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Local effects of a cosmological constant

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The main topic of this brief report is to discuss the local effects of a cosmological term in the (locally) linearized field equation. It appears the Λ imprint is not necessarily spherical, and it is argued that this could have relevant contributions on the local dynamics of clusters and superclusters.

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I. INTRODUCTION

The so-called ΛCDM model successfully interprets cosmological observations. It encompasses in a coherent framework both the late acceleration of the expansion of the Universe (revealed by Ia supernovae distances) and the properties of the relic cosmological microwave background radiation. The model describes a spatially flat isotropic and homogeneous universe [Euclidean Robertson-Walker (RW) metric] with dynamical scale factor $a(t)$ governed by general relativity with a cosmological constant Λ (ΛGR).

Despite these successes, arguments have been raised against ΛCDM. Indeed, it is tempting to interpret Λ as the vacuum energy since quantum field theory interprets vacuum as a perfect fluid with state equation $P^{(\mathrm{vac})} + \epsilon^{(\mathrm{vac})} = 0$ and stress tensor $T^{(\mathrm{vac})}_{\alpha\beta} = -\epsilon^{(\mathrm{vac})} g_{\alpha\beta}$, leading to constant $\epsilon^{(\mathrm{vac})}$. However, interpreting Λ this way results in a predicted value by far (about 120 orders of magnitude) larger than the value required to explain cosmological data. Another often raised objection is the "coincidence problem": Since the vacuum energy remains strictly constant but the matter density varies as a^{-3} during the expansion, the same order of their values today leads to the embarrassing conclusion that we are presently at a peculiar moment in the history of the Universe. However, it is not clear whether Λ must necessarily be interpreted as the vacuum energy or that the "coincidence problem" is really a problem [1]. In any case, the debate is still far from being closed (see for instance [2]).

Leaving aside this controversy, the fact is that ΛCDM is a valuable phenomenological cosmological model versus observation, and it is widely adopted by the cosmological community. As such, it is interesting to ask whether the cosmological constant could have significant effects at other scales. A specific characteristic of Λ dark energy is that it does not suffer any clustering effect. (This is not the case for a scalar-field dark energy, for instance). Hence, measuring Λ through cosmological observations gives the amplitude of its effects at all other scales and raises the question of the imprint of Λ at astrophysical scales associated with the ΛCDM scenario.

During the last decade, lots of effort has been made to investigate this problem. For instance, the matter motion has been investigated in ΛGR, in particular, in the environment of black holes [3]. Some authors claim Λ could result in nontrivial incidences on accretion disk dynamics around massive black holes. The impact of Λ on gravitational equilibrium was studied in [4,5]. At the level of the solar system, the Λ signature on the periastron shift and the geodetic precession have been estimated [6]. Considering the expansion of the Universe in our close neighborhood, some authors argue that Λ may be responsible for the local value of the Hubble flow parameter ($\sim 60\ \mathrm{km\,s^{-1}/Mpc}$), which is significantly lower than its large-scale value ($\sim 70\ \mathrm{km\,s^{-1}/Mpc}$) [7]. Identifying the impact of Λ on the lensing problem may also be understood as an aspect of the considered question (even if it often involves cosmological distances). Notice that defining the problem for this case is not obvious since the presence of the cosmological term drastically changes the space-time geometry at great distances. The issue remains an open debate [8].

Considering weak gravitational fields at intermediate scales, it is often claimed that a (positive) cosmological constant acts as a centrally symmetric "repulsive force" proportional to the distance. This is supported by the Schwarzschild-de Sitter spherical solution

$$ds^2 = -\left(1 - \frac{2m}{r} - \frac{\Lambda r^2}{3}\right)dt^2 + \left(1 - \frac{2m}{r} - \frac{\Lambda r^2}{3}\right)^{-1} dr^2 + r^2(d\theta^2 + \sin\theta^2 d\varphi^2),$$

which leads in the weak field hypothesis ($2m/r + \Lambda r^2/3 \ll 1$) to a "Newman-Seeliger potential"

$$U = \frac{1}{2}(1 + g_{00}) = \frac{m}{r} + \frac{\Lambda r^2}{6}$$

with a repulsive Λ contribution. However, since the spherical symmetry is present from the start, the resulting "force" necessarily exhibits this symmetry. The ΛRW cosmological models that are spherically symmetric for all observers enforce this interpretation in some sense (accordingly encompassing equilibrium solutions like the

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Einstein universe). However, a glance at the exact ΛGR vacuum solution (one can easily check that it satisfies $R_{\alpha\beta} = \Lambda g_{\alpha\beta}$)

$$ds^2 = -\frac{\cos^2\tilde{x}}{|\sin\tilde{x}|^{2/3}} d\tilde{t}^2 + d\tilde{x}^2 + \epsilon|\sin\tilde{x}|^{4/3}(d\tilde{y}^2 + d\tilde{z}^2)$$
$$\text{with} \quad \tilde{x}^\alpha \equiv \frac{\sqrt{3\Lambda}}{2} x^\alpha, \tag{1}$$

where $\epsilon = \pm 1$, shows that the cosmological constant may result in nonspherical effects. Indeed, the $\epsilon = +1$ solution represents a nonspherical but static [since $(K^\sigma) = (1, 0, 0, 0)$ is a Killing vector] space-time, while the $\epsilon = -1$ solution represents a space-time in anisotropic expansion/contraction as it can be seen changing (t, x) for (x, t) and the signature convention.

In this paper, we go back to the local effects of the cosmological constant without making any prior symmetry assumption on how it affects the space-time geometry. The most natural approach is to linearize the ΛGR equation around the Minkowski metric without presuming any symmetry, as is usually done in the $\Lambda = 0$ case. The main difference with $\Lambda = 0$ is that the Minkowski metric is not a ΛGR vacuum solution. However, ΛGR solutions being locally Minkowskian, such an expansion remains possible in a sufficiently limited (i.e. of size $\ll \sqrt{1/\Lambda}$) regular region of space-time. Since we are interested in applications to galactic clusters dynamics, this condition and the weak-field hypothesis are relevant as long as (1) the potential is small enough, at least in regions where test particles (gas particles, one of the galaxies, etc.) are expected to move, and (2) the region in which the linearized theory is considered (cluster's size) is small compared to $\sqrt{1/\Lambda}$. Throughout the paper we suppose that both conditions are satisfied.

II. WEAK FIELDS IN ΛGR

Let us expand the ΛGR equation

$$2R_{\alpha\beta} = 16\pi\left(T_{\alpha\beta} - \frac{1}{2}T g_{\alpha\beta}\right) + 2\Lambda g_{\alpha\beta}, \tag{2}$$

writing the metric $g_{\alpha\beta} = m_{\alpha\beta} + h_{\alpha\beta}$ with $m_{\alpha\beta} = \text{diag}\,(-1, +1, +1, +1)$ and $|h_{\alpha\beta}| \ll 1$. We get

$$-\Box h_{\alpha\beta} + \partial_\alpha\partial_\sigma \bar{h}^\sigma_\beta + \partial_\beta\partial_\sigma \bar{h}^\sigma_\alpha$$
$$= 16\pi \bar{T}_{\alpha\beta} + 2\Lambda m_{\alpha\beta} + O(h^2) + O(\Lambda h) \tag{3}$$

where $\overline{X}_{\alpha\beta} \equiv X_{\alpha\beta} - \frac{1}{2}X m_{\alpha\beta}$ and $\Box \equiv m^{\rho\sigma}\partial_\rho\partial_\sigma$ (as usual, indices are raised/lowered with the help of $m_{\alpha\beta}$). This equation was considered by [5,9], but these authors were interested in the gravitational wave aspect of the problem and did not explore the consequences on N body dynamics considered here. It was also obtained in [10], but the authors just considered a static solution with a one-point mass stress tensor. However, an appropriate gauge transform shows that their solution, while apparently nonspherical, turns out to be the (spherical) Schwarzschild-de Sitter solution in disguise. Based on this paper, [11] investigated the action of Λ on gravitational wave propagation.

Now, let us define $h'_{\alpha\beta}$ by

$$h_{\alpha\beta} = \overset{(m)}{h}_{\alpha\beta} + h'_{\alpha\beta}, \tag{4}$$

where $\overset{(m)}{h}_{\alpha\beta}$ satisfies the $\Lambda = 0$ version of (3). Hence,

$$m^{\lambda\sigma}\partial_\alpha\partial_\sigma h'_{\beta\lambda} + m^{\lambda\sigma}\partial_\beta\partial_\sigma h'_{\alpha\lambda} - \partial_\alpha\partial_\beta h' - m^{\lambda\sigma}\partial_\lambda\partial_\sigma h'_{\alpha\beta}$$
$$= 2\Lambda m_{\alpha\beta}. \tag{5}$$

This equation does not admit the trivial solution $h'_{\alpha\beta} = 0$ [Minkowski is not a ΛGR vacuum solution]. Let us look for locally quadratic solutions

$$h'_{\alpha\beta} = \Lambda K_{\alpha\beta\mu\nu} x^\mu x^\nu + o(x^2), \tag{6}$$

where $K_{\alpha\beta\mu\nu}$ are 100 constants satisfying $K_{\alpha\beta\mu\nu} = K_{\beta\alpha\mu\nu} = K_{\alpha\beta\nu\mu}$ (from the symmetries of $h_{\alpha\beta}$ and $x^\mu x^\nu$). Inserting (6) in (5) leads to 10 constraints on these 100 constants

$$m^{\lambda\sigma}K_{\beta\lambda\alpha\sigma} + m^{\lambda\sigma}K_{\alpha\lambda\beta\sigma} - m^{\lambda\sigma}K_{\lambda\sigma\alpha\beta} - m^{\lambda\sigma}K_{\alpha\beta\lambda\sigma}$$
$$= m_{\alpha\beta}. \tag{7}$$

Hence, there are 90 degrees of freedom in the resolution of this system. The seven constants K_{0000}, K_{000i}, and K_{0i00} do not enter (7). Since (7) is linear, 83 degrees of freedom remain in choosing the 93 constants that have at most two null indices. Let us emphasize that no gauge choice was made at this level. The (linearized) de Sitter metric in isotropic coordinates corresponds to the solution $K_{00kl} = \frac{1}{3}\delta_{kl}$ and $K_{ijkl} = -\frac{1}{6}\delta_{ij}\delta_{kl}$ (the others, $K_{\alpha\beta\mu\nu} = 0$) with $\overset{(m)}{h}_{\alpha\beta} = 0$.

Let us now choose the Hilbert gauge

$$\partial_\sigma \bar{h}^{\alpha\sigma} = 0 \tag{8}$$

and suppose gravitating matter is concentrated in compact regions (representing the N galaxies of a cluster) and choose for $\overset{(m)}{h}_{\alpha\beta}$ in (4)

$$\overset{(m)}{h}_{\alpha\beta} \equiv 4\int_{(\text{sources})} \bar{T}_{\alpha\beta}(\vec{X}, t - |\vec{x} - \vec{X}|)\frac{d^3X}{|\vec{x} - \vec{X}|}$$
$$\simeq \delta_{\alpha\beta}\sum_{(A)}\frac{2M_A}{r_A},$$

where X^k_A and M_A are the spatial coordinates and mass of the body A (galaxy) and $r^2_A = (x^k - X^k_A)(x^k - X^k_A)$. Since $\overset{(m)}{h}_{\alpha\beta}$ is a harmonic ($\Lambda = 0$) general relativity solution, (8) leads to

$$m^{\rho\sigma}K_{\alpha\rho\beta\sigma}=\frac{1}{2}m^{\rho\sigma}K_{\rho\sigma\alpha\beta} \quad (9)$$

i.e. 16 new constraints on $K_{\alpha\beta\mu\nu}$ (since $K_{\alpha\rho\beta\sigma}\neq K\beta\rho\alpha\sigma$). The full system (7) and (9) results in 26 constraints on the 100 constants $K_{\alpha\beta\mu\nu}$. This system reads, in expanded form, as

$$\begin{aligned}
&2K_{0k0k}-K_{kk00}-K_{00kk}=-1,\\
&K_{ik0k}+K_{0kik}-K_{kk0i}-K_{0ikk}=0,\\
&-K_{0i0j}-K_{0j0i}+K_{00ij}+K_{ij00}+K_{ikjk}+K_{jkik}-K_{kkij}-K_{ijkk}\\
&\quad=\delta_{ij},\\
&-K_{0000}+2K_{0k0k}-K_{kk00}=0,\\
&-K_{000i}+2K_{0kik}-K_{kk0i}=0,\\
&-2K_{0i00}+K_{000i}+2K_{ik0k}-K_{kk0i}=0,\\
&-2K_{0i0j}+K_{00ij}+2K_{ikjk}-K_{kkij}=0,
\end{aligned} \quad (10)$$

i.e. (1 + 3 + 6) + (1 + 3 + 3 + 10) = 26 equations (recall the last ij equation is not symmetric). This system admits $K_{0000}=-\frac{3}{2}$, $K_{kl00}=\frac{1}{2}\delta_{kl}$, $K_{00kl}=-\frac{1}{6}\delta_{kl}$, and $K_{ijkl}=-\frac{1}{6}\delta_{ij}\delta_{kl}$ (the others, $K_{\alpha\beta\mu\nu}=0$) as a solution with $\overset{(m)}{h}_{\alpha\beta}=0$, corresponding to the (linearized) de Sitter metric in harmonic coordinates.

Local dynamics

The dynamics of a test particle is governed by the geodesic equation. At the lowest order (i.e. Newtonian), the resulting acceleration $a^k\equiv\ddot{x}^k$ reads $a^k=-\Gamma^k_{00}$ since the velocity of the particle is very low with respect to the speed of light. In addition, since the material sources also have very low velocities with respect to the speed of light, the standard post-Newtonian hierarchy $|\partial_0\overset{(m)}{h}_{\alpha\beta}|\ll|\partial_i\overset{(m)}{h}_{\alpha\beta}|$ is valid. On the other hand, let us stress that such a hierarchy does not exist *a priori* when the derivative operators act on the part h' of (4). Hence, one gets

$$a^k=\frac{1}{2}\partial_k h_{00}-\partial_0 h'_{0k}. \quad (11)$$

From the vacuum (00) component of (3), one gets, thanks to the standard post-Newtonian approximation,

$$\partial_k\left(\frac{1}{2}\partial_k h_{00}-\partial_0 h'_{0k}\right)=\Lambda-\frac{1}{2}\partial_0\partial_0 h'_{kk}.$$

Developing $\partial_0\partial_0 h'_{kk}$ with the help of (6), it appears the divergence of the acceleration field (11) is simply given by

$$\partial_k a^k=\Lambda(1-K_{kk00}). \quad (12)$$

(One gets $\partial_k a^k=-\frac{1}{2}\Lambda$ for the de Sitter space-time). Using (4), the acceleration field (11) reads, in explicit form,

$$a^k=N^k t+M^{kl}x^l+\sum_{(A)}\partial_k\left(\frac{M_A}{r_A}\right). \quad (13)$$

The 12 quantities $N^k\equiv\Lambda(K_{000k}-2K_{0k00})$ and $M^{kl}\equiv\Lambda(K_{00kl}-2K_{0k0l})$ ($\neq M^{lk}$, in general) are constants characterizing the local gravitational field. The third term on the right side in (13) is the usual Newtonian interacting term, while the two first terms, proportional to Λ, are of cosmological origin. At this order, this "local cosmological field" depends only on the 12 constants N^k and M^{kl}. For the de Sitter metric, one has $N^k=0$ and $M^{kl}=-\frac{1}{6}\Lambda\delta_{kl}$.

Since Eq. (10) imposes 26 constraints on the 100 parameters $K_{\alpha\beta\mu\nu}$, solutions such that N^k and $M^{kl}=0$ are expected to exist. In this case, the cosmological term has no effect on the motion of particles (at the considered level of approximation). On the other hand, in the general case, both N^k and M^{kl} terms are nonspherical in such a way that the effects of Λ are qualitatively of a different nature than the usually considered imprints on local dynamics. [The cases where Λ just results in a radial force proportional to the distance correspond to the peculiar fields for which $N^k=0$ and $M^{kl}\propto\delta^{kl}$, that encompass the (linearized) de Sitter solution.] Equation (13) shows that Λ results essentially in two dynamical effects: (1) an acceleration $N^k t$ proportional to the cosmic time, and (2) an acceleration $M^{kl}x^l$ proportional to the vector (x^k). At any given cosmic time, the first effect results in a (local) uniform acceleration field in the direction defined by N^k. However, let us notice it has no effect on the relative motion of two particles (considered at the same time) located inside a region where the solution (6) is globally relevant (but it may have an effect when considering motions inside such a region with respect to an observer located outside this region). The second effect is not radial, in general, and has a repulsive or attractive radial component, depending on the sign of the quadratic form $M^{kl}x^k x^l$. Its detailed effect can be more closely estimated considering its impact on the relative motion [inside a region where the solution (6) is globally relevant], discarding the Newtonian terms $\partial_k(\frac{M_A}{r_A})$, i.e. considering $\ddot{\delta x}^k=M^{kl}\delta x^l$. If the matrix M^{kl} allows us to define a matrix Q^{kl} such that $Q^{kl}Q^{lm}=M^{km}$ (nine nonlinear equations constraining nine quantities Q^{kl}), the velocity field $v^k=Q^{kl}x^l$ satisfies $\ddot{\delta x}^k=M^{kl}\delta x^l$. In the case where Q^{kl} is symmetric , such a field mimics a quadrupole term in the local velocity field, plus an isotropic expansion term (since Q^{kl} is not traceless *a priori*) that contributes to the local Hubble expansion. Let us emphasize that such a quadrupole contribution is often required to fit local flow departures from Hubble expansion (see, for instance, [12]).

III. DISCUSSION

The RW representation of our Universe in which matter is comoving in the RW frame is valid at best at very large scales. At lower scales, observations show that galaxies and clusters are not at rest with respect to the RW frame. Also, the Galaxy is not at rest with respect to the cosmological microwave background radiation (identified to the RW frame). Besides, it has been known for two decades that nearby galaxies exhibit a systematic flow towards a region close to the Abell 3627 cluster, leading to the so-called great attractor problem [13], and a detailed representation of local flows exhibits even more complicated structures, including a quadrupole contribution in the velocity field [12]. In this context, it would be worthwhile to consider the possible existence of the Λ fields that generically enter (13), if the observed local dynamics has to be interpreted in the framework of ΛGR.

With this in mind, it would be useful to derive an order of magnitude of the Λ fields effect. Unfortunately, both the large number of degrees of freedom in the system (7) and the lack of exact solution without symmetry prevent us from getting such an estimate. The best we can do is conjecture that, in the generic case, the constants $|K_{\alpha\beta\mu\nu}|$ should range between zero and values of the same order of magnitude they have in the (highly symmetric) de Sitter case. This suggests the constants $K_{\alpha\beta\mu\nu}$ may be at best of the order of one-tenth to unity. Let us consider, for instance, the velocity field generated by N^k. In dimensionalized units, it satisfies $\frac{dv_N^k}{dt} = 3\Omega_\Lambda c H^2 (K_{000k} - 2K_{0k00})t$. Taking $(K_{000k} - 2K_{0k00}) \sim 1/10$, one could expect $\frac{dv_N}{dt} \sim \frac{1}{5} c H^2 t$ since $\Omega_\Lambda \sim 0.7$. Up to an additive constant, this leads to $v_N \sim \frac{1}{10}(Ht)^2 c$. Taking $Ht \sim 1/10$, i.e. t is a significant part of the age of the Universe, may result in local velocity flows of several thousand $\mathrm{km\,s^{-1}}$. This velocity is lowered, taking for K_{000k} and K_{0k00} values smaller than $1/10$. This order of magnitude suggests that these fields may take a significant part in some anisotropic streams observed in galactic clusters.

It is natural to require the ''local'' solution (4) to match with the solution describing the Universe at bigger scales, at least at the cosmological level. Since the RW metric is homogeneous and isotropic, one could expect that matching with such a metric should drastically reduce the residual degrees of freedom for admissible solutions of (7) and (9). Along these lines, one could wonder whether the large-scale isotropy constrains the Λ imprints to be isotropic at every scale in some sense. The exact ΛGR vacuum solution (1) in the case $\epsilon = +1$ suggests that such a claim is not justified. Indeed, while the vector $K^\sigma = (0, 1, 0, 0)$ is not a Killing vector of this space-time, it satisfies asymptotically the Killing condition $g_{\alpha\sigma}\partial_\beta K^\sigma + g_{\beta\sigma}\partial_\alpha K^\sigma + K^\sigma \partial_\sigma g_{\alpha\beta} = 0$ when $\tilde{x} \to \pi/2$. Since no matter is present ($T_{\alpha\beta} = 0$) in the solution (1), this shows the Λ imprints in a universe can exhibit a symmetry in some regions of space-time while this symmetry is not present elsewhere. Similarly, it is natural to consider that the large-scale isotropy of the Universe is *a priori* not incompatible with local Λ anisotropic effects. Besides, even at the cosmological scales, our Universe differs from its RW representation since it is known that the large-scale matter repartition is far from being homogeneous [14]. Whatever the case, it is worth having in mind that, besides the numerous effects listed at the beginning of this paper, the source of the accelerated expansion could also result in local effects that may take a significant part in the anisotropic dynamics observed on nearby galaxies and clusters.

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