PHYSICAL REVIEW D **85,** 063502 (2012)

Lorentz-violating dynamics in the pre-Planckian Universe

G. Salesi*

Università Statale di Bergamo, Facoltà di Ingegneria viale Marconi 5, I-24044 Dalmine (BG) and Italy Istituto Nazionale di Fisica Nucleare, Sezione di Milano via Celoria 16, I-20133 Milan, Italy

(Received 12 October 2011; revised manuscript received 5 January 2012; published 1 March 2012)

We have recently proposed a Lorentz-violating energy-momentum relation entailing an exact momentum cutoff and studied various physical applications of that dispersion law. By a simple phenomenological approach we study Lorentz violation effects on early Universe and pre-Planckian cosmological radiation. In particular, we predict an effective infinite speed of light soon after the big bang, leading to a possible solution of the horizon and flatness problems without recourse to inflation, cosmological scalar fields, or other *ad hoc* energy sources.

DOI: 10.1103/PhysRevD.85.063502 PACS numbers: 98.80.Es, 11.55.Fv, 98.80.Bp, 98.80.Jk

I. INTRODUCTION

As is well known, at the Planck scale classical and quantum approaches lead to different predictions, and we have to overcome general relativity in order to unify gravity with the other fundamental forces of nature which are well described by quantum field theory and the standard model. For example, applying general relativity to the black hole evaporation, we encounter unsolved theoretical problems or inconsistencies, such as mass loss rate divergence, baryon and lepton number nonconservation, "information paradox," etc. Other serious problems and divergences—the monopole problem, the cosmological entropy problem, the coincidence problem, the flatness problem, the horizon problem, the cosmological constant problem—arise when studying the big bang singularity and the pre-Planckian era in standard (relativistically covariant) theories. On the other hand, in the last decades Lorentz-violating (LV) theoretical approaches have been proposed (implicitly or explicitly), entailing an essentially noncontinuous, discrete spacetime where, as expected from the uncertainty relations, a Planckian energy-momentum scale naturally arises. Ultrahigh energy Lorentz violations have been proposed in many different experimental and theoretical frameworks, e.g. (see [1] and references therein), superstring and quantum gravity theories, grand-unification theories, causal dynamical triangulation, "extensions" of the standard model incorporating breaking of Lorentz and *CPT* symmetries, foamlike quantum spacetimes, classical spacetimes endowed with a noncommutative geometry or with a discrete structure at the Planck length, and theories with a variable speed of light or variable physical constants.

An interesting theoretical approach to Lorentz symmetry violation is found in "deformed" special relativity [2–4], working in k-deformed Lie-algebra noncommutative spacetimes, in which both a fundamental mass scale (depending on the particular model, it can be the Planck mass 10^{19} GeV, the grand unified theory energy 10^{15} GeV, the SUSY-breaking scale 10^{11} GeV, or the superstring energy scale, etc.) and the speed of light act as characteristic scales of a six-parameter group of spacetime four-rotations with deformed but preserved Lorentz symmetries. Deformed relativity has been generalized to curved spacetimes, as in the so-called "doubly general relativity," also called "gravity's rainbow" [5]. The resulting metric depends on both the probe energy and the gravity field, as we might expect for sub-Planckian spatial regions.

In various recent papers [6–9] we have adopted a special LV momentum-dependent metric where, analogously to the phonon motions in a crystal lattice, particles can really neglect the quantized structure of the underlying vacuum only at low energies. On the contrary, at very high energies particles can effectively feel the discretelike structure and the quantum properties of the medium crossed. A very general momentum-dependent metric can indeed be written as follows:

$$\mathrm{d}\, s^2 = f^{-2}(p)\mathrm{d}t^2 - g^{-2}(p)\mathrm{d}l^2, \tag{1}$$

where the form factors f and g are expected to be different from unity only for Planckian momenta, if the LV scale is assumed to be the Planck energy. One of the most important consequences of (1) is the modification of the ordinary momentum-energy dispersion law $E^2 - p^2 = m^2$, by means of additional terms which vanish in the low momentum limit:

$$E^2 f^2(p) - p^2 g^2(p) = m^2. \tag{2}$$

On the basis of various physical considerations, we have chosen the most simple LV metric, namely,

$$f^2(E) = 1, \qquad g^2(p) = 1 - \lambda p, \tag{3}$$

where the positive LV parameter λ is usually assumed to be of the order of the reciprocal of the Planck mass, $\lambda \sim M_{\text{Planck}}^{-1}$. This choice leads to a negative cubic correction to the ordinary covariant dispersion law,

*salesi@unibg.it

$$E^2 = p^2 + m^2 - \lambda p^3. \tag{4}$$

As a matter of fact, the cubic corrections to the dispersion law are the most recurring in the literature. Indeed, in ''noncritical''-Liouville string theory [10] we find $E^2 = p^2 + m^2 + \xi g_s p^3/M_s$, where g_s is the string coupling and M_s is a suitable mass scale; similar cubic expressions are obtained in the above-mentioned LV standard model extensions, in theories with a spacetime ''medium'' or quantum foam, in quantum gravity, as well as in deformed special relativity (e.g., in [2] we find cubic corrections: $E = p^2 + m^2 - \lambda E p \sim p^2 - \lambda p^3$ for $m = 0$ and $p \ll 1/\lambda$).

In [6] we have adopted the dispersion law (4) in order to give a simple explanation for the baryon asymmetry in the Universe. Because of the negative sign of the LV term, we were able to propose a straightforward mechanism for generating the observed matter-antimatter asymmetry through a Lorentz-breakdown energy scale of the order of the Greisen-Zatsepin-Kuzmin cutoff. In [7] our LV model leads to very specific physical predictions in the neutrino oscillations scenario, accounting for observed anomalies such as the apparently anomalous excess of low-energy ν_e-like events, reported by the MiniBooNE Collaboration, as well as the nonobservation of the corresponding anomalous excess of $\bar{\nu}_e$-like events. Upon investigating the black hole thermodynamics in a deformed relativity framework with a Planckian cutoff [8], we adopted a Schwarzschild momentum-dependent metric modified according to the above law dispersion: in such a way, obtaining net deviations of the basic thermodynamical quantities from the Hawking-Bekenstein predictions. In particular, the black hole evaporation is expected to quit at a nonzero critical mass value of the order of the Planck mass, leaving a zero temperature remnant, and avoiding any spacetime singularity. We also found [9] large deviations from the Hawking-Bekenstein predictions for the black hole time evolution, depending on the value of the Lorentz-violating parameter introduced. Actually, in that paper, we predicted a slow death of terminal black holes in the place of an infinitely fast evaporation (with a dramatic final gamma-ray burst) predicted by the Hawking theory.

Let us remark that, in the literature on deformed special relativity and gravity's rainbow, the chosen form factors f and g do not imply an *exact* Planck cutoff or a maximum momentum. By contrast, in our dispersion law with a negative term $-\lambda p$ the energy vanishes when $p = p_{\max} = \frac{1}{\lambda} \sim M_{\text{Planck}}$, which plays the role of a ''maximal momentum'' corresponding to the noncontinuous discrete ''granular'' nature of space. Actually, Eq. (4), unlike other dispersion laws put forward in the literature, is not the leading order term in a series expansion in λp but, rather, Eqs. (3) are assumed to represent the exact form of a metric endowed with a momentum cutoff. Even if other forms of LV metric with an exact Planck cutoff are possible, it is noticeable that most of our predictions quoted above seem to be model independent and are reobtained in quantum theoretical approaches to Planck-scale physics [11].

In some interesting works, e.g. in [12], it has been argued that strong Lorentz violations at the Planck scale can affect the relativistic covariance also at very lower energies because of the existence of loop corrections involving virtual particles with Planck-scale momenta. As a consequence, the implications of the modified dispersion relations might involve extremely drastic modifications of the expected particle behavior. Nevertheless, in our phenomenological approach (as, in general, in gravity's rainbow) we do assume that, after quantum and thermodynamical averages on the pre-Planckian radiation gas, primordial photons can still be described as usual semiclassical objects [9], even though adopting a modified dispersion law. Hence we shall assume, at least in the first approximation, the basic postulates of the Bose-Einstein statistics and the usual counting of the phase space available states. As a matter of fact, we think that it can be really meaningful to study the dynamics of a semiclassical cosmological fluid evolving in a curved spacetime endowed with an energy-dependent metric. This does not exclude the subsequent inclusion of quantum corrections in future calculations. Actually, as said before, the present dynamical or statistical assumptions and approximations have been very successful in describing very different physical frameworks, e.g., cosmic baryon asymmetry or terminal black holes.

II. COSMOLOGICAL OPEN PROBLEMS IN A LORENTZ-VIOLATING SCENARIO

Some authors [13] have recently claimed that the adopted momentum-dependent metric can be more rigorously classified as a (mass-dependent) *Finsler* metric rather than a Riemannian one. As is well known, the Riemannian manifolds are special cases of the Finsler manifolds, which are the most general metric spaces in theories which ''extend'' Einstein's general relativity. Nevertheless, for simplicity, we hereafter shall call the cosmological evolution equations obtained from Einstein's equations in the presence of a momentum-dependent metric ''Friedmann-Lemaitre-Robertson-Walker (FLRW) equations.'' Let us note that in the present paper all the particles are assumed to be massless radiation particles. This, as proved in [13], makes the notions of ''spacetime geometry'' and of ''tangent bundle geometry'' still meaningful in the Finsler metric, since each particle sees the same spacetime geometrical structure. On the other hand, it is possible to get the solutions of the Einstein equations for a momentum-dependent FLRW metric [5] with the assumptions of space homogeneity and isotropy, time-independent spatial components, and a zero cosmological constant Λ, considering the early Universe as a sphere filled by a photon hot gas with energy density ρ and pressure P. Actually, the LV parameters f and g, such as the ones in

Eq. (3), correspondingly "deform" one of the nonvanishing Christoffel symbols (a indicates, as usual, the cosmic scale factor),

$$\Gamma^0_{ij} = \left(\frac{f}{g}\right)^2 a\dot{a}g_{ij}, \qquad \Gamma^i_{0j} = \frac{\dot{a}}{a}\delta^i_j. \tag{5}$$

Consequently, taking into account the modified velocity four-vector $u_\mu = (f^{-1}; 0, 0, 0)$ entering the energy-momentum tensor [5], and introducing the Hubble constant $H \equiv \dot{a}/a$ and the curvature parameter k, the LV Friedmann equations with a zero cosmological constant Λ can be written as in [5]:

$$H^2 = \frac{8\pi G}{3c^2}\frac{\rho}{f^2} - k\left(\frac{g}{f}\right)^2\frac{c^2}{a^2}, \qquad \frac{\ddot{a}}{a} = -\frac{4\pi G}{3c^2}\frac{\rho + 3P}{f^2}, \tag{6}$$

which, according to our choice (3) for f and g, reduce to

$$\begin{aligned} H^2 &= \frac{8\pi G}{3c^2}\rho - k(1-\lambda p)\frac{c^2}{a^2},\\ \frac{\ddot{a}}{a} &= -\frac{4\pi G}{3c^2}(\rho + 3P). \end{aligned} \tag{7}$$

Since in this paper we adopt an approximate, essentially qualitative approach to try to solve some cosmological problems, we shall neglect the presence of the modulating factor $1 - \lambda p$ [whose mean value ranges between 1 at $T = 0$ and $\sqrt{1/5}$ at $T = \infty$, cf. (14)]. Anyway, according to the aim of the present work, in the first approximation we shall neglect any LV correction to the spacetime metric and shall refer to the usual Friedmann equations. We instead will focus on the consequences of the LV energy-momentum relation on the early cosmic expansion, in particular, on the pre-Planckian photons' mean velocity which, as we are going to show, turns out to be nonconstant and dependent on the temperature. As we will show below, another important effect of our momentum-dependent metric is a sharp modification (at very high temperatures) of the equation of state linking ρ and P. Anyhow, when applying the modified FRLW equations in studying the early evolution of the Universe, we shall not explicitly take into account the modified equation of state.

A. Modified blackbody thermodynamics

By adopting a thermal distribution and dispersion law (4) for photons ($m = 0$),

$$E = pc\sqrt{1 - \lambda p}, \tag{8}$$

the spectral energy density in the semiclassical phase space is given by

$$\mathrm{d}\rho = \frac{pc\sqrt{1-\lambda p}}{\mathrm{e}^{(pc/kT)\sqrt{1-\lambda p}} - 1}\frac{8\pi p^2}{h^3}\mathrm{d}p. \tag{9}$$

The total energy density is obtained by momentum integration,

$$\rho = \frac{U}{V} = \int_0^{1/\lambda}\frac{8\pi c}{h^3}\frac{p^3\sqrt{1-\lambda p}}{\mathrm{e}^{(pc/kT)\sqrt{1-\lambda p}} - 1}\mathrm{d}p. \tag{10}$$

In the low temperature limit, $T \ll c/k\lambda$, we can easily recover the classical Stefan-Boltzmann law $\rho = \sigma T^4 = \pi^2 k^4 T^4/15\hbar^3 c^3$. By contrast, at high temperatures, $T \gg c/k\lambda$, replacing the exponential by its first order expansion, we derive a new result, very different from the classical one:

$$\rho \simeq \frac{8\pi}{3}\frac{kT}{h^3\lambda^3}. \tag{11}$$

In the same limit the photon density is given by

$$n \simeq \lim_{T\to\infty}\int_0^{1/\lambda}\frac{8\pi p^2}{h^3}\frac{1}{\mathrm{e}^{(pc/kT)\sqrt{1-\lambda p}} - 1}\mathrm{d}p = \frac{32\pi}{3}\frac{kT}{ch^3\lambda^2}. \tag{12}$$

Even if both the total energy $U \simeq \frac{8\pi}{3}\frac{kT}{h^3\lambda^3}V$ and the total photon number $N = nV \simeq \frac{32\pi}{3}\frac{kT}{ch^3\lambda^2}V$ diverge for $T \to \infty$, the mean energy for photon is finite and (taking $\lambda \sim 1/M_{\mathrm{Planck}}c$) of the order of the Planck energy

$$\varepsilon \equiv \frac{U}{N} = \frac{c}{4\lambda}. \tag{13}$$

As a consequence, the classical energy equipartition principle $\varepsilon = \frac{1}{2}kT$ does not hold anymore. We could say that at the big bang initial instant, when the temperature was infinite, the mean energy for particle was not infinite, but of the order of the Planck energy, thus avoiding a typical divergence ($\varepsilon \to \infty$) resulting from standard cosmology.[1] Similarly, for $T \to \infty$ the photon mean momentum is finite,

$$\bar{p} = \frac{4}{5\lambda}. \tag{14}$$

The radiation pressure (Ω indicates the grand potential)

$$P = -\frac{\Omega}{V} = -\frac{8\pi kT}{h^3}\int_0^{(1/\lambda)} p^2 \ln[1 - e^{-(pc\sqrt{1-\lambda p}/kT)}]\mathrm{d}p \tag{15}$$

can be evaluated as above. At low temperatures we recover the classical result $P = \frac{1}{3}\rho = \frac{1}{3}\sigma T^4$, while at high temperatures we find a linear-logarithmic law

$$P \simeq \frac{8\pi kT}{3h^3\lambda^3}\ln\frac{\lambda kT}{c}. \tag{16}$$

[1] Analogously, in [14] different LV dispersion laws lead to hotter pre-Planckian plasma which does not contain more energetic photons at the peak of the distribution: it only contains more photons at a peak located at the same energy.

The pressure-energy ratio w in the presence of Lorentz violation is, in general, a function of the temperature. Actually, when approaching the big bang instant, with $T \gg c/k\lambda$, we have

$$w \simeq \ln\frac{\lambda kT}{c}. \tag{17}$$

In the same temperature domain the LV blackbody equation of state can be approximated as follows:

$$P \simeq \rho \ln\left(\frac{3h^3\lambda^4\rho}{8\pi}\right). \tag{18}$$

For low temperatures the entropy density goes as usual, $s \sim \frac{4}{3}\sigma T^3$, while for $T \gg c/k\lambda$ it diverges logarithmically,

$$s \equiv \frac{S}{V} = \int \frac{\partial\rho}{\partial T}\frac{\mathrm{d}T}{T} \simeq \frac{8\pi k}{3h^3\lambda^3}\ln\frac{\lambda kT}{c}. \tag{19}$$

Noticeably, in the transition from the post-Planckian age to the pre-Planckian one, we have a logarithmic correction to the classical entropy, already found in various cosmological models involving quantum corrections to general relativity predictions [15].

B. Horizon problem

The so-called "horizon problem" refers to the apparent causality violation emerging from the observed very high homogeneousness of the present Universe, which appears to be near scale invariant up to a part in 10^5. Actually, the too-fast expansion of the Hubble sphere in the early Universe soon disconnects regions which move away from each other, and one has to add, by hand, special initial conditions in order to obtain the very regular cosmic structure observed today.

We are going to show that in our model the effective Universe horizon (Hubble radius or comoving causal range) $R \equiv c/\dot{a}$ diverges at very early times, and throughout the pre-Planckian era it is much larger than the horizon radius predicted by the standard big bang theory. Subsequently, towards the end of the pre-Planckian era, the comoving distance decreases abruptly and only later increases $\propto a$ (in fact, at large times and small temperatures our predictions totally agree with the standard ones). As a matter of fact, due to dispersion relation (8), the momentum-dependent group velocity is given by

$$c(p) = \left|\frac{\mathrm{d}E}{\mathrm{d}p}\right| = \frac{|2 - 3\lambda p|}{2\sqrt{1-\lambda p}}c. \tag{20}$$

On the other hand, owing to the photon statistic distribution

$$\mathrm{d}f(p) = \frac{1}{\mathrm{e}^{(cp/kT)\sqrt{1-\lambda p}} - 1}\frac{8\pi p^2}{h^3}\mathrm{d}p, \tag{21}$$

at the big bang infinite temperature the particle momentum is of the order of the cutoff $p = 1/\lambda$ (a sort of condensation in the momentum space). Then, for the above expression of the group velocity, almost all particles in the thermalized gas are endowed with infinite speed.[2] Afterwards, the temperature decreases to the Planck one, and we find an increasing number of photons with momentum lower than the maximum one. Thus, on average, the radiation flux slowed dramatically in the pre-Planckian era, causing a decrease of the Hubble radius given by

$$R_H = \frac{v}{\dot{a}} = \frac{\tilde{c}(T)}{\dot{a}},$$

where $\tilde{c}(T)$ indicates the speed of most photons (e.g. the ones endowed with the momentum which maximizes the probability distribution density at a given temperature). This might solve the horizon problem, if we think that comoving regions of the very early Universe were causally connected at any spatial scale since the speed of the radiation particles was infinite at the beginning of the Universe expansion. The same regions became disconnected at the end of the pre-Planckian era when the photon speed decreased with the decrease of temperature: then, reentering the Hubble radius only later, for $T \ll T_{\mathrm{PL}}$, when $v = c$ for all photons and R_H, as it occurs in the Lorentz-covariant cosmology, does increase with the increase of the Universe comoving radius, $R_H \sim a$.

Besides the above considerations, there is another alternative approach to the horizon problem. The speed of sound $v_s(\rho)$ through the radiation fluid filling the early Universe is given by

$$v_s^2(\rho) \equiv c^2\frac{\partial p}{\partial\rho} = c^2\frac{\partial(w\rho)}{\partial\rho} = c^2\left(w + \rho\frac{\partial w}{\partial\rho}\right). \tag{22}$$

While for the Lorentz-invariant theory v_s is equal to $c/\sqrt{3}$, in the present LV framework for $T \gg c/k\lambda$ we have, from the above equation and from Eqs. (11) and (17),

$$v_s \simeq c\left[\ln\left(\frac{3h^2\lambda^4}{8\pi c}\rho\right)\right]^{1/2}. \tag{23}$$

Consequently, for $t \to 0$ and $\rho, T \to \infty$, the pre-Planckian speed of sound is $\gg \frac{c}{\sqrt{3}}$ since it diverges together with density and temperature. In various recent papers [17,18]

[2] Another definition, an alternative to the "frequency-dependent" one in Eq. (20), is derived from deformed Lorentz transformations, e.g., those quoted in [16], leading to a modified Lorentz factor $\tilde{\gamma}$ depending also on energy and momentum. Actually, in special relativity, after a $-v$ boost from the quiet frame to the laboratory frame, we have $p = (p' + \frac{v}{c^2}E')\gamma$ and $E = (E' + vp')\gamma$ from which, being in the quiet frame $p' = 0$ and $E' = mc^2$, it follows that $v = \frac{pc^2}{E}$ (which holds also for massless particles with $E = pc$). Analogously, in deformed special relativity, from $p = (p' + \frac{v}{c^2}E')\tilde{\gamma}$ and $E = (E' + vp')\tilde{\gamma}$, we shall have $v = \frac{pc^2}{E}$ as well: therefore, in our model $v = \frac{pc}{\sqrt{1-\lambda p}}$, implying an infinitely large speed of light at the cutoff, just like the one defined in Eq. (20).

it has been argued that if the speed of sound in the early Universe was much larger than c, a nearly scale-invariant spectrum of density fluctuations could have been produced through a process independent of the usual horizon problem solutions. As a matter of fact, besides the Hubble radius another horizon exists that is endowed with an independent dynamics, namely, the "sound horizon"

$$R_s \equiv \frac{v_s}{\dot{a}} \tag{24}$$

which is expected to grow much more than the comoving distance predicted in the classical big bang theory. As an example, in [17,19] it is shown that, in an expanding Universe, the generation of a super-Hubble scale-invariant spectrum of perturbations over a range of wavelengths consistent with observation just requires, in the absence of inflation and cosmic acceleration, a speed of sound faster than the speed of light or a super-Planckian energy density. Actually, both conditions are satisfied in our LV scenario, where the speed of sound is highly superluminal and very rapidly varying.

C. Flatness problem

Another basic problem of standard cosmology is the "flatness problem": since today the observed curvature of the Universe is close to zero, the Friedmann equations imply an infinitely vanishing curvature in the early Universe, which therefore would be very improbable and too unstable. In the absence of Lorentz violations the FRLW equation for the Hubble constant is written

$$H^2 = \frac{8\pi G}{3c^2}\rho - k\frac{c^2}{a^2}. \tag{25}$$

The previous equation can be rewritten as follows (hereafter we label present time quantities by zero),

$$H^2 = H_0^2\left[\frac{\rho}{\rho_0}\Omega_{m0} + a^{-2}\Omega_{k0}\right], \tag{26}$$

where H_0 is the Hubble constant today; the current "matter-energy density parameter"

$$\Omega_{m0} \equiv \rho_0/\rho_c \tag{27}$$

is defined as the ratio between the actual energy density ρ_0 and the critical energy density $\rho_c = 3H_0^2c^2/8\pi G$; the quantity

$$\Omega_{k0} \equiv -kc^2/H_0^2a_0^2 = -kc^2/H_0^2 \tag{28}$$

(the expansion radius a_0 in the present age is taken to be unitary) indicates today's "curvature density parameter." It is easily proved that between the two density parameters the constraint $\Omega_{m0} + \Omega_{k0} = 1$ holds. Taking into account Eq. (26), the "relative curvature," or "deviation from flatness," can be defined as

$$\mathcal{C}(a) \equiv \frac{|\Omega_{k0}|a^{-2}}{\Omega_{m0}\rho/\rho_0}. \tag{29}$$

Since in the standard FRLW model $\rho \sim a^{-3(1+w)}\rho_0$, we can also write

$$\mathcal{C}(a) = \frac{|\Omega_{k0}|a^{-2}}{\Omega_{m0}a^{-3(1+w)}} = \frac{|\Omega_{m0} - 1|a^{-2}}{\Omega_{m0}a^{-3(1+w)}} = \mathcal{C}_0 a^{1+3w}, \tag{30}$$

where

$$\mathcal{C}_0 \equiv \frac{|\Omega_{k0}|}{\Omega_{m0}} = \frac{kc^2\rho_c}{H_0^2\rho_0}$$

is the small deviation from flatness measured today. As is well known, taking into account that $a \sim T^{-1}$ and (from Wilkinson Microwave Anisotropy Probe and COsmic Background Explorer) $\Omega_{k0} < 0.1$, for the radiation case $w = 1/3$ we infer from Eq. (30) that at the Planck time C was of the order of 10^{-62}: namely, the cosmological flatness problem.

In recent years cosmologists have tried to solve the flatness problem via inflationary models in an accelerating Universe ($w < -1/3$), or by introducing a time-varying Newton "constant" $G(t)$, or even by assuming a curvature parameter $k(\rho)$ depending on the early Universe energy density. As an example, in [14] a gravity's rainbow approach is proposed, where the curvature term is multiplied times a metric form factor $g(\rho)$ depending on the Universe energy density: this choice, in turn, implies a speed of light $c(\rho)$ depending on ρ which in the pre-Planckian epoch is much larger than c. In what follows we shall not consider LV modifications to the curvature, but only an effective dependence of the speed of light on energy density. As a matter of fact, we have previously seen that the primordial speed of radiation particles, $\tilde{c}(T)$, is strongly dependent on the temperature. Taking into account that the temperature can be considered as a function of the energy density [actually, in pre-Planckian times we have that ρ and T are linearly proportional, cf. Eq. (11)], we can assume that the speed of light is a function of the density as well. Notice that our temperature- or density-dependent speed of light, $\tilde{c}(\rho(t))$ or $\tilde{c}(T(t))$, is implicitly time varying, ranging from infinite at $t = 0$ to c at $t \gg T_{\mathrm{PL}}$. Let us now rewrite the above FRLW equation in our LV scenario with time-varying speed of light $\tilde{c}(t)$,

$$H^2 = \frac{8\pi G}{3\tilde{c}^2}\rho - k\frac{\tilde{c}^2}{a^2}. \tag{31}$$

Rewriting the above equation in terms of Ω_{k0} and Ω_{m0}, after a little algebra we see that Eq. (26) can be modified as follows:

$$H^2 = H_0^2\left[\left(\frac{c}{\tilde{c}}\right)^2\frac{\rho}{\rho_0}\Omega_{m0} + \left(\frac{\tilde{c}}{c}\right)^2 a^{-2}\Omega_{k0}\right], \tag{32}$$

while, in the place of (29), we now obtain

$$\mathcal{C}(a) = \left(\frac{\tilde{c}}{c}\right)^4 \frac{|\Omega_{k0}|a^{-2}}{\Omega_{m0}\rho/\rho_0} = \left(\frac{\tilde{c}}{c}\right)^4 \frac{\rho_0}{\rho a^2}\mathcal{C}_0. \tag{33}$$

From the above equation we therefore deduce that, in contrast with classical theory, at an initial instant, when $T \gg T_{\rm PL}$, the deviation from flatness can be nonvanishing,

$$\mathcal{C}(a \ll 1) \geq \mathcal{C}_0,$$

because in (33) the very small factor $\rho_0/(\rho a^2) \sim a^{1+3w}$ can be counterbalanced by the very large ratio $(\tilde{c}/c)^4$ which is diverging in the pre-Planckian era. We then see that the superluminality of the pre-Planckian Universe, due to our LV dispersion law, is the key to the solution not only of the horizon problem but also of the flatness problem.

D. Cosmic entropy arrow

As it occurs in other cosmological models without the flatness problem, the present phenomenological model entails, as expected, a nonconservation entropy effect, even without recourse to a ‘‘reheating’’ of the Universe. One possible explanation for the apparent energy nonconservation, due to entropy nonconservation in the absence of reheating sources, can be related to the breaking of Poincaré-Lorentz symmetry (in particular, the spacetime translation invariance) in the pre-Planckian Universe [14]. Assuming Lorentz symmetry and c constant, by exploiting the Friedmann acceleration equation $\ddot{a} = -\frac{4\pi G}{3c^2}(\rho + 3P)a$ and Eq. (25), we obtain

$$\dot{\rho} + 3H(\rho + P) = 0 \tag{34}$$

which is a fluidodynamical version of the entropy conservation law for adiabatic processes,

$$S = \text{const.} \tag{35}$$

In fact, starting from the first law of thermodynamics

$$\mathrm{d}Q = T\mathrm{d}S = p\mathrm{d}V + \mathrm{d}U,$$

we can write ($V \propto a^3$, $S \equiv sV$)

$$T\mathrm{d}S = P\mathrm{d}a^3 + \mathrm{d}(\rho a^3) = (\rho + P)\mathrm{d}a^3 + a^3\mathrm{d}\rho \tag{36}$$

which, by taking the derivative with respect to time (in thermal equilibrium), just becomes

$$Ta^{-3}\frac{\mathrm{d}S}{\mathrm{d}t} = 3H(\rho + P) + \dot{\rho}. \tag{37}$$

Then, from (37) and (34), we obtain (35).

Let us now assume a nonconstant speed of light $c = c(T(t))$ and take the derivative with respect to time of the modified Friedmann equation for H, Eq. (31). Also taking into account the modified Friedmann acceleration equation

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3\tilde{c}^2}(\rho + 3P),$$

we finally get

$$Ta^{-3}\frac{\mathrm{d}S}{\mathrm{d}t} = \dot{\rho} + 3H(\rho + P) = 2\rho\dot{\tilde{c}}\tilde{c}^{-1} + \frac{3k\tilde{c}^3\dot{\tilde{c}}}{4\pi G a^2}. \tag{38}$$

Therefore, in the early LV Universe, with $\dot{\tilde{c}} \neq 0$, we have, in general, entropy nonconservation, while after the pre-Planckian era and in the present Universe, the total entropy is constant in time. In particular, as $\dot{\tilde{c}} < 0$, for $k > 0$ we always have decreasing entropy. For $k < 0$ the entropy increases if the following condition holds:

$$2\rho\dot{\tilde{c}}\tilde{c}^{-1} + \frac{3k\tilde{c}^3\dot{\tilde{c}}}{4\pi G a^2} > 0. \tag{39}$$

When $k = -1$ this constraint requires that in the pre-Planckian phase the entropy does increase if the energy density is larger than a value of the critical density corresponding to a ‘‘luminal’’ expansion speed:

$$\rho > \frac{3\tilde{c}^2}{8\pi G}\left(\frac{\tilde{c}}{a}\right)^2 \equiv \frac{3\tilde{c}^2}{8\pi G}\tilde{H}^2. \tag{40}$$

Notice also that, in contrast to the classical predictions, in the presence of Lorentz symmetry violation, we do not have constant entropy, even in the case $k = 0$, where the entropy loss rate is

$$\frac{\mathrm{d}S}{\mathrm{d}t} = \frac{2\rho\dot{\tilde{c}}\tilde{c}^{-1}a^3}{T}. \tag{41}$$

The above results are also expected on the basis of simple physical considerations [20,21]. The entropy variation cannot be ascribed to dissipation or particle production, as the cosmological fluid is perfect and the particle number is invariant. As a matter of fact, the entropy of such a special fluid as a LV primordial plasma turns out to be nonconstant as long as the speed of light is also nonconstant. For open universes the increasing entropy can be qualitatively justified since the decreasing speed of light means a narrowing of the past light cone of the observers, who hence gradually lose information [20].

III. CONCLUSIONS

Starting from a special dispersion law endowed with a net momentum cutoff already proposed and investigated in our previous papers, here we have studied various important physical consequences of Lorentz symmetry breaking on the expanding primordial radiation plasma, soon after the big bang. Taking into account Lorentz violations at the Planck scale is indeed one of the more effective ways to describe, within a mere phenomenological approach, physical domains where quantum mechanics is expected to strongly affect the general relativity predictions.

We have first found that in the presence of Lorentz violations the blackbody radiation obeys the ordinary Stefan-Boltzmann equation of state only for temperatures much lower than the Planck one. By contrast, we have proved that in the pre-Planckian era the energy density and the pressure are linearly proportional to the temperature, in contrast to the classical T^4 behavior. As a consequence, the pressure-energy ratio, as well as the entropy, is logarithmically proportional to the temperature. The logarithmic behavior of the cosmic entropy around the Planck time, emerging in the present nonquantum phenomenological framework, is sometimes found in loop quantum cosmology and in other quantum gravity applications to the big bang theory.

We have applied our phenomenological approach to the Universe expansion soon after the big bang, but before the Planck time, in order to yield a likely explanation, agreeing with experimental data from the COsmic Background Explorer and Wilkinson Microwave Anisotropy Probe, to fundamental open problems in cosmology.

We have therefore investigated the dynamical effective modifications to the initial Universe evolution due to our dispersion law, focusing on the sharp dependence of the pre-Planckian photon speed and sound speed on temperature and energy density, i.e. on time. In fact, we show that in our model both speeds are expected to be infinitely larger than c when approaching the big bang. Actually, superluminal motion of different cosmic regions and/or super-Hubble scale-invariant perturbations can prevent the horizon problem.

Analogously, the divergence of the speed of light does seem to provide a solution to the troublesome flatness problem. We exploit a first approximation to a LV version of Friedmann equations without modifying the spacetime metric but assuming a modified equation of state and a speed of light varying with temperature or density. Consequently, the vanishing pre-Planckian Universe curvature predicted by the classical big bang model is now multiplied times the fourth power of an infinitely large speed of light. In such a way, the resulting early curvature does not vanish anymore and our beginning Universe does not need to be highly fine-tuned. Thus, at the big bang instant the energy-matter density is not required to be infinitely close to the critical value (departing from it up to a part in 10^{62}) as it happens in standard cosmology.

Finally, we have briefly studied the reheating and cosmic time arrow issues, which are topics emerging in any model beyond the classical big bang theory, e.g., in inflation theories. We evaluate the nonvanishing entropy production in a Universe crossed by photons endowed with an effective time-varying speed of light: as a matter of fact, a nonconstant entropy in thermal equilibrium can, in the end, be due to the underlying Lorentz symmetry breaking.

In a forthcoming paper we shall numerically solve the FLRW equations with a momentum-dependent metric. Anyway, the present phenomenological approximation of a more exact analysis has shown that some basic serious problems in contemporary cosmology can be overcome without recourse to inflation or to new energy fields. Recalling what we said in the introduction section about the relation occurring between relativistic covariance and effective spacetime structure, we could conclude that the Planck time appears as a watershed between a hot age (characterized by spacetime discreteness and noncommutativity, Poincaré group violation, and infinite photon speed) and a cold age, where c stabilizes to the actual constant value in a commutative spacetime continuum endowed with exact relativistic symmetries.

ACKNOWLEDGMENTS

We are glad to thank A. Del Popolo and, in particular, S. Esposito for useful hints and stimulating discussions.

[1] E. di Grezia, S. Esposito, and G. Salesi, Mod. Phys. Lett. A **21**, 349 (2006).

[2] G. Amelino-Camelia and T. Piran, Phys. Rev. D **64**, 036005 (2001); G. Amelino-Camelia, Phys. Lett. B **392**, 283 (1997); Int. J. Mod. Phys. D **11**, 35 (2002); arXiv:gr-qc/0201012; arXiv:gr-qc/0309054; Phys. Lett. B **528**, 181 (2002); G. Amelino-Camelia, F. D'Andrea, and G. Mandanici, J. Cosmol. Astropart. Phys. 09 (2003) 006; A. Agostini, G. Amelino-Camelia, and F. D'Andrea, Int. J. Mod. Phys. A **19**, 5187 (2004); G. Amelino-Camelia, J. Kowalski-Glikman, G. Mandanici, and A. Procaccini, Int. J. Mod. Phys. A **20**, 6007 (2005); G. Amelino-Camelia, L. Doplicher, S. Nam, and Y. Seo, Phys. Rev. D **67**, 085008 (2003).

[3] J. Lukierski, H. Ruegg, and W. J. Zakrzewski, Ann. Phys. (N.Y.) **243**, 90 (1995); Z. Guralnik, R. Jackiw, S. Y. Pi, and A. P. Polychronakos, Phys. Lett. B **517**, 450 (2001); V. Nazaryan and C. E. Carlson, Phys. Rev. D **71**, 025019 (2005); C. E. Carlson, C. D. Carone, and R. F. Lebed, Phys. Lett. B **549**, 337 (2002); M. Hayakawa, Phys. Lett. B **478**, 394 (2000); arXiv:hep-th/9912167; A. Anisimov, T. Banks, M. Dine, and M. Graesser, Phys. Rev. D **65**, 085032 (2002); T. Kifune, Astrophys. J. Lett. **L21**, 518 (1999); W. Kluzniak, arXiv:astro-ph/9905308; R. J. Protheroe and H. Meyer, Phys. Lett. B **493**, 1 (2000); D. Bahns, S. Doplicher, K. Fredenhagen, and G. Piacitelli, Phys. Rev. D **71**, 025022 (2005); C. K. Zachos, Mod. Phys. Lett. A **19**, 1483 (2004).

[4] N. R. Bruno, G. Amelino-Camelia, and J. Kowalski-Glikman, Phys. Lett. B **522**, 133 (2001); G. Amelino-Camelia, Phys. Lett. B **510**, 255 (2001); Int. J. Mod. Phys. D **11**, 35 (2002); G. Amelino-Camelia and M. Arzano, Phys. Rev. D **65**, 084044 (2002); A. Ballesteros, F. J. Herranz, and N. R. Bruno, *Symmetry Methods in*

Physics, edited by C. Burdik, O. Navratil, and S. Posta (Joint Institute for Nuclear Research, Dubna, Russia, 2004), pp. 1–20.

[5] J. Magueijo and L. Smolin, Classical Quantum Gravity **21**, 1725 (2004).

[6] E. di Grezia, S. Esposito, and G. Salesi, Europhys. Lett. **74**, 747 (2006).

[7] S. Esposito and G. Salesi, Mod. Phys. Lett. A **25**, 597 (2010).

[8] G. Salesi and E. Di Grezia, Phys. Rev. D **79**, 104009 (2009).

[9] S. Esposito and G. Salesi, Phys. Rev. D **83**, 084043 (2011).

[10] J.R. Ellis, E. Gravanis, N.E. Mavromatos, and D.V. Nanopoulos, arXiv:gr-qc/0209108.

[11] R. Casadio, B. Harms, and Y. Leblanc, Phys. Rev. D **58**, 044014 (1998); R. Casadio and B. Harms, Int. J. Mod. Phys. A **17**, 4635 (2002); P. Nicolini, A. Smailagic, and E. Spallucci, Phys. Lett. B **632**, 547 (2006); P. Nicolini, Int. J. Mod. Phys. A **24**, 1229 (2009); A. Bonanno and M. Reuter, Phys. Rev. D **60**, 084011 (1999); M. Reuter, Phys. Rev. D **57**, 971 (1998); A. Bonanno and M. Reuter, Phys. Rev. D **62**, 043008 (2000); V.P. Frolov, W. Israel, and S.N. Solodukhin, Phys. Rev. D **54**, 2732 (1996).

[12] J. Collins, A. Perez, D. Sudarsky, L. Urrutia, and H. Vucetich, Phys. Rev. Lett. **93**, 191301 (2004).

[13] F. Girelli, S. Liberati, and L. Sindoni, Phys. Rev. D **75**, 064015 (2007).

[14] S. Alexander and J. Magueijo, *Proceedings of the XIIIrd Rencontres de Blois* (Gioi Publishers, Hanoi, Vietnam, 2004), p. 281.

[15] M. Jamil, A. Sheykhi, and M. U. Farooq, Int. J. Mod. Phys. D **19**, 1831 (2010); T. Zhu and J-R. Ren, Eur. Phys. J. C **62**, 413 (2009); M. Jamil and M. U. Farooq, J. Cosmol. Astropart. Phys. 03 (2010) 001; R. Banerjee and B. R. Majhi, Phys. Lett. B **662**, 62 (2008); J. High Energy Phys. 06 (2008) 095; B. R. Majhi, Phys. Rev. D **79**, 044005 (2009); R. Banerjee and S. K. Modak, J. High Energy Phys. 05 (2009) 063; S. K. Modak, Phys. Lett. B **671**, 167 (2009).

[16] J. Magueijo and L. Smolin, Phys. Rev. Lett. **88**, 190403 (2002); Phys. Rev. D **67**, 044017 (2003); S. Alexander, R. Brandenberger, and J. Magueijo, Phys. Rev. D **67**, 081301 (2003); S. Koh and R. H. Brandenberger, J. Cosmol. Astropart. Phys. 06 (2007) 021; C. Heuson, arXiv:gr-qc/0312034.

[17] J. Magueijo, Phys. Rev. Lett. **100**, 231302 (2008).

[18] J. Khoury and F. Piazza, J. Cosmol. Astropart. Phys. 07 (2009) 026.

[19] G. Geshnizjani, W. H. Kinney, and A. M. Dizgah, J. Cosmol. Astropart. Phys. 11 (2011) 049.

[20] L. P. Chimento, A. S. Jakubi, and D. Pavon, Phys. Lett. B **508**, 1 (2001).

[21] E. T. Jaynes, Phys. Rev. **108**, 171 (1957).