Elastic pp-scattering at $\sqrt{s} = 7$ TeV with the genuine Orear regime and the dip

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The unitarity condition unambiguously requires the Orear region to appear in between the diffraction cone at low transferred momenta and the hard parton scattering regime at high transferred momenta in hadron elastic scattering. It originates from rescattering of the diffraction cone processes. It is shown that such a region has been observed in the differential cross-section of the elastic pp-scattering at $\sqrt{s} =$ 7 TeV. The Orear region is described by exponential decrease with the scattering angle, and imposed on it damped oscillations. They explain the steepening at the end of the diffraction cone as well as the dip and the subsequent maximum observed in TOTEM data. The failure of several models to describe the data in this region can be understood as an improper account of the unitarity condition. It is shown that the real part of the amplitude can be as large as the imaginary part in this region. The overlap function is calculated and shown to be small outside the diffraction peak. Its negative sign there indicates the important role of phases in the amplitudes of inelastic processes.

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I. INTRODUCTION

The TOTEM collaboration has published [1] experimental results on the differential cross-section of the elastic pp-scattering at the total center of mass system energy $\sqrt{s} = 7$ TeV. Among the most interesting features they observe is the steepening of the diffraction cone near the squared transferred momentum 0.3 GeV^2, the dip at 0.53 GeV^2, and the maximum at 0.7 GeV^2. We explain them as resulting from the rigorous requirements of the unitarity condition. It prescribes the Orear regime characterized by exponential decrease with the scattering angle to start at transferred momenta just above the diffraction cone. The damped oscillations imposed on it lead to the dip in the differential cross-section. No particular model has been used.

At the same time, there exist several models mostly based on the reggeon approach. Their predictions are extensively cited in Ref. [1]. Being rather successful in the diffraction cone, they fail to describe the new data quantitatively beyond the diffraction peak. This demonstrates that the unitarity condition is not properly accounted there in these models. Since then, some other models have been proposed [2,3].

At the end of the 1960s, the very first experimental data on elastic pp- and πp-scattering were obtained at energies between 6.8 and 19.2 GeV in the laboratory system [4–6]. They showed that just after the diffraction cone, which behaved as a Gaussian in the scattering angle, there was observed exponential decrease in the angle behavior, which was called the Orear regime after the name of its investigator [5]. Some indications on the shoulder appearing at the beginning of this region (evolved later to the dip at the higher Intersecting Storage Rings energies) were also obtained. A special session was devoted to these findings at the 1968 Rochester Conference in Wien.

The theoretical indications about the possibility of such regime were obtained even earlier [7–9], but the results did not fit new experimental findings.

At the same time, the simple theoretical explanation based on rigorous *model-independent* consequences of the unitarity condition was proposed [10,11], and a careful fit to experimental data showed good quantitative agreement with experiment [12].

We follow these ideas to demonstrate that they are also applicable to the recent data of the TOTEM collaboration at the LHC at energies as high as 7 TeV.

II. THEORETICAL DESCRIPTION

The elastic scattering proceeds mostly at small angles. The diffraction peak has a Gaussian shape in the scattering angles or exponentially decreases as the function of the transferred momentum squared:

$$\frac{d\sigma}{dt}\Big/\left(\frac{d\sigma}{dt}\right)_{t=0} = e^{Bt} \approx e^{-Bp^2\theta^2}, \tag{1}$$

where the four-momentum transfer squared is

$$t = -2p^2(1-\cos\theta) \approx -p^2\theta^2(\theta \leq \theta_d \ll 1) \tag{2}$$

with p and θ denoting the momentum and the scattering angle, respectively, in the center of mass system, and B known as the diffraction slope.

At large energies the forward scattering amplitude has a small real part, as is known from the dispersion relations [13,14]. Therefore, in the first approximation, it is reasonable to assume that its real part is negligible within the diffraction peak $\theta \leq \theta_d$. Then the elastic scattering in this region can be described by the amplitude

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$$A(p, \theta) \approx 4ip^2\sigma_t e^{-Bp^2\theta^2/2} \tag{3}$$

with a proper optical theorem normalization of the total cross-section σ_t in the forward direction. We stress that Eq. (3) follows directly from experimental results and does not appeal to any particular model.

Let us have a look at the unitarity condition, which is

$$\mathrm{Im}\,A(p, \theta) = I_2(p, \theta) + F(p, \theta) = \frac{1}{32\pi^2}\iint d\theta_1 d\theta_2 \frac{\sin\theta_1 \sin\theta_2 A(p, \theta_1)A^*(p, \theta_2)}{\sqrt{[\cos\theta - \cos(\theta_1+\theta_2)][\cos(\theta_1-\theta_2) - \cos\theta]}} + F(p, \theta). \tag{4}$$

The region of integration in Eq. (4) is given by the conditions

$$|\theta_1 - \theta_2| \leq \theta, \qquad \theta \leq \theta_1 + \theta_2 \leq 2\pi - \theta. \tag{5}$$

The integral term represents the two-particle intermediate states of the incoming particles. The function $F(p, \theta)$ represents the shadowing contribution of the inelastic processes to the elastic scattering amplitude. Following Van Hove [7], it is called the overlap function. It determines the shape of the diffraction peak and is completely nonperturbative. Only some phenomenological models pretend to describe it (see also Ref. [15], where its shape is obtained using the unitarity relation in combination with experimental data).

Now, let us consider the integral term I_2 outside the diffraction peak. Because of the sharp falloff of the amplitude (3) with angle, the principal contribution to the integral arises from a narrow region near the line $\theta_1 + \theta_2 \approx \theta$. Therefore, one of the amplitudes should be inserted at small angles within the cone, while another one is kept at angles outside it. At the beginning, let us neglect the real parts of the amplitude both in the diffraction region and at large angles. We insert Eq. (3) for one of the amplitudes in I_2 and integrate over one of the angles. Then the linear integral equation is obtained:

$$\mathrm{Im}A(p, \theta) = \frac{p\sigma_t}{4\pi\sqrt{2\pi B}}\int_{-\infty}^{+\infty} d\theta_1 e^{-Bp^2(\theta-\theta_1)^2/2}\mathrm{Im}A(p, \theta_1) + F(p, \theta). \tag{6}$$

It can be solved analytically (for more details, see Refs. [10,11]) with the assumption that the role of the overlap function $F(p, \theta)$ is negligible outside the diffraction cone. [16] To account for the real part of the amplitude, one replaces σ_t with $\sigma_t f_\rho$ where $f_\rho = 1 + \rho_d\rho_l$, with average values of ratios of real to imaginary parts of the amplitude in and outside the diffraction cone denoted as ρ_d and ρ_l, respectively. It follows from Eq. (4) that $A_1A_2^* \to \mathrm{Im}A_1\mathrm{Im}A_2(1 + \rho_1\rho_2)$.

Using the Fourier transformation, one gets the solution

$$\mathrm{Im}\,A(p, \theta) = C_0 e^{-\sqrt{2B\ln((4\pi B)/(\sigma_t f_\rho))}p\theta} + \sum_{n=1}^{\infty} C_n e^{-(\mathrm{Re}b_n)p\theta}\cos(|\mathrm{Im}b_n|p\theta - \phi_n), \tag{7}$$

$$b_n \approx \sqrt{2\pi B|n|}(1 + i\,\mathrm{sign}n) \qquad n = \pm 1, \pm 2, \ldots \tag{8}$$

This shape has been obtained from contributions due to the pole on the real axis and a set of the pairs of complex conjugated poles. Correspondingly, it contains the exponentially decreasing with θ (or $\sqrt{|t|}$) term (Orear regime) with imposed on it damped oscillations. Let us mention Ref. [17], where nondamped oscillations were predicted in the reggeon exchange model but they are not observed in experiment.

The elastic scattering differential cross-section outside the diffraction cone (in the Orear regime region) is

$$\frac{d\sigma}{p_1 dt} = \Big(e^{-\sqrt{2B|t|\ln((4\pi B)/(\sigma_t f_\rho))}} + p_2 e^{-\sqrt{2\pi B|t|}}\cos(\sqrt{2\pi B|t|} - \phi)\Big)^2. \tag{9}$$

The first (Orear) term is exponentially decreasing with θ (or $\sqrt{|t|}$), and the second term demonstrates the damped ($n = 1$) oscillations, which are in charge of the dip-maximum structure near the diffraction cone. The omitted terms with larger n in Eq. (7) are damped more strongly because they contain $\sqrt{n}$ in exponents. Note that the exponents of the damped terms are much larger numerically than that of the Orear term if the experimentally measured values of the diffraction cone slope B and the total cross-section σ_t are inserted. Namely, B and σ_t determine mostly the shape of the elastic differential cross-section in the Orear region between the diffraction peak and the large angle parton scattering. The value of $4\pi B/\sigma_t$ is so close to 1 that the first term is very sensitive to ρ_l. Thus it becomes possible for the first time to estimate the ratio ρ_l from fits of experimental data.

Besides the overall normalization constant p_1, this formula contains the constants p_2 and ϕ, which determine the strength and the phase of the oscillation [18], respectively. They can be found from fits of experimental data. The constant p_1 is determined by the transition point from the diffraction cone to the Orear regime. The constants p_2 and ϕ define the depth of the dip and its position, respectively.

Concerning the ρ ratios, one can choose $\rho_d \approx 0.14$ as prescribed by the dispersion relations for its value at $t = 0$ [13,14] and use ρ_l as another fitted parameter which influences the exponents in Eq. (9).

Let us note that all parameters can depend on energy as well as the values of the diffraction cone slope B and the total cross-section σ_t. Surely, this is unimportant if the fit is done at a fixed energy, as in the present paper.

The unitarity condition is not a complete theory. It imposes some restrictions on its consequences, however. Its solution predicts the dependence on $p\theta \approx \sqrt{|t|}$ but not the dependence on the collision energy. Nevertheless, main exponents in Eq. (9) depend on energy. We are able to predict them at different energies if the dependence of the diffraction slope B and the total cross-section σ_t is known from experiment. In this way, different reactions (including $\bar{p}p$in particular) may be analyzed.

Apart from a comparison of theoretical predictions with experimental data, one can get some knowledge about the overlap function $F(p, \theta)$ (see Ref. [15]). It is important, in particular, to confirm the assumption about its smallness outside the diffraction peak. Then Eq. (4) is used as an expression for $F(p, \theta)$:

$$F(p, \theta) = 16p^2\left(\pi\frac{d\sigma}{dt}/(1+\rho^2)\right)^{1/2} - \frac{8p^4(1+\rho_d\rho)}{\pi\sqrt{(1+\rho_d^2)(1+\rho^2)}}\int_{-1}^{1} dz_2 \times \int_{z_1^-}^{z_1^+} dz_1\left[\frac{d\sigma}{dt_1}\cdot\frac{d\sigma}{dt_2}\right]^{1/2} K^{-1/2}(z, z_1, z_2), \qquad (10)$$

where $z_i = \cos\theta_i$; $K(z, z_1, z_2) = 1 - z^2 - z_1^2 - z_2^2 + 2zz_1z_2$, and the integration limits are $z_1^{\pm} = zz_2 \pm [(1-z^2)\times(1-z_2^2)]^{1/2}$.

At $\sqrt{s} = 7$ TeV, the angles are extremely small, so the kernel becomes very singular. K is close to 0 but integrable. The divergence is of the type $\int dz/\sqrt{z}$ and can be computed. Computing F in the diffraction cone, one uses $\rho = \rho_d$. Outside it, $\rho = \rho_l$.

Let us mention that the inhomogeneous Eq. (4) has been solved [11] by iterations with the overlap function approximated by $F/s\sigma_{\mathrm{inel}} = \exp[-B_{\mathrm{in}}p^2\theta^2/2]$. Its more precise approximation is required to get accurate results, but it is important that the conclusion about the phase ϕ remains valid.

Below we show and discuss the obtained results.

III. A FIT OF THE EXPERIMENTAL DATA

Having at our disposal Eq. (9), we try to fit experimental distribution of elastic pp-scattering at $\sqrt{s} = 7$ TeV. The experimental values of $B = 20.1$ GeV^{-2} and $\sigma_t = 98.3$ mb were used in Eq. (9). We expect that Eq. (9) must be applicable from the end of the diffraction cone at $|t| = 0.3$ GeV^2 to the beginning of hard parton processes at $|t| > 1$ GeV^2. The result is shown in Fig. 1.

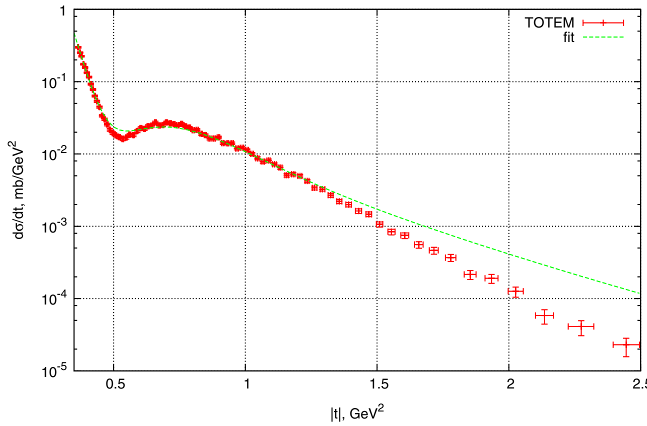

FIG. 1 (color online). Fit of experimental distribution of elastic pp-scattering at $\sqrt{s} = 7$ TeV.

It is seen that the fit is quite successful in the expected applicability region. First of all, we notice the steeper decrease in the region $0.3 < |t| < 0.36$ GeV^2 compared to the slope of the diffraction cone at $|t| < 0.3$ GeV^2 as observed in experiment. It is explained here as the negative contribution of the oscillating term in Eq. (9). That determines the phase ϕ. The dip develops at $|t| = 0.53$ GeV^2, where the cosine in the second term is close to -1. Then this term increases, becomes positive, and leads to the maximum at $|t| \approx 0.7$ GeV^2. The positions of the dip and of the subsequent maximum are uniquely determined by the period of oscillations $\Delta t = 2\pi/B$, which is predicted by the unitarity condition and depends only on the well-measured slope of the diffraction peak B. The damping exponent in front of the cos term becomes so strong at larger $|t|$ that the simple Orear regime with the first term in Eq. (9) prevails. Let us note that the exponent in this term is rather small because the ratio $4\pi B/\sigma_t$ is very close to 1 [19]. Therefore, it is extremely sensitive to the parameter ρ_l. That helps determine this parameter.

Hardly any oscillations will be observed at large $|t|$. The exponent in the oscillation term is very large and strongly damps it. One could pretend to observe the next weak oscillation at $|t| \approx 0.9$–1.0 GeV^2. However, it would require very high precision. It is interesting to note that the damping increases with energy due to increase of the slope B. At the same time, the shrinkage of the cone leads to the shift of the Orear regime (and the dip) to smaller angles at higher energies, so that the oscillations are still noticeable there.

Let us list and discuss the parameters in Eq. (9), which we found by the fitting procedure: $p_1 = 18.71$; $p_2 = 115.6$; $\phi = -0.845$; $\rho_l \approx -2$. The large value of ρ_l demonstrates that the dip is well-pronounced in the data. Up to now, the only possible model-independent estimate of the ratio of real to imaginary parts of the elastic scattering amplitude was available from the dispersion relations at $t = 0$. It is for the first time that it is done at large $|t|$ in a model-independent way, and it shows that this ratio is of the order of 1 there. Surely, there are many models where this ratio is calculated in a wide range of t values. There is no common consensus about their validity, however. The parameter ϕ is so close to its theoretical estimate that it was not even necessary to use it as a free one.

Now we discuss the role of the parameters.

(1) The parameter p_1 is in charge of the overall normalization and, consequently, of the smooth transition from the diffraction cone to the Orear region.

(2) The parameter p_2 defines the amplitude of the oscillations and, consequently, the depth of the dip. In combination with ϕ, it leads to the steepened slope at $0.3 < |t| < 0.36\ \mathrm{GeV}^2$.

(3) The phase ϕ determines the position of the dip and the beginning of the steepened slope. Actually, it was shown in Ref. [11] that it can be obtained from the iterative solution of the nonlinear Eq. (4). It is almost independent of the form of $F(p, \theta)$, so that $|\phi| \approx \pi/4$. Nevertheless, this problem asks for further studies.

(4) The parameter ρ_l in f_ρ is in charge of the exponential slope at $|t|$ above the maximum [together with B and σ_t in the first term of Eq. (9)]. It is negative and rather large (in the absolute value).

(5) The relative position of the dip and the maximum (the period of oscillations) is determined only by the diffraction cone slope B [the second term in (9)].

IV. THE OVERLAP FUNCTION

As follows from experiment, the inelastic cross-section is much larger than the cross-section of elastic scattering at high energies. Therefore, the overlap function is much larger than the integral term in the unitarity relation at small t. To see what the contribution of inelastic processes is to the unitarity relation at any values of t, it is instructive to calculate the overlap function according to Eq. (10). In Ref. [15] that has been done at the assumption of ratios of real to imaginary parts ρ equal to zero both at small and large t. Now, with the above estimate of ρ_l, we can take it into account. Nevertheless, the calculations were done with and without account of ρ to compare with previous results and to estimate the role of ρ. The results are shown in Fig. 2.

There are several distinctive features observed. First of all, as expected, the overlap function drops very quickly with increase of the transferred momentum $|t|$, and it determines the shape of the diffraction cone. Second, it crosses the abscissa axis at $|t| \approx 0.3$ and becomes negative. Namely, there the Orear regime starts working. If compared to low energies [15], the overlap function becomes narrower at higher energies. Third, it is small and changes very slowly outside the diffraction cone, similarly to the low energy behavior. Intuitively, this smallness may be understood as a consequence of strong destructive interference between amplitudes of inelastic processes with very different kinematics. In one of these amplitudes, the final state must be turned to the large angle θ relative to the direction of initial particles. Thus the overlap of these two processes is small. Fourth, the account of ρ does not change qualitatively this conclusion in general, even though it somewhat changes the numerical estimates diminishing $|F|$ further. This follows from a better fit of experimental data with ρ_l different from zero. Fifth, the negative sign of F imposes a severe problem to theorists because it shows the important role of the phases of matrix elements of inelastic processes and their strong interference when trying to reconstruct elastic scattering from two inelastic processes turned by t one to another.

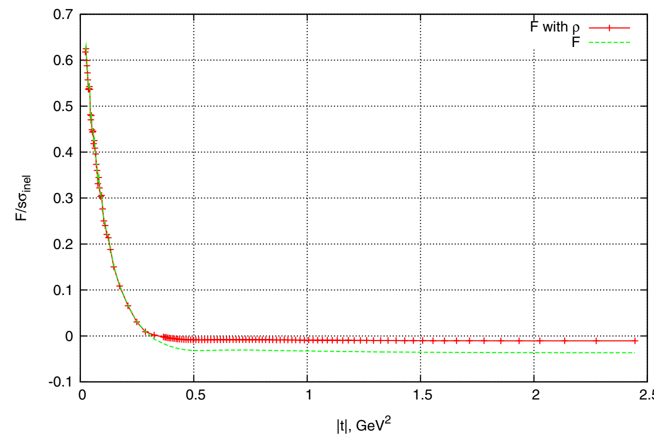

FIG. 2 (color online). The overlap functions calculated with $\rho_d = \rho_l = 0$ and with $\rho_d = 0.14$; $\rho_l = -2$ (closest to the abscissa axis).

V. CONCLUSIONS

Thus we conclude:

(i) At intermediate angles between the diffraction cone and hard parton scattering region, the unitarity condition predicts the Orear regime with exponential decrease in angles and imposes on it damped oscillations. Earlier, this solution was helpful in explaining this regime at lab energies 8–20 GeV.

(ii) The experimental data on elastic pp differential cross-section at $\sqrt{s} = 7$ TeV in this region are fitted by it with a well-described position of the dip at $|t| \approx 0.53\ \mathrm{GeV}^2$, its depth and subsequent damped oscillations with the predicted period about $0.3\ \mathrm{GeV}^2$. The large amplitude of the oscillations and their negative sign explain the steepened slope at $0.3 < |t| < 0.36\ \mathrm{GeV}^2$. The positive sign of the oscillating term at $|t| \approx 0.7\ \mathrm{GeV}^2$ leads to the maximum. Strong damping of the oscillations at higher values of $|t|$ results in clear signature of the simple exponential (in $\sqrt{|t|}$) behavior observed first by Orear, which extends up to $|t| \approx 1.5\ \mathrm{GeV}^2$.

(iii) A good fit allows, without using any definite model, for the first time to estimate the ratio of real to imaginary parts of the elastic scattering amplitude in this region ($\rho_l \approx -2$) far from forward direction $t = 0$.

(iv) The overlap function at 7 TeV has been calculated using only the experimental differential cross-section and the previously described estimate of the ratio of real to imaginary parts. As at low energies, it is small and negative in the Orear region. That confirms the assumption used when solving the unitarity equation, and it shows that the phases of inelastic amplitudes become crucial in any model of inelastic processes.

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[19] This happens at all energies.