

Light propagation with nonminimal couplings in a two-component cosmic dark fluid with an Archimedean-type force, and unlighted cosmological epochs

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During the evolution of the universe there are at least two epochs during which electromagnetic waves cannot scan the universe's internal structure to bring information to outside observers. The first epoch is when photons are in local thermodynamic equilibrium with other particles, and the second is when photon scattering by charged particles is strong. One can call these two periods of cosmological time as standard unlighted epochs. After the last scattering surface, photons become relic photons and turn into a source of information about the universe. Unlighted cosmic epochs can also appear when one considers nonminimal theories, i.e., theories in which the electromagnetic field is coupled in an intricate way with the cosmological gravitational field. By considering a cosmological model where the dark sector, i.e., the dark energy and dark matter, self-interacts via an Archimedean-type force, and taking into account a nonminimal coupling theory for the electromagnetic field, we discuss the appearance of unlighted epochs. In the framework of our nonminimal theory, a three-parameter nonminimal Einstein-Maxwell model, the curvature coupling can be formulated in terms of an effective refraction index $n(t)$. Then, taking advantage of a well-known classical analogy, namely, in a medium with $n^2 < 0$ electromagnetic waves do not propagate and their group velocity, i.e., energy transfer velocity, has zero value at the boundary of the corresponding zone, one can search for the unlighted epochs arising in the interacting dark fluid cosmological model. We study here, both analytically and numerically, cosmological models admitting unlighted epochs.

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I. INTRODUCTION

A. The cosmology, two-component dark fluid and Archimedean-type interaction

In order to examine important features happening within the universe, such as light propagation, one has to set up from the start a model for the dynamics of the universe as a whole, smoothing out all irregularities. General relativity tells that both geometry and matter are important in the dynamics of the universe and, moreover, through Einstein's equations one finds that geometry guides the matter and matter changes the geometry.

In relation to geometry, we now have a good idea of the spacetime geometry of the universe. It is governed by a cosmological time t , which is the proper time of the fundamental particles, usually considered as galaxies, of the substratum. Slices of this time yield, by the use of the cosmological principle, a homogeneous and isotropic spatial geometry, so that the metric is the Friedmann-

Lemaître-Robertson-Walker (FLRW) metric. From observations the universe is expanding, which is then taken into account by a single function, the cosmological scale factor $a(t)$, which once known, also yields the rate of expansion, i.e., the Hubble function $H(t) \equiv \frac{\dot{a}}{a}$, and the rate of acceleration, i.e., the acceleration parameter $-q(t) \equiv \frac{\ddot{a}}{aH^2}$. Moreover, the spatial slices are flat, or almost flat, simplifying the problem even further (see, e.g., [1]).

In relation to matter, we also have now a good idea of the matter content of the universe. There are three main components, namely, dark energy which accounts for 70% of the matter, dark matter which accounts for around 25%, and the rest, which includes baryonic matter, radiation, and other forms such as black holes, which accounts for 5%. Dark energy and dark matter are essential building blocks in any cosmological model. Their precise nature, both of the dark energy and dark matter, is unknown, but this does not preclude a good understanding of the gross features of the dynamics of the universe. Dark energy [2–4] is essential in the explanation of the observed accelerated expansion of the universe [5,6]. Dark matter provides an excellent explanation for the flat velocity curves seen in the outskirts of a galaxy and for the dispersion velocities of

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galaxies in clusters of galaxies [7,8]. Given there are these two important matter components that essentially drive the dynamics, it is worth contemplate them as a single whole. There are various manners in which this could be performed. One intriguing possibility is considering both components as manifestations of a single dark fluid [9–13]. Other possibility is to postulate an interaction between them so that the two components although really distinct have to be treated in a broader unified scheme, see e.g., [14,15].

In [16,17] this latter possibility of a unified interaction scheme between dark energy and dark matter has been followed. The background for the interaction itself was given by relativistic hydrodynamics theory for the dark energy component and a relativistic kinetics framework for the dark matter component. Baryonic matter is negligible in the context of cosmological dynamics and so it has been left out in the scheme. The interaction itself between both components has been provided by an Archimedean-type four-force, a direct generalization of the Archimedean buoyancy law, in which the three-force is proportional to the gradient of the pressure. The dark energy pressure is of the same order as the dark energy-density and for this reason, the influence of the Archimedean-type force on the dark matter component can be important. Now, the dark energy pressure can be negative, in which case the Archimedean-type force can be negative, instead of positive as in the usual buoyancy force case. In the case the pressure changes sign as the universe evolves, there appear several different stages in its acceleration, in which the acceleration itself can change sign. It was further shown in [16,17] that the Archimedean-type force is able to distribute between the two fluid components the corresponding energy content of the universe, which in turn guides the whole evolution of the universe, in its one or several stages of acceleration. Multistage universes have also appeared in [18].

B. Nonminimal coupling

The way in which the electromagnetic field couples to gravity is an open question. Usually it is assumed a minimal coupling where simple flat spacetime derivatives involving the field are replaced by covariant derivatives but this might not be so. For instance, one can replace the flat spacetime derivatives by covariant derivatives along with terms involving the curvature tensor and its contractions, yielding a nonminimal coupled theory. In a gravitational strong regime, like in the vicinity of black holes or in a cosmological context, these additional terms, if present, can be felt and are important (see, e.g., original papers and reviews [19–40]).

A simple and most fruitful nonminimal theory, a non-minimal Einstein-Maxwell theory, possesses an action with a Lagrangian which includes a linear combination of three cross-invariants, namely, $q_1 R F_{mn} F^{mn}$, $q_2 R^{ik} F_{im} F_{kn} g^{mn}$, and $q_3 R^{ikmn} F_{ik} F_{mn}$, where q_1 , q_2 , and

q_3 are free parameters. The scalars appearing in the Lagrangian are linear in the Riemann tensor $R^i{}_{kmn}$, Ricci tensor R_{kn} , and Ricci scalar R , and are quadratic in the Maxwell tensor F_{ik} . An interesting fact about this non-minimal theory is that the corresponding nonminimal electrodynamic equations have the same form as the equations for the electrodynamics of anisotropic inhomogeneous continuous media (see, e.g., [41–43]). This circumstance makes it possible to consider nonminimal analogs of well-known phenomena in classical electrodynamics of continuous media, such as birefringence induced by curvature [30], anomalous behavior of electric and magnetic fields nonminimally coupled to the gravitational radiation [44], curvature-induced Cherenkov effects [45], and optical activity in a vacuum [46]. Of course, the variety of non-minimal effects depends on the spacetime of the model and its symmetries.

Since electromagnetic waves, from radio waves, to light, to gamma rays, are a very important source of information about the universe, it is worthwhile to study the changes that might operate in a universe with a nonminimal coupling between the electromagnetic and gravitational fields.

C. Nonminimal electromagnetic wave propagation in an expanding universe with a cosmic dark fluid with Archimedean-type interaction, and unlighted epochs

1. Prologue

In a generic gravitational background field, electromagnetic waves nonminimally coupled propagate with a velocity which differs from the velocity of light in a vacuum. These waves, influenced by tidal interactions induced by curvature, do not travel along null geodesics of the background spacetime. Thus, nonminimal coupling may produce significant changes in the propagation of electromagnetic waves. It is therefore worthwhile to study this coupling in a cosmological context.

In cosmology, in particular, when one deals with a spatially homogeneous isotropic expansion for the universe, one is faced with electromagnetic effects, possibly nonminimal, of two types: first, the phase and group wave velocities depend on the cosmological time, and second, the amplitude and energy-density of the electromagnetic waves decrease with time. Different aspects of these two phenomena have already been studied within a nonminimal setting. For instance, in [38] the deviation of the photon velocity from the velocity of light in vacuum was estimated in the framework of Drummond and Hathrell's approach [21].

2. Motivation

We are interested here in developing the study of the propagation of electromagnetic waves nonminimally coupled to the gravitational field in an isotropically expanding universe. Since cosmological models with dark fluid interaction are of great interest, such as those studied

in [16,17], there are now new motives to study anew the propagation of these electromagnetic waves nonminimally coupled to the gravitational field in an isotropically expanding universe governed by a dark fluid. We mention three such motives.

The first motive is related with the synergy between nonminimal electromagnetic wave propagation and a cosmology in which a dark fluid (i.e., a fluid with two components, namely, dark energy plus dark matter) plays a major role [16,17]. In our nonminimal theory there are three coupling constants q_1 , q_2 , and q_3 . These nonminimal constants can be determined directly from a more fundamental theory, as was done in the case of quantum electrodynamics vacuum polarization effects in a curved background by Drummond and Hathrell [21], or they can be considered as phenomenological inputs. Of course, the results for q_1 , q_2 , and q_3 , in [21] should be thought of as providing a portion of the whole values to the three constants, since other effects can provide curvature-induced interactions between the fields, and all the effects should be summed together. Our idea here is that a dark fluid can also support nonminimal interactions. Thus, we consider the parameters q_1 , q_2 , and q_3 to be phenomenologically introduced and do trust that a mechanism of nonminimal coupling between electromagnetism and gravity via a dark fluid can be clarified in the future. In other words, if a dark fluid, as a medium with unusual properties, can act as a mediator of nonminimal coupling between photons and gravitons, we expect that the coupling parameters q_1 , q_2 , and q_3 will be estimated theoretically and established from observations.

The second motive is related to the existence of unlighted epochs in our models. The nonminimal coupling of the electromagnetic waves with the gravitational field can be codified in terms of an effective time-dependent refraction index $n(t)$, where t denotes cosmological time. From this, we will show that for specific values of the nonminimal parameters q_1 , q_2 , and q_3 , $n^2(t)$ can be negative during some finite time interval, and can either vanish or take infinite values at some transition points $t_{(s)}$, i.e., points where the sign of $n^2(t)$ changes, the subscript s , labeling the several possible transition points $s = 1, 2, \dots$ and denoting sign change. This means that the universe can pass through epochs when electromagnetic waves cannot propagate as their phase velocity is a pure imaginary quantity during this period of time. Taking into account that in order to read the history of the universe one should reconstruct the sequence of the events by using the whole spectrum of electromagnetic radiation, we coin such epochs as unlighted epochs, since portraits of the universe of those epochs cannot be available. Moreover, the equations for trapped surfaces in the universe have here the form $t = t_{(s)}$, and they are time-like in contrast to the standard space-like ones that appear in black hole formation. The use of the term unlighted epoch is then well fit, since it distinguishes clearly the two situations, namely

unlighted epochs in the universe versus trapped surfaces in black holes.

The third motive is determined by the necessity to divide properly the history of the universe into epochs according to various physical scenaria. Since the transition points in the universe history are fixed by the critical temperatures $T_{(c)}$ of some basic physical processes, one should link the temperature evolution $T(t)$ and the cosmological time t by using the cosmological scale factor $a(t)$. When the cosmological time scale is defined upon using relic cosmic microwave background photons traveling along null geodesic lines, the corresponding law is very simple, $T(t)a(t) = T(t_0)a(t_0)$, where t_0 is some reference time, like now, say. On the other hand, when we take into account a nonminimal coupling between photons and gravitons, the corresponding link between the temperature $T(t)$ and the scale factor $a(t)$ is much more sophisticated. Indeed, it is determined not only by $a(t)$, but also by its first and second derivatives, $\dot{a}(t)$ and $\ddot{a}(t)$, respectively. In other words, the temperature $T(t)$ is tied to the scale factor $a(t)$, the Hubble function $H(t) \equiv \frac{\dot{a}}{a}$ and the acceleration parameter $-q(t) \equiv \frac{\ddot{a}}{aH^2}$. This fact, of course, should induce novel and interesting features.

3. This work

In [16,17] a cosmological model based on a dark fluid made of two components, the dark energy, considered as the fluid substratum, and the dark matter, described in the framework of kinetic theory, interacting via an Archimedean-type force was considered. The evolution models for the universe were classified in terms of the admissible transition points, i.e., the points in which an accelerated expansion of the universe is changed to a decelerated expansion and vice versa. This classification is based on the analysis of the function acceleration parameter $-q(t) \equiv \frac{\ddot{a}}{aH^2}$. Now, we find here that the function $n^2(t)$, describing the behavior of the effective refraction index, also includes the functions $H(t)$ and $-q(t)$. Thus, in order to analyze non-minimal light propagation, it is useful to use $n^2(t)$ instead of $-q(t)$. In this work we study, analytically and numerically, the propagation of electromagnetic waves nonminimally coupled, along with their phase and group velocities, in six cosmological models based on a dark fluid which self-interacts through an Archimedean-type force [16,17]. This work is thus an interesting sequel of [16,17].

4. The organization of the paper

The paper is organized as follows. In Sec. II, we introduce the cosmological model. In Sec. II A, we give the cosmological model itself and recall the basic formulas of the Archimedean-type model, which are necessary for further numerical analysis. In Sec. II B, we give the basic setup for nonminimal interaction and light propagation. In

Sec. II C, using the master equations of nonminimal electrodynamics we reconstruct the effective dielectric and magnetic permeabilities, the effective refraction index $n(t)$, the effective (optical) metric in terms of the scale factor $a(t)$, the Hubble function $H(t)$, acceleration parameter $-q(t)$, and the three nonminimal coupling phenomenological constants q_1 , q_2 , and q_3 . We also discuss the refraction index $n(t)$, focus on the derivation of the expression for the group velocity of the waves (i.e., the velocity of energy transfer) using the analogy with electrodynamics of continua, and define unlighted epochs. In Sec. III, we make an analytical study of the unlighted epochs. In Sec. III A, using the Kohlrausch stretched exponential functions we give the cosmological models fit for the study, in Sec. III B, we provide the choice for the nonminimal models, and in Sec. III C, we give some analytical cosmological examples describing unlighted epochs. In Sec. IV, we consider the results of numerical analysis for the refraction index $n(t)$, phase and group velocities and effective lengths of the photon trips for six basic Archimedean-type submodels. In Sec. IV A, we discuss the cosmology, in Sec. IV B, we provide the choice for the nonminimal models, and in Sec. IV C, the models are analyzed in detail, including the submodels of perpetually accelerated universes, periodic and quasiperiodic universes, and various submodels with one, two, and three transition points. We also study the photon path length, the true duration of epochs, and the universe lifetime for these models. In Sec. V, we draw conclusions. In the Appendix, we analyze a number of one-parameter examples for which the coupling constants are linked by two relations motivated physically and geometrically.

II. THE COSMOLOGICAL MODEL

A. Cosmological context

1. The Cosmology

We start from the action functional

$$S = \int d^4x \sqrt{-g} \left(\frac{R}{2\kappa} + L^{\text{matter}} \right), \quad (1)$$

where R is the Ricci scalar, g is the determinant of the four-dimensional spacetime metric g_{ik} , L^{matter} is the matter Lagrangian which we assume includes a cosmological constant Λ , and $\kappa = \frac{8\pi G}{c^4}$ with G being Newton's constant, and c the velocity of the light. Using the standard variation procedure with respect to the metric g^{ik} one can obtain Einstein's equations. Latin indices run from 0 to 3. In the above we assume that L^{matter} , and so the corresponding energy-momentum tensor, T_{ik}^{matter} , comes from both the dark energy and dark matter, which together make up most of the total energy of the universe. Thus, the master equations for the gravity field are assumed to be the usual Einstein's equations,

$$R_{ik} - \frac{1}{2}Rg_{ik} = \kappa T_{ik}^{\text{matter}}, \quad (2)$$

where R_{ik} is the Ricci tensor, given by the contraction of the Riemann curvature tensor R_{imk}^l .

We use the spatially homogeneous flat metric of Friedmann-Lemaître-Robertson-Walker (FLRW) type given by

$$ds^2 = dt^2 - a^2(t)(dx^2 + dy^2 + dz^2), \quad (3)$$

where we chose units with $c = 1$. The energy-momentum tensor is described by the functions $\rho(t)$ and $\Pi(t)$, and $E(t)$, and $P(t)$. The functions $\rho(t)$ and $\Pi(t)$ describe the energy-density and pressure of the dark energy, respectively. The functions $E(t)$ and $P(t)$ are the energy-density and pressure of the dark matter, respectively. The cosmological constant Λ is incorporated into the dark energy state functions, i.e., the term $\frac{\Lambda}{8\pi G}$ is included into ρ and Π [16]. With these assumptions the equations coming out of Eq. (2) for the gravitational field read

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}(\rho + E), \quad (4)$$

$$\left(\frac{\dot{a}}{a}\right) + \left(\frac{\dot{a}}{a}\right)^2 = -\frac{4\pi G}{3}[(\rho + E) + 3(\Pi + P)], \quad (5)$$

with the dot denoting a derivative with respect to time.

2. Two-component dark fluid: Archimedean-type interaction between dark energy and dark matter

In order to describe the evolution of several quantities, in particular, of the refraction index $n(t)$, as this is the quantity we are interested in, we need the Hubble function $H(t)$, defined as

$$H(t) \equiv \frac{\dot{a}}{a}, \quad (6)$$

and the acceleration parameter $-q(t)$,

$$-q(t) \equiv \frac{\ddot{a}}{aH^2}. \quad (7)$$

Sometimes it is useful to swap $-q(t)$ for \dot{H} , given by,

$$\dot{H}(t) = -H^2(t)[1 + q(t)]. \quad (8)$$

Now, we obtain these functions using an Archimedean-type interaction between the dark energy and dark matter model [16,17]. The function $H(t)$ can be found from Eq. (6) and Einstein's Eq. (4), yielding

$$H^2(t) = \frac{8\pi G}{3}(\rho + E). \quad (9)$$

The acceleration parameter $-q(t)$ can be found from Eq. (7) and Einstein's Eq. (5), yielding

$$-q(t) = -\frac{1}{2} \left[1 + 3 \left(\frac{\Pi + P}{\rho + E} \right) \right]. \quad (10)$$

As for the function $\dot{H}(t)$, one uses Eqs. (8)–(10) to find

$$\dot{H}(t) = -4\pi G(\rho + E + \Pi + P). \quad (11)$$

Thus, we need to find the following state functions: the energy-density of the dark energy $\rho(t)$ and its pressure $\Pi(t)$, the energy-density of the dark matter $E(t)$ and its pressure $P(t)$. The search scheme for these quantities is the following. First of all introducing a new convenient variable

$$x \equiv \frac{a(t)}{a(t_0)}, \quad (12)$$

and using the auxiliary formulas

$$\frac{d}{dt} = xH(x)\frac{d}{dx}, \quad t - t_0 = \int_0^{(a(t)/a(t_0))} \frac{dx}{xH(x)}, \quad (13)$$

we find, through some manipulation of Einstein's equations a key equation for $\Pi(x)$

$$\xi x^2 \Pi''(x) + x \Pi'(x)(4\xi + \sigma) + 3(1 + \sigma)\Pi + 3\rho_0 = \mathcal{J}(x), \quad (14)$$

a second-order differential equation, linear in the first and second derivatives, $\Pi' \equiv \frac{d}{dx}\Pi$ and Π'' , and non-linear in the function $\Pi(x)$. The parameters ξ , σ , and ρ_0 are the coupling constants involved into the assumed linear inhomogeneous equation of state, namely,

$$\rho(x) = \rho_0 + \sigma\Pi(x) + \xi x \frac{d}{dx}\Pi(x). \quad (15)$$

This equation links the pressure to the energy-density of the dark energy (see [16] for details). The source-term $\mathcal{J}(x) = \mathcal{J}(x, \Pi - \Pi(1), \Pi')$ in the right-hand side of (14) is given by the integral

$$\mathcal{J}(x) = -\sum_{(a)} E_{(a)} \frac{[x^2 F_{(a)}(x)]'}{2x^4} \int_0^\infty \frac{y^4 dy e^{-\lambda_{(a)}\sqrt{1+y^2}}}{\sqrt{1+y^2} F_{(a)}(x)}, \quad (16)$$

where we used the definitions

$$F_{(a)}(x) = \frac{1}{x^2} \exp\{2\mathcal{V}_{(a)}[\Pi(1) - \Pi(x)]\}, \quad (17)$$

$$E_{(a)} \equiv \frac{N_{(a)} m_{(a)} \lambda_{(a)}}{K_2(\lambda_{(a)})}, \quad \lambda_{(a)} \equiv \frac{m_{(a)}}{k_B T_{(a)}}, \quad (18)$$

$$K_\nu(\lambda_{(a)}) \equiv \int_0^\infty dz \cosh \nu z \exp[-\lambda_{(a)} \cosh z]. \quad (19)$$

Here $\mathcal{V}_{(a)}$ are the constants of the Archimedean-type coupling (see [16] for details), $K_\nu(\lambda_{(a)})$ are the modified Bessel functions of second kind of order ν (we are interested in the $\nu = 2$ case), $\Pi(1) \equiv \Pi(t_0)$ is the initial value of the dark energy pressure, k_B is the Boltzmann constant, and $T_{(a)}$, $m_{(a)}$, and $N_{(a)}$ are the partial temperature, the mass, and the number of particles per unit volume of the sort (a),

respectively. When the quantity $\Pi(x)$ is found, other state functions can be calculated using the integrals

$$E(x) = \sum_{(a)} \frac{E_{(a)}}{x^3} \int_0^\infty y^2 dy \sqrt{1+y^2} F_{(a)}(x) e^{-\lambda_{(a)}\sqrt{1+y^2}}, \quad (20)$$

$$P(x) = \sum_{(a)} \frac{E_{(a)}}{3x^3} \int_0^\infty \frac{F_{(a)}(x) y^4 dy}{\sqrt{1+y^2} F_{(a)}(x)} e^{-\lambda_{(a)}\sqrt{1+y^2}}. \quad (21)$$

Thus we have formulated our integration scheme. From here, we want to work out novel features that appear in such a cosmological context.

B. Nonminimal coupling: setup

Now we want to study light propagation envisaged as a perturbation in the background cosmological manifold. This means that the electromagnetic field propagates in a given geometry, without modifying the geometry itself. The electromagnetic field is a test field. This is well justified since it is known that the energy-density of radiation is thoroughly negligible in relation to the dark energy, dark matter, and baryonic matter. We also assume that the field is nonminimally coupled and write the electromagnetic action functional as,

$$S^{\text{electromag}} = \frac{1}{4} \int d^4x \sqrt{-g} (F_{mn} F^{mn} + \mathcal{R}^{ikmn} F_{ik} F_{mn}). \quad (22)$$

Using the standard variation procedure with respect to the electromagnetic potential four-vector A_i one can obtain the nonminimally extended Maxwell equations (see, e.g., [43], see also [34]). Here $F_{ik} = \partial_i A_k - \partial_k A_i$ is the Maxwell tensor. Then, the nonminimal Maxwell equations have the following form:

$$\nabla_k H^{ik} = 0, \quad \nabla_k F^{*ik} = 0, \quad (23)$$

where the excitation tensor H^{ik} is linked with the Maxwell tensor F_{mn} by a linear constitutive equation

$$H^{ik} = C^{ikmn} F_{mn}, \quad (24)$$

with the linear response tensor C^{ikmn} given by

$$C^{ikmn} = \frac{1}{2} (g^{im} g^{kn} - g^{in} g^{km}) + \mathcal{R}^{ikmn}. \quad (25)$$

The dual tensor $F^{*ik} \equiv \frac{1}{2} \epsilon^{ikmn} F_{mn}$ is defined in a standard way through the Levi-Civita tensor ϵ^{ikmn} . The tensor \mathcal{R}^{ikmn} is the nonminimal susceptibility tensor decomposed as

$$\begin{aligned} \mathcal{R}^{ikmn} &\equiv \frac{q_1 R}{2} (g^{im} g^{kn} - g^{in} g^{km}) \\ &+ \frac{q_2}{2} (R^{im} g^{kn} - R^{in} g^{km} + R^{kn} g^{im} - R^{km} g^{in}) \\ &+ q_3 R^{ikmn}, \end{aligned} \quad (26)$$

where q_1 , q_2 , and q_3 are the phenomenological parameters of the nonminimal coupling, and R^{ikmn} , R^{im} , and R , are the Riemann tensor, the Ricci tensor, and the Ricci scalar of the background cosmological manifold, respectively. It is appropriate here to recall that the contracted susceptibility tensor \mathcal{R}^{im} satisfies

$$\mathcal{R}^{im} \equiv g_{kn} \mathcal{R}^{ikmn} = \frac{1}{2} R g^{im} (3q_1 + q_2) + R^{im} (q_2 + q_3), \quad (27)$$

and so vanishes in a generic curved spacetime when the nonminimal coupling parameters are linked through two relations $3q_1 + q_2 = 0$ and $q_2 + q_3 = 0$. Analogously, the scalar \mathcal{R} given by

$$\mathcal{R} \equiv g_{im} g_{kn} \mathcal{R}^{ikmn} = g_{im} \mathcal{R}^{im} = R(6q_1 + 3q_2 + q_3), \quad (28)$$

has zero value in a generic curved spacetime when $6q_1 + 3q_2 + q_3 = 0$.

C. Nonminimal light propagation in a cosmological context: Optical metric and refraction index, phase and group velocities, and unlighted epochs

1. Optical metric and refraction index

The optical metric—Based on the metric (3) and on the symmetries of the spacetime one obtains that the nonvanishing components of the nonminimal susceptibility tensor \mathcal{R}^{ik}_{mn} , see Eq. (26), have the following form:

$$\begin{aligned} -\mathcal{R}^{1t}_{1t} &= -\mathcal{R}^{2t}_{1t} = -\mathcal{R}^{3t}_{1t} \\ &= (3q_1 + 2q_2 + q_3) \frac{\ddot{a}}{a} + (3q_1 + q_2) \left(\frac{\dot{a}}{a} \right)^2, \end{aligned} \quad (29)$$

$$\begin{aligned} -\mathcal{R}^{12}_{12} &= -\mathcal{R}^{13}_{13} = -\mathcal{R}^{23}_{23} \\ &= (3q_1 + q_2) \frac{\ddot{a}}{a} + (3q_1 + 2q_2 + q_3) \left(\frac{\dot{a}}{a} \right)^2, \end{aligned} \quad (30)$$

where the indices 1, 2, 3 correspond to x , y , z , respectively. In addition, due to the spacetime isotropy the linear response tensor C^{ikmn} can be rewritten in the standard multiplicative form

$$C^{ikmn} = \frac{1}{2\mu(t)} (g_*^{im} g_*^{kn} - g_*^{in} g_*^{km}), \quad (31)$$

where

$$g_*^{im} = n^2(t) \delta^i_t \delta^m_t - \frac{1}{a^2(t)} (\delta^i_1 \delta^m_1 + \delta^i_2 \delta^m_2 + \delta^i_3 \delta^m_3), \quad (32)$$

is the so-called associated metric (see [47]), and the function $n^2(t)$ is defined as

$$n^2(t) \equiv \varepsilon(t) \mu(t), \quad (33)$$

with $\varepsilon(t)$ and $\mu(t)$ given by

$$\varepsilon(t) \equiv 1 + 2\mathcal{R}^{1t}_{1t}, \quad \frac{1}{\mu(t)} \equiv 1 + 2\mathcal{R}^{12}_{12}. \quad (34)$$

Taking into account that in a cosmological context the global velocity four-vector is $U^i = \delta^i_t$, one can see that the associated metric (32) is in fact an optical metric [48,49] given by

$$g_*^{im} = g^{im} + [n^2(t) - 1] U^i U^m. \quad (35)$$

Its inverse is then

$$g_{*km} = g_{km} + \left[\frac{1}{n^2(t)} - 1 \right] U_k U_m, \quad (36)$$

with

$$g_*^{im} g_{*km} = \delta^i_k, \quad (37)$$

holding. The quantity $n(t)$ is then interpreted as an effective refraction index due to its correspondence, in conjunction with the optical metric g_*^{im} , to the effective refraction in the geometrical optics approximation framework.

Let us see this in detail. In the geometrical optics approximation, the electromagnetic potential A_k and the field strength F_{kl} can be put as follows:

$$\begin{aligned} A_k &= \tilde{A}_k e^{i\Psi}, & F_{kl} &= i[p_k \tilde{A}_l - p_l \tilde{A}_k] e^{i\Psi}, \\ p_k &= \nabla_k \Psi, \end{aligned} \quad (38)$$

for some amplitude \tilde{A}_k and phase function Ψ . In the leading order approximation the Maxwell equations (23) reduce to $C^{ikmn} p_k p_m \tilde{A}_n = 0$, i.e.,

$$[g_*^{im} g_*^{kn} - g_*^{in} g_*^{km}] p_k p_m \tilde{A}_n = 0. \quad (39)$$

Using the optical metric property $g_*^{im} U_m = n^2 U^i$, the Landau gauge $U^m \tilde{A}_m = 0$, and the requirement that the frequency $\omega \equiv U^m p_m$ is nonvanishing, one can show that Eq. (39) leads to the dispersion equation

$$g_*^{km} p_k p_m = 0. \quad (40)$$

Thus, the propagation of photons nonminimally coupled to the gravity field is equivalent to their motion along a null geodesic line in an effective spacetime with metric (35) (see, e.g., [48,49] for details). In our setting, the refraction index $n(t)$ determines the effective phase velocity of light in our FLRW-type spacetime, namely, $V_{\text{ph}} \equiv \frac{c}{n(t)}$. Thus, let us consider in more detail the properties of the quantity $n^2(t)$.

The expression for the refraction index—The square of the effective refraction index can be obtained from Eqs. (33) and (34) and is given by

$$n^2(t) = \frac{1 - 2(3q_1 + 2q_2 + q_3) \frac{\ddot{a}}{a} - 2(3q_1 + q_2) \left(\frac{\dot{a}}{a} \right)^2}{1 - 2(3q_1 + q_2) \frac{\ddot{a}}{a} - 2(3q_1 + 2q_2 + q_3) \left(\frac{\dot{a}}{a} \right)^2}. \quad (41)$$

Notably, it can be put as a function of the Hubble function $H(t)$ and the acceleration parameter $-q(t)$, defined in (6) and (7), as well as of two effective nonminimal coupling constants, Q_1 and Q_2 given by,

$$Q_1 \equiv -2(3q_1 + 2q_2 + q_3), \quad Q_2 \equiv -2(3q_1 + q_2). \quad (42)$$

As a function of these quantities Eq. (41) can be rewritten as

$$n^2(t) = \frac{1 + [Q_2 - Q_1 q(t)]H^2(t)}{1 + [Q_1 - Q_2 q(t)]H^2(t)}. \quad (43)$$

For the cosmological models with a de Sitter-type final stage (i.e., $-q(t \rightarrow \infty) \rightarrow 1$), Eq. (43) yields $n^2 \rightarrow 1$. The signs and the values of the parameters Q_1 and Q_2 are not yet established, but one can discuss several phenomenological possibilities using geometric analogies and physical motivations, see the Appendix.

2. Phase velocity and group velocity (or energy transfer velocity)

The phase velocity of an electromagnetic wave is defined as ($c = 1$)

$$V_{\text{ph}} \equiv \frac{\omega}{k} = \frac{1}{n(t)}. \quad (44)$$

This definition is standard and follows directly from the dispersion relation [50].

On the other hand, the definition of group velocity, or energy transfer velocity, V_* , is connected with the definition of the electromagnetic field energy flux, and this problem requires a preliminary discussion. First of all, one should state that formally the propagation of electromagnetic waves takes place in isotropic backgrounds with a refraction index equal to one, but under the influence of a nonminimal coupling the interaction can be reformulated in terms of an effective refraction index $n(t)$, which depends on time through the Riemann tensor components. Since, as we have seen, the nonminimal coupling of photons to gravity is equivalent to the consideration of some isotropic dispersive medium, it seems reasonable to take the relevant stress-energy tensor of the electromagnetic field for the definition of the energy transfer velocity V_* . Now, there is a number of definitions for the electromagnetic stress-energy tensor in a medium, the best known are the ones by Minkowski [51], Abraham [52,53], Grot-Engingen-Israel-Maugin [54–57], Hehl-Obukhov [58], and de Groot-Suttorp [59]. The energy flux four-vectors in these definitions differ from one another. Thus for us a choice between the several definitions is considered as an ansatz.

We follow the Hehl-Obukhov definition [58], according to which the stress-energy tensor formally is as in vacuum, $T_{ik}^{\text{electromag}}$, i.e., $T_{ik}^{\text{electromag}} \equiv \frac{1}{4}g_{ik}F^{mn}F_{mn} - F_{im}F_k^m$, and

thus is symmetric, traceless, and does not depend on the macroscopic velocity of the medium. Nevertheless, the ponderomotive force

$$F^l \equiv \nabla_k T^{\text{electromag}kl} = F^l_m \nabla_k (H^{km} - F^{km}) \quad (45)$$

is nonvanishing. In contrast to the vacuum case, this stress-energy tensor is not a conserved quantity, its four-divergence is equal to the ponderomotive force. According to this definition [58], the energy flux I^i is

$$I^i \equiv (g^{ip} - U^i U^p) T_{pq}^{\text{electromag}} U^q = \eta^{ipq} B_p E_q, \quad (46)$$

where

$$\eta^{ipq} \equiv \epsilon^{ipqs} U_s, \quad B_p \equiv F_{pj}^* U^j, \quad E_q \equiv F_{qj} U^j, \quad (47)$$

and the energy-density scalar W is

$$W \equiv U^p T_{pq}^{\text{electromag}} U^q = -\frac{1}{2}(E^m E_m + B^m B_m). \quad (48)$$

When we deal with test electromagnetic waves in an isotropic universe, the necessary quantities can be found as follows. The nonminimally extended Maxwell equation can be rewritten now in the form

$$\nabla_k \left[\frac{1}{\mu} F^{ik} + \frac{n^2 - 1}{\mu} (F^i_m U^k - F^k_m U^i) U^m \right] = 0, \quad (49)$$

where $\mu(t)$ and $n^2(t)$ are given by (33) and (34). Let the direction along which the electromagnetic wave propagates be x^1 , say. Then Eq. (49) admits the following solution for the potential four-vector,

$$A_i(t, x^1) = \delta_i^2 b(t) \cos \Psi, \quad (50)$$

with

$$\Psi = \Psi(t_0) + k_1 [x^1 - f(t)], \quad (51)$$

and where k_1 is a constant wave-vector, $b(t)$ the amplitude factor, and $f(t)$ the frequency function. These satisfy the following equations:

$$\frac{df}{dt} = \frac{1}{an} \sqrt{1 + \left(\frac{a\mu}{bk_1^2} \right) \frac{d}{dt} \left[\left(\frac{an^2}{\mu} \right) \frac{db}{dt} \right]}, \quad (52)$$

$$\frac{d}{dt} \left[\left(\frac{b^2 an^2}{\mu} \right) \frac{df}{dt} \right] = 0. \quad (53)$$

In the geometrical optics approximation, i.e., for short waves, one has ($k_1 \rightarrow \infty$) and (52) gives $\dot{f} = \frac{1}{an}$, i.e., the quantity $\omega(t) \equiv k_1 \dot{f} = \frac{k_1}{an}$ plays the role of a time-dependent frequency, the quantity $k(t) \equiv \frac{k_1}{a}$ is

a wave-vector modulus, and the dispersion relation takes the form $\omega(t) = \frac{k(t)}{n(t)}$. In this approximation, Eq. (53) gives $b(t) = b_0 \sqrt{\frac{\mu(t)}{n(t)}}$.

From a physical point of view (see, e.g., [50]) it is reasonable to calculate the nonvanishing energy flux four-vector component and the energy-density scalar averaged over a wave period, in which case one can use $\langle \cos^2 \Psi \rangle = \langle \sin^2 \Psi \rangle = \frac{1}{2}$ and $\langle \cos \Psi \sin \Psi \rangle = 0$, where $\langle \ \rangle$ denotes average over a period. In the geometrical optics approximation the energy flux I^i and the energy-density scalar W have the form

$$\langle I^1 \rangle = -\frac{1}{a^4} \langle F_{12} F_{12} \rangle = \frac{b^2 k_1^2}{2na^5}, \quad (54)$$

and

$$\langle W \rangle = \frac{1}{2a^2} \langle F_{12}^2 \rangle + \frac{1}{2a^4} \langle F_{12}^2 \rangle = \frac{b^2 k_1^2 (n^2 + 1)}{4a^4 n^2}. \quad (55)$$

Thus, the physical component of the energy transfer velocity V_* is

$$V_* = \frac{\sqrt{-g_{11} \langle I^1 \rangle \langle I^1 \rangle}}{\langle W \rangle} = \frac{2n}{n^2 + 1}. \quad (56)$$

The function $V_*(n(t)) = \frac{2n}{n^2 + 1}$ is appropriate for the description of the energy transfer velocity, since depending on the refraction index n it vanishes in the limits $n \rightarrow 0$ and $n \rightarrow \infty$. In addition, its value is maximum in a pure vacuum since $V_* = 1$ when $n = 1$, and generically satisfies the inequality $|V_*(n(t))| \leq 1$. Below, to simplify terminology, we indicate this energy transfer velocity as group velocity V_{gr} , i.e., $V_{\text{gr}} \equiv V_*$, and so

$$V_{\text{gr}} = \frac{2n}{n^2 + 1}. \quad (57)$$

3. Unlighted cosmological epochs: definition

It is remarkable that the square of the effective refraction index can take negative values, $n^2(t) < 0$, for some set of the parameters Q_1 and Q_2 . We call the time intervals for which $n^2(t) < 0$ as unlighted epochs, since during these periods of time the refraction index is a pure imaginary quantity, and the phase and group velocities of the electromagnetic waves are not defined. The function $n^2(t)$ can change sign at the moments $t_{(s)}$ of the cosmological time when the numerator or the denominator in Eq. (43) vanish. When the numerator vanishes, one has $n(t_{(s)}) = 0$, $V_{\text{ph}}(t_{(s)}) = \infty$, and $V_{\text{gr}}(t_{(s)}) = 0$. When the denominator vanishes, one has $n(t_{(s)}) = \infty$, $V_{\text{ph}}(t_{(s)}) = 0$, and $V_{\text{gr}}(t_{(s)}) = 0$. In both cases the unlighted epochs appear

and disappear when the group velocity of the electromagnetic waves vanishes, i.e., at these points there is no energy transfer. In our terminology the times $t_{(s)}$ are the unlighted epochs boundary points. For this reason, the condition $V_{\text{gr}}(t_{(s)}) = 0$ sets the criterion for the unlighted epoch appearance or disappearance. In other words, the unlighted epochs start and finish, when the associated metric, i.e., the optical metric, given in Eq. (32) becomes singular.

In our cosmological context we distinguish three types of unlighted epochs:

- (1) Unlighted epochs of the first type: These epochs start at $t_{(1)} = 0$ with $n^2(0) < 0$ and finish with $n^2(t_{(2)}) = 0$.
- (2) Unlighted epochs of the second type: These epochs start at $t_{(1)} > 0$ with $n^2(t_{(1)}) = 0$ and finish at $t_{(2)} > t_{(1)}$ with $n^2(t_{(2)}) = 0$.
- (3) Unlighted epochs of the third type: These epochs appear when at least one boundary point has a refraction index with an infinite value, i.e., $n^2(t_{(1)}) = \infty$ or $n^2(t_{(2)}) = \infty$. Of course, in this type both quantities may be infinite.

Clearly, only one unlighted epoch of the first type can exist, whereas the number of unlighted epochs of the second and third types are predetermined by the guiding parameters Q_1 and Q_2 .

III. UNLIGHTED COSMOLOGICAL EPOCHS: ANALYTICAL STUDY

A. The cosmology

In order to illustrate analytically the physics of unlighted epochs, let us consider a scale factor $a(t)$ given by a stretched exponential function, namely,

$$a(t) = a_0 \exp\{(\mathcal{H}t)^\nu\}, \quad (58)$$

for some constant \mathcal{H} and exponent ν . This function was introduced by Kohlrausch [60] in 1854 and now is systematically used in various physical and mathematical contexts (see, e.g., papers concerning the applications of the Kohlrausch-Williams-Watts (KWW) function [61,62]). When $\nu = 1$ the function (58) coincides with the standard de Sitter exponent. When $\nu = 2$ one deals with an anti-Gaussian function obtained in [16] as an exact solution of a model with Archimedean-type interaction between dark energy and dark matter. This stretched exponent also appeared in [40] in the context of generalized Chaplygin gas models. For the function (58) one obtains from Eqs. (6)–(8),

$$\begin{aligned} H(t) &= \nu \mathcal{H}^\nu t^{\nu-1}, & -q(t) &= 1 + \frac{\nu-1}{\nu} (\mathcal{H}t)^{-\nu}, \\ \dot{H}(t) &= \nu(\nu-1) \mathcal{H}^\nu t^{\nu-2}. \end{aligned} \quad (59)$$

It is reasonable to assume that ν is positive in order to guarantee that $a(t \rightarrow \infty) \rightarrow \infty$ and $-q(t \rightarrow \infty) \rightarrow 1$. Clearly, when $0 < \nu < 1$, \dot{H} is negative, and the Hubble function $H(t)$ vanishes at $t \rightarrow \infty$. Our ansatz is that $\nu \leq 2$. Note that for $\nu < 2$ one has $\dot{H}(t \rightarrow \infty) \rightarrow 0$, and thus, $n^2(t \rightarrow \infty) \rightarrow 1$. Let us focus on the properties of the function $n^2(t)$ for a scale factor of the Kohlrausch type Eq. (58),

$$n^2(t) = \frac{(\mathcal{H}t)^{2-\nu} + \nu^2 \mathcal{H}^2(Q_1 + Q_2)(\mathcal{H}t)^\nu + \nu(\nu-1)\mathcal{H}^2 Q_1}{(\mathcal{H}t)^{2-\nu} + \nu^2 \mathcal{H}^2(Q_1 + Q_2)(\mathcal{H}t)^\nu + \nu(\nu-1)\mathcal{H}^2 Q_2}. \quad (60)$$

When $\nu = 1$, there are no unlighted epochs, since $n^2(t) \equiv 1$. When $\nu = 2$, both the numerator and the denominator in (60) are quadratic functions of t , which means that depending on the values of the parameters Q_1 and Q_2 , one can find zero, one, two, three, or four boundary points $t_{(s)}$. In other words, the number of unlighted epochs can be zero, one, or two. These models represent perpetually accelerated universes since $-q(t)$ is non-negative for $t \geq 0$. When ν is a perfect rational $\nu = \frac{m_1}{m_2}$ with m_1 and m_2 natural numbers and $m_1 < m_2$, the numerator and denominator of the function (60) can be rewritten as a polynomial of order $2m_2 - m_1$. Such a polynomial has $2m_2 - m_1$ roots, part of them, say k , can be real and positive defining the corresponding number of unlighted epochs.

B. The choice for the nonminimal parameters

In order to develop the unlighted epochs in the cosmological models of interest one has to choose the nonminimal parameters. Here we give two examples which are enough to have a feeling of the physics. In one case we put $Q_1 = -Q_2$, and define $Q \equiv Q_1$. In the other case we choose $Q_1 = -\frac{81}{800\mathcal{H}^2}$ and $Q_2 = -\frac{729}{800\mathcal{H}^2}$.

C. Two cosmological models and their unlighted epochs

1. Universe with $\nu < 2$ and $Q_1 = -Q_2 \equiv Q$

We now put $Q_1 = -Q_2$ and define $Q = Q_1$. Then, the square of the refraction index in Eq. (60) has now the form

$$n^2(t) = \frac{(\mathcal{H}t)^{2-\nu} + \nu(\nu-1)\mathcal{H}^2 Q}{(\mathcal{H}t)^{2-\nu} - \nu(\nu-1)\mathcal{H}^2 Q}. \quad (61)$$

There is now only one time moment $t = t_{(1)}$ when n^2 is either equal to zero or infinite. Equivalently, there is only one moment when the group velocity takes zero value. When $(\nu-1)Q < 0$, the denominator is positive, $n^2(0) = -1 < 0$, and the numerator vanishes at $t_{(1)} = \frac{1}{\mathcal{H}} \times [\nu\mathcal{H}^2|(\nu-1)Q|]^{1/(2-\nu)}$; clearly, we deal with an unlighted epoch of the first type, see the dashed line in Fig. 1 left-hand panel. When $(\nu-1)Q > 0$, the numerator is positive, $n^2(0) = -1 < 0$, and the denominator vanishes at $t_{(1)} = \frac{1}{\mathcal{H}} [\nu(\nu-1)\mathcal{H}^2 Q]^{1/(2-\nu)}$; clearly, we deal with an unlighted epoch of the third type with $n^2(t_{(1)}) = \infty$, see the solid line in Fig. 1 left-hand panel.

2. Universe with $\nu = \frac{2}{3}$ and $Q_1 = -\frac{81}{800\mathcal{H}^2}$, $Q_2 = -\frac{729}{800\mathcal{H}^2}$

Using the auxiliary variable $x \equiv (27\mathcal{H}t)^{2/3}$ we can rewrite the square of the refraction index in Eq. (60) in the following form:

$$n^2(t) = \frac{x^2 + 4\mathcal{H}^2(Q_1 + Q_2)x - 18\mathcal{H}^2 Q_1}{x^2 + 4\mathcal{H}^2(Q_1 + Q_2)x - 18\mathcal{H}^2 Q_2}. \quad (62)$$

It is easy to find general conditions when the denominator has no roots and the numerator has two positive

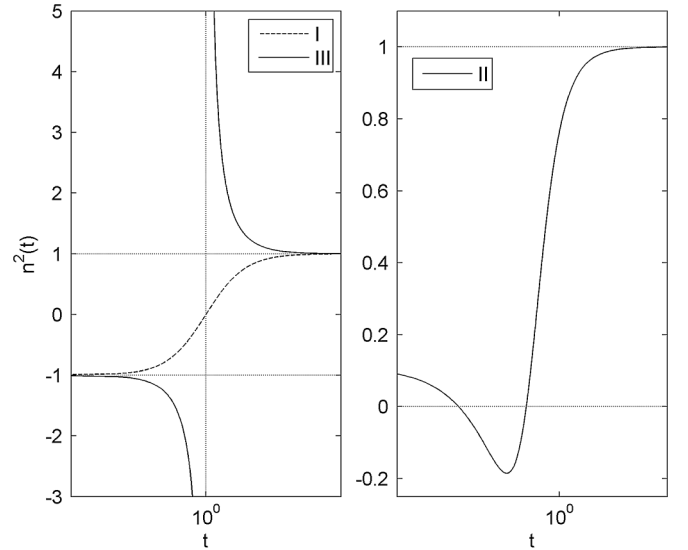


FIG. 1. A plot of $n^2(t)$ as a function of time t in the Kohlrausch-type model. Left panel: Contains sketches of unlighted epochs of the first and third types. For both types the Kohlrausch parameter is set to $\nu = 1.00001$. The nonminimal parameters are the following: $Q_2 = -Q_1 = 10000$ for the model of the first type, and $Q_2 = -Q_1 = -10000$ for the model of the third type. The unlighted epoch of the first type appears in the plot as a dashed curve. It is continuous, it starts with a negative value, $n^2(0) = -1$, and has one zero (the point in which the phase velocity is infinite and the group velocity is vanishing). The unlighted epoch of the third type appears in the plot as a solid curve. It is discontinuous, the function $n^2(t)$ has one infinite jump (in this point the phase and group velocities are vanishing). Right panel: Illustrates an unlighted epoch of the second type with parameters $\nu = 2/3$, $Q_1 = -81/800$, $Q_2 = -729/800$. There are two points, in which the refraction index and group velocity vanish and the phase velocity becomes infinite.

roots. For instance, Fig. 1 right-hand panel, illustrates the case with $Q_1 = -\frac{81}{800\mathcal{H}^2}$ and $Q_2 = -\frac{729}{800\mathcal{H}^2}$. We deal now with an unlighted epoch of the second type, which starts at $t_{(1)} = \frac{1}{\mathcal{H}} \left[\frac{3}{40} (3 - \sqrt{5}) \right]^{3/2}$, and finishes at $t_{(2)} = \frac{1}{\mathcal{H}} \left[\frac{3}{40} (3 + \sqrt{5}) \right]^{3/2}$.

IV. UNLIGHTED COSMOLOGICAL EPOCHS: NUMERICAL STUDY. THE REFRACTION INDEX, PHASE AND GROUP VELOCITIES FOR SIX MODELS AND THE EFFECTIVE PHOTON PATH LENGTH

A. The cosmology

In [16,17] several cosmological models were presented through numerical results. Here we study nonminimal light propagation in six such models. They include, perpetually accelerated universes, periodic universes, universes with one transition, two transition and three transition points, and quasiperiodic universes.

B. The choice for the nonminimal parameters

To study nonminimal light propagation in the universes indicated above, there is a plethora of possibilities to make a choice of the nonminimal parameters. A sample of those possibilities is presented in the Appendix. For our study we make one choice, bearing in mind that the other choices will reproduce qualitatively the same results. We choose that the nonminimal susceptibility scalar \mathcal{R} vanishes, $\mathcal{R} = 0$. From Eq. (28) this means $6q_1 + 3q_2 + q_3 = 0$ and thus, from Eq. (42), one has $Q_1 + Q_2 = 0$. Putting $Q \equiv Q_1 = -Q_2$, one obtains in this one-parameter example the following expression for $n^2(t)$:

$$n^2(t) = \frac{1 - Q\mathcal{H}^2(t)[1 + q(t)]}{1 + Q\mathcal{H}^2(t)[1 + q(t)]} = \frac{1 + Q\dot{H}(t)}{1 - Q\dot{H}(t)}. \quad (63)$$

Now that we see an explicit expression for $n^2(t)$ our choice can be motivated as follows. First, it is an example that best illustrates the several unlighted epochs. This is because only one function, \dot{H} , guides the behavior of the effective refraction index. Second, when $t \rightarrow \infty$, $\dot{H} \rightarrow 0$ and thus $n^2 \rightarrow 1$ providing $V_{\text{ph}} \rightarrow V_{\text{gr}} \rightarrow 1$ in our late-time Universe, as it should be. We will consider this one-parameter example as the one which will provide a substratum for our numerical analysis.

C. Six cosmological models, their unlighted epochs, and the photon path length

The results of our numerical calculations for the six basic cosmological models in which an Archimedean-type interaction between dark energy and dark matter plays a main role are presented in Figs. 2–7. The panels (a) of the figures display the plots of $\dot{H}(t)$. The panels (b) display the plots of the square of the refraction index $n^2(\tau)$, where τ is a logarithmic time defined by $\tau \equiv \log x$, with $x \equiv \frac{a(t)}{a(t_0)}$. For

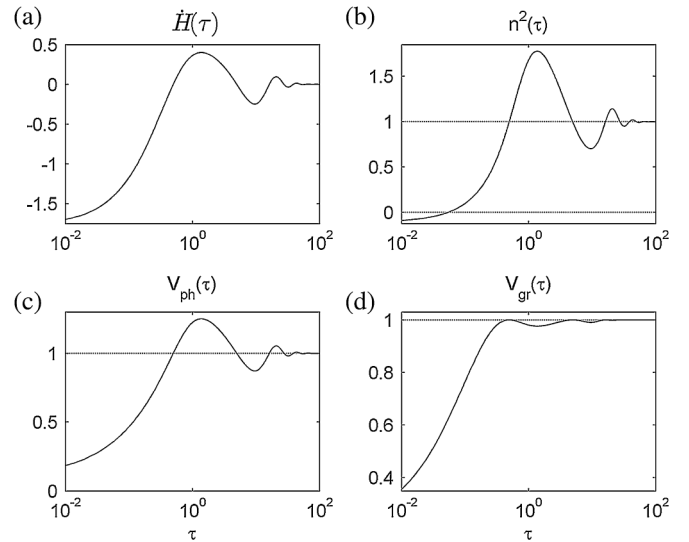


FIG. 2. Perpetually accelerated universe. The plots in panels (a), (b), (c), and (d) are presented for a typical model with the following parameters: $\xi = 0.35$, $\sigma = -0.99$ (i.e., $3\xi + \sigma \approx 0.06 > 0$), $\mathcal{V}_{(0)} = 1$, $E_{(0)} = 0.0205$, $\lambda_{(0)} = 1$, $\rho_* = 0.333 \cdot 10^{-4}$, and $\Pi'(1) = -1$. Panel (a): Illustrates the behavior of the function $\dot{H}(t)$, the plot of which has a finite number of visible damped oscillations, and tends to zero asymptotically; it relates to the case of the non-negative acceleration parameter $-q(t)$, i.e., there are no transition points in this model, there is no partition of the universe history into epochs, but there are a few eras, the start and finish of which are marked by the extrema of the function \dot{H} . Panel (b): The plot of the square of the refraction index $n^2(\tau)$ is presented in the panel (b) for the parameter $Q = 0.7$. It contains at least three eras, in which the refraction index exceeds one, three eras with $n < 1$, and $n^2(\tau)$ tends asymptotically to one at $t \rightarrow \infty$. There is an unlighted epoch of the first kind, which is characterized by negative values of the function $n^2(t)$; it starts at $t = 0$ and extends to the middle of the first era. At the end of this unlighted epoch the refraction index takes zero value, and starting from this point electromagnetic waves can propagate and transfer information into the universe. Panel (c): This panel displays the phase velocity of the electromagnetic waves as a function of time; the plot relates to a parameter $Q = -0.55$; this choice guarantees that $n^2(t)$ is positive, and V_{ph} is a real function everywhere. The plot of the phase velocity reflects the history of universe, i.e., it has extrema just at the moments when one era is changing into another. Clearly, the waves move more slowly in the early universe, when the curvature is large, then there are few eras with oscillations of the phase velocity near the vacuum speed of light, and finally, V_{ph} tends to one asymptotically. Panel (d): The last panel shows the behavior of the group velocity of the electromagnetic waves; the calculations are made for the same values of the guiding nonminimal parameters as for the phase velocity. Clearly, V_{gr} does not exceed one, it tends to one asymptotically, and an energy transfer takes place slowly in the early universe.

the calculations we use formula (63) putting there the values of the guiding parameter Q for which the square of the refraction index $n^2(t)$ can be negative for some period of time. The panels (c) display the plots of the

electromagnetic wave phase velocity. Here we use values of Q , for which $n^2(t) > 0$ and the phase velocity happens to be a pure real function. The panels (d) display the plots of the group velocity for the same values of the guiding parameters Q . Additional internal windows clarify fine details of the plots.

1. Perpetually accelerated universe

The first class of models is the class of perpetually accelerated universes. This class arises when $-q(t)$ is non-negative for $t \geq 0$ and so there are no points in which $-q(t)$ could change sign. In this sense the history of this class of universes includes only one epoch.

In this model the dark energy-density ρ is always non-negative and the dark energy Π is nonpositive, see [17] for more details. Although there is only one epoch for these universes, one can divide this epoch into eras. The extrema of the functions $-q(t)$, $H(t)$, $\rho(t)$, and $\Pi(t)$ give the boundary points between the eras, see [17] for more details. We do not plot $-q(t)$, $H(t)$, $\rho(t)$, and $\Pi(t)$, see [17] for such plots.

We are interested in light propagation and unlighted epochs. So we plot $\dot{H}(t)$, the square of the refraction index $n^2(t)$, the phase velocity $V_{\text{ph}}(t)$, and for the group velocity $V_{\text{gr}}(t)$. Figure 2 shows that indeed for the particular case chosen there are at least six eras which appear visually.

The first era is an era of superacceleration. During this era the function $\dot{H}(t)$ grows monotonically and reaches its global maximum, as it is shown in the panel (a) of Fig. 2. The same behavior happens for the square of the refraction index n^2 , for the phase velocity $V_{\text{ph}}(t)$, and for the group velocity $V_{\text{gr}}(t)$, as it is shown in panels (b), (c), and (d) of Fig. 2, respectively.

The second, third, and following eras appear one after the other as the parameters $-q(t)$, $H(t)$, $\rho(t)$, and $\Pi(t)$ of the universe relax to a state with asymptotically constant positive values, namely, $-q_\infty$, H_∞ , ρ_∞ , and negative Π_∞ . For the parameters that guide the properties of light propagation we see from Fig. 2 that $\dot{H}(t)$ asymptotically vanishes, see panel (a), and the refraction index, the phase velocity and the group velocity tend quasiperiodically asymptotically to one, see panel (b), (c), and (d), respectively.

Other features can be mentioned: (i) There are at least three eras, in which the refraction index exceeds one, $n > 1$, and three eras with $n < 1$. (ii) There is one unlighted epoch of the first type, which is characterized by negative values of the function $n^2(t)$. This unlighted epoch starts at $t = 0$ with a nonzero value of n^2 , and extends up to the middle of the first era. At the end of this unlighted epoch the refraction index takes zero value (the corresponding effective phase velocity would be infinite), and starting from this point the electromagnetic waves can propagate and transfer information into the universe. (iii) Clearly, the waves move more slowly in the early universe, when the

curvature is large. (iv) Let us note that the plot of the phase velocity properly reflects the history of the universe, i.e., it has extrema just at the moments when one era is changing into another.

2. Periodic universe

The second class of models is the class of periodic universes. This class arises when the equation $q(t) = 0$ has an infinite number of roots, and the history of the universe splits into an infinite number of identical epochs with accelerated and decelerated expansions.

The acceleration parameter $-q(t)$, and the Hubble function $H(t)$, oscillate with fixed frequency and amplitude after the second transition point [17]. These oscillations are reflected in the oscillations of the energy-density and pressure of the dark energy. We do not plot $-q(t)$, $H(t)$, $\rho(t)$, and $\Pi(t)$, see [17] for such plots.

We are interested in light propagation and unlighted epochs. From the plots of Fig. 3 we see that $\dot{H}(t)$ also

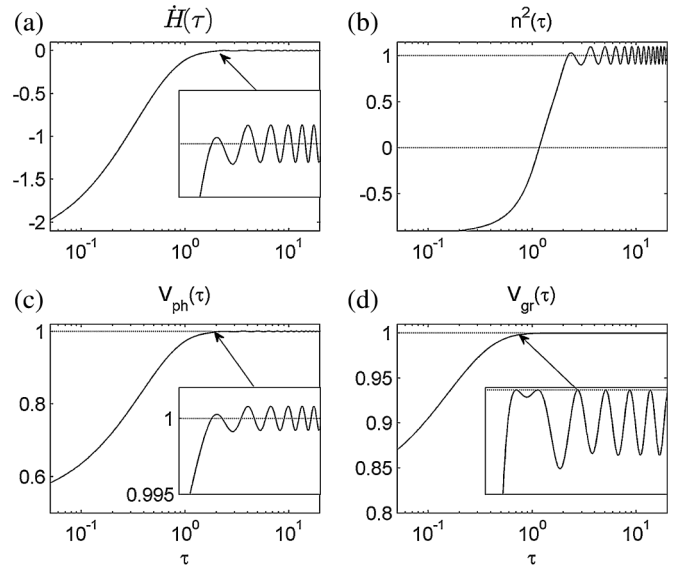


FIG. 3. Periodic universe. The plots in panels (a), (b), (c), and (d) are presented for a typical model with the following parameters: $\xi = 0.1$, $\sigma = -0.299999$ (i.e., $3\xi + \sigma \approx 10^{-6} > 0$), $\mathcal{V}_{(0)} = 1$, $E_{(0)} = 0.0205$, $\lambda_{(0)} = 1$, $\rho_* = 0.333 \cdot 10^{-4}$, and $\Pi'(1) = 0.01$. The model is characterized by an infinite number of transition points, in which the acceleration parameter $-q$ changes sign. Panel (a): The function $\dot{H}(t)$ (for $Q = 15$) grows monotonically in the early universe and then oscillates near zero value. Panel (b): The plot of the function $n^2(t)$ displays, first, an unlighted epoch of the first type, then grows monotonically up to the end of the first era, and then oscillates near the value $n = 1$. Panel (c): This is a plot of the phase velocity of the electromagnetic waves (for $Q = -0.25$). The curve follows the plot of \dot{H} , indicating the starting and finishing points of the corresponding epochs in the history of the universe. Panel (d): This is a plot of the group velocity. It shows it also grows monotonically during the first era, then starts to oscillate so that its maximal value reaches the speed of light in a vacuum.

oscillates, which in turn reflects again in the behavior of the refraction index $n^2(t)$, the phase velocity $V_{\text{ph}}(t)$, and the group velocity $V_{\text{gr}}(t)$.

Other features can be mentioned: The plot of the function $n^2(t)$, see panel (b) of Fig. 3, displays an unlighted epoch of the first kind, which extends from $t = 0$ into the middle of the first era.

3. One transition point universe

The third class of models is the class of universes with one transition point during their evolution. This class arises when the acceleration parameter $-q(t)$ is a deformed Heaviside step-function, or a hyperbolic tangent (see [17]).

The acceleration parameter $-q(t)$ has the property that the change of a deceleration epoch into an acceleration epoch takes place only once in a narrow period of time, and the plot looks like a typical plot for a phase transition. The

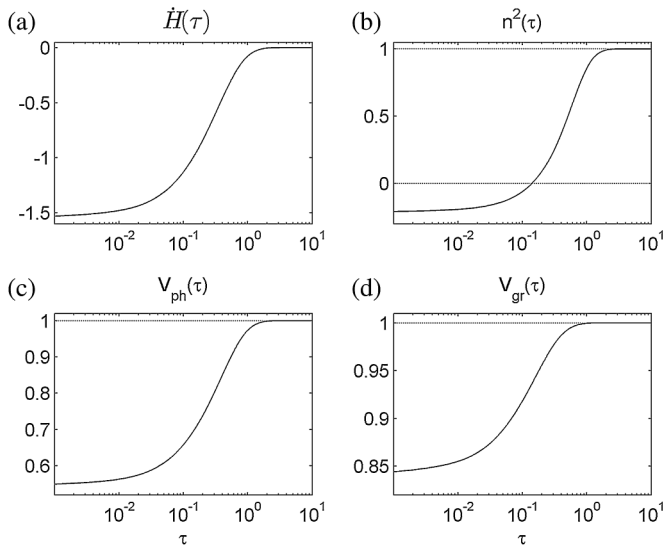


FIG. 4. One transition point universe. This is an example of a universe with one transition point in its evolution. The plots in panels (a), (b), (c), and (d) are presented for a typical model with the following parameters: $\xi = 0.1$, $\sigma = 50$, $\mathcal{V}_{(0)} = 1$, $E_{(0)} = 0.0205$, $\lambda_{(0)} = 1$, $\rho_* = 0.333 \cdot 10^{-4}$, and $\Pi'(1) = -5$. The plots of all functions presented in the panels (a), (b), (c), and (d), look like the plot for the hyperbolic tangent. The plots have no extrema and illustrate the one-fold transition from a universe expanding with negative acceleration to a universe with accelerated expansion. The history of this universe is clearly divided into two epochs of acceleration/deceleration without distinguished eras inside. This is clearly seen in panel (a) for \dot{H} . During the first epoch of deceleration an unlighted epoch can arise, see panel (b) for which $Q = 1$ was chosen, and both phase and group velocities are less than the speed of light in a vacuum. The second epoch can be characterized by parameters which are close to cosmological values measured nowadays. In particular, the phase and group velocities during all the second epoch, see panels (c) and (d) where $Q = -0.35$ was chosen, are close to one.

second epoch, characterized by an accelerated expansion, looks like the de Sitter-type stage with $\rho + \Pi = 0$ and vanishing E and P . The first and second epochs are not divided into eras within this model. We do not plot $-q(t)$, $H(t)$, $\rho(t)$, and $\Pi(t)$, see [17] for such plots. We are interested in light propagation and unlighted epochs. Figure 4 illustrates this class of models.

4. Two transition points universe

The fourth class of models is the class of universes with two transition points and one extremal point during their evolution. This class arises when the acceleration parameter $-q(t)$ has a double root, i.e., an extremal point, inside the second epoch of acceleration. This model is also distinguished by the fact that the dark matter pressure reaches a maximum at the end of the first era of the first acceleration epoch. We do not plot $-q(t)$, $H(t)$, $\rho(t)$, and $\Pi(t)$, see [17] for such plots.

We are interested in light propagation and unlighted epochs. Figure 5 illustrates this class of models. The sophisticated behavior that appears in this model is reflected

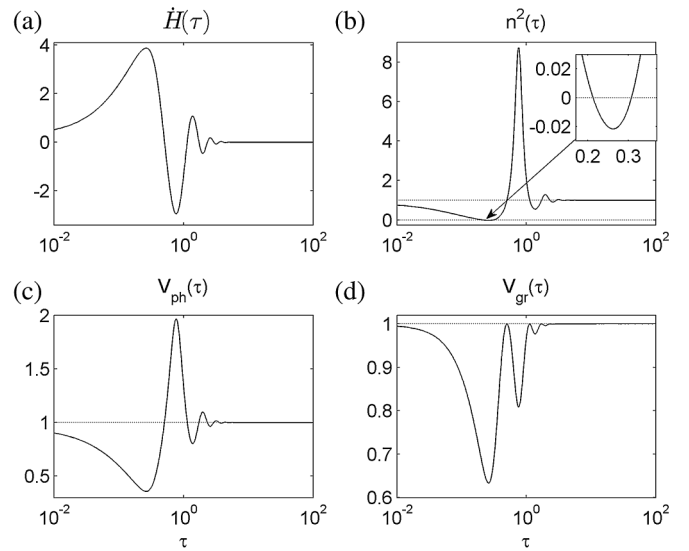


FIG. 5. Two transition points universe. This gives an example of a universe evolution with two transition points and one extremal point. The plots in panels (a), (b), (c), and (d) are presented for a typical model with the following parameters: $\xi = 0.1$, $\sigma = -0.08$, $\mathcal{V}_{(0)} = 1$, $E_{(0)} = 0.0205$, $\lambda_{(0)} = 1$, $\rho_* = 0.333 \cdot 10^{-4}$, and $\Pi'(1) = -15$. Panel (a) displays the function \dot{H} . In panel (b) one can see that the plot for the square of the refraction index $n^2(t)$ contains an unlighted epoch of the second type since the function $n^2(t)$, here for $Q = -0.28$, vanishes and then takes negative values at the end of the first era of the first epoch, in contrast to the case, when an unlighted epoch appears at $t = 0$. In panel (c) the phase velocity is plotted. Other new feature is that in the early universe the group velocity, calculated here for $Q = 0.2$, is close to the vacuum speed of light, and then this velocity reaches the same value asymptotically at $t \rightarrow \infty$, see panel (d).

explicitly in the form of the plot $\dot{H}(t)$, see Fig. 5. In this scenario there is an unlighted epoch of the second kind, where the function $n^2(t)$ vanishes at some t , with $t > 0$, then takes negative values at the end of the first era of the first epoch, and then vanishes again. Thus this unlighted epoch is separated by two points, where the refraction index vanishes and the effective phase velocity takes infinite values.

5. Three transition points universe

The fifth class of models is the class of universes with two epochs of decelerated expansion and two epochs of accelerated evolution. This class arises when the acceleration parameter $-q(t)$ is of the so-called \mathcal{N} -type [17]. This model is also distinguished by the fact that the start of the universe's expansion relates to the deceleration epoch, which is then replaced by a short acceleration epoch. The second and final accelerated epoch is of the de Sitter type. We do not plot $-q(t)$, $H(t)$, $\rho(t)$, and $\Pi(t)$, see [17] for such plots. We are interested in light propagation and unlighted epochs. Figure 6 illustrates this class of models. The unlighted epoch harbors now the first and second epochs of deceleration, as well as the first epoch of accelerated expansion. Clearly, this is a new feature.

6. Quasiperiodic universe

The sixth class of models is the class of universes with a large albeit finite number of transition points. It is a

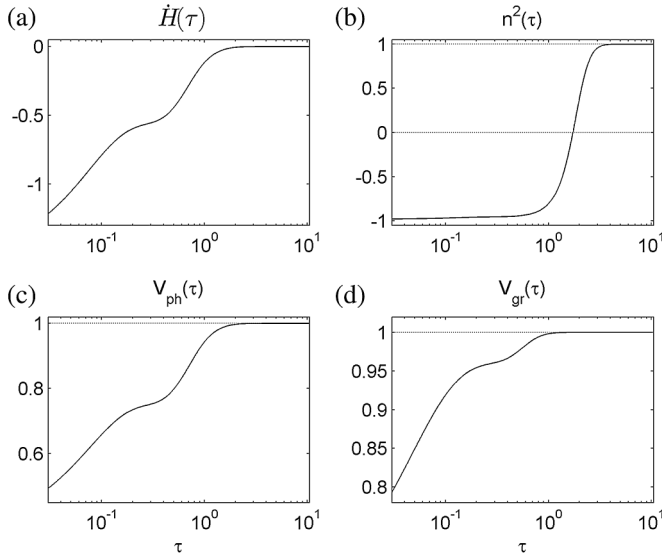


FIG. 6. Three transition points universe. The parameters of the model are the following: $Q = 80$ for panels (a) and (b), $Q = -0.5$ for panels (c) and (d), $\xi = 0.1$, $\sigma = 1$, $\mathcal{V}_{(0)} = 1$, $E_{(0)} = 0.0205$, $\lambda_{(0)} = 1$, $\rho_* = 0.333 \cdot 10^{-4}$, and $\Pi'(1) = -5$. The universe passes through two epochs of deceleration and two epochs of accelerated expansion. The unlighted epoch harbors now the first and second epochs of deceleration, as well as the first epoch of accelerated expansion.

quasiperiodic universe. This model is intermediate between the model with two transition points and the periodic model.

This class arises when, starting from some transition point, the curve $-q(t)$ remains above the line $q = 0$. This means that at a later time the universe's expansion is accelerated. We do not plot $-q(t)$, $H(t)$, $\rho(t)$, and $\Pi(t)$, see [17] for such plots.

We are interested in light propagation and unlighted epochs. Figure 7 illustrates this class of models. The behavior of the functions $\dot{H}(t)$, $n^2(t)$, $V_{\text{ph}}(t)$ and $V_{\text{gr}}(t)$ is quasiperiodic and the amplitudes of their oscillations decrease asymptotically. In addition to the unlighted epoch of the first type, one can see now one point with $n^2 = 0$, i.e., a compressed unlighted epoch of the second type. In between them the square of the refraction index is positive. The behavior of the phase velocity is unusual in the early universe since it is greater than the speed of light in a vacuum during the first era of the first epoch, and then tends to the asymptotic value equal to one in a quasioscillatory manner.

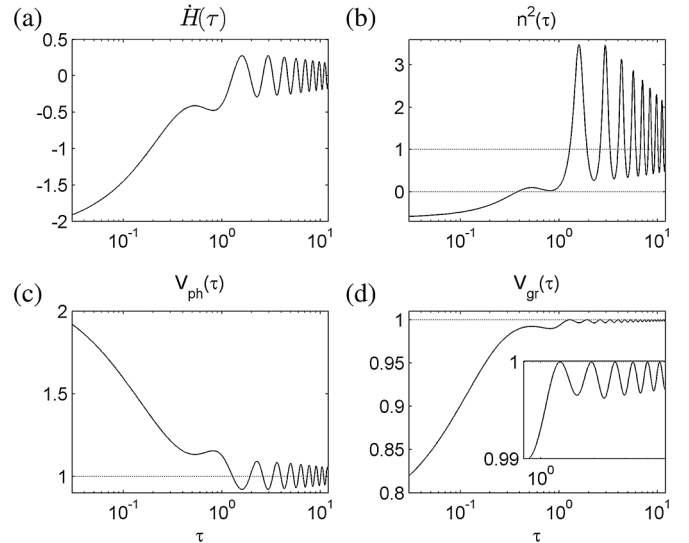


FIG. 7. Quasiperiodic universe. The universe history is divided into a great number of epochs by a finite number of transition points. Panel (a) of $\dot{H}(t)$ shows clearly this behavior. The plot of $n^2(t)$, see the panel (b) with $Q = 2$, demonstrates that in addition to an unlighted epoch of the first type, arising at $t = 0$, an unlighted epoch of the second type compressed into a point also appears. This model shows also an unusual behavior of the phase velocity in the early universe, namely, it is greater than the speed of light in a vacuum during the first era of the first epoch, in contrast to the models described above, and then tends to the asymptotic value equal to one in the quasioscillatory regime. The behavior of the group velocity is of a different kind, namely, in the early universe it grows monotonically, and then tends asymptotically to one in a quasioscillatory regime, see panels (c) and (d) with $Q = 0.3$.

7. Generic features of the six models: The photon path length, the true duration of epochs and the universe lifetime

Let us define two dimensionless functions,

$$\Gamma_{\text{ph}}(t) \equiv \frac{1}{t} \int_0^t dt' V_{\text{ph}}(t'), \quad (64)$$

and

$$\Gamma_{\text{gr}}(t) \equiv \frac{1}{t} \int_0^t dt' V_{\text{gr}}(t'). \quad (65)$$

The first of them, $\Gamma_{\text{ph}}(t)$, gives the ratio between two quantities: the length traveled during a time t by an electromagnetic wave with phase velocity $V_{\text{ph}}(t)$ and the length traveled during the same time t by a photon moving with the speed of light in a pure vacuum. In the second quantity, $\Gamma_{\text{gr}}(t)$, the phase velocity is replaced by the group velocity. Alternatively, one can consider these two functions as mean values of the phase and group velocities averaged over time t , respectively. We have calculated these two functions for the models discussed above. The results are

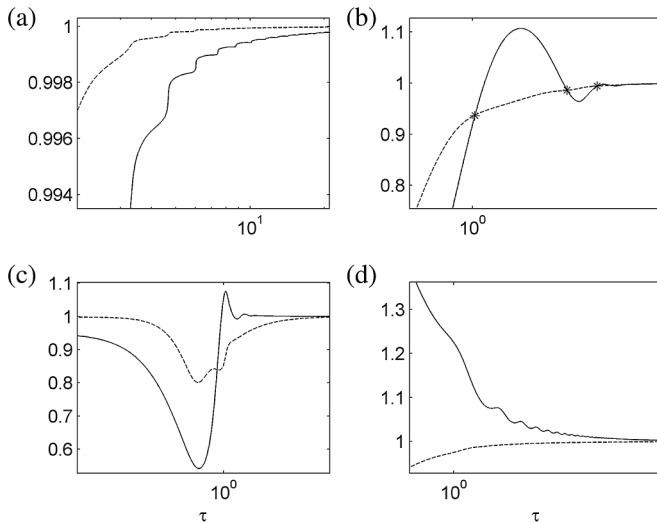


FIG. 8. Solid lines illustrate the behavior of $\Gamma_{\text{ph}}(t)$ and dashed lines illustrate the behavior of $\Gamma_{\text{gr}}(t)$. $\Gamma_{\text{ph}}(t)$ gives the ratio between the phase length, which the electromagnetic wave runs during a time t , if we take into account its phase velocity, and the length traveled by a photon moving at the speed of light in a pure vacuum. For $\Gamma_{\text{gr}}(t)$, the phase velocity is replaced by the group velocity. Panel (a) illustrates the periodic model, panel (b) relates to the model with perpetual acceleration, panel (c) contains the illustration to the model with one transition point, and panel (d) illustrates the quasiperiodic model. For the periodic and quasiperiodic models the plots of the functions $\Gamma_{\text{ph}}(t)$ and $\Gamma_{\text{gr}}(t)$ have no intersection points. The other two models show the presence of crossing points. Generally, in the late-time universe the quantities $\Gamma_{\text{ph}}(t)$ and $\Gamma_{\text{gr}}(t)$ give practically the same results, the ratios are close to one. The difference between these quantities is, however, essential in the early universe.

presented in the Fig. 8. A general feature of all presented plots is that as $t \rightarrow \infty$ both these functions tend to one $\lim_{t \rightarrow \infty} \Gamma_{\text{ph}}(t) = 1$ and $\lim_{t \rightarrow \infty} \Gamma_{\text{gr}}(t) = 1$. This is not surprising, since when $t \rightarrow \infty$ the contributions of the periods with $V_{\text{ph}} \neq 1$ and $V_{\text{gr}} \neq 1$, which in themselves are short, become vanishingly small. Thus, for the estimation of the total life time of the universe we can assume that, on average, photons propagate with speed of light in a pure vacuum. On the other hand, when we have to calculate the duration of epochs and eras in the early universe, the behavior of Γ_{ph} and Γ_{gr} , as well as the estimation of the photon path lengths, depend on the type and parameters of the model, and on whether one uses the phase velocity or the group velocity.

V. CONCLUSIONS

We can draw several conclusions.

1. The standard, concordant, cosmological model deals with at least two epochs, during which electromagnetic waves cannot scan the universe's internal structure to bring information to observers. The first epoch is when photons are in local thermodynamic equilibrium with other particles, and the second is when photon scattering by charged particles is strong. One can call these two periods of cosmological time as standard unlighted epochs. After the last scattering surface, photons become relic photons and turn into a source of information about the universe.

Now, if one takes into account electromagnetic interactions with the dark sector, i.e., with the dark energy and dark matter, one can expect that unlighted epochs of a new type can appear. Here we described one possible example, namely, the unlighted epochs produced by the nonminimal coupling of gravitational and electromagnetic fields. Since in the framework of the nonminimal three-parameter Einstein-Maxwell model, the curvature coupling can be formulated in terms of an effective refraction index $n(t)$, as we did, we can take advantage of the well-known classical analogy, namely, in a medium with $n^2 < 0$ electromagnetic waves do not propagate and their group velocity, i.e., energy transfer velocity, has zero value at the boundary of the corresponding zone. This analogy is the key ingredient of our analysis. Indeed, using systematically the refraction index $n^2 < 0$, we have studied, both analytically and numerically, cosmological models admitting unlighted epochs, the photon coupling to the spacetime curvature being the key element of the models.

2. We established a formula for the group velocity, or energy transfer velocity, of an electromagnetic wave non-minimally coupled to the gravity field, namely, $V_{\text{gr}}(t) = \frac{2n(t)}{n^2(t)+1}$, with $|V_{\text{gr}}(t)| \leq 1$. Since unlighted epochs with a negative effective refraction index squared can start and finish only when $n^2(t_{(*)}) = 0$ or $n^2(t_{(**)}) = \infty$, the group velocity is zero at the unlighted epochs boundary points. At these points the electromagnetic energy transfer stops, and

thus the condition $V_{\text{gr}}(t) = 0$ is the condition to be employed as the criterion for recognizing the beginning and the end of the unlighted epochs.

We have distinguished three types of unlighted epochs: the first one starts at $t = 0$ with negative effective refraction index squared and finishes when $n^2 = 0$; the second one starts at $t = t_{(1)} > 0$ and finishes at $t = t_{(2)} > t_{(1)}$ with $n^2(t_{(1)}) = n^2(t_{(2)}) = 0$; the third one is characterized by the fact that at least at one of the two boundaries n^2 has an infinite value. The phase velocity $V_{\text{ph}}(t)$, another important physical characteristic of an electromagnetic wave, is such that $V_{\text{ph}}(t) = \frac{1}{n(t)}$, and so, when $n^2 = 0$ it can take infinite values at the beginning or at the end of unlighted epochs. The appearance of unlighted epochs of all three types were illustrated analytically, using a cosmological model with a scale factor of the Kohlrausch type, i.e., a stretched exponential scale factor.

The unlighted epochs of the first and second types were then described numerically using our Archimedean-type interaction model between dark energy and dark matter. Clearly, the appearance or absence of unlighted epochs is connected with the signs and absolute values of the non-minimally coupling parameters q_1 , q_2 and q_3 . Thus, fingerprints of the unlighted epochs in the cosmic microwave background data could give some constraints on these nonminimal coupling parameters. We hope to discuss this problem in another work.

3. The appearance of unlighted epochs caused by the nonminimal coupling of photons to the gravitational field adds new features into the history of the universe as written by electromagnetic fields. Note that if the universe passes through an unlighted epoch of the second type, we know for certain that information scanned by electromagnetic waves during the preceding epochs is washed out, i.e., such unlighted epochs act as informational laundry.

In this connection there is a very interesting question: When and why unlighted epochs of the second type can appear? Let us note, that quite generally, the behavior of the function $n^2(t)$ inherits the features of the function $\dot{H}(t)$; to see this compare the panels (a) and (b) in Figs. 2–7. In this sense, the example of the model with two transition points, see Fig. 5, shows explicitly that the appearance of an unlighted epoch of the second type is a consequence of large quasioscillations of the function $\dot{H}(t)$ and thus a consequence of large quasioscillations of the spacetime curvature. Such quasioscillations, in their turn, are the result of the dark matter and dark energy energy-momentum redistribution, induced by the Archimedean-type coupling [16,17].

4. The time span of different eras in the universe's history can be estimated using optical information. From these durations one can estimate the distance traveled by the electromagnetic waves through their velocity of propagation. But then, one should clarify what is the electromagnetic wave propagation velocity which should be used

in the interpretation of the observational data. Is it the group velocity, the phase velocity, or the velocity of light in a pure vacuum? As we have seen these quantities differ in the early universe in an essential way. From Fig. 8 it is seen, for instance, that the group velocity and the corresponding traveled distance are less sensitive to the details of the dynamics of the universe.

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APPENDIX

As mentioned in Sec. II C 1, here we give several examples of specific nonminimal theories by motivating possible choices for the nonminimal coupling parameters q_1 , q_2 and q_3 . Using the two effective nonminimal coupling constants, Q_1 and Q_2 given by

$$Q_1 \equiv -2(3q_1 + 2q_2 + q_3), \quad Q_2 \equiv -2(3q_1 + q_2), \quad (\text{A1})$$

we have shown we can write the square of the refraction index in terms of Hubble function $H(t)$ and acceleration parameter $-q(t)$ as

$$n^2(t) = \frac{1 + [Q_2 - Q_1 q(t)]H^2(t)}{1 + [Q_1 - Q_2 q(t)]H^2(t)}. \quad (\text{A2})$$

Of course $n^2(t)$ depends on Q_1 and Q_2 , and thus on q_1 , q_2 , and q_3 . However, independently of this choice, for the de Sitter universe, one obtains $n^2(t) = 1$. Indeed, since the acceleration parameter is constant and is equal to one, $-q(t) = 1$, we obtain that $\varepsilon(t) = \frac{1}{\mu(t)} = 1 + (Q_2 + Q_1)H^2(t)$. Although $\varepsilon(t) \neq 1$ and $\mu(t) \neq 1$, one has $n^2(t) \equiv \varepsilon(t)\mu(t) = 1$ for arbitrary coupling parameters.

In general, the cosmological models are sensitive to the signs and the values of the parameters Q_1 and Q_2 . There is an infinite variety for the choice. However, six phenomenological possibilities using geometric analogies and physical motivations can be implemented.

1. Vanishing nonminimal susceptibility scalar ($\mathcal{R} = 0$)

When $6q_1 + 3q_2 + q_3 = 0$ and thus, from Eq. (28), $\mathcal{R} = 0$, one has $Q_1 + Q_2 = 0$. Putting $Q \equiv Q_1 = -Q_2$, one obtains in this one-parameter example the following expression:

$$n^2(t) = \frac{1 - QH^2(t)[1 + q(t)]}{1 + QH^2(t)[1 + q(t)]} = \frac{1 + Q\dot{H}(t)}{1 - Q\dot{H}(t)}. \quad (\text{a3})$$

This example has attracted our attention and we have used it in our numerical calculations in Sec. IV. Indeed, the interest in this choice is that only one function, \dot{H} , guides the behavior of the effective refraction index, with $n^2 \rightarrow 1$, when $\dot{H} \rightarrow 0$. From a physical point of view this means that asymptotically electromagnetic waves coupled non-minimally to the gravitational field propagate with a phase velocity equal to the speed of light in a vacuum.

2. Gauss-Bonnet-type relation

When one imposes that the differential equations forming the nonminimal Einstein-Maxwell system are of second order (see, e.g., [20,28]), then one should use $q_1 + q_2 + q_3 = 0$ and $2q_1 + q_2 = 0$. In this case the non-minimal susceptibility tensor \mathcal{R}_{ikmn} is proportional to the double dual Riemann tensor ${}^*R_{ikmn}^*$, i.e., $\mathcal{R}_{ikmn} = \gamma {}^*R_{ikmn}^*$ for some constant γ [43]. Then we have $Q_1 = 0$, $Q_2 = -2q_1$, and

$$n^2(t) = \frac{1 - 2q_1 H^2(t)}{1 + 2q_1 q(t) H^2(t)}. \quad (\text{A4})$$

If the nonminimal parameter q_1 is negative, then during the acceleration epoch (i.e., $-q(t) > 0$) the square of the refraction index is positive, and there are no unlighted epochs.

3. First Weyl-type relation

Assume now that $\mathcal{R}^{mn} = 0$, in which case one can take $3q_1 + q_2 = 0$ and $q_2 + q_3 = 0$. Then, the susceptibility tensor is proportional to the Weyl tensor \mathcal{C}_{ikmn} , i.e., $\mathcal{R}_{ikmn} = \omega \mathcal{C}_{ikmn}$, and $Q_1 = 0$, $Q_2 = 0$. In this case the effective permittivity scalars are equal to the unity, $\varepsilon(t) = 1$ and $\mu(t) = 1$. Thus $n^2(t) = 1$ in any epoch.

4. Second Weyl-type relation

Take $3q_1 + q_2 = 0$ and $q_3 = 0$. Then the susceptibility tensor is proportional to the difference between the Riemann and Weyl tensors, i.e., $\mathcal{R}_{ikmn} = \Omega[R_{ikmn} - \mathcal{C}_{ikmn}]$. In this example $Q_1 = 6q_1$, $Q_2 = 0$, and thus

$$n^2(t) = \frac{1 - 6q_1 q(t) H^2(t)}{1 + 6q_1 H^2(t)}. \quad (\text{A5})$$

Now the square of the refraction index is positive during the acceleration epoch ($-q(t) > 0$) when the nonminimal parameter q_1 is positive.

5. Symmetry relation with respect to the left and right dualizations

If one imposes that the left and right dual tensors coincide ${}^* \mathcal{R}_{ikmn} = \mathcal{R}^*{}_{ikmn}$, then one gets the example in which $q_2 + q_3 = 0$. In this example $Q_1 = Q_2$, and thus $n^2(t) = 1$ for arbitrary epochs, although the dielectric permittivity and magnetic permeability

$$\varepsilon(t) = \frac{1}{\mu(t)} = 1 - 2(3q_1 + q_2) H^2[1 - q(t)] \quad (\text{A6})$$

differ from unity when $3q_1 + q_2 \neq 0$.

6. Drummond-Hathrell-type relation

An example of a calculation for the three coupling parameters based on one-loop corrections to quantum electrodynamics in curved spacetime has been considered by Drummond and Hathrell [21]. In this example the non-minimal coupling parameters are connected by the relations $q_1 \equiv -5\tilde{Q}$, $q_2 = 13\tilde{Q}$, $q_3 = -2\tilde{Q}$. The parameter \tilde{Q} is positive, and constructed by using the fine-structure constant α and the Compton wavelength of the electron λ_e , i.e., $\tilde{Q} \equiv \frac{\alpha \lambda_e^2}{180\pi}$. So, $Q_1 = -18\tilde{Q}$, $Q_2 = 4\tilde{Q}$, yielding

$$n^2(t) = \frac{1 + 2\tilde{Q} H^2(t)[2 + 9q(t)]}{1 - 2\tilde{Q} H^2(t)[9 - 2q(t)]}. \quad (\text{A7})$$

This example was discussed in [38].

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