Deciphering nonfemtoscopic two-pion correlations in $p + p$ collisions with simple analytical models

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A simple model of nonfemtoscopic particle correlations in proton-proton collisions is proposed. The model takes into account correlations induced by the conservation laws as well as correlations induced by minijets. It reproduces well the two-pion nonfemtoscopic correlations of like-sign and unlike-sign pions in proton-proton collision events at $\sqrt{s} = 900$ GeV analyzed by the ALICE Collaboration. We also argue that similar nonfemtoscopic correlations can appear in the hydrodynamic picture with event-by-event fluctuating nonsymmetric initial conditions that are typically associated with nonzero higher-order flow harmonics.

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I. INTRODUCTION

The method of identical particle correlation femtoscopy gives us the possibility to measure length and time scales with an accuracy of more than 10^{-14} m and 10^{-22} sec, respectively. It is widely used now for the determination of sizes and lifetimes of the sources of particle emission such as the systems created in heavy ion, hadron, and lepton collisions (for reviews see, e.g., Ref. [1]). The method is grounded on the Bose-Einstein or Fermi-Dirac symmetry properties of quantum states and, in fact, measures correlations between numbers of identical particles with close energy and momentum.

In the method of correlation femtoscopy the space-time structure of the systems is usually represented in terms of the interferometry radii that are the result of a Gaussian fit of a two-particle correlation function depending on the momentum difference in the pair. For bosons such correlations, reflecting spatiotemporal scales of an extended source, are caused by the quantum Bose-Einstein statistics. These correlation functions can be obtained from the ratio of the two- (identical) particle momentum spectra to the product of the single-particle ones. The former typically need to be corrected for the Coulomb and strong final state interactions (FSI) that depend on space-time points of the last collisions for the detected pair (see, e.g., the recent review [2] and references therein). In pioneering papers [3] the measured interferometry radii were interpreted as geometrical sizes of the systems. Later on it was found [4,5] that because the typical systems formed in experiments with heavy ion collisions are expanding and have inhomogeneous structures, the above geometrical interpretation is not complete, and the interpretation of interferometry radii as homogeneity lengths in the systems was proposed [6]. It was demonstrated that the homogeneity lengths carry information about the dynamical properties (rate of expansion, lifetime, etc.) of the source and depend on the mean momentum of pairs [5,6]. In particular, it is firmly established that specific transverse momentum dependence of femtoscopy scales—interferometry radii—in heavy ion collisions is mostly caused by a collective (hydrodynamical) transverse expansion of the systems formed in these collisions [1].

The situation is more complicated, and the above method has to be modified for elementary particle collisions, like $p + p$, which have smaller spatiotemporal scales as compared to heavy ion collisions. It became clear [7–9] that for relatively small systems the additional two-particle correlations affect the correlation functions in the kinematic region where quantum statistical (QS) and FSI correlations are usually observed. The well-known example of such additional correlations is the correlation induced by total energy and momentum conservation laws (see, e.g., Ref. [10]). As opposed to the QS and FSI correlations, which are familiar from the correlation femtoscopy method and so are sometimes called femtoscopy correlations, these correlations are not directly related to the spatiotemporal scales of the emitter and are therefore called nonfemtoscopic correlations. Since the latter noticeably affect correlation functions for small systems, the interferometry radii extracted from the complete correlation function in $p + p$ collisions depend strongly on the assumption about the so-called correlation baseline—the strength and momentum dependence of the nonfemtoscopic correlations [7–9]. It has an influence on the interpretation of the momentum dependence of the interferometry radii in $p + p$ collisions, where the possibility of hydrodynamic behavior of matter is questionable. Therefore, for successful and unambiguous applications of the correlation femtoscopy method to elementary particle collisions, one needs to know the mechanisms of nonfemtoscopic correlations to separate the femtoscopic and nonfemtoscopic correlations.

Recently, the ALICE Collaboration utilized some event generators, which do not include effects of quantum statistics, for an estimate of the correlation baseline (i.e., nonfemtoscopic correlation function of identical pions) under the Bose-Einstein peak at LHC energies [7,8].

It was motivated by a reasonable agreement of the corresponding event generator simulations with the experimental data for correlation functions of oppositely charged pions in $p + p$ collisions at the same energy [7,8]. The calculated correlation baseline has been utilized by the ALICE Collaboration to extract femtoscopic correlations from measured identical pion two-particle correlation functions [7].

Because the utilized event generators account for energy-momentum conservation and emission of minijets, it was conjectured in Refs. [7,8] that some specific peculiarities of the unlike-sign pion correlations as well as like-sign nonfemtoscopic pion correlations can be caused by the jetlike and energy-momentum conservation induced correlations. In what follows, we support this conjecture. We develop a simple analytical model with the minimal number of parameters for the two-pion correlations induced by minijets and transverse momentum conservation law, and show that this model can fit the correlations of unlike-sign pion pairs at $\sqrt{s} = 900$ GeV $p + p$ collisions measured by the ALICE Collaboration [7]. Also, with a reasonable change of parameters, the model can fit nonfemtoscopic correlations of like-sign pion pairs obtained from the event generator simulations of $p + p$ collision events at the same energy that are reported in Ref. [7].[1] Our model is simple and analytical, and clearly demonstrates the interplay between minijet and conservation law induced correlations in the formation of the nonfemtoscopic correlations.

But this is not the whole story. Despite the fact that the event generators do reproduce the unlike pion correlations as has been verified by the ALICE Collaboration, there is enough room to doubt whether the nonfemtoscopic correlations of like-sign pions are properly simulated by them. Indeed, none of the utilized event generators can reproduce the LHC data on the multiplicities and momentum spectra well (see, e.g., [12] and references therein). This suggests that some essential ingredients may be missing in these event generators. The possible candidate for missed dynamics is hydrodynamics. The latter describes well the dynamics of heavy ion collisions; see, e.g., Ref. [13] and references therein. If hydrodynamics is applied also for $p + p$ collisions with high multiplicities [14], then the other mechanisms of nonfemtoscopic correlations should be taken into account. In what follows, we present some heuristic arguments, based on an illustrative analytical model, that unlike-sign two-pion correlation functions calculated in hydrodynamics with event-by-event asymmetrically fluctuating initial densities can be qualitatively similar at relatively low $q_{\rm inv}$ to the ones calculated in PHOJET-like event generators, where these correlations for relatively low $q_{\rm inv}$ are mainly caused by the minijets. Finally, we briefly discuss what these results can mean for modeling of the correlation baseline (i.e., identical pion nonfemtoscopic correlations) and for applications of the correlation femtoscopy method to $p + p$ collisions.

[1]For convenience, we compare results of our model with the PHOJET event generator [11] simulations reported in Ref. [7]. Note that the simulations carried out by the ALICE Collaboration gave similar results for all utilized event generators [7,8].

II. DEFINITIONS AND PARAMETRIZATIONS OF TWO-PARTICLE CORRELATIONS

The two-particle correlation function is defined as

$$C(p_1, p_2) = \frac{P(p_1, p_2)}{P(p_1)P(p_2)}, \tag{1}$$

where $P(p_1, p_2)$ is the probability of observing two particles with momenta $\mathbf{p}_1$ and $\mathbf{p}_2$, while $P(p_1)$ and $P(p_2)$ designate the single-particle probabilities. Experimentally, the two-particle correlation function is defined as the ratio of the distribution of particle pairs from the same collision event to the distribution of pairs with particles taken from different events. In heavy ion collisions almost all the correlations between identical pions with low relative momentum are due to quantum statistics and final state interactions. In this case

$$C(p_1, p_2) = C_{\rm F}(\mathbf{p}, \mathbf{q}), \tag{2}$$

where $\mathbf{p} = (\mathbf{p}_1 + \mathbf{p}_2)/2$, $\mathbf{q} = \mathbf{p}_2 - \mathbf{p}_1$, and C_F denotes the femtoscopic correlation function. For identical bosons C_F is often parametrized (after corrections for FSI correlations) by the Gaussian form, which for the one-dimensional parametrization looks like

$$C_{\rm F}(|\mathbf{p}|, q_{\rm inv}) = 1 + \lambda \exp(-R_{\rm inv}^2 q_{\rm inv}^2). \tag{3}$$

Here λ describes the correlation strength, $R_{\rm inv}$ is the Gaussian "invariant" interferometry radius, and $q_{\rm inv} = \sqrt{(\mathbf{p}_2 - \mathbf{p}_1)^2 - (E_2 - E_1)^2}$ is equal to the modulus of the three-momentum difference in the pair rest frame.

In elementary particle collisions additional (nonfemtoscopic) correlations, like those arising from jet/string fragmentation and from energy and momentum conservation (see, e.g., Refs. [7–9]), can also give a significant contribution. Then, assuming the factorization property,

$$C(p_1, p_2) = C_{\rm F}(\mathbf{p}, \mathbf{q}) C_{\rm NF}(\mathbf{p}, \mathbf{q}). \tag{4}$$

Here $C_{\rm NF}$ denotes the nonfemtoscopic correlation function, and in the simplest case the nonfemtoscopic effects can be parametrized as, e.g., second order polynomials,

$$C_{\rm NF}(|\mathbf{p}|, q_{\rm inv}) = a + b q_{\rm inv} + c q_{\rm inv}^2. \tag{5}$$

This form can be used together with some parametrization of $C_{\rm F}$ [e.g., with (3)] in order to fit the correlation function $C(p_1, p_2)$ for small systems, as has been done, for example, by the STAR Collaboration for two-pion correlation functions in $p + p$ collisions at $\sqrt{s} = 200$ GeV [9]. At $c > 0$ the phenomenological parametrization (5) explicitly reproduces the well-known effect of positive correlations between particles with large relative momenta $|\mathbf{q}|$ caused by the energy-momentum conservation laws; see the energy and momentum conservation-induced correlation model for $C_{\rm NF}$ [10]. Note that a, b, and c in Eq. (5) depend, in general, on $|\mathbf{p}|$.

III. PROBABILITY DENSITIES OF DISTINGUISHABLE EQUIVALENT PARTICLES

Aiming to estimate the nonfemtoscopic pion correlations, we consider emitted pions as distinguishable, yet equivalent noninteracting particles with symmetrical probability density functions, thereby excluding femtoscopic QS and FSI correlations. To a certain extent, this corresponds to the (quasi)classical approximation used in current event generators, like PHOJET.

Let us assume that N bosons of the same species (say, pions) are produced with momenta $\mathbf{p}_1, \ldots, \mathbf{p}_N$ in $(N + X)$ multiparticle production events. Then the N-particle probability density $P_N(p_1, \ldots, p_N)$ is a symmetrical function for all $N!$ permutations of the particle momenta p_i. For convenience, it is normalized by

$$\int d\Omega_p P_N(p_1, \ldots, p_N) = 1, \tag{6}$$

where $d\Omega_p = \frac{d^3p_1}{E_1} \ldots \frac{d^3p_N}{E_N}$. Then the N'-pion probability density, $N' < N$, is defined as

$$P_N(p_1, \ldots, p_{N'}) = \int d\Omega_{p^*} E_i^* \delta^{(3)}(\mathbf{p}_1 - \mathbf{p}_1^*) \ldots E_{N'}^* \times \delta^{(3)}(\mathbf{p}_{N'} - \mathbf{p}_{N'}^*) P_N(p_1^*, \ldots, p_N^*). \tag{7}$$

In what follows, we use an assumption of distinguishability of equivalent particles, which means the absence of quantum interference between possibilities corresponding to all $N!$ permutations of the particle momenta p_i. Therefore, the symmetrized N-particle probability density is defined as

$$P_N(p_1, \ldots, p_N) = \frac{1}{N!} \sum_{i \neq \ldots \neq k=1}^{N} \int d\Omega_{p^*} E_i^* \times \delta^{(3)}(\mathbf{p}_1 - \mathbf{p}_i^*) \ldots E_k^* \times \delta^{(3)}(\mathbf{p}_N - \mathbf{p}_k^*) \hat{P}_N(p_1^*, \ldots, p_N^*), \tag{8}$$

where the nonsymmetrized N-particle probability density $\hat{P}_N(p_1, \ldots, p_N)$ is normalized to unity, and $N!$ in the denominator is required to guarantee the normalization condition (6). Taking into account Eq. (7), we see that the single-particle probability $P_N(p_1)$ and the two-particle probability $P_N(p_1, p_2)$ can be written as

$$P_N(p_1) = \frac{1}{N} \sum_{i=1}^{N} \int d\Omega_{p^*} E_i^* \delta^{(3)}(\mathbf{p}_1 - \mathbf{p}_i^*) \hat{P}_N(p_1^*, \ldots, p_N^*), \tag{9}$$

$$P_N(p_1, p_2) = \frac{1}{N(N-1)} \sum_{i \neq j=1}^{N} \int d\Omega_{p^*} E_i^* \delta^{(3)}(\mathbf{p}_1 - \mathbf{p}_i^*) \times E_j^* \delta^{(3)}(\mathbf{p}_2 - \mathbf{p}_j^*) \hat{P}_N(p_1^*, \ldots, p_N^*). \tag{10}$$

The nonsymmetrized N-pion probability density in such events reads

$$\hat{P}_N(p_1, \ldots, p_N) = \frac{1}{K} \sum_X \int d\Omega_k \delta^{(4)}\Big(p_a + p_b - \sum_{i=1}^{N} p_i - \sum_{j=1}^{X} k_j\Big) \times |M_{N+X}(p_1, \ldots, k_X)|^2, \tag{11}$$

where $M_{N+X}(p_1, \ldots, k_X)$ is the nonsymmetrized $(N + X)$-particle production amplitude, p_a and p_b are four-momenta of colliding particles (protons in $p + p$ collision events), and K is the normalization factor,

$$K = \sum_X \int d\Omega_k d\Omega_p \delta^{(4)}\Big(p_a + p_b - \sum_{i=1}^{N} p_i - \sum_{j=1}^{X} k_j\Big) \times |M_{N+X}(p_1, \ldots, k_X)|^2. \tag{12}$$

Expression (11) for $\hat{P}_N(p_1, \ldots, p_N)$ is rather complicated because, in particular, it depends on X particles that are produced in addition to N pions. Production of additional particles also means that one can hardly expect that the total energy or momentum of the pion subsystem are constants in the system's center of mass; instead, one can expect that they fluctuate from event to event. Then, motivated by Eq. (11), we assume that a nonsymmetrized N-pion probability density can be written as

$$\hat{P}_N(p_1, \ldots, p_N) = \frac{1}{K} \delta(p_1, \ldots, p_N) F_N(p_1, \ldots, p_N), \tag{13}$$

where $F_N(p_1, \ldots, p_N)$ is a nonsymmetrized function of pionic momenta and $\delta(p_1, \ldots, p_N)$ denotes the average constraints on the N-pion states that appear due to energy and momentum conservation in multiparticle production events. Then the normalization factor is

$$K = \int d\Omega_p \delta(p_1, \ldots, p_N) F_N(p_1, \ldots, p_N). \tag{14}$$

If the only correlations are the correlations associated with energy and momentum conservation, we have

$$F_N(p_1, \ldots, p_N) = f(p_1) f(p_2) \ldots f(p_{N-1}) f(p_N), \tag{15}$$

and calculations in the large N limit of single-particle and two-particle probability densities result in the special case of the energy and momentum conservation-induced correlation parametrization [10] of the correlations induced by the energy and momentum conservation laws. However, such a simple prescription cannot result in the unlike-sign pion correlations measured by the ALICE Collaboration

[7]. Also, it does not reproduce the nonfemtoscopic like-sign pion correlations generated by the PHOJET event generator [7].

IV. ANALYTICAL MODEL FOR THE TWO-PION CORRELATIONS INDUCED BY MINIJETS AND MOMENTUM CONSERVATION

The ALICE Collaboration has analyzed the unlike-sign pion correlations [7] and found that they can be well reproduced by event generators which account for, among others factors, total energy-momentum conservation and minijet production. In what follows, we assume that just these two factors induce the observed behavior of the unlike-sign pion correlations and are responsible for the nonfemtoscopic correlations in like-sign ones. In our simple model we start from zero total transverse momentum of the system, keeping in mind that for a subsystem this statement should be weakened. Also, we neglect the constraints conditioned by the conservation of energy and longitudinal momentum, supposing that the system under consideration is an N-pion subsystem in a small midrapidity region of the total system. Then in the first approximation

$$\delta(p_1, \dots, p_N) = \delta^{(2)}(\mathbf{p}_{T1} + \mathbf{p}_{T2} + \dots + \mathbf{p}_{TN}), \qquad (16)$$

where $\mathbf{p}_{T1}, \mathbf{p}_{T2}, \dots \mathbf{p}_{TN}$ are transverse components of the momenta of the N particles.

Let us assume that there are no other correlations in the production of N-pion states except the correlations induced by the transverse momentum conservation and cluster (minijet) structures in momentum space. For the sake of simplicity we assume here that only the two-particle clusters appear. Then one can write for fairly large $N \gg 1$

$$F_N(p_1, \dots, p_N) = f(p_1)\dots f(p_N)Q(p_1, p_2)\dots Q(p_{N-1}, p_N), \qquad (17)$$

where $Q(p_i, p_j)$ denotes the jetlike correlations between momenta $\mathbf{p}_i$ and $\mathbf{p}_j$; the existence of such correlations means that F_N cannot be expressed as a product of one-particle distributions. Then, utilizing the integral representation of the δ-function by means of the Fourier transformation, $\delta^{(2)}(\mathbf{p}_T) = (2\pi)^{-2} \int d^2 r_T \exp(i\mathbf{r}_T\mathbf{p}_T)$, and accounting for Eqs. (9), (13), (16), and (17), the single-particle probability reads

$$P_N(p_1) = \frac{1}{(2\pi)^2 K} \int d^2 r_T G_N(\mathbf{p}_1, \mathbf{r}_T), \qquad (18)$$

where

$$G_N(\mathbf{p}_1, \mathbf{r}_T) = \int d\Omega_{p^*} E_1^* \delta^{(3)}(\mathbf{p}_1 - \mathbf{p}_1^*) e^{i\mathbf{r}_T(\mathbf{p}_{T1}^* + \dots + \mathbf{p}_{TN}^*)} \times F_N(p_1^*, \dots, p_N^*). \qquad (19)$$

A possibility of different cluster configurations of particles means, in particular, that registered particles with momenta $\mathbf{p}_1$ and $\mathbf{p}_2$ can belong either to different minijets or to the same minijet. Then, taking into account Eqs. (10), (13), (16), and (17), we get

$$P_N(p_1, p_2) = \frac{N}{N(N-1)} P_N^{1\text{jet}}(p_1, p_2) + \frac{N(N-1) - N}{N(N-1)} P_N^{2\text{jet}}(p_1, p_2), \qquad (20)$$

where

$$P_N^{1\text{jet}}(p_1, p_2) = \frac{1}{(2\pi)^2 K} \int d^2 r_T G_N^{1\text{jet}}(\mathbf{p}_1, \mathbf{p}_2, \mathbf{r}_T), \qquad (21)$$

$$P_N^{2\text{jet}}(p_1, p_2) = \frac{1}{(2\pi)^2 K} \int d^2 r_T G_N^{2\text{jet}}(\mathbf{p}_1, \mathbf{p}_2, \mathbf{r}_T), \qquad (22)$$

and

$$G_N^{1\text{jet}}(\mathbf{p}_1, \mathbf{p}_2, \mathbf{r}_T) = \int d\Omega_{p^*} E_i^* \delta^{(3)}(\mathbf{p}_1 - \mathbf{p}_1^*) E_j^* \times \delta^{(3)}(\mathbf{p}_2 - \mathbf{p}_2^*) e^{i\mathbf{r}_T(\mathbf{p}_{T1}^* + \dots + \mathbf{p}_{TN}^*)} F_N, \qquad (23)$$

$$G_N^{2\text{jet}}(\mathbf{p}_1, \mathbf{p}_2, \mathbf{r}_T) = \int d\Omega_{p^*} E_i^* \delta^{(3)}(\mathbf{p}_1 - \mathbf{p}_1^*) E_j^* \times \delta^{(3)}(\mathbf{p}_2 - \mathbf{p}_3^*) e^{i\mathbf{r}_T(\mathbf{p}_{T1}^* + \dots + \mathbf{p}_{TN}^*)} F_N. \qquad (24)$$

Here $F_N \equiv F_N(p_1^*, \dots, p_N^*)$. The first term on the right-hand side of Eq. (20) is associated with events where the two registered particles belong to the same minijet, and the second term corresponds to events where the particles are from different minijets. Evidently, the former is relatively rare; however, notice that the first term can be significant for small systems with not very large N.

Now let us check whether this model can reproduce, with reasonable parameters, the correlation functions of unlike-sign pions measured by the ALICE Collaboration [7] and nonfemtoscopic correlations of like-sign pions that are generated in the PHOJET simulations and utilized as the correlation baseline by the ALICE Collaboration [7]. Calculations within the model will deliberately be as simple as possible just to demonstrate its viability. Here we do not use the approximate methods like the saddle point approach; instead, we utilize appropriate analytical parametrizations of the functions of interest, namely,

$$f(p_i) = E_i \exp\left(-\frac{\mathbf{p}_{i,T}^2}{T_T^2}\right) \exp\left(-\frac{\mathbf{p}_{i,L}^2}{T_L^2}\right) \qquad (25)$$

and

$$Q(p_i, p_j) = \exp\left(-\frac{(\mathbf{p}_i - \mathbf{p}_j)^2}{\alpha^2}\right), \qquad (26)$$

where T_T, T_L, and α are some parameters, and in what follows, we assume that $T_L \gg T_T$. In accordance with the ALICE baseline obtained from the PHOJET event generator simulations, we assume that only $q_{\rm inv}$ is measured for each $\mathbf{p}_T$ bin. Assuming that longitudinal components of the registered particles are equal to zero, $p_{1L} = p_{2L} = 0$, we approximate $q_{\rm inv}^2$ as

$$q_{\rm inv}^2 \approx \mathbf{q}_T^2\left(\frac{m^2 + \mathbf{p}_T^2\sin^2\phi}{m^2 + \mathbf{p}_T^2}\right), \qquad (27)$$

where ϕ denotes the unregistered angle between $\mathbf{p}_T$ and $\mathbf{q}_T$, $\mathbf{p}_T\mathbf{q}_T = |\mathbf{p}_T||\mathbf{q}_T|\cos\phi$. Then

$$C_{\rm NF}(|\mathbf{p}_T|, q_{\rm inv}) = \frac{\int_0^{2\pi} d\phi P_N(p_1, p_2)}{\int_0^{2\pi} d\phi P_N(p_1)P_N(p_2)}, \qquad (28)$$

and, taking into account Eq. (20), we get

$$C_{\rm NF}(|\mathbf{p}_T|, q_{\rm inv}) = \frac{N-2}{N-1}\left(C_N^{\rm 2jet}(|\mathbf{p}_T|, q_{\rm inv}) + \frac{1}{N-2}C_N^{\rm 1jet}(|\mathbf{p}_T|, q_{\rm inv})\right), \qquad (29)$$

where

$$C_N^{\rm 2jet}(|\mathbf{p}_T|, q_{\rm inv}) = \frac{\int_0^{2\pi} d\phi P_N^{\rm 2jet}(p_1, p_2)}{\int_0^{2\pi} d\phi P_N(p_1)P_N(p_2)}, \qquad (30)$$

$$C_N^{\rm 1jet}(|\mathbf{p}_T|, q_{\rm inv}) = \frac{\int_0^{2\pi} d\phi P_N^{\rm 1jet}(p_1, p_2)}{\int_0^{2\pi} d\phi P_N(p_1)P_N(p_2)}. \qquad (31)$$

It is well known (see, e.g., Ref. [10]) that the influence of exact conservation laws on single-particle and two-particle momentum probability densities at the N-particle production process depends on the value of N and disappears at $N \to \infty$. Since one considers a subsystem of N pions but not the total system, to weaken the influence of the total transverse momentum conservation on pions we shall consider $C_M^{\rm 1jet}$ and $C_M^{\rm 2jet}$ with $M > N$ instead of $C_N^{\rm 1jet}$ and $C_N^{\rm 2jet}$ in Eq. (29). This is the simplest way to account for a weakened conservation law in our model. At the same time, the factor $1/(N-2)$ in (29) remains the same since it is associated with the combinatorics of the distribution of particles between clusters in momentum space (''minijets''), which happens whether or not one weakens the total momentum conservation law. Also, for more exact fitting of the data points in each average transverse momentum bin, we utilize the auxiliary factors Λ. Then Eq. (29) gets the form

$$C_{\rm NF}(|\mathbf{p}_T|, q_{\rm inv}) = \Lambda(|\mathbf{p}_T|)\left(C_M^{\rm 2jet}(|\mathbf{p}_T|, q_{\rm inv}) + \frac{1}{N-2}C_M^{\rm 1jet}(|\mathbf{p}_T|, q_{\rm inv})\right). \qquad (32)$$

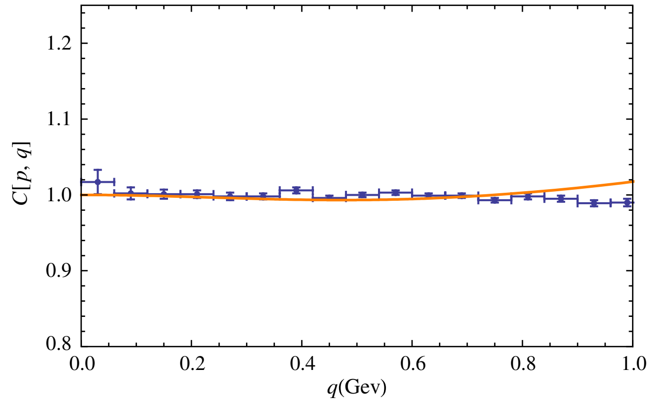

FIG. 1 (color online). The nonfemtoscopic correlation functions of like-sign pions in the $0.1 < p_T < 0.25$ GeV bin from a simulation using PHOJET [7,15] (solid dots) and those calculated from the analytical model: minijets + momentum conservation (solid line). See the text for details.

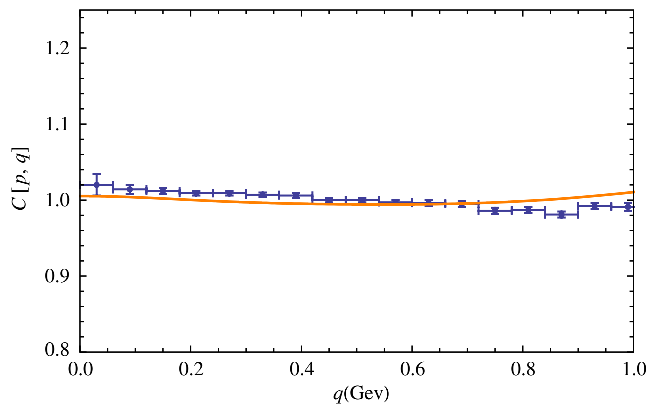

FIG. 2 (color online). The same as Fig. 1 but in the $0.25 < p_T < 0.4$ GeV bin.

The results of our calculations of the nonfemtoscopic correlation functions $C_{\rm NF}$ are shown in Figs. 1–10 in comparison with correlation functions reported by the ALICE Collaboration [7] for different transverse momenta of pion pairs (actually, we performed calculations for the mean value in each bin). The auxiliary factors $\Lambda(|\mathbf{p}_T|)$ differ from unity only slightly.[2] The data for unlike-sign pion correlations measured by the ALICE Collaboration as well as for the PHOJET simulations of like-sign two-pion nonfemtoscopic correlation functions at midrapidity for the total charged multiplicity $N_{\rm ch} \geq 12$ bin in $p + p$ collisions at $\sqrt{s} = 900$ GeV are taken from Refs. [7,15]. Note that correlations of nonidentical pions measured by the

[2]Namely, they are 0.95, 0.96, 0.98, 0.99, 0.99 for like-sign pions (in Figs. 1–5, respectively), and 0.92, 0.92, 0.93, 0.91, 0.82 for unlike-sign pions (in Figs. 6–10, respectively).

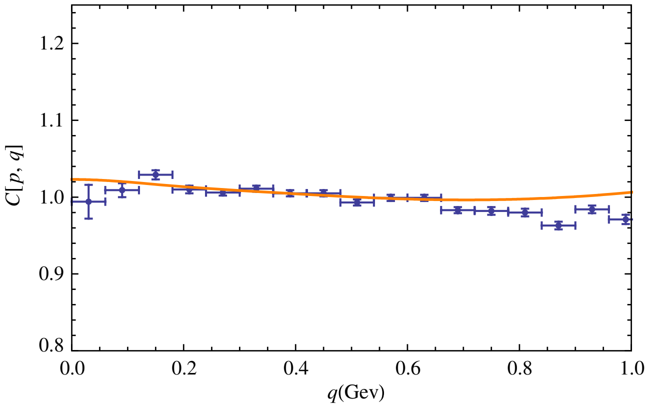

FIG. 3 (color online). The same as Fig. 1 but in the $0.4 < p_T < 0.55$ GeV bin.

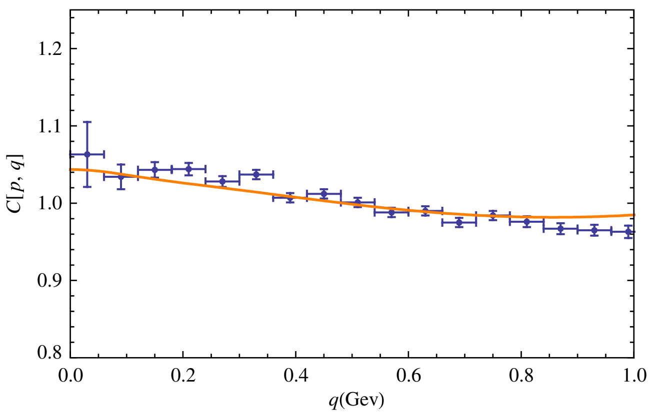

FIG. 4 (color online). The same as Fig. 1 but in the $0.55 < p_T < 0.7$ GeV bin.

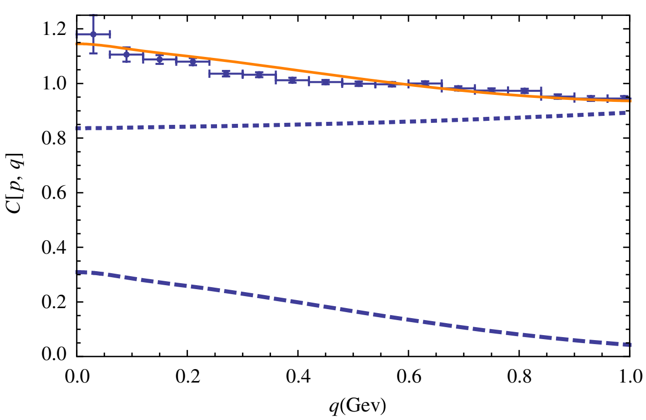

FIG. 5 (color online). The nonfemtoscopic correlation functions of like-sign pions in the $0.7 < p_T < 1.0$ GeV bin from a simulation using PHOJET [7,15] (solid dots) and those calculated from the analytical model (solid line). The contributions to the nonfemtoscopic correlation function from the first term of Eq. (32) (dotted line) and from the second one (dashed line) are also presented.

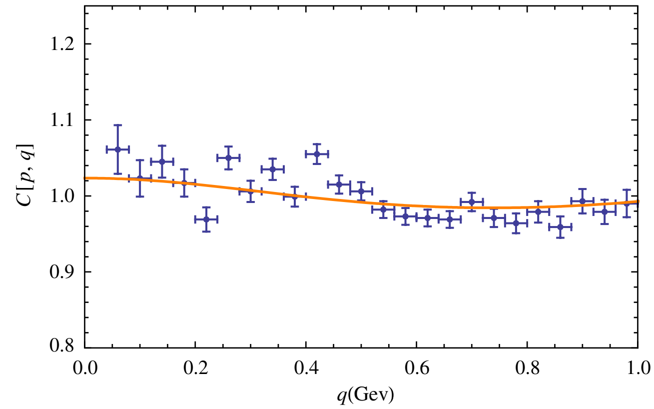

FIG. 6 (color online). The correlation functions of unlike-sign pions in the $0.1 < p_T < 0.25$ GeV bin measured by the ALICE Collaboration from Refs. [7,15] (solid dots) and those calculated from the analytical model: minijets + momentum conservation (solid line). See the text for details.

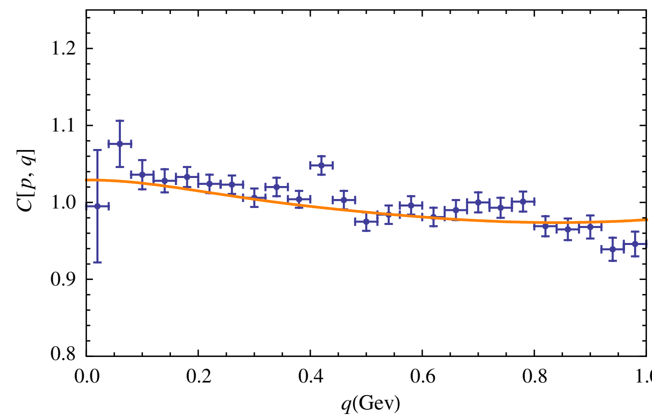

FIG. 7 (color online). The same as Fig. 6 but in the $0.25 < p_T < 0.4$ GeV bin.

ALICE Collaboration, as well as the PHOJET simulations of identical two-pion correlation functions, demonstrate Coulomb FSI correlations at the lowest $q_{\rm inv}$ bin and peaks coming from resonance decays. These Coulomb FSI, as well as contributions from resonance production, are not taken into account and so are not reproduced in our model.

The presented results are obtained for $M = 50$, $T_T = \alpha = 0.65$ GeV (to minimize the number of fit parameters, we fixed $T_T = \alpha$ for all calculations),[3] and the fitted values of N are different for like-sign and unlike-sign pion pairs, namely, $N^{\pm\pm} = 20$ for the former and $N^{+-} = 11$ for the latter. The relatively high value of M can be interpreted as a residual effect on the pion subsystem of total energy-momentum conservation in a multiparticle production

[3]Note that with these parameter values the mean transverse momentum $\langle p_T \rangle$ is about 0.58 GeV.

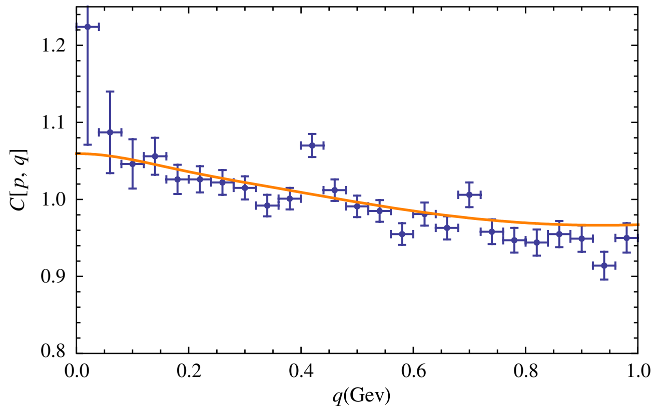

FIG. 8 (color online). The same as Fig. 6 but in the $0.4 < p_T < 0.55$ GeV bin.

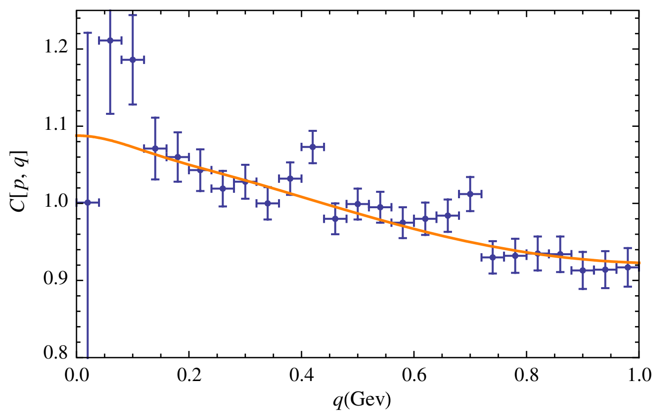

FIG. 9 (color online). The same as Fig. 6 but in the $0.55 < p_T < 0.7$ GeV bin.

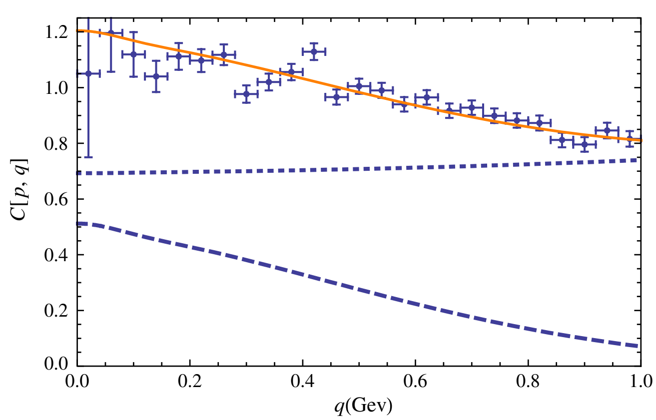

FIG. 10 (color online). The correlation functions of unlike-sign pions in the $0.7 < p_T < 1.0$ GeV bin measured by the ALICE Collaboration from Refs. [7,15] (solid dots) and those calculated from the analytical model (solid line). The contributions to the nonfemtoscopic correlation function from the first term of Eq. (32) (dotted line) and from the second one (dashed line) are also presented.

process. The relation $N^{+-} < N^{\pm\pm}$ between fitted N values means that the magnitude of the correlations induced by a minijet for unlike-sign pion pairs is higher than for like-sign ones. This happens because in the former there is no local charge conservation constraint for the production of oppositely charged pion pairs and, therefore, one can expect less identically charged pion pairs from the fragmenting minijets than oppositely charged ones.

One can see from the figures that the behavior of the nonfemtoscopic correlation functions of pions, $C_{\rm NF}$, is reproduced well despite the simplicity of our model. This is a result of the competition of the two trends: an increase of the correlation function with $q_{\rm inv}$ because of momentum conservation and a decrease of it due to fragmentation of one minijet into the registered pion pair. Figures 5 and 10 also demonstrate the relative contribution of the first and second terms in Eq. (32) to the nonfemtoscopic correlation functions.

V. ANALYTICAL MODEL FOR EVENT-BY-EVENT MOMENTUM SPECTRA FLUCTUATIONS

As it follows from our previous discussion, the lower magnitude of the nonfemtoscopic correlations at relatively low $q_{\rm inv}$ for like-sign pion pairs as compared to the correlations of unlike-sign pions is natural if pions are produced through minijet fragmentation. However, this cannot be the case for other production mechanisms that do not include noticeable production of minijets. For example, if thermalization takes place in $p + p$ collisions and hydrodynamical evolution forms the "soft" momentum spectra,[4] then the production of minijets at relatively low p_T is, typically, reduced. In this case utilization of the nonfemtoscopic correlations of like-sign pion pairs obtained in the PHOJET and similar event generators as a correlation baseline for the femtoscopic correlations [see Eq. (4)] can be in doubt. Then the question arises whether the unlike two-pion correlations in $p + p$ collisions, which are reproduced in PHOJET-like models, are ultimately caused by minijets and conservation laws only, or whether a similar behavior can be attributed to hydrodynamics also.

First, note that there are no correlations induced by the exact global energy-momentum conservation in hydrodynamic models, and corresponding conservation laws are satisfied only in average for particles that are produced at some hypersurface where hydrodynamics is switched off. The global energy-momentum conservation constraints can be added on an event-by-event basis if the post-hydrodynamical hadronic stage is calculated by means of some hadronic cascade model (the so-called "hybrid" model). One more source of the nonfemtoscopic correlations in such models are event-by-event fluctuations of initial conditions for the hydrodynamical stage. These

[4]Hydrodynamic models, perhaps, can give a reasonable description of elementary particle collisions; see, e.g., [16].

fluctuations result in fluctuations of the two-particle and single-particle momentum spectra, and, as usual, the effect of the fluctuations is more pronounced for small systems. Then, an important question is whether such correlations can be similar to minijet induced correlations that, as we know, may reproduce unlike-sign pion correlations at relatively low $q_{\rm inv}$.

Let us give an illustrative example of nonfemtoscopic correlations that appear because of event-by-event fluctuating initial conditions for the hydrodynamic stage (in hybrid models this stage is matched with the subsequent hadronic cascade stage) and are similar to the ones produced by minijets. Suppose that the N-particle probability density is defined as

$$P_N(p_1, p_2, \ldots, p_N) = \sum_i w(u_i) P_N(p_1, p_2, \ldots, p_N; u_i), \quad (33)$$

where $P_N(p_1, p_2, \ldots, p_N; u_i)$ is the N-particle probability density for some u_i type of the initial conditions, and $w(u_i)$ denotes the distribution over initial conditions, $\sum_i w(u_i) = 1$. To analyze the possible effect of fluctuating initial conditions, here we neglect conservation law constraints and the production of minijets. Because we assume uncorrelated particle emissions for each specific initial condition, one can write

$$P_N(p_1, p_2, \ldots, p_N; u_i) = f(p_1; u_i) f(p_2; u_i) \ldots f(p_{N-1}; u_i) f(p_N; u_i), \quad (34)$$

where we normalize $f(p; u_i)$ as $\int \frac{d^3p}{E} f(p; u_i) = 1$, and then $K = 1$; see Eqs. (13) and (14). The two-particle nonfemtoscopic correlation function $C_{\rm NF}$ then reads

$$C_{\rm NF}(p_1, p_2) = \frac{\sum_i w(u_i) f(p_1; u_i) f(p_2; u_i)}{\sum_i w(u_i) f(p_1; u_i) \sum_j w(u_j) f(p_2; u_j)}. \quad (35)$$

Evidently, the different type of fluctuation, i.e., the form of the distribution $w(u_i)$, leads to a different behavior of the nonfemtoscopic correlations. To illustrate that fluctuations can lead to the nonfemtoscopic correlation functions that are similar to the ones induced by minijets, let us consider the toy model where

$$w(\mathbf{u}_T) = \frac{\alpha^2}{\pi} \exp(-\mathbf{u}_T^2 \alpha^2), \quad (36)$$

$$f(p; \mathbf{u}_T) = \frac{\beta^2 \gamma}{\pi^{3/2}} E \exp(-(\mathbf{p}_T - \mathbf{u}_T)^2 \beta^2) \exp(-p_L^2 \gamma^2), \quad (37)$$

and normalization is chosen in such a way that $\int d^2 u_T w(\mathbf{u}_T) = 1$ and $\int \frac{d^3p}{E} f(p; \mathbf{u}_T) = 1$. The main feature of such a model is that event-by-event single-particle transverse momentum spectra have a maximum for event-by-event fluctuating $\mathbf{p}_T$ values. Such momentum spectrum fluctuations could take place, e.g., in hydrodynamics with a highly inhomogeneous initial energy density profile without cylindrical or elliptic symmetry.[5] One can easily see that in this case $C_{\rm NF}$ decreases with q_T^2,

$$C_{\rm NF}(p, q) \sim \exp\left(-\frac{\beta^4}{2(\alpha^2 + \beta^2)} q_T^2\right), \quad (38)$$

and this means [after taking into account (27) and (28)] that $C_{\rm NF}$ decreases with $q_{\rm inv}^2$ too, which is similar to the behavior of $C_{\rm NF}$ if the nonfemtoscopic correlations are induced by minijets. At the same time, unlike the latter, the hydrodynamical fluctuations lead to similar correlations for like-sign and unlike-sign pion pairs. Then, our analysis suggests that, up to different resonance yields, the value of the slope of the correlation baseline at relatively low $q_{\rm inv}$ can be somewhere between pure hydrodynamic (i.e., the same as for nonidentical pion pairs) and pure minijet (i.e., lower than for nonidentical pion pairs) scenarios.

VI. CONCLUSIONS

We presented here a simple analytical model that takes into account correlations induced by the total transverse momentum conservation as well as correlations induced by the minijets. It is shown that the model gives a reasonable description of the correlations of nonidentical pions measured by the ALICE Collaboration [7] in $p + p$ collisions at $\sqrt{s} = 900$ GeV, and also the nonfemtoscopic correlations of identical pions generated in the PHOJET simulations of $p + p$ collisions and utilized by the ALICE Collaboration [7] as the correlation baseline. We conclude that the cluster (minijet) structures in the final momentum space of produced particles can result in noticeable nonfemtoscopic two-pion correlation functions that decrease when $q_{\rm inv}$ grows at relatively low $q_{\rm inv}$, while the global energy-momentum conservation constraints typically result in an increase with $q_{\rm inv}$ for fairly high $q_{\rm inv}$. Our model can be utilized for simple estimates of the nonfemtoscopic correlations induced by minijets and conservation laws that contribute to the total two-particle correlation functions.

There can be different types of multiparticle production mechanisms, and some of them could result in qualitatively similar nonfemtoscopic correlation functions. We presented heuristic arguments that the two-pion nonfemtoscopic correlation functions calculated in hydrodynamics with event-by-event fluctuating initial conditions can be qualitatively similar at relatively low $q_{\rm inv}$ to the ones calculated in the PHOJET-like generators, where the nonfemtoscopic correlations for low $q_{\rm inv}$ are mainly caused by minijets. It is worth noting an important difference between the nonfemtoscopic correlations induced by minijets and hydrodynamical fluctuations: while the former lead to a higher magnitude of the nonfemtoscopic correlations for unlike-sign pion pairs as compared to like-sign pions, the

[5]It seems that this is the case in heavy ion collisions, where nonsymmetrical fluctuations of initial conditions lead to nonzero v_3 and higher flow harmonics (see, e.g., Ref. [17]).

latter result in a similar (up to the resonance contributions) strength of the nonfemtoscopic correlations for identical and nonidentical pions. Then, if the applicability of hydrodynamics to $p + p$ collisions is supported, such an analysis allows one to estimate the correlation baseline and, so, to extract the femtoscopic scales in these collisions by means of tuning the hydrokinetic model to reproduce the experimental unlike-sign pion correlations.

Because the particle production mechanisms in $p + p$ are still unclear, the nature of nonfemtoscopic correlations in these collisions is also an open question. Different dynamical models, that reproduce unlike pion correlations, can give different estimates of the correlation baseline and, so, lead to different results for the correlation femtoscopy analysis of the space-time scales of the collision process. In our opinion, this difficulty can be overcome if the correlation analysis is applied not to femtoscopic correlation functions, but to experimental data reported for the total correlation functions of pion pairs. Then the space-time scales can be estimated by means of the dynamic model that will also be able to reproduce, among other observables, these complete correlations.

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