

Determination of the η - η' mixing angle

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We extract η - η' mixing angle and the ratios of decay constants of light pseudoscalar mesons π^0 , η , and η' using recently available *BABAR* measurements on η -photon and η' -photon transition form factors and more accurate experimental data for the masses and two-photon decay widths of the light pseudoscalar mesons.

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Determining the composition of η and η' mesons has attracted continuous interest in hadronic physics. The idea of η and η' containing gluonic and intrinsic $|c\bar{c}\rangle$ components has long been employed in explaining many experimental results, including recent observations of large branching ratios for some decay processes of J/ψ and B mesons into pseudoscalar mesons [1].

There are three charge neutral states in the nonet of pseudoscalar mesons in the $SU(3)_F$ quark model: π^0 , η_8 and η_1 . The latter two mix to give the physical particles η and η' ,

$$\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \eta_8 \\ \eta_1 \end{pmatrix}. \quad (1)$$

Alternatively, one could use the quark-flavor basis mixing scheme [2],

$$\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} \eta_q \\ \eta_s \end{pmatrix}, \quad (2)$$

with $|\eta_q\rangle = \frac{1}{\sqrt{2}}(|u\bar{u}\rangle + |d\bar{d}\rangle)$ and $|\eta_s\rangle = |s\bar{s}\rangle$. The mixing angles in the two schemes are related via $\theta = \phi - \arctan\sqrt{2} \approx \phi - 54.7^\circ$. A two-mixing-angle scheme has also been suggested in the study of the mixing of decay constants [3].

The mixing angle can neither be calculated from first principles in QCD nor measured directly—it has to be determined phenomenologically. There are a lot of studies on this subject using different methods and a number of different processes, including various decay processes involving the light pseudoscalar mesons [1–6].

One important source of information in determining the mixing angle is the transition processes, $\gamma\gamma^* \rightarrow \eta, \eta'$, for which the transition form factors (TFFs), $F_{\eta\gamma}(Q^2)$ and

$F_{\eta'\gamma}(Q^2)$, with Q^2 being the virtuality of the off-mass-shell photon, are defined. The usual procedure [7] using the TFFs to evaluate the η - η' mixing angle is to calculate the Q^2 dependence of these transition form factors and compare with the experimental data, which are given at a certain range of Q^2 [8–10]. However, theoretical calculations for the TFFs at finite Q^2 suffer sizable corrections and are sensitive to the nonperturbative model used for the distribution amplitude of the mesons, which results in large uncertainties in determining the mixing angle.

Two analytical constraints on the η - η' mixing were obtained in Ref. [11] by considering the two-photon decays of η and η' and the asymptotic behavior of the η and η' TFFs in the limit $Q^2 \rightarrow \infty$, together with the fact that the asymptotic behavior of the meson TFFs is firmly predicted by QCD [12]. Newly available *BABAR* data [10] extend the measurements for the η and η' TFFs to higher Q^2 and to a much larger range of Q^2 , and thus provide new information for the η and η' TFFs at $Q^2 \rightarrow \infty$. At the same time, experimental information on the masses and two-photon decay widths of mesons involved have improved considerably over the last decade. These new experimental data shall have an impact on the determining of the η - η' mixing parameters.

In this paper, we extract the $Q^2 \rightarrow \infty$ behavior of the η and η' TFFs from the *BABAR* data. Using this new information and updated experimental data about the two-photon decays $\eta \rightarrow \gamma\gamma$ and $\eta' \rightarrow \gamma\gamma$ [13], we determine the η - η' mixing angle and the ratios of decay constants in the two mixing schemes [see Eqs. (1) and (2)] using the method of Ref. [11].

The analytical expressions obtained in Ref. [11] for the mixing angle θ and the ratio of the decay constants of the η_1 and η_8 states $r = f_1/f_8$ are

$$\tan\theta = \frac{-(1+c^2)(\rho_1+\rho_2) + \sqrt{(1+c^2)^2(\rho_1+\rho_2)^2 + 4(c^2-\rho_1\rho_2)(1-c^2\rho_1\rho_2)}}{2(c^2-\rho_1\rho_2)}, \quad (3)$$

$$r = \frac{(1+c^2)(\rho_1-\rho_2) + \sqrt{(1+c^2)^2(\rho_1-\rho_2)^2 + 4c^2(1+\rho_1\rho_2)^2}}{2c(1+\rho_1\rho_2)}, \quad (4)$$

where $c = \sqrt{8}$ and

$$\rho_1 = \left[\frac{\Gamma_{\eta \rightarrow \gamma\gamma} m_{\eta'}^3}{\Gamma_{\eta' \rightarrow \gamma\gamma} m_{\eta}^3} \right]^{1/2}, \quad (5)$$

$$\rho_2 = \frac{F_{\eta\gamma}(Q^2 \rightarrow \infty)}{F_{\eta'\gamma}(Q^2 \rightarrow \infty)}. \quad (6)$$

One advantage of determining the mixing parameters from Eqs. (3)–(6) is that both the theoretical uncertainty incurred in calculating the TFFs at finite Q^2 and the experimental uncertainty are minimized by considering the ratios of the decay widths for the two-photon decay processes and the ratios of the transition form factors at large Q^2 .

Furthermore, considering the ratio of the decay widths for the $\pi^0 \rightarrow \gamma\gamma$ and $\eta \rightarrow \gamma\gamma$ processes, we can also determine the ratios f_8/f_π and f_1/f_π ,

$$\frac{f_8}{f_\pi} = \rho_0 \left[\frac{c_8}{c_\pi} \cos\theta - \frac{1}{r} \frac{c_1}{c_\pi} \sin\theta \right], \quad (7)$$

$$\frac{f_1}{f_\pi} = \rho_0 \left[\frac{c_8}{c_\pi} r \cos\theta - \frac{c_1}{c_\pi} \sin\theta \right], \quad (8)$$

where $c_\pi = 1$, $c_8 = 1/\sqrt{3}$, $c_1 = 2\sqrt{2}/\sqrt{3}$, and

$$\rho_0 = \left[\frac{\Gamma_{\pi^0 \rightarrow \gamma\gamma} m_{\eta}^3}{\Gamma_{\eta \rightarrow \gamma\gamma} m_{\pi^0}^3} \right]^{1/2}. \quad (9)$$

The above analysis can be easily applied to the quark-flavor basis mixing scheme [see Eq. (2)] by replacing the parameters $c = c_1/c_8$, $r = f_1/f_8$, c_8 , and c_1 with $c' = c_s/c_q = \sqrt{2}/5$, $r' = f_s/f_q$, $c_q = 5/3$, and $c_s = \sqrt{2}/3$, respectively [11].

The parameters ρ_0 and ρ_1 can be fixed by using the masses and two-photon decay widths of π^0 , η and η' . We employ the data given by the 2010 Particle Data Group [13],

$$\begin{aligned} \Gamma_{\pi^0 \rightarrow \gamma\gamma} &= 7.74 \pm 0.46 \text{ eV}, \\ \Gamma_{\eta \rightarrow \gamma\gamma} &= 0.510 \pm 0.026 \text{ keV}, \\ \Gamma_{\eta' \rightarrow \gamma\gamma} &= 4.28 \pm 0.19 \text{ keV}, \end{aligned} \quad (10)$$

$$\begin{aligned} m_{\pi^0} &= 134.9766 \pm 0.0006 \text{ MeV}, \\ m_{\eta} &= 547.853 \pm 0.024 \text{ MeV}, \\ m_{\eta'} &= 957.78 \pm 0.06 \text{ MeV}. \end{aligned} \quad (11)$$

We will use the CLEO [9] and BABAR [10] data for the TFFs at large Q^2 to determine the parameter ρ_2 . The CLEO Collaboration [9] measured $F_{\eta\gamma}(Q^2)$ and $F_{\eta'\gamma}(Q^2)$ in the Q^2 regions up to 20 and 30 GeV², respectively, and presented the data in a monopole form proposed in Ref. [14]:

$$|F_{P\gamma}(Q^2)|^2 = \frac{1}{(4\pi\alpha)^2} \frac{64\pi\Gamma_{P \rightarrow \gamma\gamma}}{m_P^3} \frac{1}{(1 + Q^2/\Lambda_P^2)^2}, \quad (12)$$

where $\alpha \simeq 1/137$ is the QED fine coupling constant and Λ_P is the pole mass parameter.

The BABAR Collaboration [10] recently measured the η -photon and η' -photon transition form factors in the Q^2 range from 4 to 40 GeV². The results were not presented in the monopole form [Eq. (12)], partially because their results for the pion-photon transition form factor exhibit a very quick growth for $Q^2 > 15$ GeV² [15], which is very hard to explain in QCD [16]. However, this trend of fast growth is noticeably missing from the BABAR data for the η -photon and η' -photon transition form factors, and thus the BABAR data for the η -photon and η' -photon transition form factors are consistent with perturbative QCD calculations for the form factors and shall be described with the monopole form as given by Eq. (12).

We use QCD-motivated monopole form Eq. (12) to fit experimental data. The values of Λ_η and $\Lambda_{\eta'}$ in Eq. (12) that were determined using the CLEO data, BABAR data, and the combined data are presented in Table I. We have combined the statistical and systematic errors for the CLEO data in quadrature, since the BABAR data are presented with only combined errors. The values of $\chi^2/\text{d.o.f}$ given in the table provide further justification for the use of Eq. (12) in describing these data. The values of Λ_η and $\Lambda_{\eta'}$ determined with the CLEO and BABAR data agree within their uncertainties, but the BABAR data greatly improve the accuracy in determining the values of Λ_η and $\Lambda_{\eta'}$. Using the combined data in the fitting changes the results slightly.

The parameter Λ_P has a natural explanation as the pole mass of vector meson in the vector meson-dominated

TABLE I. Λ_P and $\chi^2/\text{d.o.f}$ in fitting the data for the TFFs with Eq. (12).

	Λ_η (MeV)	η $\chi^2/\text{d.o.f}$	$\Lambda_{\eta'}$ (MeV)	η' $\chi^2/\text{d.o.f}$
CLEO	775 ± 12	0.95	856 ± 10	0.88
BABAR	787 ± 7	0.99	861 ± 4	1.04
CLEO + BABAR	784 ± 6	0.96	849 ± 6	0.88

TABLE II. The mixing parameters determined for the η_8 - η_1 mixing scheme.

	θ	f_1/f_8	f_8/f_π	f_1/f_π
CLEO	-16.26 ± 0.86	1.162 ± 0.053	0.955 ± 0.042	1.109 ± 0.053
BABAR	-16.54 ± 0.71	1.146 ± 0.045	0.966 ± 0.041	1.107 ± 0.050
CLEO + BABAR	-16.84 ± 0.72	1.128 ± 0.044	0.979 ± 0.042	1.105 ± 0.050

TABLE III. The mixing parameters determined for the quark-flavor basis mixing scheme.

	ϕ	f_s/f_q	f_q/f_π	f_s/f_π
CLEO	38.11 ± 0.79	1.197 ± 0.065	1.076 ± 0.044	1.29 ± 0.10
BABAR	37.90 ± 0.70	1.177 ± 0.054	1.077 ± 0.044	1.268 ± 0.088
CLEO + BABAR	37.66 ± 0.70	1.156 ± 0.054	1.078 ± 0.044	1.246 ± 0.087

model for the TFFs. The values we obtained, $\Lambda_\eta \sim 780$ MeV and $\Lambda_{\eta'} \sim 860$ MeV, are very close to the masses of $\rho(770$ MeV) and $K^*(890$ MeV).

The results for the mixing angle and decay constants determined using the CLEO data, BABAR data, and the combined data, together with the two-photon decay widths, are presented in Tables II and III for the η_8 - η_1 mixing scheme and quark-flavor basis mixing scheme, respectively. The mixing angle obtained in this work, $\phi \simeq 37^\circ \sim 38^\circ$, is slightly smaller than the central value of 39.8° obtained in Ref. [11]. This is mainly due to an increase in the estimation for the $\Gamma_{\eta \rightarrow \gamma\gamma}$ by the 2010 Particle Data Group. This increase also affects slightly the results for the ratios of decay constants. The uncertainties for the mixing angles and the ratios f_1/f_8 and f_s/f_q obtained in this work are considerably smaller than that given in Ref. [11] due to the new, more accurate experimental data for the meson masses, the two-photon decay widths, and the meson-photon transition form factors. The uncertainties for the other ratios of decay constants, f_8/f_π and f_1/f_π in the η_8 - η_1 mixing scheme and f_q/f_π and f_s/f_π in the quark-flavor basis mixing scheme, are also generally smaller than that estimated in Ref. [11].

Our results for the mixing angle are in agreement with recent results of $\phi \simeq 37^\circ \sim 42^\circ$ obtained with other methods [6]. The value of f_8/f_π obtained in this work is smaller

than that obtained with Chiral Perturbation Theory at next-to-leading order [4] and some phenomenological analyses [6], but is larger than the result reported in Ref. [5]. We note that the Chiral Perturbation Theory result may be alerted by higher-order corrections. As it has been pointed out in Ref. [11], in the previous studies either the questionable assumption that the decay constants and the particle states share the same mixing scheme or the two-mixing-angle scheme was adopted. The relations between the mixing parameters involved in the two-mixing-angle scheme and those that appear in our model remain to be studied further.

In summary, understanding the composition of the light pseudoscalar mesons η and η' is of great importance in the study of many hadron processes involved these mesons. Employing the two analytical constraints on the η - η' mixing proposed by us in a previous work, we extracted the η - η' mixing angle and the ratios of decay constants in two widely used mixing schemes using recently available BABAR measurements on the η -photon and η' -photon transition form factors and more accurate experimental data for the masses and two-photon decay widths of the light pseudoscalar mesons.

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