

Voyage to Alpha Centauri: Entanglement degradation of cavity modes due to motion

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(Received 25 May 2011; revised manuscript received 10 August 2011; published 15 March 2012)

We propose a scheme to investigate whether nonuniform motion degrades entanglement of a relativistic quantum field that is localized both in space and in time. For a Dirichlet scalar field in a cavity in Minkowski space, in small but freely adjustable acceleration of finite but arbitrarily long duration, degradation of observable magnitude occurs for massless transverse quanta of optical wavelength at Earth gravity acceleration and for kaon mass quanta already at microgravity acceleration. We outline a space-based experiment for observing the effect and its gravitational counterpart.

DOI: [10.1103/PhysRevD.85.061701](https://doi.org/10.1103/PhysRevD.85.061701)

PACS numbers: 03.67.Mn, 03.65.Yz, 04.62.+v

I. INTRODUCTION

A common way to implement quantum information tasks involves storing information in cavity field modes. How the motion of the cavities affects the stored information is a question that could be of practical relevance in space-based experiments [1].

In this paper we analyze the degradation of initially maximal entanglement between scalar field modes in two Dirichlet cavities in Minkowski space, one inertial and the other undergoing motion that need not be stationary. Our analysis combines, to our knowledge for the first time in relativistic quantum information theory, the explicit confinement of a quantum field to a finite size cavity and a freely adjustable time-dependence of the cavity's acceleration. This allows observers within the cavities to implement quantum information protocols in a way that is localized both in space and in time [2]. In particular, our system-environment split is manifestly causal and invokes no horizons or other teleological notions that would assume acceleration to persist into the asymptotic, post-measurement future (cf. [3]). By the equivalence principle, the analysis can be regarded as a model of gravity effects on entanglement.

Our main results are:

- (i) In generic motion, particle creation in the moving cavity causes the entanglement to depend on time. This is in stark contrast to the previously analyzed relativistic situations (see [4,5] for a small sample) where uniform acceleration is assumed to persist into the asymptotic future and the entanglement between inertial and accelerated observers is argued to be preserved in time.
- (ii) We give an analytic method for computing the entanglement for trajectories that consist of inertial and uniformly linearly accelerated segments in the small acceleration approximation. An advantage over the small amplitude approximations customary with the dynamical Casimir effect [6] is that the

segment durations and the travelled distances are arbitrary. The entanglement remains constant within each segment, but we find that it does depend on the *changes* in the acceleration and on the time intervals between these changes. We present explicit results for three sample scenarios for a massless field in $(1 + 1)$ dimensions, finding, in particular, that in this approximation any degradation caused by the accelerated segments can be undone by fine-tuning the durations of the intermediate inertial segments.

- (iii) In $(3 + 1)$ dimensions, the entanglement degradation has an observable magnitude for massless transverse quanta of optical wavelength at Earth gravity acceleration and for kaon mass quanta already at microgravity acceleration. The effect should hence be detectable in space-based experiments, where it would, in particular, test whether an equivalence principle between acceleration and a gravitational field holds also in the context of quantum information.

II. CAVITY PROTOTYPE CONFIGURATION

Let Alice and Rob ("Relativistic Bob" [4]) be observers in $(1 + 1)$ -dimensional Minkowski spacetime, each carrying a cavity that contains an uncharged scalar field of mass $\mu \geq 0$ with Dirichlet boundary conditions. Alice and Rob are initially inertial with vanishing relative velocity, and each cavity has length $\delta > 0$. The field modes in each cavity are discrete, indexed by the quantum number $n \in \{1, 2, \dots\}$ and having the frequencies $\omega_n := \sqrt{M^2 + \pi^2 n^2} / \delta$ where $M := \mu \delta$ (we set $c = \hbar = 1$).

Let Alice and Rob initially prepare their two-cavity system in the maximally entangled Bell state $|\Psi\rangle = \frac{1}{\sqrt{2}} \times (|0\rangle_A |0\rangle_R + |1_k\rangle_A |1_k\rangle_R)$, where the subscripts *A* and *R* identify the cavity, $|0\rangle$ is the vacuum and $|1_k\rangle$ is the one-particle state with quantum number *k*. Experimentally, $|\Psi\rangle$ might be prepared by allowing a single atom to emit an excitation of frequency ω_k over a flight through the two cavities [7], and the assumption of a single *k* is experimentally justified if δ is so small that cavity's frequency separation

*Previously known as Fuentes-Guridi and Fuentes-Schuller.

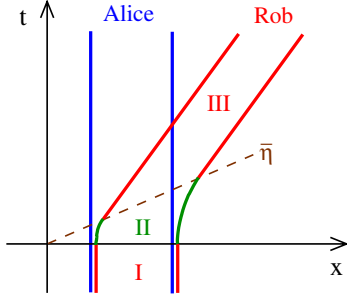


FIG. 1 (color online). Cavity trajectories in Minkowski space. Alice's cavity remains inertial. Rob's cavity is inertial in region I, accelerates in region II and is again inertial in region III. The figure shows the prototype case where Rob's acceleration is to the right and uniform throughout region II, and $\bar{\eta}$ is the duration of the acceleration in Rindler time $\text{atanh}(t/x)$.

$\omega_{n+1} - \omega_n$ is large compared with the frequency separations of the atom.

We then allow Rob to undergo arbitrary acceleration for a finite interval of his proper time. After the acceleration Rob's cavity is again inertial and has proper length δ in its new rest frame. Figure 1 shows the prototype case where Rob's acceleration is uniform while it lasts.

Let U_n , $n = 1, 2, \dots$, denote Rob's field modes that are of positive frequency ω_n with respect to his proper time before the acceleration. Let \bar{U}_n , $n = 1, 2, \dots$, similarly denote Rob's field modes that are of positive frequency ω_n with respect to his proper time after the acceleration. The two sets of modes are related by the Bogoliubov transformation $\bar{U}_m = \sum_n (\alpha_{mn} U_n + \beta_{mn} U_n^*)$, where the star denotes complex conjugation and the coefficient matrices α and β are determined by the motion of the cavity during the acceleration [8]. Working in the Heisenberg picture, the state $|\Psi\rangle$ does not change in time, but for late time observations the early time states $|0\rangle_R$ and $|1_k\rangle_R$ need to be expressed in terms of Rob's late time vacuum $|\bar{0}\rangle_R$ and the late time excitations on it, by formulas that involve the Bogoliubov coefficients [9]. In this sense, the acceleration creates particles in Rob's cavity.

We regard the late time system as tripartite between Alice's cavity, the (late time) frequency ω_k in Rob's cavity and the (late time) frequencies $\{\omega_n \mid n \neq k\}$ in Rob's cavity. As any excitations in the $n \neq k$ frequencies are entirely due to the acceleration, we regard these frequencies as the environment. We ask: Has the entanglement between Alice's cavity and the frequency ω_k in Rob's cavity been degraded, from the maximal value it had before Rob's acceleration?

We quantify the entanglement by the negativity, the widely used entanglement monotone defined by $\mathcal{N}(\rho) := -\sum_{\lambda_i < 0} \lambda_i$, where the reduced density matrix ρ is obtained by tracing the full density matrix $|\Psi\rangle\langle\Psi|$ over Rob's late time frequencies $\{\omega_n \mid n \neq k\}$ and λ_i are the eigenvalues of the partial transpose of ρ . \mathcal{N} has the advantage of being

easy to compute in bipartite systems of arbitrary dimension [10]. The closely related logarithmic negativity, $E_{\mathcal{N}} := \ln(1 + \mathcal{N})$, is an upper bound on the distillable entanglement E_D and is operationally interpreted as the entanglement cost E_C under operations preserving the positivity of partial transpose [11]. In this respect, the entanglement quantification based on negativity nicely interpolates between the two canonical (yet generically difficult to compute) extremal entanglement measures E_D and E_C [12]. A sample of recent entanglement analyses utilizing negativity can be found in [13].

The situation covering all the scenarios below is when $\alpha = \text{diag}(z_1, z_2, \dots) + h\alpha^{(1)} + O(h^2)$ and $\beta = h\beta^{(1)} + O(h^2)$, where h is a small parameter, the first-order coefficient matrices $\alpha^{(1)}$ and $\beta^{(1)}$ are off diagonal, and each z_n has unit modulus. We find that the first correction to ρ occurs in order h^2 , the partial transpose is to this order an 8×8 matrix with exactly one negative eigenvalue, and the order h^2 formula for the negativity reads

$$\mathcal{N} = \frac{1}{2} - h^2 \sum'_n \left(\frac{1}{2} |\alpha_{nk}^{(1)}|^2 + |\beta_{nk}^{(1)}|^2 \right), \quad (1)$$

where the prime on the sum means that the term $n = k$ is omitted.

III. MASSLESS FIELD

We now specialize to a massless field. Let I, II and III denote, respectively, the initial inertial region, the region of acceleration and the final inertial region. As a first travel scenario, let the acceleration in region II be uniform, in the sense that Rob's cavity is dragged to the right by the boost Killing vector $\xi := x\partial_t + t\partial_x$ as shown in Fig. 1. Let the proper acceleration at the center of the cavity be h/δ , where the dimensionless positive parameter h must satisfy $h < 2$ in order that the proper acceleration at the left end of the cavity is finite. In region II, the field modes that are positive frequency with respect to ξ are then a discrete set V_n with $n \in \{1, 2, \dots\}$ and their frequencies with respect to the proper time τ at the center of the cavity are $\tilde{\Omega}_n = (\pi h n) / [2\delta \text{atanh}(h/2)]$.

The Bogoliubov transformation from $\{U_n\}$ to $\{V_n\}$ can be computed by standard techniques [8] at the junction $t = 0$. The coefficient matrices, which we denote by ${}_{\circ}\alpha$ and ${}_{\circ}\beta$, have small h expansions that begin

$${}_{\circ}\alpha_{nn} = 1 - \frac{1}{240} \pi^2 n^2 h^2 + O(h^4), \quad (2a)$$

$${}_{\circ}\alpha_{mn} = \sqrt{mn} \frac{(-1 + (-1)^{m-n})}{\pi^2 (m-n)^3} h + O(h^2) \quad (m \neq n), \quad (2b)$$

$${}_{\circ}\beta_{mn} = \sqrt{mn} \frac{(1 - (-1)^{m-n})}{\pi^2 (m+n)^3} h + O(h^2). \quad (2c)$$

The Bogoliubov transformation from region I to region III can now be written as the composition of three individual

transformations. The first is with the coefficient matrices $(\circ\alpha, \circ\beta)$ from I to II. The second is with the coefficient matrices $(\text{diag}(p, p^2, p^3, \dots), \text{diag}(p^{-1}, p^{-2}, p^{-3}, \dots))$, where $p := \exp(i\tilde{\Omega}_1\tilde{\tau})$ and $\tilde{\tau}$ is the duration of the acceleration in τ : this undoes the phases that the modes V_n develop over region II. The third is the inverse of the first, from II to III, with the coefficient matrices $(\circ\alpha^\dagger, -\circ\beta^T)$. Collecting, we find from (1) that the negativity \mathcal{N}_1 is given to order h^2 by

$$\begin{aligned}\mathcal{N}_1 &= \frac{1}{2} - h^2 \sum_{r=0}^{\infty} a_{kr} |p^{1+2r} - 1|^2 \\ &= \frac{1}{2} - 2[Q(k, 1) - Q(k, p)]h^2,\end{aligned}\quad (3)$$

where

$$\begin{aligned}Q(n, z) &:= \frac{4n^2}{\pi^4} \text{Re}\left(\text{Li}_6(z) - \frac{1}{64} \text{Li}_6(z^2)\right) + \frac{6n}{\pi^4} \\ &\times \sum_{r=[n/2]}^{\infty} \text{Re}(z^{1+2r}) \left(\frac{1}{(1+2r)^5} - \frac{n}{(1+2r)^6}\right)\end{aligned}\quad (4)$$

and a_{nr} are the coefficients in the expansion $Q(n, z) = \sum_{r=0}^{\infty} a_{nr} \text{Re}(z^{1+2r})$. Here Li_6 is the polylogarithm [14], the square brackets in the lower limit of the sum in (4) denote the integer part, and a_{nr} are all strictly positive. The sum term in (4) is $O(n^{-3})$ as $n \rightarrow \infty$, and numerics shows that the sum term contribution to $Q(n, z)$ is less than 1.1% already for $n = 1$ and less than 0.25% for $n \geq 2$.

\mathcal{N}_1 (3) is periodic in $\tilde{\tau}$ with period $2\pi\tilde{\Omega}_1^{-1}$ and attains its unique minimum at half-period. A plot is shown in Fig. 2. The reason for the periodicity is that the full time evolution of the field in Rob's cavity during the accelerated segment is periodic with this period since the frequencies $\tilde{\Omega}_n$ are integer multiples of the fundamental frequency $\tilde{\Omega}_1$. \mathcal{N}_1 is therefore periodic not just in the small h approximation of (3) but exactly for arbitrary h . More generally, the same periodicity occurs for all cavity trajectories that contain a uniformly accelerated segment. We note that the period can be written as $2\delta(h/2)^{-1} \text{atanh}(h/2)$: this is the proper time elapsed at the center of the cavity between

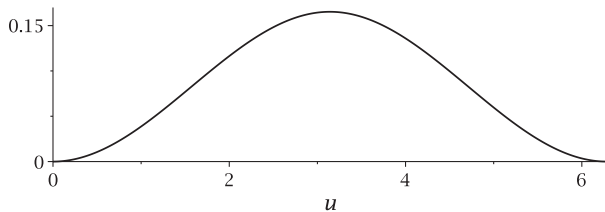


FIG. 2. The plot shows $(\frac{1}{2} - \mathcal{N}_1)h^{-2}$ with $k = 1$ as a function of $u := \tilde{\Omega}_1\tilde{\tau}$ over the full period $0 \leq u \leq 2\pi$.

sending and recapturing a light ray that bounces off each wall once.

As a second travel scenario, suppose that Rob blasts off as above, coasts inertially for proper time $\tilde{\tau}'$ and then performs a braking manoeuvre that is the reverse of the initial acceleration, bringing him to rest (at, say, Alpha Centauri). Composing the segments as above, we see that the negativity \mathcal{N}_2 is periodic in $\tilde{\tau}'$ with period 2δ . Noting that for leftward acceleration (2) holds with negative h , we find to order h^2 the formula

$$\begin{aligned}\mathcal{N}_2 &= \frac{1}{2} - h^2 \sum_{r=0}^{\infty} a_{kr} |p^{1+2r} - 1|^2 |(pp')^{1+2r} - 1|^2 \\ &= \frac{1}{2} - 2[2Q(k, 1) - 2Q(k, p) + Q(k, p') \\ &\quad - 2Q(k, pp') + Q(k, p^2p')]h^2,\end{aligned}\quad (5)$$

where $p' := \exp(i\pi\tilde{\tau}'/\delta)$. In addition to displaying the periodicities in $\tilde{\tau}$ and $\tilde{\tau}'$, (5) shows that the coefficient of h^2 vanishes iff $p = 1$ or $pp' = 1$. This implies that to order h^2 , any entanglement degradation caused by the accelerated segments can be cancelled by fine-tuning the duration of the coasting segment. A plot is shown in Fig. 3.

As a third scenario, suppose Rob travels to Alpha Centauri as above, rests there for proper time $\tilde{\tau}''$ and then returns to Alice by reversing the outward manoeuvres. Again composing the segments, we see that the negativity \mathcal{N}_3 is periodic in $\tilde{\tau}''$ with period 2δ , and to order h^2 we find

$$\begin{aligned}\mathcal{N}_3 &= \frac{1}{2} - h^2 \sum_{r=0}^{\infty} a_{kr} |p^{1+2r} - 1|^2 |(pp')^{1+2r} - 1|^2 \\ &\quad \times |(p^2p'p'')^{1+2r} - 1|^2,\end{aligned}\quad (6)$$

where $p'' := \exp(i\pi\tilde{\tau}''/\delta)$, and the sum can be expressed as a sum of 14 Q s if desired [cf. the final expressions in (3) and (5)]. The periodicities in $\tilde{\tau}$, $\tilde{\tau}'$ and $\tilde{\tau}''$ are manifest in (6). The coefficient of h^2 vanishes iff $p = 1$, $pp' = 1$ or $p^2p'p'' = 1$, so that to order h^2 any entanglement degradation caused by the accelerated segments can be cancelled

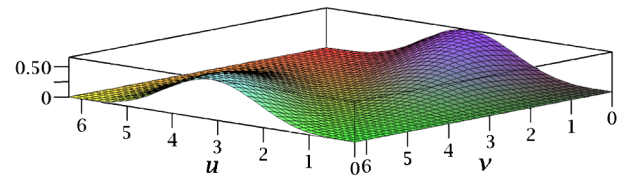


FIG. 3 (color online). The plot shows $(\frac{1}{2} - \mathcal{N}_2)h^{-2}$ with $k = 1$ as a function of $u := \tilde{\Omega}_1\tilde{\tau}$ and $v := \pi\tilde{\tau}'/\delta$ over the full periods $0 \leq u \leq 2\pi$ and $0 \leq v \leq 2\pi$. Note the zeroes at $u \equiv 0 \pmod{2\pi}$ and at $u + v \equiv 0 \pmod{2\pi}$.

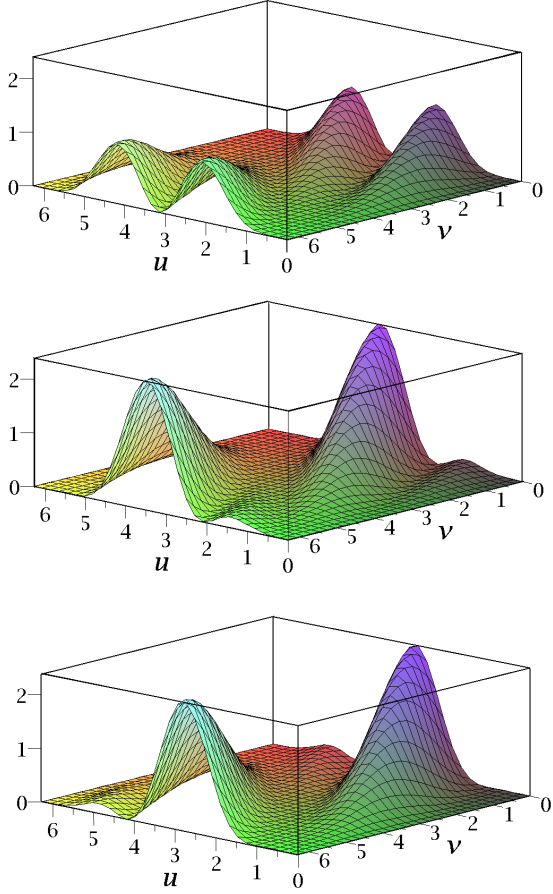


FIG. 4 (color online). The plots show $(\frac{1}{2} - \mathcal{N}_3)h^{-2}$ with $k = 1$ as a function of $u := \bar{\Omega}_1 \bar{\tau}$ and $v := \pi \bar{\tau}' / \delta$ for $0 \leq u \leq 2\pi$ and $0 \leq v \leq 2\pi$ when (from top to bottom) $\bar{\tau}'' = 0, 2\delta/3$ and $4\delta/3$.

by fine-tuning the duration of either of the independent inertial segments. Selected plots are shown in Fig. 4.

Four comments are in order. First, should one wish to consider noninertial initial or final states for Rob, our small h analysis is applicable whenever the assumptions leading to formula (1) still hold. For example, in a kickstart

scenario that contains just regions I and II of Fig. 1, so that Rob's final state is uniformly accelerating, we find $\mathcal{N}_{\text{kick}} = \frac{1}{2} - Q(k, 1)h^2$.

Second, the validity of our perturbative treatment requires the negativity to remain close to its initial value $\frac{1}{2}$, which in our scenarios happens when $kh \ll 1$. As the expansions (2) are not uniform in the indices, the treatment could potentially have missed even in this regime effects due to very high energy modes. However, we have verified that when the h^2 terms are included in the expansions (2), these expansions satisfy the Bogoliubov identities [8] perturbatively to order h^2 and the products of the order h matrices in the identities are unconditionally convergent. This gives confidence in our order h^2 negativity formulas, whose infinite sums come from similar products of order h matrices.

Third, the matrices (2) can be self-consistently truncated to the lowest 2×2 block provided the rows and columns are renormalized by suitable factors of the form $1 + O(h^2)$ to preserve the Bogoliubov identities to order h^2 . Taking Rob's initial excitation to be in the lower frequency, we find that all the above negativity results hold with the replacement $Q(1, z) \rightarrow a_{10}\text{Re}(z) + \frac{1}{2}a_{11}\text{Re}(z^3)$, and the error in this replacement is less than 0.7%. The high frequency effects on the entanglement are hence very strongly suppressed.

Fourth, the analysis can be adapted to a fermionic field [15] and to scenarios where mode entanglement is generated from an initially unentangled state [16].

IV. MASSIVE FIELD

For a massive field the frequencies are not uniformly spaced and the negativity is no longer periodic in the durations of the inertial and uniformly accelerated segments. The massive counterparts of the expansions (2) can be found using uniform asymptotic expansions of modified Bessel functions [14,17], with the result

$$\circ\alpha_{nn} = 1 - \left(\frac{\pi^2 n^2}{240} + \frac{M^2}{120} + \frac{M^2(M^2 - 5)}{240\pi^2 n^2} + \frac{M^2(M^2 - 24)}{96\pi^4 n^4} \right) h^2 + O(h^4), \quad \circ\beta_{nn} = O(h^2), \quad (7a)$$

$$\circ\alpha_{mn} + \circ\beta_{mn} = \frac{2mn(-1 + (-1)^{m-n})[\pi^2(n^2 + 3m^2) + 4M^2](M^2 + \pi^2 n^2)^{1/4}}{\pi^4(m^2 - n^2)^3(M^2 + \pi^2 m^2)^{1/4}} h + O(h^2) \quad (m \neq n), \quad (7b)$$

$$\circ\alpha_{mn} - \circ\beta_{mn} = \frac{2mn(-1 + (-1)^{m-n})[\pi^2(m^2 + 3n^2) + 4M^2](M^2 + \pi^2 m^2)^{1/4}}{\pi^4(m^2 - n^2)^3(M^2 + \pi^2 n^2)^{1/4}} h + O(h^2) \quad (m \neq n), \quad (7c)$$

and we have again verified that the Bogoliubov identities are satisfied perturbatively to order h^2 . The perturbative treatment is now valid provided $h \ll 1$ and $hM^2 \lesssim 100$, allowing the possibility that M may be large. When $k \ll M$, a qualitatively new feature is that the order h contribution in $\circ\alpha$ is proportional to M^2 , resulting in an overall enhancement factor M^4 in the negativity. In the travel scenario with one accelerated segment, the negativity takes in this limit the form

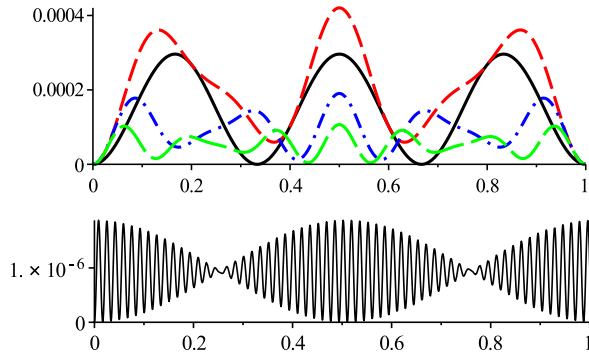


FIG. 5 (color online). The plots show $(\frac{1}{2} - \mathcal{N}_1)h^{-2}M^{-4}$ (8) for $M = 10^3$ as a function of $\pi\bar{\tau}/(4M\delta)$, in the upper figure with $k = 1, 2, 3, 4$ (solid, dashed, dash-dotted, wide-dashed) and in the lower figure with $k = 30$.

$$\mathcal{N}_1 = \frac{1}{2} - h^2 M^4 \times \frac{256k^2}{\pi^8} \sum_n'' \frac{n^2}{(k^2 - n^2)^6} \times \left\{ 1 - \cos[(\sqrt{M^2 + \pi^2 k^2} - \sqrt{M^2 + \pi^2 n^2})(\bar{\tau}/\delta)] \right\}, \quad (8)$$

where the double prime means that the sum is over positive n with $n \equiv k + 1 \pmod{2}$. \mathcal{N}_1 (8) is approximately periodic in $\bar{\tau}$ with period $4M\delta/\pi$, but it contains also significant higher frequency components. Plots are shown in Fig. 5.

V. DISCUSSION: (3 + 1) DIMENSIONS

The above (1 + 1)-dimensional entanglement degradation analysis extends immediately to linear acceleration in (3 + 1)-dimensional Minkowski space, where the transverse momentum merely contributes to the effective (1 + 1)-dimensional mass. For a massless field in a cavity of length $\delta = 10$ m and acceleration 10 ms^{-2} , an effect of

observable magnitude can be achieved by trapping quanta of optical wavelengths provided the momentum is highly transverse so that $k \ll M \approx 10^8$. Were it possible to trap and stabilise massive quanta of kaon mass $\mu = 10^{-27}$ kg in a cavity of length $\delta = 10$ cm, the effect would become observable already at the extreme microgravity acceleration of 10^{-10} ms^{-2} .

These estimates suggest that experimental verification of the effect is feasible, but they also suggest that accelerations of gravitational origin should be properly accounted for, both in the theoretical analysis and in the experimental setup. On the theoretical side, we anticipate that the core properties of our analysis extend to nonlinear acceleration and to weakly curved spacetime. To achieve experimental control, especially on accelerations of gravitational origin, the experiment may need to be performed by entangling cavities in spaceships. An experiment within a single spaceship could test the relative acceleration effect analyzed in this paper. An experiment using cavities in two and possibly widely separated spaceships would test whether an equivalence principle between acceleration and a gravitational field holds also in the context of quantum information. An experimental confirmation that gravity degrades entanglement would indeed provide a novel addition to the currently scarce experimental evidence on quantum phenomena involving gravity [18].

ACKNOWLEDGMENTS

We thank G. Adesso, A. Dragan, D. Faccio, N. Friis, L. Hackermüller, A. Lee, R. Mann and E. Martín-Martínez for helpful discussions and correspondence. J.L. thanks F. Olver for correspondence on a misprint in formula (4.15) in [17]. I.F. thanks EPSRC [CAF Grant No. EP/G00496X/2] for financial support. J.L. was supported in part by STFC (UK).

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