing the chameleon scalar $\phi$ along with other fields represented collectively by $\psi$. A metric with signature $(+,-,-,-)$ is used and $A(\phi) = e^{\beta\kappa\phi} = \exp(\beta\phi/M_0)$, with $\beta$ a dimensionless coupling parameter, assumed to be of order unity, $\kappa = \sqrt{8\pi G} = 1/M_0$, where $M_0$ is the reduced Planck mass. The EF metric $g_{\mu\nu}$ is related to the Jordan frame (JF) metric $\tilde{g}_{\mu\nu}$ by $\tilde{g}_{\mu\nu} = A^2 g_{\mu\nu}$. The matter portion of the action is

$$\begin{aligned} S_m &= \int d^4x\sqrt{\tilde{g}}\,\tilde{\mathcal{L}}_m(\tilde{g}_{\mu\nu}, \psi) \\ &= \int d^4x\sqrt{g}\,\mathcal{L}_m[A^2(\phi)g_{\mu\nu}, \psi]. \end{aligned} \tag{3.2}$$

The action Eq. (3.1) gives rise to the equations of motion (EoM)

$$\begin{aligned} &\mathcal{R}_{\mu\nu} - \frac{1}{2}g_{\mu\nu}\mathcal{R} = -\kappa^2\mathcal{T}_{\mu\nu} = -\kappa^2[\mathcal{T}^\phi_{\mu\nu} + \mathcal{T}^m_{\mu\nu}] \\ &\Box\phi + \frac{\partial V}{\partial\phi} - \sigma = 0; \\ &\frac{du^\nu}{ds} + \Gamma^\nu_{\alpha\beta}u^\alpha u^\beta - \frac{1}{m}\partial_\mu m[g^{\mu\nu} - u^\mu u^\nu] = 0, \end{aligned} \tag{3.3}$$

where $\sigma \equiv \frac{\partial\mathcal{L}_m}{\partial\phi}$. For nonrelativistic matter, $\sigma = -\beta\kappa\rho_{\mathrm{EF}} = -\beta\kappa\bar{\rho}A(\phi)$, where $\rho_{\mathrm{EF}} = \mathcal{T}^m$ is the EF matter energy density and $\bar{\rho} = \rho_{\mathrm{EF}}A^{-1}(\phi)$ is a $\phi$ independent, conserved energy density in the EF. The EoM for $\phi$ can therefore be written in the form

$$\Box\phi + \frac{\partial V}{\partial\phi} + \beta\kappa\bar{\rho}A(\phi) = 0 \tag{3.4}$$

and there is an effective potential $V_{\mathrm{eff}}(\phi) = V(\phi) + \bar{\rho}A(\phi) = V(\phi) + \bar{\rho}e^{\beta\kappa\phi}$. Assuming a positive value of $\beta$, and $V(\phi)$ to be a runaway potential, say of the form $V = M^5/\phi$ (see, e.g., Ref. [5]), $V_{\mathrm{eff}}$ develops a minimum at a value of $\phi_{\min}$, which depends on the local energy density $\bar{\rho}$. A large $\bar{\rho}$ results in a large chameleon mass $m_\phi^2 = V''_{\mathrm{eff}}(\phi_{\min})$, while a small $\bar{\rho}$ results in a small mass $m_\phi$.

Also note that there is an extra term involving $\partial_\mu(\ln m) = \partial_\mu(\ln A(\phi))$ in the "geodesic" equation above.

This arises from the fact that a test mass having a constant value $m_0$ in the JF corresponds to a $\phi$-dependent mass $m = m_0 A(\phi)$ in the EF [11].

We will consider $\phi = \phi(r)$ to be a static weak field with a dependence upon the radial distance from some source mass $M$, which generates a Schwarzschild metrical gravity field, with $g_{00} = (1 - \frac{r_S}{r}) = (1 - \frac{2GM}{r})$. The EoM in the Newtonian limit (weak field, static limit, with nonrelativistic particle motion) yield

$$\nabla^2 h_{00} = 2\kappa^2\Big(\mathcal{T}_{00} - \frac{1}{2}\mathcal{T}^{\lambda}_{\lambda}\Big)$$
$$\nabla^2\phi - \frac{\partial V}{\partial\phi} - \beta\kappa\bar{\rho}A(\phi) = 0 \quad (3.5)$$
$$\frac{d^2\vec{x}}{dt^2} = -\frac{1}{2}\nabla h_{00} - \nabla(\ln A)$$

*Remarks.—*

(1) There are two contributions to the acceleration $\vec{a}$ of a test mass: the metric or Newtonian part, $\vec{a}_N = -\nabla(\frac{1}{2}h_{00})$, and the scalar chameleon part, $\vec{a}_c = -\nabla(\ln A) = -\beta\kappa\nabla\phi$. So from the geodesic equation above, $\vec{a} = \vec{a}_N + \vec{a}_c$.

(2) Assuming a chameleon-type model as described by Khoury and Weltman [5,6], where $V(\phi)$ is a decreasing function of $\phi$ and $A(\phi)$ is an increasing function, the vacuum value $\phi_c$ gets shifted to smaller values when $\bar{\rho}$ increases. The chameleon mass is given by $m_\phi^2 = V''_{\rm eff}(\phi) = V''(\phi) + \beta^2\kappa^2\bar{\rho}e^{\beta\kappa\phi}$. The mass $m_\phi$ is large where $\bar{\rho}$ is large (and therefore the $\phi$ field is short-ranged), but where $\bar{\rho}$ becomes very small, $m_\phi$ is very small, and the $\phi$ field becomes nearly massless and long-ranged. Therefore, Earth-based gravity differs from deep space–based gravity.

(3) For a Schwarzschild metric, the Newtonian gravitational field is

$$\vec{a}_N = -\frac{1}{2}\nabla g_{00} = -\frac{1}{2}\nabla\Big(1 - \frac{2GM}{r}\Big)$$
$$= -\frac{GM}{r^2}\hat{r} = -\frac{r_S}{2r^2}\hat{r} \quad (3.6)$$

and the chameleon "anomaly" is

$$\vec{a}_c = -\beta\kappa\nabla\phi = -\beta\kappa(\partial_r\phi)\hat{r},$$
$$\kappa = \sqrt{8\pi G} = 1/M_0. \quad (3.7)$$

(4) For a central mass $M$ located at $\vec{x} = 0$, the matter part of $\mathcal{T}_{\mu\nu}$ is $\mathcal{T}^m_{\mu\nu} = \delta^0_\mu\delta^0_\nu M\delta^{(3)}(\vec{x})$ and the chameleon field part is $\mathcal{T}^\phi_{\mu\nu} = \partial_\mu\phi\partial_\nu\phi - \eta_{\mu\nu}[\frac{1}{2}\eta^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi - V(\phi)]$, with $\mathcal{T}^{\phi\mu}_{\mu} = (\nabla\phi)^2 + 4V(\phi)$. Then the $\phi$ contribution to the right side of the first equation in Eq. (3.5) is

$$\mathcal{T}^{\phi}_{00} - \frac{1}{2}\mathcal{T}^{\phi\lambda}_{\lambda} = -V(\phi) \quad \text{for } \frac{r_S}{r} \ll 1. \quad (3.8)$$

Inputting the Schwarzschild metric means that the stress-energy of the chameleon field is assumed to have a negligible effect on the metric outside of a source, like the Sun, where $r \gg r_S = 2GM$. This is expected to be the case for small $\bar{\rho}$, large $\phi$, and small $V$.

### B. The Chameleon Field

We adopt the chameleon model proposed by Khoury and Weltman in Refs. [5,6] and borrow their results. We consider a compact uniform spherical mass $M_c$ with radius $R_c$. The exterior solution for the chameleon field (see Eq. (26) in Ref. [5]) is approximately given by

$$\phi = -\frac{C}{r}e^{-m_\infty(r-R_c)} + \phi_\infty, \quad (3.9)$$

where $m_\infty$ is the chameleon mass outside of the object, $\phi_\infty$ is the value of $\phi$ that minimizes $V_{\rm eff}$ outside of the object, and the density profile is given by

$$\bar{\rho} = \begin{Bmatrix} \bar{\rho}_c, & r < R_c \\ \bar{\rho}_\infty, & r > R_c \end{Bmatrix}. \quad (3.10)$$

Inside the object, $V_{\rm eff}$ is minimized by $\phi_c$ and the chameleon mass is $m_c$. The constant $C$ takes a value

$$C = \frac{\beta\kappa M_c}{4\pi}\begin{Bmatrix} \Big(\frac{3\Delta R_c}{R_c}\Big), & \text{thin shell, } \frac{\Delta R}{R} \ll 1 \\ 1, & \text{thick shell, } \frac{\Delta R}{R} > 1 \end{Bmatrix}. \quad (3.11)$$

We define $\Delta_c = \Delta R_c/R_c$, which is given by (see Eq. (16) of Ref. [5])

$$\Delta_c = \frac{\Delta R_c}{R_c} = \frac{\phi_\infty - \phi_c}{6\beta M_0\Phi_c}, \quad (3.12)$$

where $\Phi_c = M_c/8\pi M_0^2 R_c = GM_c/R_c$ is the Newtonian potential at the surface of the sphere, and $\phi \approx \phi_c$ well inside the object, near the core, where the chameleon mass $m$ is large, $m_c \gg m_\infty$. We then have

$$\partial_r\phi = \Big(m_\infty + \frac{1}{r}\Big)C\Big[\frac{e^{-m_\infty(r-R_c)}}{r}\Big]. \quad (3.13)$$

As pointed out in Refs. [5,6], a thin-shelled object (like a planet or a star) has a value of $C$, and hence the spatially varying part of $\phi$, suppressed by a factor of $\Delta_c \ll 1$ compared to that of a thick-shelled object (like a small satellite).

### C. Acceleration

We define the radial component of acceleration by $\mathcal{A} = \hat{r}\cdot\vec{a} = a_r$. From Eqs. (3.6), (3.7), and (3.13), we have the Newtonian and chameleonic accelerations

$$\mathcal{A}_N = -\frac{GM_c}{r^2} \tag{3.14a}$$

$$\mathcal{A}_c = -\beta\kappa\partial_r\phi = -\beta\kappa C\left(m_\infty + \frac{1}{r}\right)\left[\frac{e^{-m_\infty(r-R_c)}}{r}\right]. \tag{3.14b}$$

Both accelerations are directed radially inward ($\beta > 0$), with $a_r = \mathcal{A}_N + \mathcal{A}_c$, and the chameleonic acceleration acts as an anomalous acceleration, i.e., a deviation from the Newtonian acceleration.

We now consider the case where $m_\infty r \ll 1$, $m_\infty(r - R_c) \ll 1$, and $r_S/r \ll 1$; that is, for distances well outside a compact body of mass $M_c$ and radius $R_c$. For a very small mass $m_\infty$, these can be satisfied for distances $r \gg R_c$ so that $R_c \ll r \ll 1/m_\infty$. Assuming this to be the case, the chameleonic acceleration of a (thick-shelled) test mass is approximately

$$\mathcal{A}_c \approx -\beta\kappa C\frac{1}{r^2}. \tag{3.15}$$

Comparing this to the radial part of the Newtonian acceleration,

$$\frac{\mathcal{A}_c}{\mathcal{A}_N} \approx 2\beta^2\begin{Bmatrix} 3\Delta_c, & \text{thin shell}, \ \Delta_c \ll 1 \\ 1, & \text{thick shell}, \ \Delta_c > 1 \end{Bmatrix}, \tag{3.16}$$

where Eq. (3.11) has been used and $\Delta_c$ is given by Eq. (3.12). Therefore, for a large thin-shelled source, like the Sun, with $\Delta_c \ll 1$, the chameleonic acceleration of a small thick-shelled test mass is a very small fraction of the Newtonian acceleration, with $\mathcal{A}_c \approx -6\beta^2\Delta_c GM_c/r^2$, with $R_c \ll r \ll 1/m_\infty$.

If the test mass is actually a thin-shelled object, there is an additional factor of $3\Delta$ for the test mass (see Sec. VII A of Ref. [5]), so that the chameleonic acceleration of the thin-shelled test mass due to a thin-shelled source is

$$\frac{\mathcal{A}_c}{\mathcal{A}_N} \approx 2\beta^2(3\Delta_1)(3\Delta_2) = 18\beta^2\Delta_1\Delta_2. \tag{3.17}$$

This would describe the chameleon acceleration of a planet due to the Sun, for example, since both objects are thin-shelled, and this acceleration is suppressed by an additional $\Delta$ factor compared to that describing the chameleon acceleration of a small thick-shelled object, such as a small satellite or spacecraft.

To summarize, for the condition $R_c \ll r \ll 1/m_\infty$ there are three cases:

$$\text{(i)}\ \frac{\mathcal{A}_c}{\mathcal{A}_N} \approx 2\beta^2(3\Delta_S) = 6\beta^2\Delta_S$$

$$\text{(ii)}\ \frac{\mathcal{A}_c}{\mathcal{A}_N} \approx 2\beta^2(3\Delta_1)(3\Delta_2) = 18\beta^2\Delta_1\Delta_2$$

$$\text{(iii)}\ \frac{\mathcal{A}_c}{\mathcal{A}_N} \approx 2\beta^2, \tag{3.18}$$

where for the cases (i)–(iii) we have (i) $S$ = source, thin-shelled, $\Delta_S \ll 1$; test particle is thick-shelled, $3\Delta \to 1$

(ii) both source and test object are thin-shelled; $\Delta_{1,2} \ll 1$

(iii) both source and test particle are thick-shelled, $(3\Delta_1)(3\Delta_2) \to 1$.

## IV. THE PIONEER ANOMALY AND THE CHAMELEON EFFECT

The Pioneer anomaly is associated with the observed deviations from predicted Newtonian accelerations of the Pioneer 10 and 11 spacecrafts after passing about 20 AU from the Sun, leaving the Solar System. This anomalous acceleration is very small and often explained by unmodeled mundane causes, but it has some interesting features that seem compatible with the existence of a chameleon effect. Some of these features, mentioned in the Introduction, are listed here.

### A. Pioneer Anomaly Features

(1) There is a small apparent acceleration that was previously assumed constant with a magnitude of $a_P = 8.74 \pm 1.33 \times 10^{-10}$ m/s$^2$, for distances of ~20–70 AU from the Sun, with $a_P/a_N \ll 1$. However, it has recently been argued that the magnitude of $a_P$ has a (decreasing) time dependence [4]. Specifically, a linear model with $a_P(t) = a_P(t_0) + \dot{a}_P(t - t_0)$ contains a jerk term $\dot{a}_P$, with a reported value of $\dot{a}_P \approx -1.7 \times 10^{-11}$m/s$^2$/yr [4].

(2) It seems to be directed inward toward the Sun. (However, the authors of Ref. [4] report that the direction of the acceleration $\vec{a}_P$ remains imprecisely determined, with no support for an inward direction toward the Sun over a direction toward the Earth.)

(3) The same anomalous acceleration is seen for both spacecrafts.

(4) Such anomalies are not observed in planetary motions, disfavoring a gravitational explanation, unless a modified theory of gravity operates where small objects such as spacecraft are affected differently than planets.

### B. Chameleon Effect Features

Let us consider the simple interaction between a small, thick-shelled spacecraft and the massive, thin-shelled sun.

(1) Case (i) of Eq. (3.18) gives a chameleon acceleration of

$$\mathcal{A}_c \approx 6\beta^2\Delta_S\mathcal{A}_N = -6\beta^2\left(\frac{\Delta R_S}{R_S}\right)\frac{GM_S}{r^2}, \tag{4.1}$$

where $\Delta_S = (\Delta R_S/R_S) \ll 1$ for a thin-shelled sun with radius $R_S$. This satisfies $\mathcal{A}_c/\mathcal{A}_N \ll 1$ for

$\beta \sim O(1)$ for a large thin-shelled sun and a small thick-shelled spacecraft. But for $r = r(t)$, Eq. (4.1) indicates that a time dependence is present, $\mathcal{A} = \mathcal{A}(t)$. The time rate of change of $|\mathcal{A}_c|$, for a radial trajectory with velocity $v_r = dr/dt$, is

$$\frac{d|\mathcal{A}_c|}{dt} = v_r \frac{d|\mathcal{A}_c|}{dr} \approx -\frac{2}{r} v_r |\mathcal{A}_c(r)|, \quad \text{or} \quad \frac{d(\ln|\mathcal{A}_c|)}{dt} \approx -\frac{2v_r}{r}. \tag{4.2}$$

This is very small for a nonrelativistic speed $v_r \ll 1$ and large distances $r$, so we find $\mathcal{A}_c/\mathcal{A}_N \ll 1$, with $|\mathcal{A}_c(t)|$ a slightly decreasing function of time for an increasing $r(t)$.

(2) The direction of $\mathcal{A}_c$ is radially inward, toward the Sun.

(3) For two thick-shelled spacecraft, the chameleon acceleration $\mathcal{A}_c$ is the same for a given $r$, as seen in (4.1). For very mildly varying $\mathcal{A}(r)$, the two Pioneer spacecrafts should have chameleon accelerations nearly the same, $\mathcal{A}_{c,10} \approx \mathcal{A}_{c,11}$.

(4) The chameleon acceleration of a large thin-shelled planet due to its interaction with the thin-shelled sun is suppressed by the planet's factor of $3\Delta_{\text{planet}} = 3\Delta R_{\text{planet}}/R_{\text{planet}}$, so that from case (ii) of Eq. (3.18), we have

$$\frac{\mathcal{A}_{c,\text{planet}}}{\mathcal{A}_N} \approx 18\beta^2 \Delta_S \Delta_{\text{planet}} = 3\Delta_{\text{planet}} \frac{\mathcal{A}_{c,P}}{\mathcal{A}_N} \ll \frac{\mathcal{A}_{c,P}}{\mathcal{A}_N}, \tag{4.3}$$

where $\mathcal{A}_{c,P}$ is the chameleon acceleration of a Pioneer spacecraft. The deviation from Newtonian acceleration is greatly suppressed for a planet, and as pointed out in Refs. [5,6] allows the chameleon mechanism to easily pass all Solar System tests of gravity.

### C. Numerical Estimates

Here we make some approximate estimates based upon the original chameleon model of Khoury and Weltman. [5] The mass of the Sun is $M_S = 1.99 \times 10^{33}$ g and the distance of the Earth from the Sun is taken to be $r_E = 1$ AU $= 1.5 \times 10^{13}$ cm, which would give a Newtonian acceleration of the Earth toward the Sun of $\mathcal{A}_{N,E} = -GM_S/r_E^2 = -5.9 \times 10^{-3}$ m/s$^2$. Therefore the Newtonian acceleration of an object at a distance $r$ from the Sun is

$$\mathcal{A}_N = \mathcal{A}_{N,E}\left(\frac{r_E}{r}\right)^2. \tag{4.4}$$

The Newtonian accelerations at distances of 20 AU and 70 AU are, respectively,

$$\mathcal{A}_N^{20} = \mathcal{A}_{N,E}\left(\frac{1}{20}\right)^2 = -1.5 \times 10^{-5}\ \text{m/s}^2; \quad \mathcal{A}_N^{70} = \mathcal{A}_{N,E}\left(\frac{1}{70}\right)^2 = -1.2 \times 10^{-6}\ \text{m/s}^2. \tag{4.5}$$

The change in the magnitude of $\vec{a}_N$ between 20 AU and 70 AU is

$$\Delta|\vec{a}_N| = \Delta|\mathcal{A}_N| = |\mathcal{A}_N^{70}| - |\mathcal{A}_N^{20}| = -1.4 \times 10^{-5}\ \text{m/s}^2. \tag{4.6}$$

These results will be useful in estimates of space and time rates of change of $\mathcal{A}_c$.

*Chameleon parameters.*— From Eq. (4.1) we have a chameleonic acceleration given by

$$\frac{\mathcal{A}_c}{\mathcal{A}_N} \approx 6\beta^2 \Delta_S, \tag{4.7}$$

provided that the Pioneer is thick-shelled, i.e., $\Delta_P = \Delta R_P/R_P > 1$. We will assume this to be the case, so that an upper bound on $\mathcal{A}_c$ is established with Eq. (4.7). If the Pioneer were actually thin-shelled, with $\Delta_P \ll 1$, there would be an additional suppression factor of $3\Delta_P$ leading to a chameleon acceleration much smaller than that of $6\beta^2\Delta_S$. We will also take $\beta \sim 1$.

The upper bound on the thin-shell factor $\Delta_E$ for the Earth, proposed by Khoury and Weltman in Ref. [6] [see Eq. (15) in that article] is given as

$$\Delta_E < 10^{-7} \tag{4.8}$$

We can use this in our estimate for the shell factor $\Delta_S$ for the Sun that appears in Eq. (4.7). The shell factor $\Delta_S$ for the Sun and the shell factor $\Delta_E$ for the Earth are taken to be

$$\Delta_S = \frac{\Delta R_S}{R_S} = \frac{\phi_G - \phi_S}{6\beta M_0 \Phi_S}, \qquad \Delta_E = \frac{\Delta R_E}{R_E} = \frac{\phi_G - \phi_E}{6\beta M_0 \Phi_E}, \tag{4.9}$$

where $\phi_{S(E)}$ is the value of $\phi_c$ inside the Sun (Earth), $\phi_G$ is the value of $\phi_\infty$ in our galaxy, and $\Phi_{S(E)}$ is the Newtonian potential at the surface of the Sun (Earth), $\Phi = GM/R$. Now, take $\rho_E \sim 5.7$ g/cm$^3$, $\rho_S \sim 1.4$ g/cm$^3 \sim \frac{1}{4}\rho_E$; we take these to be roughly equal for simplicity, $\rho_S \sim \rho_E$, and since $\phi_c$ is determined by the density $\bar{\rho}$, we therefore take $\phi_S \approx \phi_E$. (For a high-density contrast, $\bar{\rho}_c \gg \bar{\rho}_\infty$, we have $\phi_c \ll \phi_\infty$, so that $\phi_G - \phi_{S(E)} \approx \phi_G$, and consequently $\Delta_S/\Delta_E \approx \Phi_E/\Phi_S$.) For the Newtonian potentials,

$$\frac{\Phi_S}{\Phi_E} = \frac{M_S}{M_E}\frac{R_E}{R_S} \approx 3 \times 10^3; \qquad \Phi_S \approx 3 \times 10^3 \Phi_E. \tag{4.10}$$

From Eq. (4.9),

$$\frac{\Delta_S}{\Delta_E} \sim \frac{\Phi_E}{\Phi_S} \approx 3 \times 10^{-4} \sim 10^{-4}; \quad \Delta_S \sim 10^{-4}\Delta_E. \tag{4.11}$$

Using $\Delta_E \sim 10^4 \Delta_S$, Eqs. (4.11) and (4.8) give

$$\Delta_S < 10^{-11}. \tag{4.12}$$

*Chameleon acceleration.*— From Eqs. (4.7) and (4.12),

$$\frac{\mathcal{A}_c}{\mathcal{A}_N} \approx 6\beta^2 \Delta_S \lesssim 6\beta^2 \times 10^{-11}. \tag{4.13}$$

The average contribution to the Pioneer anomalous acceleration would be roughly

$$\frac{\langle|\mathcal{A}_c|\rangle}{a_P} \approx 6\beta^2 \Delta_S \frac{\langle|\mathcal{A}_N|\rangle}{a_P}. \tag{4.14}$$

We can estimate a spatial average of the Newtonian acceleration,

$$\langle|\mathcal{A}_N|\rangle = \langle a_N\rangle = \frac{1}{\Delta r}\int_{r_1}^{r_2} \frac{GM_S}{r^2} dr = \frac{GM_S}{\Delta r}\frac{\Delta r}{r_1 r_2} = \frac{GM_S}{r_1 r_2}, \tag{4.15}$$

and taking $r_1 = 20r_E = 20$ AU and $r_2 = 70r_E = 70$ AU, we get

$$\langle|\mathcal{A}_N|\rangle = \langle a_N\rangle \approx 4.2 \times 10^{-6}\ \mathrm{m/s^2}. \tag{4.16}$$

Eqs (4.12) and (4.14) then give

$$\frac{\langle|\mathcal{A}_c|\rangle}{a_P} \sim 6\beta^2 \Delta_S \left[\frac{4.2 \times 10^{-6}\ \mathrm{m/s^2}}{9 \times 10^{-10}\ \mathrm{m/s^2}}\right] = \beta^2 \Delta_S (2.8 \times 10^4) < (2.8 \times 10^{-7})\beta^2. \tag{4.17}$$

Taking $a_P \sim 10^{-9}$ m/s$^2$ and $\beta \approx 1$, from Eq. (4.17) it appears that a chameleon acceleration, if it existed, would be undetectably small, with an average value estimated as

$$\langle|\mathcal{A}_c|\rangle \lesssim 10^{-16}\ \mathrm{m/s^2}. \tag{4.18}$$

If the Pioneer spacecraft were actually thin-shelled, there would be an additional suppression factor of $3\Delta_P \ll 1$, according to case (ii) of Eq. (3.18), reducing the chameleonic acceleration even further, so that Eq. (4.18) serves as an upper bound on $\langle|\mathcal{A}_c|\rangle$.

*Spatial and temporal variation.*— We can use a linear model to estimate an average space rate of change $\Delta|\mathcal{A}_c|/\Delta r$, for a change in distance of 50 AU from $r_1 = 20$ AU to $r_2 = 70$ AU and using Eq. (4.6):

$$\frac{\Delta|\mathcal{A}_c|}{\Delta r} \approx 6\beta^2 \Delta_S \frac{\Delta|\mathcal{A}_N|}{\Delta r} \sim 6\beta^2 \Delta_S \left[\frac{|\mathcal{A}_N^{70}| - |\mathcal{A}_N^{20}|}{50\ \mathrm{AU}}\right] \sim 6\beta^2 \Delta_S (-2.7 \times 10^{-7}\ \mathrm{m/s^2/AU}). \tag{4.19}$$

Therefore, Eq. (4.13) gives

$$\left|\frac{\Delta|\mathcal{A}_c|}{\Delta r}\right| < (6\beta^2 \times 10^{-11})(2.7 \times 10^{-7}\ \mathrm{m/s^2/AU}) \sim 10^{-17}\ \mathrm{m/s^2/AU}. \tag{4.20}$$

So the estimate of

$$\left|\frac{\Delta|\mathcal{A}_c|}{\Delta r}\right| < 10^{-17}\ \mathrm{m/s^2/AU} \tag{4.21}$$

is negligible in size in comparison to an estimate of

$$\left|\frac{\Delta a_P}{\Delta r}\right| \sim |\dot{a}_P| \frac{\Delta t}{\Delta r} \sim (.17 \times 10^{-10}\ \mathrm{m/s^2/yr}) \frac{30\ \mathrm{yr}}{50\ \mathrm{AU}} \sim 10^{-11}\ \mathrm{m/s^2/AU} \tag{4.22}$$

obtained using the jerk term $\dot{a}_P$ in Ref. [4].

A chameleonic jerk term $\dot{a}_c$ can be estimated using $\dot{a}_c = -\dot{\mathcal{A}} \approx \Delta|\mathcal{A}|/\Delta t$, with $\Delta|\mathcal{A}| \approx |\mathcal{A}_{70}| - |\mathcal{A}_{20}|$ and $\Delta t \sim 30$ yr; this will be a negative number since $r$ decreases with time for an outward-bound trajectory and $|\mathcal{A}| \propto a_N \propto 1/r^2$. We have

$$|\dot{a}_c| \sim 6\beta^2 \Delta_S \left|\frac{\Delta|a_N|}{\Delta t}\right| \lesssim (6\beta^2 \times 10^{-11}) \frac{(1.4 \times 10^{-5}\ \mathrm{m/s^2})}{30\ \mathrm{yr}} \sim 2.7 \times 10^{-17} \mathrm{m/s^2/yr}. \tag{4.23}$$

The value of the Pioneer jerk term (using the linear model) reported in Ref. [4] is $|\dot{a}_P| = .17 \times 10^{-10}$ m/s$^2$/yr, so that $\dot{a}_c/\dot{a}_P \lesssim 10^{-6}$. From Eqs. (4.17), (4.18), (4.21), (4.22), and (4.23), we conclude that within the context of the original Khoury-Weltman model, the chameleon effect, if it exists, is too small to account for the anomalous Pioneer acceleration or its spatial or temporal rate of change reported in Ref. [4].

## V. SOLAR SYSTEM CONSTRAINTS

In the previous section we assumed, as in the original chameleon model of Khoury and Weltman, that $\beta \approx 1$, and we used their results to obtain the estimate for the thin-shell factor for the Sun $\Delta_S < 10^{-11}$. We see [e.g., from Eq. (4.17)] that the chameleonic contribution to the Pioneer anomaly is controlled by the factor $\beta^2 \Delta_S$. We now use this result, but relax our assumption that $\beta \sim 1$ and abandon our estimate of $\Delta_S$, and instead, we obtain a fix on the factor $\beta^2 \Delta_S$ by using the results obtained in the recent analysis by Hees and Füzfa [7], wherein an upper limit of this factor can be obtained from the PPN parameter $\gamma$ obtained from Solar System constraints of the Cassini mission [8]:

$$\gamma - 1 = (2.1 \pm 2.3) \times 10^{-5}. \tag{5.1}$$

Hees and Füzfa (HF) use slightly different notations for the scalar field and chameleon parameters, but we can readily build a simple translation dictionary by noting that HF write the action in a form (using our metric signature)

$$S = \int d^4x \sqrt{g} \left\{ \frac{m_P^2}{16\pi} \mathcal{R}[g_{\mu\nu}] + \frac{1}{2} m_P^2 g^{\mu\nu} \partial_\mu \hat{\phi} \partial_\nu \hat{\phi} - \hat{V}(\hat{\phi}) \right\} + S_m[\hat{A}^2(\hat{\phi}) g_{\mu\nu}, \psi], \tag{5.2}$$

where the hat notation denotes the fields and functions used by HF, $m_P = 1/\sqrt{G}$, $\hat{A}(\hat{\phi}) = e^{k\hat{\phi}}$, and the JF metric $\tilde{g}_{\mu\nu}$ and EF metric $g_{\mu\nu}$ are related by $\tilde{g}_{\mu\nu} = \hat{A}(\hat{\phi})g_{\mu\nu}$. Comparison with Eq. (3.1) then shows that

$$\hat{\phi} = \phi/m_P, \qquad \hat{A}(\hat{\phi}) = e^{k\hat{\phi}} = A(\phi) = e^{\beta\kappa\phi},$$
$$k = \sqrt{8\pi}\beta, \qquad \hat{V}(\hat{\phi}) = V(\phi). \tag{5.3}$$

It should be noted, however, that the authors of Ref. [7] use a different form of the effective potential, as they choose to use the Jordan frame energy density $\tilde{\rho}$ as a constant rather than the conventionally chosen density $\bar{\rho} = \rho_{\rm EF}A^{-1}(\phi)$, which is a $\phi$ independent quantity in the Einstein frame representation of the theory [5,6]. Thus, the HF effective potential is written as

$$\hat{V}_{\rm eff}(\hat{\phi}) = \hat{V}(\hat{\phi}) + \frac{1}{4}\tilde{\rho}e^{4k\hat{\phi}} \tag{5.4}$$

instead of our conventionally chosen effective potential [see Eq. (3.4)]

$$V_{\rm eff}(\hat{\phi}) = \hat{V}(\hat{\phi}) + \bar{\rho}e^{k\hat{\phi}} = V(\phi) + \bar{\rho}e^{\beta\kappa\phi}. \tag{5.5}$$

This difference can be largely ignored, however, as it does not qualitatively change the results obtained [7]. More specifically, we borrow the result from Ref. [7] that $k\hat{\phi}_\infty \lesssim 2\times 10^{-12} \ll 1$, where $\hat{\phi}_\infty$ minimizes the effective potential at $r = \infty$, so that for $\tilde{\rho} \approx \bar{\rho}$ and fields of interest where $\hat{\phi} \leq \hat{\phi}_\infty$, we have the ratio

$$\frac{\frac{1}{4}\tilde{\rho}\hat{A}^4}{\bar{\rho}\hat{A}} = \frac{1}{4}\frac{\tilde{\rho}}{\bar{\rho}}e^{3k\hat{\phi}} \approx O(1), \tag{5.6}$$

showing that we have reasonable confidence in using our effective potential along with an application of the results obtained in Ref. [7].

### A. Cassini Bounds

Hees and Füzfa [7] obtain the result relating the effective coupling constant $k_{\rm eff}$, the thin-shell factor $\epsilon = \Delta_S$ for the Sun, and the PPN parameter $\gamma$

$$(\gamma - 1) = -\frac{2kk_{\rm eff}}{4\pi + kk_{\rm eff}} \approx -6\frac{\epsilon k^2}{4\pi + 3\epsilon k^2}. \tag{5.7}$$

In order for the chameleon mechanism to account for a nonzero value of $(\gamma - 1)$, we see that $(\gamma - 1)$ must be negative, so that from Eq. (5.1),

$$|\gamma - 1| \leq |\gamma - 1|_{\max} = 2\times 10^{-6}. \tag{5.8}$$

Inverting Eq. (5.7) to obtain $\epsilon k^2$, we have

$$(\epsilon k^2)_{\max} \approx \frac{4\pi}{3}\left(\frac{|\gamma - 1|_{\max}}{2 - |\gamma - 1|_{\max}}\right) \approx 4.2\times 10^{-6}. \tag{5.9}$$

In terms of our original Khoury and Weltman parameters $\beta$ and $\Delta_S$, this translates into

$$\beta^2\Delta_S \lesssim 3.3\times 10^{-7}. \tag{5.10}$$

We note that this is in accord with our previous estimates based upon $\beta \approx 1$ and $\Delta_S < 10^{-11}$.

### B. Estimates Based Upon the Cassini Bounds

*Chameleonic acceleration.—* We can now simply use Eq. (5.10) without any assumptions for the values of $\beta$ and $\Delta_S$ to obtain estimates of the maximum chameleonic contribution to the Pioneer anomaly. For example, using Eq. (5.10) in Eq. (4.17) yields

$$\frac{\langle|\mathcal{A}_c|\rangle}{a_P} \lesssim 5.5\times 10^{-2}, \tag{5.11}$$

indicating that a chameleonic acceleration could account for no more than 5.5% of the Pioneer acceleration.

*Spatial and temporal variation.—* In a similar manner, referring to Eqs. (4.19), (4.20), (4.21), (4.22), and (4.23), the application of Eq. (5.10) gives a spatial variation

$$\left|\frac{\Delta|\mathcal{A}_c|}{\Delta r}\right| \lesssim 5.4\times 10^{-13}\ {\rm m/s^2/AU} \tag{5.12}$$

and

$$\left|\frac{|\Delta\mathcal{A}_c|/\Delta r}{|\Delta a_P|/\Delta r}\right| \lesssim 5.4\times 10^{-2} \tag{5.13}$$

and a time variation (jerk term)

$$|\dot{a}_c| \lesssim 9\times 10^{-13}\ {\rm m/s^2/yr} \tag{5.14}$$

with

$$\frac{\dot{a}_c}{\dot{a}_P} \lesssim 5.3\times 10^{-2}. \tag{5.15}$$

Again, apparently the chameleonic jerk term is no more than about 5.3% of the reported Pioneer jerk term.

## VI. SUMMARY

The chameleon model proposed in Refs. [5,6] has basic features that seem to be compatible, in a natural way, with the prominent features exhibited by the Pioneer anomaly. A small, thick-shelled spacecraft can have a much more pronounced deviation from a Newtonian acceleration than can a large, massive, thin-shelled planet. Therefore, the anomaly seen by the Pioneer 10 and 11 spacecrafts does not become manifest in any anomalous planetary motions.

Furthermore, the chameleon effect produces an acceleration which is small in comparison to the Newtonian acceleration if the spacecraft is thick-shelled, and this acceleration is directed sunward, i.e., toward the gravitational source. The chameleonic acceleration is found to have a $1/r^2$ dependence, so that for an outward-bound journey the chameleon "anomaly" decreases in magnitude.

We have estimated the chameleonic acceleration and its spatial and temporal rates of change, and we conclude that the chameleon effect cannot account for the Pioneer anomalous acceleration or jerk term recently reported by Ref. [4]. Specifically, using the original Khoury-Weltman chameleon model and results [5,6], we find that $\langle|\mathcal{A}_c|\rangle/a_P \lesssim 10^{-7}$, $\frac{\Delta|\mathcal{A}_c|/\Delta r}{\Delta a_P/\Delta r} \lesssim 10^{-6}$, and $\dot{a}_c/\dot{a}_P \lesssim 10^{-6}$.

However, more general considerations simply based upon Solar System constraints (specifically the constraints from the Cassini bounds on the PPN parameter $\gamma$), lead to maximum contributions $\langle|\mathcal{A}_c|\rangle/a_P \lesssim 5.5 \times 10^{-2}$, $\frac{\Delta|\mathcal{A}_c|/\Delta r}{\Delta a_P/\Delta r} \lesssim 5.4 \times 10^{-2}$, and $\dot{a}_c/\dot{a}_P \lesssim 5.3 \times 10^{-2}$. We conclude that Solar System constraints allow possible chameleonic effects to account for no more than a few percent of those that are observed. We suspect that an inverse square component seen in the anomalous acceleration is more likely due to an unmodeled reaction force from solar radiation pressure rather than a chameleon field effect.

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*Note added in proof*—Two papers, Refs. [12,13], which have recently come to our attention, suggest that the Pioneer anomaly can be completely accounted for when solar radiation pressure and thermal recoil are taken into account. We thank C. Lammerzahl for drawing our attention to Ref. [12].

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