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Lorentz gauge theory as a model of emergent gravity

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We consider a class of Lorentz gauge gravity theories within Riemann-Cartan geometry that admits a topological phase in the gravitational sector. The dynamic content of such theories is determined only by the contortion part of the Lorentz gauge connection. We demonstrate that there is a unique Lagrangian that admits propagating spin-one mode in correspondence with gauge theories of other fundamental interactions. Remarkably, despite the R^2 type of the Lagrangian and noncompact structure of the Lorentz gauge group, the model possesses rather a positive-definite Hamiltonian. This has been proved in the lowest order of perturbation theory. This implies further consistent quantization and leads to renormalizable quantum theory. It is assumed that the proposed model describes possible mechanism of emergent Einstein gravity at very early stages of the Universe due to quantum dynamics of contortion.

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I. INTRODUCTION

The idea that a Lorentz gauge approach can lead to a consistent quantum theory of gravity has been developed for the last 50 years since the seminal paper by Utiyama [1]. The exhausting list of references can be found in reviews on this topic (see, for instance, [2,3]). Among early works devoted to Lorentz gauge theory with Yang-Mills-type Lagrangian, one should mention the papers [4–9] where main features of classical and quantum theory were studied. Extension of the Lorentz gauge approach to the case of general Lorentz connection including contortion was widely explored as well [2,3,10,11]. The most general Lagrangian quadratic in Riemann-Cartan curvature and with the Einstein-Hilbert term was considered in [12]. Recently, a Lorentz gauge gravity model with the contortion part in the Lorentz gauge connection that admits a topological phase for gravitation has been proposed [13]. We assume that such a topological phase can be possibly realized at very early stages of our Universe close to or before the big bang. The standard gravity is supposed to be an effective theory that is induced during phase transition due to quantum dynamics of contortion. The idea that Einstein gravity is an effective theory and can be deduced from some more fundamental theory is not new; it was sounded by Zel'dovich and Sakharov in the 1970s [14,15]. Possible mechanisms of inducing the Einstein theory via quantum corrections were proposed in the past by many physicists in various approaches: conformal invariance breaking schemes [16,17], nonlinear realizations of the Lorentz group [18,19], models with spontaneous symmetry breaking [20–24], superstring models, loop quantum gravity [25,26], and others [27,28]. In order to capture the nature of gravity, thermodynamic approaches have been also developed [29–31]. Recently, it was conjectured that the gravity could be regarded as the entropic force through the holographic principle [32]. In most of these approaches, the Einstein-Hilbert term is induced by quantum corrections due to interaction with the matter field.

Our approach is based on the gauge principle that was successfully realized in formulating the theories of electroweak and strong interactions. We consider the local Lorentz symmetry as an appropriate gauge symmetry for constructing a generalized theory of gravity in geometric framework since it reflects the equivalence principle, which is a cornerstone of general relativity. This introduces naturally the contortion as a part of general Lorentz gauge connection. Whether or not the contortion (torsion) is relevant to our real world is discussed in detail in [33].

We consider theories with a Lagrangian containing only Riemann-Cartan curvature-squared terms. We do not introduce terms quadratic in torsion since we treat the contortion as a part of Lorentz gauge connection, not as a tensor. By this way, we keep the gauge structure of the considered Lorentz gauge gravity models close to standard gauge approach. It has been shown [13] that there is a model with a special R^2-type Lagrangian that

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admits a topological phase for the gravitation whereas contortion still possesses dynamical degrees of freedom. An interesting feature of the model is that the number of dynamical degrees of freedom of torsion is the same as the number of physical degrees of the metric tensor. This gives a hint that torsion may play a role of quantum counterpart to the classical metric of Einstein gravity, which is supposed to be an effective theory generated by the quantum dynamics of torsion [34]. The analysis of dynamic content of the model in [13] has been performed at the lowest linearized level in the contortion part and in the presence of constant Riemann curvature space-time background. Because of these limitations, several important issues in this model remain unclear, especially whether the dynamical properties of torsion are intrinsic properties or they depend on the presence of the background metric.

As it is known, theories with R^2-type Lagrangian suffer from a serious problem related to nondefiniteness of the Hamiltonian due to noncompact structure of the Lorentz gauge group. This has been the main obstacle toward consistent quantization and defining a physical unitary S matrix. One possible way to overcome this problem is based on Euclidean gravity formalism [35–37]. One should notice that presence of higher-derivative terms in the Lagrangian still may cause problems with unitarity and ghosts in the graviton propagator in Euclidean gravity [36,38].

In the present paper, we study dynamical properties of the topological gravity model with torsion in the limit of flat space-time metric. We have found that Lorentz gauge connection has dynamic degrees of freedom with a Lagrangian specified by the same set of parameters in the initial Lagrangian as in the case of the presence of background constant Riemannian curvature space-time. This proves that contortion possesses genuine dynamical properties independently on the metric. It is unexpected, we have demonstrated in the lowest order of perturbation theory, that the model has a positive-definite Hamiltonian. This allows to define stable quantum vacuum and perform consistent quantization preserving unitarity in the theory.

In Sec. II, we present the principal ideas lying in the basis of the model of quantum gravity with contortion. In Sec. III, we study the dynamic content of the theory by solving equations of motion in Lagrange formalism. All equations of motion are solved in linearized approximation by using decomposition of the Lorentz connection around fixed classical solution corresponding to constant torsion background. In Sec. IV, we prove the positive definiteness of the Hamiltonian in the linearized approximation. The last section contains discussion of possible physical implications.

II. LORENTZ GAUGE THEORY WITH TOPOLOGICAL GRAVITY

Lorentz gauge theory on curved space-time can be described naturally within Riemann-Cartan geometrical formalism. Let us start first with the main outlines of Riemann-Cartan geometry. The basic geometric objects are the vielbein e_a^m and the general Lorentz affine connection A_{mcd}, which can be identified with the Lorentz gauge potential. The infinitesimal Lorentz transformation of the vielbein e_a^m is given by

$$\delta e_a^m = \Lambda_a{}^b e_b^m, \tag{1}$$

where $\Lambda_{ab}(= -\Lambda_{ba})$ is the Lorentz gauge parameter. We use $m, n, \ldots$ to denote world indices, and $a, b, \ldots$ for Lorentz frame indices. We assume that the vielbein is invertible and the metric $\eta_{ab}(= e_a^m e_{mb})$ has Lorentz signature $\eta_{ab} = \mathrm{diag}(-, +, +, +)$.

The covariant derivative with respect to the Lorentz group transformation is defined in a standard manner

$$D_a = e_a^m(\partial_m + g\mathbf{A}_m), \tag{2}$$

where $\mathbf{A}_m \equiv A_{mcd}\Omega^{cd}$ is the affine connection taking values in the Lorentz Lie algebra whose generator is given by Ω^{cd}, and g is a new gravitational gauge coupling constant. For brevity of notation, we will use a redefined connection that absorbs the coupling constant. The original Lorentz gauge transformation of the connection $\mathbf{A}_m$ has the form

$$\delta\mathbf{A}_m = -\partial_m\mathbf{\Lambda} - [\mathbf{A}_m, \mathbf{\Lambda}], \tag{3}$$

where $\mathbf{\Lambda} = \Lambda_{cd}\Omega^{cd}$. The Lorentz gauge connection A_{ma}^b can be rewritten as the sum

$$A_{mab} = \varphi_{mab}(e) + K_{mab}, \tag{4}$$

where K_{mab} is a contortion and $\varphi_{mab}(e)$ is a Levi-Civita spin connection given in terms of the vielbein

$$\varphi_{mab}(e) = -\tfrac{1}{2}(e_b^n\partial_m e_{na} - e_a^n e_m^c \partial_b e_{nc} + \partial_a e_{mb} - (a \leftrightarrow b)). \tag{5}$$

The torsion and curvature tensors are defined in a standard way

$$[D_a, D_b] = T_{ab}{}^c D_c + R_{abcd}\Omega^{cd}, \tag{6}$$

where the torsion components in the unholonomic basis can be expressed in terms of contortion, and conversely

$$T_{ab}{}^c = K_{ab}{}^c - K_{ba}{}^c, \qquad K_{abc} = \tfrac{1}{2}(T_{abc} - T_{bca} + T_{cab}). \tag{7}$$

The most general quadratic in Riemann-Cartan curvature Lagrangian reads

$$\mathcal{L} = c_1 R_{abcd}R^{abcd} + c_2 R_{abcd}R^{cdab} + c_3 R_{ab}R^{ab} + c_4 R_{ab}R^{ba} + c_5 R^2 + c_6 A^2_{abcd}, \tag{8}$$

where the last term is an additional invariant that appears in Riemann-Cartan space-time. The tensor A_{abcd} is defined as follows [12],

$$A_{abcd} \equiv \tfrac{1}{6}(R_{abcd} + R_{acdb} + R_{adbc} + R_{bcad} + R_{bdca} + R_{cdab}). \tag{9}$$

In Riemannian space-time, the tensor A_{abcd} vanishes due to the Jacobi cyclic identity

$$R_{abcd} + R_{acdb} + R_{adbc} = 0. \tag{10}$$

A careful analysis of gravity models including the Einstein term in the Lagrangian was done in [12]. We do not consider the Einstein term since we treat the Einstein gravity as an effective theory that should not be quantized and that is induced from a more general theory, in our case from Riemann-Cartan gravity. So, only contortion represents quantum dynamical degree of freedom in a special Riemann-Cartan gravity model. In general, the Lagrangian (8) contains propagating modes for both fields, metric and contortion. So, formally the metric can still be considered as a quantum field as well as the contortion. This is not merely satisfactory because metric and contortion represent different geometric objects. The metric plays a role of kinematic variable in description of the space-time geometry, whereas the contortion, as a part of gauge connection, plays a role of gauge potential that represents dynamic object in gauge theories of electroweak and strong interactions. To keep only the contortion as a quantum variable, we conjecture that a generalized Riemann-Cartan gravity may admit a phase where the metric describes a pure topological structure of the space-time. So, the metric does not satisfy any equations of motion and it cannot be quantized in principle. This is our main idea. We are looking for such a Lagrangian in Riemann-Cartan space-time that reduces to topological Gauss-Bonnet density in the limit of Riemannian geometry.

In Riemann-Cartan geometry, the proper generalization of the topological Gauss-Bonnet invariant (Euler characteristic) is given by the Bach-Lanczos (BL) density [39,40]

$$I_{\rm BL} = R_{abcd}R^{cdab} - 4R_{ab}R^{ba} + R^2. \tag{11}$$

The properties of the Bach-Lanczos invariant are described in a detail in [41]. A proper Lagrangian can be derived from the general expression (8) by fitting the parameters c_i as follows,

$$\mathcal{L} = -\frac{1}{32}\{\alpha R^2_{abcd} + (1-\alpha)R_{abcd}R^{cdab} - 4\beta R^2_{bd} - 4(1-\beta)R_{bd}R^{db} + R^2 + 6\gamma A^2_{abcd}\}, \tag{12}$$

where the parameters α, β, γ remain arbitrary. One can check that the Lagrangian reduces to the Gauss-Bonnet density in the limit of Riemannian space-time, i.e., when contortion is set to be zero. One can rewrite the Lagrangian in a more simple form

$$\mathcal{L} = -\tfrac{1}{32}\{(\alpha+\gamma)R^2_{abcd} - (\alpha-\gamma)R_{abcd}R^{cdab} + 4\gamma R_{abcd}R^{acdb} - 4\beta(R^2_{bd} - R_{bd}R^{db}) + I_{\rm BL}\}. \tag{13}$$

It has been shown that the model described by the Lagrangian (13) admits dynamical degrees of freedom for the contortion only for the special values of the parameters, $\beta = 0$, $\gamma = -3\alpha$ with overall normalization factor α [13]. The result has been obtained from the analysis of linearized equations of motion for contortion in the presence of constant Riemann curvature space-time background. Therefore, the principal question arises whether contortion will keep its properties in the flat Riemannian space-time. In other words, whether the dynamics of torsion represents its intrinsic properties independent of the metric. If the contortion still possesses dynamical properties in flat space-time, then another important question arises, at which values of the parameters α, β, γ it will happen.

In the present paper, we will mainly concentrate on flat metric limit, i.e., a pure Lorentz gauge theory with the Lagrangian of type (13). The field strength (curvature tensor) in flat space-time takes a simple form

$$R_{mncd} = \partial_m A_{ncd} + A_{mce}A_{ned} - (m \leftrightarrow n). \tag{14}$$

Since the background vielbein is flat, there is no difference between the world and Lorentzian indices. Our study will be constrained by a special choice of the parameter, $\beta = 0$, which is a necessary condition of existence of propagating vector mode in the presence of constant curvature space-time [13].

The theory described by the Lagrangian (13) is highly nonlinear and belongs to degenerate theories [42]. Application of canonical formalism to such theories is quite complicated due to the presence of constraints of higher orders. Therefore, to study the dynamical structure of the theory we will use Lagrange formalism and apply linearized approximation method, which is effective in the analysis of nonlinear equations of motion. We will split the Lorentz gauge connection into classical background field B_{acd} (which plays a role of the mean field) and fluctuating part q_{acd} as follows,

$$A_{acd} = B_{acd} + q_{acd}. \tag{15}$$

Under the decomposition (15), the general field strength is split into two parts as follows,

$$\begin{aligned} R_{abcd} &= \mathcal{R}_{abcd}(B) + \tilde{R}_{abcd}(q), \\ \mathcal{R}_{abcd}(B) &= \partial_a B_{bcd} + B_{ace}B_{bed} - (a \leftrightarrow b), \\ \tilde{R}_{abcd}(q) &= \mathcal{D}_{\underline{a}} q_{b\underline{c}\underline{d}} + q_{ace}q_{bed} - (a \leftrightarrow b), \end{aligned} \tag{16}$$

where $\mathcal{D}_a$ is a background covariant derivative containing the classical field B_{acd}, and the underlined indices stand for indices over which the covariantization is performed.

There are two gauge nonequivalent representations for gauge potentials leading to the same constant field strength in $SU(2)$ Yang-Mills theory: Abelian type and non-Abelian type [43–45]. In the case of constant curvature space-time, the Abelian type of gravitational field has been used for spin connection [13]. The calculations are crucially simplified using normal coordinate decomposition of the metric. In the present case of flat space-time, it is more convenient to choose a constant background field of non-Abelian type defined by the following Lorentz gauge potential, B_{acd},

$$B_{0cd} = 0, \qquad B_{\alpha\beta\gamma} = \epsilon_{\alpha\beta\gamma}H, \qquad B_{\alpha 0\beta} = \delta_{\alpha\beta}G, \tag{17}$$

where Greek indices run through the space components and $\epsilon_{123} = +1$. The constant field is determined by two number parameters G, H which correspond to rank two of the Lorentz group. The corresponding field strength reads

$$\begin{aligned} &\mathcal{R}_{0\alpha cd} = 0,\\ &\mathcal{R}_{\alpha\beta 0\delta} = -2\epsilon_{\alpha\beta\delta}HG,\\ &\mathcal{R}_{\alpha\beta\gamma\delta} = (H^2 - G^2)(\delta_{\alpha\gamma}\delta_{\beta\delta} - \delta_{\alpha\delta}\delta_{\beta\gamma}). \end{aligned} \tag{18}$$

We will analyze the equations of motion in detail for the case of constant background $G = 0$, $H \neq 0$, which is one of the background solutions.

III. EQUATIONS OF MOTION IN LAGRANGE FORMALISM

The classical theory with the Lagrangian (13) is degenerate. This implies that the number of equations of motion in free theory is less than the number of field degrees of freedom. So, one has to consider nonlinear equations of motion to determine the dynamic content of all fields. The degeneracy of the quadratic Lagrangian (13) manifests in appearance of additional local symmetries. One symmetry is similar to $U(1)$ gauge symmetry

$$\delta_{U(1)}q_{acd} = \tfrac{1}{3}(\eta_{ac}\partial_d\lambda - \eta_{ad}\partial_c\lambda), \qquad \delta_{U(1)}q^a{}_{ad} = \partial_d\lambda, \tag{19}$$

and it implies that only transverse degrees of freedom of the vector field $q^a{}_{ad}$ can be propagating. Another symmetry with a constrained parameter χ_{bc} has the following form,

$$\delta_\chi q_{acd} = \partial_c\chi_{da} - \partial_d\chi_{ca}, \tag{20}$$

where $\chi_{bc} = \chi_{cb}$, $\chi^c{}_c = 0$, and $\partial^c\chi_{cd} = 0$. These symmetries reduce essentially the number of dynamical component fields in the contortion.

Let us consider linearized equations of motion corresponding to the Lagrangian (13)

$$\begin{aligned} \frac{\delta\mathcal{L}}{\delta q_{ncd}} \equiv{}& (\alpha+\gamma)\mathcal{D}^{\underline{m}}(\mathcal{D}_m q_{n\underline{c}\underline{d}} - \mathcal{D}_n q_{m\underline{c}\underline{d}})\\ &- (\alpha-\gamma)\mathcal{D}^{\underline{m}}(\mathcal{D}_{\underline{c}} q_{\underline{d}mn} - \mathcal{D}_{\underline{d}} q_{\underline{c}mn})\\ &+ \gamma\mathcal{D}^{\underline{m}}(\mathcal{D}_m q_{\underline{c}\underline{d}n} - \mathcal{D}_{\underline{c}} q_{m\underline{d}n} - \mathcal{D}_n q_{\underline{c}\underline{d}m}\\ &+ \mathcal{D}_{\underline{c}} q_{n\underline{d}m} - (c \leftrightarrow d))\\ ={}& 0, \end{aligned} \tag{21}$$

where covariant derivatives inside the brackets act on the last two indices of q_{ncd}, and for the second covariant derivatives, $\mathcal{D}^{\underline{m}}$, the covariantization is performed over underlined indices. One has 24 equations of motion; six equations among them represent Noether identities due to local Lorentz symmetry. One has to impose six gauge-fixing conditions that will be chosen in consistence with equations of motion.

It is convenient to make the following decomposition of the Lorentz gauge connection q_{acd} into irreducible parts $(q_{00\mu}, q_{0\mu\nu}, q_{\mu\gamma\delta}, q_{\mu 0\rho})$ where

$$\begin{aligned} q_{\mu\gamma\delta} ={}& \epsilon_{\gamma\delta\rho}\overset{\top\top}{S}_{\mu\rho}\left(+\frac{1}{2}\left(\delta_{\mu\rho} - \frac{\partial_\mu\partial_\rho}{\Delta}\right)\overset{\top}{S}\right.\\ &+ (\partial_\mu S_\rho + \partial_\rho S_\mu) + \epsilon_{\mu\rho\sigma}A_\sigma),\\ q_{\mu 0\rho} ={}& \overset{\top\top}{R}_{\mu\rho} + \frac{1}{2}\left(\delta_{\mu\rho} - \frac{\partial_\mu\partial_\rho}{\Delta}\right)\overset{\top}{R} + (\partial_\mu R_\rho + \partial_\rho R_\mu)\\ &+ \epsilon_{\mu\rho\sigma}Q_\sigma. \end{aligned} \tag{22}$$

We define $\Delta = \partial_\alpha\partial_\alpha$, and the superscript "$\top$" stands for traceless components and "$\top\top$" denotes traceless and transverse irreducible part. The decomposition is similar to that used for the metric tensor in canonical formalism of Einstein gravity [46]. Note that the fields $\overset{\top}{S}$, $\overset{\top}{R}$ and longitudinal components $A^{\rm l}_\alpha = \frac{\partial_\alpha\partial_\beta}{\Delta}A_\beta$, $Q^{\rm l}_\alpha = \frac{\partial_\alpha\partial_\beta}{\Delta}Q_\beta$ do not transform under Lorentz gauge transformations.

We will solve all equations of motion in component form. Let us start with the equation

$$\begin{aligned} \frac{\delta\mathcal{L}}{\delta q_{00\delta}} \equiv{}& \Delta q_{00\delta} - \partial_\mu\partial_\delta q_{00\mu} + \partial_\mu\partial_0(q_{\delta 0\mu} - q_{\mu 0\delta})\\ &+ 4H\epsilon_{\mu\delta\epsilon}\partial_\mu q_{00\epsilon} + 2H\epsilon_{\mu\delta\varphi}\partial_0 q_{\mu\varphi 0} - 4H^2 q_{00\delta}\\ ={}& 0. \end{aligned} \tag{23}$$

The equation represents a constraint that can be solved exactly

$$H^2\partial_\delta q_{00\delta} = H\partial_0\partial_\delta Q_\delta, \tag{24}$$

$$q^{\rm tr}_{00\delta} = \frac{1}{\Delta + 4H^2}\partial_0(-2\epsilon_{\delta\mu\phi}\partial_\mu Q^{\rm tr}_\phi + 4HQ^{\rm tr}_\delta). \tag{25}$$

The constraint allows to express the field $q_{00\delta}$ in terms of Q_α. Notice that we cannot impose a gauge-fixing condition to eliminate the field $Q^{\rm l}_\alpha$ since it is gauge invariant under the Lorentz gauge transformation. In Eq. (24), we keep H-terms explicitly to show that this constraint vanishes identically in the limit $H \to 0$. Further, we will assume that H is a small parameter to justify our perturbative analysis of equations of motion.

The equation $\delta\mathcal{L}/\delta q_{0\gamma\delta}$ contains a part with time derivatives of first order. It is convenient to use the Lorentz gauge freedom and impose a gauge-fixing condition that makes these terms vanished,

$$(\alpha + \gamma)\partial_\mu q_{\mu\gamma\delta} - \gamma\partial_\mu(q_{\gamma\mu\delta} - q_{\delta\mu\gamma}) - \alpha H(\epsilon_{\gamma\mu\varphi}q_{\mu\varphi\delta} - \epsilon_{\delta\mu\varphi}q_{\mu\varphi\gamma}) = 0. \tag{26}$$

The gauge-fixing condition can be written in terms of component fields as follows,

$$2(\alpha + \gamma)\partial_\alpha S_\alpha + \gamma\overset{\top}{S} - \frac{2\alpha}{\Delta}H\partial_\alpha A_\alpha = 0, \tag{27}$$

$$\Delta S^{\mathrm{tr}}_\alpha - \epsilon_{\alpha\gamma\delta}\partial_\gamma A^{\mathrm{tr}}_\delta - 2HA^{\mathrm{tr}}_\alpha = 0. \tag{28}$$

The last equation allows to express the pseudovector field S^{tr}_α in terms of the physical vector field A^{tr}_α. Since one has six gauge degrees of freedom due to the Lorentz gauge symmetry, one can impose another three gauge-fixing conditions. We will impose them later; for the present moment, it is difficult to determine which conditions should be imposed in a consistent manner with all equations of motion. With this, the equation $\delta\mathcal{L}/\delta q_{0\gamma\delta}$ results in a constraint

$$\begin{aligned}\frac{\delta\mathcal{L}}{\delta q_{0\gamma\delta}} &\equiv (\alpha + \gamma)\Delta q_{0\gamma\delta} + \gamma\partial_\mu(\partial_\gamma q_{0\delta\mu} - \partial_\delta q_{0\gamma\mu}) + \alpha\partial_\mu(\partial_\gamma q_{\delta 0\mu} - \partial_\delta q_{\gamma 0\mu}) - 2\gamma\epsilon_{\gamma\delta\rho}\Delta Q_\rho + 2\gamma\epsilon_{\gamma\mu\rho}\partial_\delta\partial_\mu Q_\rho \\ &\quad - 2\gamma\epsilon_{\delta\mu\rho}\partial_\gamma\partial_\mu Q_\rho + H\{2\alpha\partial_\gamma Q_\delta + 2(\alpha + 2\gamma)\epsilon_{\gamma\mu\epsilon}\partial_\mu q_{0\delta\epsilon} + \gamma\epsilon_{\gamma\delta\epsilon}\partial_\mu(q_{0\epsilon\mu} - q_{\mu\epsilon 0}) + \gamma\epsilon_{\gamma\mu\epsilon}\partial_\delta q_{0\epsilon\mu} \\ &\quad + 2(-\alpha + \gamma)\epsilon_{\gamma\mu\epsilon}\partial_\mu q_{\delta\epsilon 0} - (\gamma \leftrightarrow \delta)\} - H^2\{2\alpha q_{0\gamma\delta} + \alpha(q_{\gamma 0\delta} - q_{\delta 0\gamma})\} \\ &= 0.\end{aligned} \tag{29}$$

For our purpose to determine the dynamic content of the theory, we will need the solution to this equation up to order H^2,

$$\begin{aligned}q_{0\gamma\delta} = &-\left(1 + \frac{2H^2}{\Delta}\right)\partial_\gamma R^{\mathrm{tr}}_\delta - \frac{4\alpha - 2\gamma}{\alpha + \gamma}H\epsilon_{\gamma\delta\alpha}R^{\mathrm{l}}_\alpha - \frac{\alpha - \gamma}{\alpha + \gamma}H\epsilon_{\gamma\delta\alpha}\frac{\partial_\alpha}{\Delta}\overset{\top}{R} + \frac{1}{2}\epsilon_{\gamma\delta\alpha}Q^{\mathrm{tr}}_\alpha + \frac{2H}{\Delta}\partial_\gamma Q^{\mathrm{tr}}_\delta \\ &+ \frac{\gamma}{\alpha + \gamma}\epsilon_{\gamma\delta\alpha}Q_\alpha + \frac{\alpha(\alpha + 3\gamma)}{(\alpha + \gamma)^2}\frac{H^2}{\Delta}\epsilon_{\gamma\delta\alpha}Q^{\mathrm{l}}_\alpha - (\gamma \leftrightarrow \delta) + \mathcal{O}(H^{n\geq 3}).\end{aligned} \tag{30}$$

The next equation of motion, $\delta\mathcal{L}/\delta q_{\nu 0\nu}$, represents a constraint that allows to express the component field $\overset{\top}{R}$ in terms of other fields

$$\begin{aligned}\frac{\delta\mathcal{L}}{\delta q_{\nu 0\nu}} &\equiv \alpha\Delta\overset{\top}{R} - 2\alpha\partial_0\partial_\mu A_\mu + H\left\{2(3\gamma - \alpha)\partial_0 T + \left(6(\alpha + 2\gamma) + \frac{4\gamma(2\alpha - 3\gamma)}{\alpha + \gamma}\right)\partial_\mu Q_\mu\right\} \\ &\quad - 4H^2\left\{\frac{2\alpha - 3\gamma}{\alpha + \gamma}(2(2\alpha - \gamma)\partial_\mu R_\mu - (\alpha - \gamma)\overset{\top}{R}) + 2\gamma\overset{\top}{R} + 4\gamma\partial_\mu R_\mu\right\} + \mathcal{O}(H^{n\geq 3}) \\ &= 0,\end{aligned} \tag{31}$$

where we introduce a useful notation T for the irreducible totally antisymmetric part of $q_{\alpha\beta\gamma}$,

$$q_{(\alpha\beta\gamma)} \equiv q_{\alpha\beta\gamma} + q_{\beta\gamma\alpha} + q_{\gamma\alpha\beta} = \epsilon_{\alpha\beta\gamma}T, \qquad T \equiv \tfrac{1}{2}\epsilon_{\mu\delta\varphi}q_{\mu\delta\varphi} = \overset{\top}{S} + 2\partial_\gamma S_\gamma. \tag{32}$$

Let us consider the following equation of motion,

$$\begin{aligned}\frac{\delta\mathcal{L}}{\delta q_{\nu\nu\delta}} &\equiv \alpha(\Box q_{\nu\nu\delta} - \partial_\delta\partial_\mu q_{\nu\nu\mu} - \partial_\mu\partial_\nu q_{\mu\nu\delta} + \partial_0(\partial_\nu q_{0\nu\delta} + \partial_\delta q_{\nu\nu 0} + \partial_\nu q_{\delta 0\nu}) - 2H^2 q_{\nu\nu\delta}) \\ &\quad + H\{\alpha\epsilon_{\delta\epsilon\beta}\partial_\mu q_{\mu\epsilon\beta} - 3\alpha\epsilon_{\delta\beta\epsilon}\partial_\mu q_{\beta\epsilon\mu} - 4\alpha\epsilon_{\delta\mu\epsilon}\partial_\mu q_{\nu\nu\epsilon} + 3(\alpha + 2\gamma)\epsilon_{\mu\beta\epsilon}\partial_\mu q_{\beta\epsilon\delta} - (2\alpha - 3\gamma)\epsilon_{\beta\mu\epsilon}\partial_\mu q_{\delta\epsilon\beta} \\ &\quad + \alpha\epsilon_{\delta\nu\epsilon}\partial_0 q_{\nu\epsilon 0} + (\alpha - 3\gamma)\epsilon_{\epsilon\mu\beta}\partial_\delta q_{\epsilon\mu\beta} - \alpha\epsilon_{\delta\epsilon\beta}\partial_0 q_{0\epsilon\beta}\} \\ &= 0,\end{aligned} \tag{33}$$

where $\Box \equiv -\partial_0\partial_0 + \partial_\alpha\partial_\alpha$. The transverse part of the equation leads to a propagation equation for the transverse part of the vector field $A_\delta = -q_{\nu\nu\delta}/2$,

$$\Box q_{\nu\nu\delta} - \frac{\partial_\delta\partial_\mu}{\Delta}q_{\nu\nu\mu} + \mathcal{O}(H^{n\geq 1}) = 0. \tag{34}$$

The longitudinal part of the equation at the lowest order H^0 coincides with the lowest-order part of the Eq. (31), so that a nontrivial part of the equation appears at the next order in H:

$$H\left\{(\alpha+\gamma)\Delta T-(\alpha-3\gamma)\partial_0\partial_0 T+\frac{\alpha}{\gamma}(\alpha+5\gamma)\partial_0 W\right\}$$
$$+\mathcal{O}(H^{n\geq 2})=0, \tag{35}$$

where the field W corresponds to a scalar irreducible part of $q_{0\gamma\delta}$,

$$W\equiv\frac{1}{2}\epsilon_{\alpha\beta\gamma}\partial_\alpha q_{0\beta\gamma}=\frac{2\gamma}{\alpha+\gamma}\partial_\alpha Q_\alpha+\mathcal{O}(H).$$

As we will see below, the fields T and W represent propagating scalar modes corresponding to the longitudinal field components S^{l}_α, Q^{l}_α. Notice that one has arbitrariness in choosing a set of independent field variables in the theory. The Eq. (35) contains fields $\overset{\top}{S}$, S^{l}_α which satisfy the gauge-fixing condition (27) including the field A^{l}_α. So, it is appropriate (and consistent with all other equations of motion) to treat the constraint (35) as a nonlinear equation for A^{l}_α.

Let us now consider the equation $\delta\mathcal{L}/\delta q_{\beta0\delta}$,

$$\begin{aligned}\frac{\delta\mathcal{L}}{\delta q_{\beta0\delta}}\equiv{}&\alpha\Delta q_{\beta0\delta}+\gamma\Delta(q_{\beta0\delta}-q_{\delta0\beta})-\alpha\partial_\mu\partial_\beta q_{\mu0\delta}-\alpha\partial_0\partial_0(q_{\beta0\delta}-q_{\delta0\beta})-\gamma\Delta q_{0\beta\delta}+\alpha\partial_0\partial_\beta q_{00\delta}-\alpha\partial_0\partial_\delta q_{00\beta}\\&-\alpha\partial_0\partial_\mu q_{\delta\mu\beta}+\gamma\partial_0\partial_\mu q_{(\mu\beta\delta)}+\gamma\partial_\mu(-\partial_\beta q_{\mu0\delta}+\partial_\delta q_{\mu0\beta}+\partial_\beta q_{\delta0\mu}-\partial_\delta q_{\beta0\mu})+\alpha\partial_\mu\partial_\delta q_{0\mu\beta}\\&-\gamma\partial_\mu\partial_\delta q_{0\mu\beta}+\gamma\partial_\mu\partial_\beta q_{0\mu\delta}+H\{2\alpha\epsilon_{\beta\delta\epsilon}\partial_0 q_{00\epsilon}-\alpha\epsilon_{\delta\epsilon\mu}\partial_0 q_{\epsilon\mu\beta}+\gamma\epsilon_{\delta\epsilon\mu}\partial_0 q_{(\epsilon\mu\beta)}\\&+2(\alpha+2\gamma)\epsilon_{\delta\mu\epsilon}\partial_\mu q_{\beta\epsilon\mu}+2\gamma\epsilon_{\delta\mu\epsilon}\partial_\mu q_{\epsilon0\beta}-(\alpha+2\gamma)\epsilon_{\delta\mu\phi}\partial_\beta q_{\mu\phi0}+(\alpha+2\gamma)\epsilon_{\beta\delta\epsilon}\partial_\mu q_{\mu\epsilon0}\\&+(\alpha-2\gamma)\epsilon_{\beta\delta\epsilon}\partial_\mu q_{0\epsilon\mu}+2\gamma\epsilon_{\beta\mu\epsilon}\partial_\mu q_{\delta0\epsilon}+2(\alpha-2\gamma)\epsilon_{\delta\mu\epsilon}\partial_\mu q_{0\epsilon\beta}+2\gamma\epsilon_{\beta\mu\epsilon}\partial_\mu q_{0\epsilon\delta}-\gamma\epsilon_{\delta\phi\mu}\partial_\beta q_{0\phi\mu}\}\\&+H^2\{-(3\alpha+2\gamma)q_{\beta0\delta}+(\alpha-2\gamma)\delta_{\beta\delta}q_{\nu0\nu}-\alpha q_{0\beta\delta}\}\\={}&0.\end{aligned} \tag{36}$$

To solve this equation for all its irreducible parts, one has to take into account terms up to order H^2 because some irreducible components of this equation vanish at the lower-order expansion in H. Let us start with the equation obtained by contraction with the antisymmetric tensor $\epsilon_{\alpha\beta\delta}$. This equation produces two constraints. The first one corresponds to the longitudinal projection of the contracted equation, and it can be simplified to the following constraint,

$$-2\gamma\partial_\alpha W-\alpha\partial_0\partial_\alpha\overset{\top}{S}+2\gamma\partial_0\partial_\alpha T+2(\alpha+2\gamma)\Delta Q^{\mathrm{l}}_\alpha$$
$$-4\alpha\partial_0\partial_0 Q^{\mathrm{l}}_\alpha+\mathcal{O}(H^{n\geq 1})=0. \tag{37}$$

This equation provides a propagation equation for Q^{l}_α. The transverse part of the antisymmetrized Eq. (36) vanishes at the lowest order H^0 and leads to a nonlinear relationship between fields R^{tr}_δ and A^{tr}_δ,

$$H\partial_0 A^{\mathrm{tr}}_\alpha+H^2\epsilon_{\alpha\beta\delta}\partial_\beta R^{\mathrm{tr}}_\delta-\left(2+\frac{\gamma}{\alpha}\right)H^2 Q^{\mathrm{tr}}_\alpha+\mathcal{O}(H^{n\geq 3})=0. \tag{38}$$

Let us now consider the Eq. (36) symmetrized over its indices. The divergence of the symmetrized part $\partial_\beta\delta\mathcal{L}/\delta q_{\{\beta0\delta\}}$ implies two equations. First, one does not vanish only at order H^1, and it produces the same constraint as (37). The second equation is

$$(2\alpha+3\gamma)H\Delta Q^{\mathrm{tr}}_\delta-2(3\alpha+\gamma)H^2(\epsilon_{\delta\gamma\rho}\partial_\gamma Q^{\mathrm{tr}}_\rho+\Delta R^{\mathrm{tr}}_\delta)$$
$$+\mathcal{O}(H^{n\geq 3})=0. \tag{39}$$

Because of the relationship (38) between the fields R^{tr}_α and A^{tr}_α, the last constraint implies, in general, a vanishing condition for both fields R^{tr}_α, A^{tr}_α and absence of any propagating modes in the model. There is only one special case where our model admits dynamical vector field, namely, we choose a condition on the parameters

$$\gamma=-3\alpha \tag{40}$$

that excludes the field R^{tr}_α from the equation. With this, the field A^{tr}_α remains dynamical. Our careful analysis shows that this condition is consistent with all other equations of motion and with Noether identities. Notice the constraint on the parameters is exactly the same as in the case of the model of the gravity with contortion in the presence of constant curvature space-time background [13]. This is an unexpected result because we have different equations of motion in the models with flat and nonflat metric.

At this moment, we can choose the remaining three gauge-fixing conditions in a suitable manner. From the last constraint and previous solutions to the equations of motion, one can verify that the fields Q^{tr}_δ and R^{l}_δ do not affect the solution structure in principle. It is convenient to choose vanishing conditions for Q^{tr}_δ and R^{l}_δ that are consistent with equations of motion and simplify further calculations. So, from now on we impose the gauge-fixing conditions

$$Q^{\mathrm{tr}}_\delta=0,\qquad R^{\mathrm{l}}_\delta=0. \tag{41}$$

With the previously imposed gauge conditions (26), the Lorentz gauge symmetry has been fixed completely.

The remaining equation corresponding to the traceless and transverse part of the equation $\delta\mathcal{L}/\delta q_{\{\beta 0\delta\}}$ gives a relationship for spin two modes

$$\Delta \overset{\top\top}{R}_{\beta\delta} = \tfrac{1}{2}\partial_0(\epsilon_{\alpha\delta\rho}\partial_\alpha \overset{\top\top}{S}_{\beta\rho} + \epsilon_{\alpha\beta\rho}\partial_\alpha \overset{\top\top}{S}_{\delta\rho}) + \mathcal{O}(H). \tag{42}$$

The last equation of motion is given by $\frac{\delta\mathcal{L}}{\delta q_{\beta\gamma\delta}}$. It is convenient to rewrite this equation in a dual form

$$\begin{aligned}\Phi_{\alpha\beta} &\equiv \epsilon_{\alpha\gamma\delta}\frac{\delta\mathcal{L}}{\delta q_{\beta\gamma\delta}} \\ &= \alpha\Box\overset{*}{q}_{\beta\alpha} + \gamma\delta_{\alpha\beta}\overset{*}{q}_{\nu\nu} + (\gamma-\alpha)\partial_\beta\partial_\mu\overset{*}{q}_{\mu\alpha} - \gamma(\Delta\overset{*}{q}_{\beta\alpha} - \partial_\alpha\partial_\rho\overset{*}{q}_{\beta\rho} + \delta_{\alpha\beta}\partial_\mu\partial_\nu\overset{*}{q}_{\mu\nu} + \partial_\alpha\partial_\beta\overset{*}{q}_{\nu\nu}) \\ &\quad - (\alpha-\gamma)\epsilon_{\alpha\gamma\rho}\partial_\gamma\partial_\mu q_{\rho\mu\beta} + \alpha\epsilon_{\alpha\gamma\rho}\partial_0\partial_\gamma q_{\rho 0\beta} - 2\gamma\partial_0(\delta_{\alpha\beta}\partial_i Q_i - \partial_\beta Q_\alpha) - 2\gamma\partial_0\partial_\beta Q_\alpha \\ &\quad + \frac{1}{2}(\alpha+\gamma)\epsilon_{\alpha\gamma\delta}\partial_0\partial_\beta q_{0\gamma\delta} + \gamma\epsilon_{\alpha\gamma\rho}\partial_0\partial_\gamma q_{0\rho\beta} + H\{2\gamma\partial_\mu q_{\mu\alpha\beta} - (\alpha+2\gamma)\partial_\mu q_{\beta\alpha\mu} - \alpha\partial_\beta q_{\nu\nu\alpha} \\ &\quad + 2(\alpha-\gamma)\partial_\alpha q_{\nu\nu\beta} - \alpha\partial_\mu q_{\alpha\mu\beta} - \alpha\partial_0 q_{0\alpha\beta} + (\alpha-2\gamma)\delta_{\alpha\beta}\partial_0 q_{\nu\nu 0} - \alpha\partial_0 q_{\beta\alpha 0} - (\alpha-2\gamma)\delta_{\alpha\beta}\partial_\mu q_{\nu\nu\mu} \\ &\quad + 2\gamma\partial_\mu(\epsilon_{\mu\beta\alpha}\overset{*}{q}_{\nu\nu} - \epsilon_{\mu\beta\gamma}\overset{*}{q}_{\gamma\alpha})\} + H^2\{4\gamma(\overset{*}{q}_{\beta\alpha} - \overset{*}{q}_{\alpha\beta}) - (4\gamma+\alpha)\epsilon_{\alpha\beta\gamma}q_{\nu\nu\gamma}\},\end{aligned} \tag{43}$$

where $\overset{*}{q}_{\beta\alpha} \equiv \frac{1}{2}\epsilon_{\alpha\gamma\delta}q_{\beta\gamma\delta}$. The trace part of the equation, $\Phi_{\alpha\alpha}$, yields an equation that can be simplified using the condition $\gamma = -3\alpha$,

$$-4\Box\partial_\alpha S_\alpha + 2\Delta\partial_\alpha S_\alpha - 3\partial_0\partial_\alpha Q_\alpha + \mathcal{O}(H) = 0. \tag{44}$$

The Eqs. (44) and (37) imply that the longitudinal components of the vector fields S_α, Q_α become propagating. Defining a scalar field corresponding to the longitudinal component of S_α,

$$\psi = -\tfrac{2}{3}\partial_\alpha S_\alpha, \tag{45}$$

one can rewrite the equations of motion as follows,

$$\begin{aligned}&\Box\partial_\alpha Q_\alpha + \Delta(\partial_\alpha Q_\alpha + \partial_0\psi) + \mathcal{O}(H) = 0,\\ &\Box\psi - \partial_0(\partial_\alpha Q_\alpha + \partial_0\psi) + \mathcal{O}(H) = 0.\end{aligned} \tag{46}$$

Explicit expressions for propagating solutions to these equations will be given in the next section.

The remaining equations of motion corresponding to the vector irreducible parts of $\Phi_{\alpha\beta}$ do not produce new independent equations. The irreducible part of the equation $\epsilon_{\delta\alpha\beta}\Phi_{\alpha\beta}$ coincides with (33). The divergence of the Eq. (43), $\partial_\alpha\Phi_{\alpha\beta}$, reproduces the same propagating equation for A^{tr}_μ as in (34) and the constraint (37). The divergence of the Eq. (43) with respect to the second index, $\partial_\beta\Phi_{\alpha\beta}$, reflects the Noether identity structure. One can verify that the transverse part of this equation leads to a nontrivial equation at order $H^{n\geq 1}$,

$$\alpha H\{\Delta A^{\mathrm{tr}}_\alpha - H\epsilon_{\alpha\beta\gamma}\partial_0\partial_\beta R^{\mathrm{tr}}_\delta\} + \mathcal{O}(H^{n\geq 3}) = 0, \tag{47}$$

which is consistent with the constraint (38). The longitudinal part of the equation $\partial_\beta\Phi_{\alpha\beta}$ can be simplified by using the constraint (31),

$$H\frac{\partial_0\partial_0}{\Delta}\partial_\alpha A_\alpha + \mathcal{O}(H^{\geq 2}) = 0. \tag{48}$$

The component field A^{l}_α has been already defined by the Eq. (35). The Eq. (48) does not represent a new independent equation but reflects the structure of the solution of (35). Namely, the equation contains a second-order time derivative that indicates the possibility of the existence of wavelike (soliton) solutions for A^{l}_α in the full nonlinear theory beyond the linearized approximation given by decomposition (15).

The last irreducible component of the equation $\Phi_{\alpha\beta}$ is given by its symmetric traceless part. Substituting the irreducible field $\overset{\top\top}{R}_{\alpha\beta}$ from (42) and using a useful identity

$$\overset{\top\top}{S}_{\alpha\beta} + \epsilon_{\alpha\gamma\delta}\epsilon_{\beta\nu\rho}\frac{\partial_\gamma\partial_\nu}{\Delta}\overset{\top\top}{S}_{\delta\rho} = 0, \tag{49}$$

one results in the following equation at order H^2,

$$H^2\bigg\{(\alpha+\gamma)\frac{\partial_0\partial_0}{\Delta}\overset{\top\top}{S}_{\alpha\beta}\bigg\} + \mathcal{O}(H^{n\geq 3}) = 0. \tag{50}$$

The equation contains second-order time derivative, which means there might be a spin-two propagating-solution-like soliton due to nonlinearity of the initial equations of motion. The difference of the equations of motion for $\overset{\top\top}{S}_{\alpha\beta}$ in the case of constant torsion background and in the case of the gravitational space-time background [13] is that Eq. (50) does not represent a standard D'Alembert equation due to the absence of a term proportional to $H^2\overset{\top\top}{S}_{\alpha\beta}$ that would produce the D'Alembert equation.

Finally, we have demonstrated that the Lorentz gauge theory with Lagrangian (13) with parameters $\gamma = -3\alpha$, $\beta = 0$ admits two transverse propagating modes for the vector field A^{tr}_α and two scalar propagating modes Q^{l}_α, S^{l}_α. The spin-one mode A^{l}_α and spin-two mode $\overset{\top\top}{S}_{\alpha\beta}$ might have propagating modes only due to nonlinear structure of full equations of motion. Our result that the Lagrangian has exactly the same structure, $\gamma = -3\alpha$, $\beta = 0$, as the Lagrangian for the gravity with torsion in the presence of the background metric [13] confirms that the propagating spin-one mode exists independently on the background metric at hand and it is a feature of the Lorentz gauge model itself.

IV. POSITIVE DEFINITENESS OF THE HAMILTONIAN

Lorentz gauge theories with quadratic R^2-type Lagrangian suffer from the nonpositiveness problem of the Hamiltonian, which has its origin in the noncompact structure of the Lorentz group. This leads to the problem of defining a stable vacuum in quantum theory. Let us consider this problem starting with the free Lagrangian (13). Using solutions from the previous section, one can express all components of contortion q_{acd} in terms of three independent fields $A^{\rm tr}_\alpha$, $S^{\rm l}_\alpha$, and $Q^{\rm l}_\alpha$,

$$\mathcal{L}^{(2)} = \tfrac{1}{2}[A^{\rm tr}_\alpha \Box A^{\rm tr}_\alpha + (\partial_\alpha Q^{\rm l}_\alpha + \partial_0 \psi)^2 + \psi \Box \psi - Q^{\rm l}_\alpha \Box Q^{\rm l}_\alpha]. \tag{51}$$

Since the vector fields $A^{\rm tr}_\alpha$ and $S^{\rm tr}_\alpha$ are related by the Eq. (28), one can treat the scalar field $\psi = -\frac{2}{3}\partial_\alpha S_\alpha$ as a longitudinal component of S_α, or as a dual longitudinal component of the field $A^{\rm tr}_\alpha$. The field $Q^{\rm l}_\alpha$ originates from the contortion part $q_{\alpha 0 \delta}$, which corresponds to boost generators of the Lorentz group. The terms with $Q^{\rm l}_\alpha$ in the Lagrangian are potentially dangerous since they may give negative energy contribution destabilizing the vacuum. We concentrate on a part of the total Hamiltonian that includes the scalar modes ψ and $Q^{\rm l}_\alpha$. The Hamiltonian is defined in a standard manner

$$\mathcal{H}(Q^{\rm l}_\alpha, \psi) = \tfrac{1}{4}(\pi - \partial_\alpha Q^{\rm l}_\alpha)^2 - \tfrac{1}{2}\pi_\alpha^2 + \tfrac{1}{2}(\partial_\alpha \psi)^2 - (\partial_\alpha Q^{\rm l}_\alpha)^2, \tag{52}$$

where canonical momentums π and π_α are defined by

$$\pi = \frac{\partial \mathcal{L}}{\partial \partial_0 \psi} = 2\partial_0 \psi + \partial_\alpha Q^{\rm l}_\alpha, \qquad \pi_\alpha = \frac{\partial \mathcal{L}}{\partial \partial_0 Q^{l}_\alpha} = -\partial_0 Q^{\rm l}_\alpha. \tag{53}$$

Notice that the fields ψ and $Q^{\rm l}_\alpha$ have correct canonical dimension and they are treated as initial independent field variables. We will solve the Euler-Lagrange equations of motion for the fields ψ, $Q^{\rm l}_\alpha$, (46), in lowest-order approximation. For convenience, let us rewrite the equations (46) in the following form,

$$2\partial_0^2 \psi - \Delta \psi + \partial_0 \partial_\alpha Q^{\rm l}_\alpha = 0, \qquad \partial_0^2 Q^{\rm l}_\alpha - 2\Delta Q^{\rm l}_\alpha - \partial_0 \partial_\alpha \psi = 0. \tag{54}$$

The system of equations (54) cannot be factorized into decoupled equations. Let us consider possible solutions in the form of plane waves

$$\psi(k) = b(k) e^{i(-\vec{k}\vec{x} + k_0 t)}, \qquad Q^{\rm l}_\alpha(k) = c_\alpha(k) e^{i(-\vec{k}\vec{x} + k_0 t)}, \tag{55}$$

where $\vec{k}\vec{x} = k_\alpha x_\alpha$. Substitution of the plane waves into (54) gives a system of homogeneous equations that has a nontrivial solution if the following characteristic equation is satisfied,

$$(k_0^2 - \vec{k}^2)^2 = 0. \tag{56}$$

The equation is degenerated and it implies the dispersion relationship

$$k_0 = \pm \omega, \quad \text{with} \quad \omega \equiv \sqrt{\vec{k}^2}. \tag{57}$$

The coefficient functions b, c_α are related by the following equation,

$$c_\alpha(k) = \frac{k_0 k_\alpha}{\omega^2} b(k). \tag{58}$$

The corresponding solution for ψ, $Q^{\rm l}_\alpha$ can be written as a sum of positive and negative frequency modes

$$\begin{aligned}\psi(\vec{x}, t) &= \int \frac{d^3\vec{k}}{(2\pi)^4} b^+(\vec{k}) e^{i(-\vec{k}\vec{x} + \omega t)} + \int \frac{d^3\vec{k}}{(2\pi)^4} b^-(\vec{k}) e^{-i(\vec{k}\vec{x} + \omega t)}, \\ \vec{Q}^{\rm l}(\vec{x}, t) &= \int \frac{d^3\vec{k}}{(2\pi)^4} \frac{b^+(\vec{k})\vec{k}}{\omega} e^{i(-\vec{k}\vec{x} + \omega t)} - \int \frac{d^3\vec{k}}{(2\pi)^4} \frac{b^-(\vec{k})\vec{k}}{\omega} e^{-i(\vec{k}\vec{x} + \omega t)}.\end{aligned} \tag{59}$$

Using the solutions and calculating the canonical momentums π and π_α, one can easily check the identities

$$\tfrac{1}{4}(\pi - \partial_\alpha Q^{\rm l}_\alpha)^2 - (\partial_\alpha Q^{\rm l}_\alpha)^2 = 0, \qquad -\tfrac{1}{2}\pi_\alpha^2 + \tfrac{1}{2}(\partial_\alpha \psi)^2 = 0, \tag{60}$$

which imply immediately that the Hamiltonian (52) vanishes identically.

Since the Eq. (56) is degenerated, the general solution to the equations of motion (54) includes another couple of wavelike solutions. Fourier modes of the solutions can be found in the form that is suitable in further making Lorentz-invariant decomposition into positive and negative frequency parts

$$\psi(k) = (\vec{k}\vec{x} + k_0 t) a(k) e^{i(-\vec{k}\vec{x} + k_0 t)}, \qquad \vec{Q}^{\rm l}(k) = ((\vec{k}\vec{x} + k_0 t)\vec{a}(k) + i\vec{d}(k)) e^{i(-\vec{k}\vec{x} + k_0 t)}. \tag{61}$$

Substituting this ansatz into equations of motion produces the same dispersion relation (56) and the following relations for the coefficient functions,

$$\vec{a} = \frac{k_0 \vec{k}}{\omega^2} a, \qquad \vec{d} = -6\vec{a} = -\frac{6 k_0 \vec{k}}{\omega^2} a. \tag{62}$$

The general solution for ψ and $\vec{Q}^{\rm l}$ can be represented as Fourier integral over all momentum $\vec{k}$, k_0. Performing integration over k_0 using the dispersion relation (57) leads to the final expressions

$$\psi(\vec{x},t) = \int \frac{d^3\vec{k}}{(2\pi)^4}(\vec{k}\vec{x}+\omega t)a^+(\vec{k})e^{i(-\vec{k}\vec{x}+\omega t)} + \int \frac{d^3\vec{k}}{(2\pi)^4}(\vec{k}\vec{x}-\omega t)a^-(\vec{k})e^{-i(\vec{k}\vec{x}+\omega t)},$$
$$\vec{Q}^1(\vec{x},t) = \int \frac{d^3\vec{k}}{(2\pi)^4}((\vec{k}\vec{x}+\omega t)-6i)\frac{\vec{k}}{\omega}a^+(\vec{k})e^{i(-\vec{k}\vec{x}+\omega t)} - \int \frac{d^3\vec{k}}{(2\pi)^4}((\vec{k}\vec{x}-\omega t)-6i)\frac{\vec{k}}{\omega}a^-(\vec{k})e^{-i(\vec{k}\vec{x}+\omega t)}. \tag{63}$$

As usual, the Fourier functions $a^\pm(\vec{k})$, $b^\pm(\vec{k})$ turn into creation and annihilation operators during quantization procedure. It is convenient to split the Hamiltonian $\mathcal{H}(Q^1_\alpha, \psi)$ into two parts

$$\mathcal{H} = \mathcal{H}_1 + \mathcal{H}_2,$$
$$\mathcal{H}_1 \equiv \tfrac{1}{4}(\pi - \partial_\alpha Q^1_\alpha)^2 - (\partial_\alpha Q^1_\alpha)^2, \tag{64}$$
$$\mathcal{H}_2 \equiv -\tfrac{1}{2}\pi^2_\alpha + \tfrac{1}{2}(\partial_\alpha\psi)^2.$$

This allows to separate contributions P_{01}, P_{02} of the fields Q^1_α, ψ to the total energy functional

$$P_0 = \int d^3x\mathcal{H} = P_{01} + P_{02}. \tag{65}$$

Substituting the solution (63) into the last equation and performing integration over configuration space $\vec{x}$ and one of two momentum $\vec{k}$, $\vec{k}'$ corresponding to Fourier components of ψ, $\vec{Q}^1$, one can verify that the contributions from the fields Q^1_α and ψ are mutually canceled due to following relations,

$$P^{+-}_{01} = \int \frac{d^3\vec{k}}{(2\pi)^4}48\omega^2 a^+(\vec{k})a^-(-\vec{k}),$$
$$P^{+-}_{02} = -P^{+-}_{01},$$
$$P^{++}_{01} = -\int \frac{d^3\vec{k}}{(2\pi)^4}(8\omega^2(3+i\omega t))a^+(\vec{k})a^+(-\vec{k})e^{2i\omega t},$$
$$P^{++}_{02} = -P^{++}_{01},$$
$$P^{--}_{01} = -P^{--}_{02}. \tag{66}$$

So, the total contribution of the scalar modes to the energy functional vanishes identically.

It is worth stressing that the mutual exact cancellation of all contributions of scalar modes in the energy functional is not occasional. This indicates the presence of an additional symmetry in the defining equations (54). It is easy to see such a symmetry in a simple case of 1 + 1-dimensional space-time. After changing variable $\partial_x Q^1_x \to \partial_0\chi$, the system of equations (54) can be rewritten in the form

$$2\partial_0^2\psi - \partial_x^2\psi + \partial_0\partial_0\chi = 0,$$
$$\partial_0^2\chi - 2\partial_x^2\chi - \partial_x^2\psi = 0. \tag{67}$$

It is clear that the system is invariant under the following symmetry transformations,

$$x \leftrightarrow \pm t, \qquad \psi \leftrightarrow \pm\chi. \tag{68}$$

Because of this, energy contributions of scalar modes in (51) are mutually canceled. We expect that in 3 + 1 dimensions there should be a similar symmetry that provides the positive-definite energy on mass shell.

V. DISCUSSION

We have studied the dynamic content of the class of Lorentz gauge theories admitting topological phase in the gravitational sector. It has been shown that in the special choice of the parameters $\alpha = 1$, $\beta = 0$, $\gamma = -3$ the corresponding model possesses dynamical contortion. Surprisingly, the existence of propagating modes for spin-one and zero-contortion component fields is provided by the same Lagrangian in both cases, in presence of constant gravitational background and in presence of constant contortion background field. Additional spin-one and spin-two propagating modes may appear only due to full nonlinear structure of the equation of motion. At the lowest order of perturbation theory, we have proved that the Hamiltonian is positively defined. This implies that perturbative quantization can be performed straightforward. In practical calculation, it is much more convenient to use the covariant quantization formalism based on functional integral. The quantization can be performed straightforward in a similar manner as in [13]. It has been proved that quantum gravity model with a general R^2-type Lagrangian is renormalizable [47–50]. Since the initial Lagrangian (13) is expressed in terms of gauge-invariant tensors and there is no dimensional coupling constants, the proposed model of Lorentz gauge gravity belongs to renormalizable type.

The important question is whether our model leads to a quantum vacuum condensate of torsion that can provide generation of the Einstein term in the effective action of gravity. This mechanism is similar to dynamical symmetry breaking in quantum chromodynamics where one has a gluon condensate while the gluon itself is not observable at classical level. The possibility that torsion may not be observable as a classical object was pointed out in [51]. Generation of the vacuum torsion condensate due to appearance of a nontrivial minimum in the quantum effective potential would lead to an effective Einstein gravity. Suppose the vacuum condensate has a Lorentz-invariant form $\langle\mathcal{R}_{abcd}\rangle = M^2(\eta_{ac}\eta_{bd} - \eta_{ad}\eta_{bc})$. Substituting it into the initial classical Lagrangian (13), one can obtain the lowest-order terms in the effective Lagrangian of gravity

$$\mathcal{L}_{\rm eff} = -\tfrac{3}{4}M^4 + \tfrac{3}{8}M^2\hat{R} - \tfrac{1}{32}(\hat{R}^2_{abcd} - 4\hat{R}^2_{ab} + \hat{R}^2) + \mathcal{O}(\hat{R}^{n\geq 3}), \qquad (69)$$

where the terms quadratic in Riemann curvature represent the integral density for the Euler characteristic

$$\chi = \frac{1}{8\pi^2}\int d^4x\sqrt{-g}(\hat{R}^2_{abcd} - 4\hat{R}^2_{ab} + \hat{R}^2). \qquad (70)$$

To provide the correct sign of the Einstein term, the condensate parameter M^2 should be negative. This is opposite to the case of the gravity model with Yang-Mills-type Lagrangian [34] where the Einstein-Hilbert term and cosmological constant are induced when the torsion condensate corresponds to a positive constant Riemann-Cartan curvature, i.e., $M^2 > 0$. Notice that the cosmological term proportional to M^4 is reproduced with a correct sign. Another feature of our model is that the Euler characteristic enters the effective Lagrangian with a negative sign. The corresponding vacuum-to-vacuum transition amplitude is proportional to (in Euclidean space-time)

$$\langle 0|0\rangle \simeq {\rm e}^{-S_E} = {\rm e}^{+(\pi^2/4)\chi}. \qquad (71)$$

It is reasonable to consider summation over all topologies of the four-dimensional manifolds described by fiber bundles with a compact two-dimensional base space. In that case, the Euler characteristic is determined by the genus g of the base space, $\chi = 2 - 2g$, and the total vacuum-vacuum amplitude remains finite after summation over all topologies.

The possibility that the Lorentz gauge gravity may have a positive-definite classical Hamiltonian bounded from below implies that torsion can be observable not only in the form of quantum vacuum condensate but also in the form of a classical configuration. This implies an attractive possibility that torsion can be responsible for the cold dark matter since it does not interact to photon in a minimal interaction scheme. The quantum properties and possible physical implications of our model will be considered in a separate paper.

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