Scaling of the CKM matrix in the 5D MSSM

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We discuss a five-dimensional minimal supersymmetric standard model compactified on a S^1/Z_2 orbifold, looking at, in particular, the one-loop evolution equations of the Yukawa couplings for the quark sector and various flavor observables. Different possibilities for the matter fields are discussed; that is, where they are in the bulk or localized to the brane. The two possibilities give rise to quite different behaviors. By studying the implications of the evolution with the renormalization group of the Yukawa couplings and of the flavor observables, we find that, for a theory that is valid up to the unification scale, the case where fields are localized to the brane, with a large $\tan\beta$, would be more easily distinguishable from other scenarios.

I. INTRODUCTION

The masses of the quarks and charged leptons are determined in the standard model (SM) via Yukawa couplings to the Higgs boson. The origin of their structure (masses and mixing angles) has no explanation within the SM and presents as one of the major challenges for physics beyond the SM. Among these models, those with extra spatial dimensions offer many possibilities for model building and TeV-scale physics scenarios which can now be explored or constrained at the Large Hadron Collider (LHC). Extra-dimensional models allow, for example, a way to generate electroweak symmetry breaking or supersymmetry breaking through the choice of appropriate boundary conditions (for a review, see Ref. [1]). In addition, for the case of flat extra dimensions, the presence of towers of excited Kaluza-Klein (KK) states induces a power-law enhancement of the gauge couplings, leading to possible low-scale unification [2,3]. This effect can be applied to other couplings, such as Yukawa couplings, giving an original way to generate mass hierarchies [2,4]. The study of the renormalization group equations (RGEs) provides a way by which partial explorations of the physics implications at a high-energy scale is possible, as the theories at asymptotic energies may reveal new symmetries or other interesting properties which may lead to deeper insights into the physical content. As such, in order to understand these as yet unexplained hierarchies of mixing

angles and fermion masses, a great deal of work has gone into analyzing the RGEs [5–12]. Therefore, the behavior of the well-known quark sector's flavor mixings in the charged current, as described by the Cabibbo-Kobayashi-Maskawa (CKM) matrix, shall be a focus of this paper.

By using the RGEs, we will study the asymptotic properties of the Lagrangian parameters like the Yukawa coupling constants and mixing angles and explore the possibility of a model in which the CKM matrix might have a simple, special form at asymptotic energies. This quest will, however, be limited by the fact that extra-dimensional theories are only effective ones, limited by a cutoff in their physical description of fundamental phenomena. Therefore, the following study can only be used as an indication of the behavior of couplings and mixing parameters at an intermediate scale between the electro-weak scale (at which these parameters are measured) and the higher scale at which the effective theory ceases to be valid. However, in the range of the LHC energies (of the order of a few TeV) and beyond, one can indeed test if the departure from the usual behavior of the coupling evolution can be seen in precision flavor measurements. In order to discuss the implications of an effective five-dimensional supersymmetric theory, we first recall the usual formalism in the SM, where the CKM matrix has four observable parameters including three mixing angles and one phase. In the standard parametrization, it has the form:

$$V_{\rm CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}, \quad (1)$$

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where $s_{12} = \sin\theta_{12}$, $c_{12} = \cos\theta_{12}$, etc., are the sines and cosines of the three mixing angles θ_{12}, θ_{23} and θ_{13}, and δ is the CP-violating phase.

The aim of this paper is to therefore study these effects explicitly in the case of one extra dimension within a minimal supersymmetric model, which will be referred to as 5D MSSM. Note that, as pointed out in Ref. [13], a supersymmetric and extra-dimensional extension to the SM can be considered simultaneously, with the advantages that though higher dimensional field theories are nonrenormalizable (requiring the existence of a UV completion), superstring theories (which contain supersymmetry) provide the main hope in this context. In fact, such a point of view has already been explored through AdS/CFT correspondences in accounting for both ''little'' and ''big'' hierarchies [14]. Also, embedding the SM in an extra-dimensional space-time does not stabilize instabilities due to quantum corrections in the scalar potential, while supersymmetry may ameliorate this. Finally, most 4-dimensional supersymmetric models lack a simple mechanism for supersymmetry breaking, something the extra dimensions may offer. To be precise, we will discuss a five-dimensional $\mathcal{N} = 1$ supersymmetric model compactified on the S^1/Z_2 orbifold as a simple testing ground for the effects of the extra dimension on the quark Yukawa couplings and the CKM matrix observables. We will not discuss in the present paper the RGE evolution of the sparticle mixing parameters sector. This interesting question will be explored elsewhere as it requires more detailed assumptions on the sparticle spectrum and corresponding soft supersymmetry breaking (SSB) parameters, where at least some SSB parameter matrices can be nondiagonal in the basis in which quarks are diagonal (this implies flavor-violating decays, giving rise to the well-known flavor problem in the MSSM and its extensions; even without considering the RGE evolution of the SSB parameters).

The paper is organized as follows: In Sec. II, we shall develop our model and calculate the various beta functions required to determine the evolution equations for the Yukawa couplings. We shall look at two cases: where the matter fields are localized on the brane and where they are free to propagate in the bulk. In Sec. III, we shall then determine the evolutions of fermion Yukawa couplings and the CKM matrix which follows from them. In Sec. IV, we then numerically analyze the evolution behaviors of the physical observables, including the Yukawa couplings, quark flavor mixings and the *CP*-violation Jarlskog parameter. Our results and conclusions are presented in Sec. V.

II. THE 5D MSSM AND BETA FUNCTIONS

In this universal extra-dimensional formalism, the compactification S_1/Z_2 implies that the five-dimensional (5D) Lorentz invariance is broken to the usual four-dimensional (4D) one; however, a remnant of momentum conservation along the fifth coordinate implies that the KK number is conserved at tree level and that KK parity is conserved at loop level. The KK-parity invariance has two main

implications in phenomenology: the contributions of the KK modes to electroweak precision observables arise only through loops, and the exact KK parity implies that the lightest KK mode is stable and can be a cold dark matter candidate.

To begin our calculation, we note that the beta functions can be derived more easily in the superfield formalism, where we shall now briefly discuss $\mathcal{N} = 1$ supersymmetry in a five-dimensional Minkowski space and its description in terms of 4D superfields. Note that more details of this approach can be found in Refs. [15–19], where, as a matter of notation, we shall label our space-time coordinates by (x^μ, y).

As an introductory example to this formalism, consider the gauge sector, described by a 5D $\mathcal{N} = 1$ vector supermultiplet which consists (on-shell) of a 5D vector field A^M, a real scalar S, and two gauginos, λ and λ'; the action for which can be given by

$$
S_g = \int \mathrm{d}^5x \frac{1}{2kg^2} \mathrm{Tr}\left[-\frac{1}{2} F^{MN}F_{MN} - D^M S D_M S \right.
$$
$$
\left. - i\bar{\lambda}\Gamma^M D_M \lambda - i\bar{\lambda}'\Gamma^M D_M \lambda' + (\bar{\lambda} + \bar{\lambda}')[S, \lambda + \lambda'] \right],
$$

(2)

with $D_M = \partial_M + iA_M$ and $\Gamma^M = (\gamma^\mu, i\gamma^5)$. $F^{MN} = -\frac{i}{g}[D^M, D^N]$ and k normalizes the trace over the generators of the gauge groups. One can also write these fields in terms of a $\mathcal{N} = 2$, 4D vector supermultiplet, $\Omega = (V, \chi)$, where V is a $\mathcal{N} = 1$ vector supermultiplet containing A^μ and λ, and χ is a chiral $\mathcal{N} = 1$ supermultiplet containing λ' and $S' = S + iA^5$. This form of writing the fields follows from the decomposition of the 5D supercharge (which is a Dirac spinor) into two Majorana-type supercharges, which constitute a $\mathcal{N} = 2$ superalgebra in 4D. Both V and χ (and their component fields) are in the adjoint representation of the gauge group $\mathcal{G}$. Using the supermultiplets, one can write the original 5D $\mathcal{N} = 1$ supersymmetric action Eq. (2) in terms of $\mathcal{N} = 1$ 4D superfields and the covariant derivative in the y direction [18]:

$$
S_g = \int \mathrm{d}^5x \mathrm{d}^2\theta \mathrm{d}^2\bar{\theta} \frac{1}{4kg^2} \mathrm{Tr}\left[\frac{1}{4}(W^\alpha W_\alpha \delta(\bar{\theta}^2) + \mathrm{H.c})\right.
$$
$$
\left. + (e^{-2gV}\nabla_y e^{2gV})^2 \right],
$$

(3)

with $W^\alpha = -\frac{1}{4}\bar{D}^2 e^{-2gV} D_\alpha e^{2gV}$. D_α is the covariant derivative in the 4D $\mathcal{N} = 1$ superspace (see Refs. [20,21]) and $\nabla_y = \partial_y + \chi$. This action can be expanded and quantized to find the Feynman rules to a given order in the gauge coupling g. The details of this procedure can be found in Ref. [15]. Because of the nonrenormalization theorem [22], the beta functions for the couplings of the operators in the superpotential are governed by the wave-function renormalization constants $Z_{ij} = 1 + \delta Z_{ij}$. The
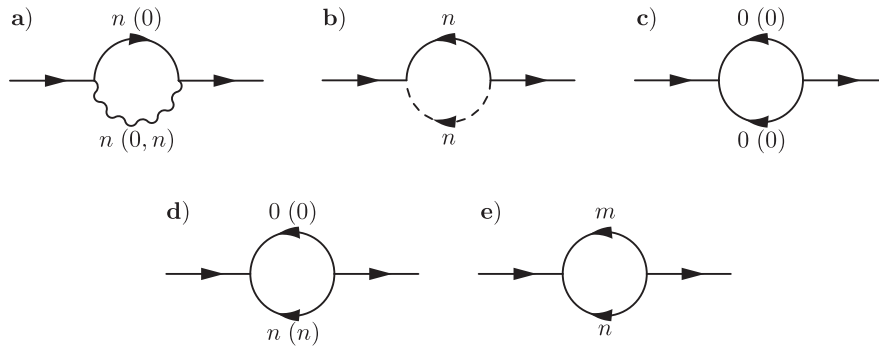
FIG. 1. The one-loop diagrams related to the wave-function renormalization of the matter superfields, in which diagrams 1(a)–1(e) refer to the case where all the matter fields are in the bulk, and the excited KK states are labeled by the number without the bracket; whereas diagrams a), c), and d) are related to the brane-localized matter fields case, in which the KK states are labeled by the number inside the bracket [15].

Feynman diagrams related to the wave-function renormalization are given in Fig. 1.

At this point, we can note that Higgs superfields and gauge superfields will always propagate into the fifth dimension in our model. However, different possibilities of localization for the matter superfields can be studied by taking the two limiting cases of superfields containing the SM fermions in the bulk or all superfields containing SM fermions restricted to the brane, respectively. For the case where all fields can propagate in the bulk, the action for the matter fields would be [15]

$$S_{\text{matter}} = \int d^8 z dy \{ \bar{\Phi}_i \Phi_i + \Phi_i^c \bar{\Phi}_i^c + \Phi_i^c \partial_5 \Phi_i \delta(\bar{\theta})$$
$$- \bar{\Phi}_i \partial_5 \bar{\Phi}_i^c \delta(\theta) + \tilde{g}(2\bar{\Phi}_i V \Phi_i - 2\Phi_i^c V \bar{\Phi}_i^c$$
$$+ \Phi_i^c \chi \Phi_i \delta(\bar{\theta}) + \bar{\Phi}_i \bar{\chi} \bar{\Phi}_i^c \delta(\theta)) \}. \tag{4}$$

Again, this action can be expanded and quantized, as detailed in Ref. [15]. Similarly, we can do the same for the case where all superfields containing SM fermions are restricted to the brane; in which case the part of the action involving only gauge and Higgs fields is not modified, whereas the action for the superfields containing the SM fermions becomes

$$S_{\text{matter}} = \int d^8 z dy \delta(y) \{ \bar{\Phi}_i \Phi_i + 2\tilde{g} \bar{\Phi}_i V \Phi_i \}. \tag{5}$$

Before proceeding further, we shall recall that the Yukawa couplings in the bulk are forbidden by the 5D $\mathcal{N} = 1$ supersymmetry. However, they can be introduced on the branes, which are 4D subspaces with reduced supersymmetry [15]:

$$S_{\text{brane}} = \int d^8 z dy \delta(y) \left\{ \frac{1}{6} \tilde{\lambda}_{ijk} \Phi_i \Phi_j \Phi_k \delta(\bar{\theta}) + \text{H.c.} \right\}, \tag{6}$$

Returning to Figs. 1(a)–1(e), these lead to the usual minimal supersymmetric standard model if all the extra-dimensional effects were excluded; that is, where the zero modes in Fig. 1(a) and 1(c) give the beta functions of the usual 4D MSSM. Figure 1(b) is a new

extra-dimensional effect related to the coupling to the chiral superfield originating from the fifth component of the gauge fields and where the higher KK modes in Figs. 1(a)–1(e) are also contributing additional extra-dimensional effects. Note that the mass dimension for the Yukawa couplings in 5D is $-\frac{1}{2}$. Therefore, the power-law dependence of their evolution, which results from these radiative corrections, becomes significantly important. Also, it should be noted at this point that though the matter fields do not normally receive a wave-function renormalization in $\mathcal{N} = 2$ supersymmetry, that is the KK conserved diagrams would not contribute to the Yukawa evolution, $\mathcal{N} = 2$ supersymmetry has been explicitly broken by the compactification [15].

As such, following the procedures outlined in Ref. [15], the RGEs for the Yukawa couplings in the 5D MSSM, for all three generations propagating in the bulk, are expressed as

$$16\pi^2 \frac{dY_d}{dt} = Y_d (3\text{Tr}(Y_d^\dagger Y_d) + \text{Tr}(Y_e^\dagger Y_e))$$
$$+ 3Y_d^\dagger Y_d + Y_u^\dagger Y_u) \pi S(t)^2$$
$$- Y_d \left(\frac{7}{15} g_1^2 + 3g_2^2 + \frac{16}{3} g_3^2 \right) S(t),$$

$$16\pi^2 \frac{dY_u}{dt} = Y_u (3\text{Tr}(Y_u^\dagger Y_u) + 3Y_u^\dagger Y_u + Y_d^\dagger Y_d) \pi S(t)^2$$
$$- Y_u \left(\frac{13}{15} g_1^2 + 3g_2^2 + \frac{16}{3} g_3^2 \right) S(t),$$

$$16\pi^2 \frac{dY_e}{dt} = Y_e (3\text{Tr}(Y_d^\dagger Y_d) + \text{Tr}(Y_e^\dagger Y_e) + 3Y_e^\dagger Y_e) \pi S(t)^2$$
$$- Y_e \left(\frac{9}{5} g_1^2 + 3g_2^2 \right) S(t). \tag{7}$$

That is, when the energy scale $\mu > 1/R$ or when the energy scale parameter $t = \ln(\mu/M_Z) > \ln(\frac{1}{M_Z R})$ (where we have set M_Z as the renormalization point and use $S(t) = e^t M_Z R$), the beta functions become as above, replacing the usual 4D MSSM RGEs. Note that, since we compactify with the symmetry $y \to -y$ (S_1/Z_2 symmetry), the integral

of the extra dimension is from $0 \rightarrow \pi R$, not from $0 \rightarrow 2\pi R$. As such, the X_δ parameter is, in our case, only half of the volume of the δ-dimensional unit sphere (see Ref. [2]), i.e. $X_1 = 1$, and this leads to a factor of 2 difference (due to the compactification) with respect to other Refs. [2,15].

Furthermore, when $0 < t < \ln(\frac{1}{M_Z R})$ (that is, $M_Z < \mu < 1/R$), the Yukawa evolution equations at low energy are dictated by the usual MSSM, where the above equations reduce to the 4D MSSM equations:

$$16\pi^2 \frac{dY_d}{dt} = Y_d(3\text{Tr}(Y_d^\dagger Y_d) + \text{Tr}(Y_e^\dagger Y_e) + 3Y_d^\dagger Y_d + Y_u^\dagger Y_u)$$
$$- Y_d\left(\frac{7}{15}g_1^2 + 3g_2^2 + \frac{16}{3}g_3^2\right),$$

$$16\pi^2 \frac{dY_u}{dt} = Y_u(3\text{Tr}(Y_u^\dagger Y_u) + 3Y_u^\dagger Y_u + Y_d^\dagger Y_d) \quad (8)$$
$$- Y_u\left(\frac{13}{15}g_1^2 + 3g_2^2 + \frac{16}{3}g_3^2\right),$$

$$16\pi^2 \frac{dY_e}{dt} = Y_e(3\text{Tr}(Y_d^\dagger Y_d) + \text{Tr}(Y_e^\dagger Y_e) + 3Y_e^\dagger Y_e)$$
$$- Y_e\left(\frac{9}{5}g_1^2 + 3g_2^2\right).$$

Note also that from the KK-number conserved one-loop diagram in Fig. 1(a), at each KK level, the new excited states exactly mirror the zero-mode ground state, where their contributions to the anomalous dimensions are exactly the same as those in the usual 4-dimensional MSSM. As a result, when the energy $\mu = 1/R$, or $S(t) = 1$, the part in the beta function associated with the gauge fields in Eqs. (7) reduce to the normal 4D MSSM formalism in Eqs. (8).

For the second case, we shall consider, that of the matter fields localized on the brane, we have (in the Feynman diagrams, only Fig. 1(a), 1(c), and 1(d) will contribute to the wave function of the matter fields, while the nonzero integer numbers inside the bracket labeling the KK states of the gauge and Higgs superfields, respectively)

$$16\pi^2 \frac{dY_d}{dt} = Y_d(3\text{Tr}(Y_d^\dagger Y_d) + \text{Tr}(Y_e^\dagger Y_e)$$
$$+ (6Y_d^\dagger Y_d + 2Y_u^\dagger Y_u)S(t))$$
$$- Y_d\left(\frac{19}{30}g_1^2 + \frac{9}{2}g_2^2 + \frac{32}{3}g_3^2\right)S(t),$$

$$16\pi^2 \frac{dY_u}{dt} = Y_u(3\text{Tr}(Y_u^\dagger Y_u) + (6Y_u^\dagger Y_u + 2Y_d^\dagger Y_d)S(t)) \quad (9)$$
$$- Y_u\left(\frac{43}{30}g_1^2 + \frac{9}{2}g_2^2 + \frac{32}{3}g_3^2\right)S(t),$$

$$16\pi^2 \frac{dY_e}{dt} = Y_e(3\text{Tr}(Y_d^\dagger Y_d) + \text{Tr}(Y_e^\dagger Y_e) + 6Y_e^\dagger Y_e S(t))$$
$$- Y_e\left(\frac{33}{10}g_1^2 + \frac{9}{2}g_2^2\right)S(t).$$

As such, if we write the evolution of the gauge couplings in 4-dimensions as

$$16\pi^2 \frac{dg_i}{dt} = b_i g_i^3, \quad (10)$$

where in the 4D MSSM, the parameters b_i read $(b_1, b_2, b_3) = (\frac{33}{5}, 1, -3)$ [7], using an $SU(5)$ normalization. If we consider our 5D theory as effective up to a scale Λ, then between the scale R^{-1} (the compactification scale of the single flat extra dimension), where the first KK states are excited, and the cutoff scale Λ, there are finite quantum corrections from the ΛR number of KK states for the gauge couplings. As a result, once the KK states are excited, these couplings exhibit power-law dependencies on Λ. This can be illustrated if $\Lambda R \gg 1$, to a very good accuracy, with the generic 4D beta function of the gauge couplings with the power-law evolution behavior [4,5]

$$\beta^{4D} \rightarrow \beta^{4D} + (S(\mu) - 1)\tilde{\beta}, \quad (11)$$

where $\tilde{\beta}$ is a generic contribution from a single KK level, and where its coefficient is not a constant but instead $S(\mu) = \mu R$, with $\mu^{\max} = \Lambda$, reflecting the power-law running behavior. Therefore, in terms of the scale parameter t, the evolution of the gauge couplings can be written as

$$16\pi^2 \frac{dg_i}{dt} = [b_i + (S(t) - 1)\tilde{b}_i]g_i^3. \quad (12)$$

Next, we consider the beta functions of the gauge couplings in the 5D MSSM. In fact, after compactification of the 5D MSSM, we have two 4D $\mathcal{N} = 1$ chiral supermultiplets, Φ and Φ^c, where the zero modes of Φ give us the normal matter fields and Higgs fields as well as their super partners, while Φ^c is a new supermultiplet. In the component field formalism, at each KK level, aside from the quantum corrections that mirror those of the 4D MSSM, the only new one-loop contributions to the A_μ Feynman diagrams are from the wave-function renormalization of A_μ (which contribute via the coupling of A_μ to the complex scalar field and its superpartner in the superfield χ and the coupling of A_μ with the new fermion field and its superpartner in the superfield Φ^c associated with the two doublets of the Higgs fields and the matter fields, respectively, in the bulk). This then gives rise to the master beta functions of the gauge couplings in the 5D MSSM as follows:

$$(\tilde{b}_1, \tilde{b}_2, \tilde{b}_3) = (\tfrac{6}{5}, -2, -6) + 4\eta, \quad (13)$$

where η represents the number of generations of fermions which propagate in the bulk. So, in the two cases we shall consider, that of all fields propagating in the bulk, i.e. $\eta = 3$, we have [4]

$$\tilde{b}_i = (\tfrac{66}{5}, 10, 6). \quad (14)$$

Similarly, for all our matter fields localized to the 3-brane (that is, $\eta = 0$), we have
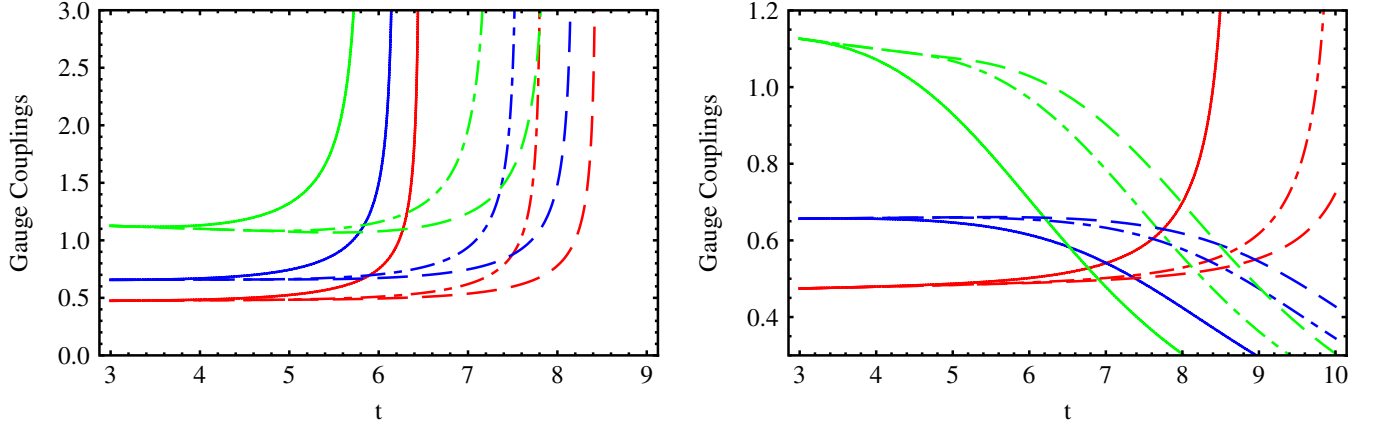
FIG. 2 (color online). Gauge couplings g_1 (red), g_2 (blue), g_3 (green) with: in the left panel, all matter fields in the bulk; and the right panel for all matter fields on the brane; for three different values of the compactification scales [2 TeV (solid line), 8 TeV (dot-dashed line), 15 TeV (dashed line)] as a function of the scale parameter t.

$$\tilde{b}_i = \left(\frac{6}{5}, -2, -6\right). \quad (15)$$

In Fig. 2, we have plotted the evolutions of the brane localized and bulk field cases for several choices of compactification scales for the extra-dimension (R). From these plots and the discussion given in Ref. [4], we find that for the three gauge coupling constants to approach a small region at some value of t requires an extremely large value of $1/R$, which is of no phenomenological interest at present. For the case of our fields being brane-localized, the extra dimensions naturally lead to gauge coupling unification at an intermediate mass scale for the compactification radii considered here.

III. SCALING OF THE CKM MATRIX

The RGEs are an important tool for investigating the properties of the quark masses and the CKM matrix at different energy scales. This, therefore, strongly constrains the possible symmetries or textures at the grand unification scale. Having derived a set of equations for the evolutions of the Yukawa and gauge couplings in the previous section, we now develop the evolutions of the CKM matrix in the 5D MSSM for our two cases. Consider first the case of all the matter fields propagating in the bulk. From the Yukawa evolution equations [Eqs. (7)], we can convert them to the following form:

$$16\pi^2 \frac{dY_d}{dt} = Y_d\{T_d \pi S^2 - G_d + (3Y_d^\dagger Y_d + Y_u^\dagger Y_u)\pi S^2\},$$

$$16\pi^2 \frac{dY_u}{dt} = Y_u\{T_u \pi S^2 - G_u + (3Y_u^\dagger Y_u + Y_d^\dagger Y_d)\pi S^2\},$$

$$16\pi^2 \frac{dY_e}{dt} = Y_e\{T_e \pi S^2 - G_e + (3Y_e^\dagger Y_e)\pi S^2\}, \quad (16)$$

where $T_d = 3\text{Tr}(Y_d^\dagger Y_d) + \text{Tr}(Y_e^\dagger Y_e)$, $G_d = (\frac{7}{15}g_1^2 + 3g_2^2 + \frac{16}{3}g_3^2)S(t)$, $T_u = 3\text{Tr}(Y_u^\dagger Y_u)$, $G_u = (\frac{13}{15}g_1^2 + 3g_2^2 + \frac{16}{3}g_3^2)S(t)$,

$T_e = 3\text{Tr}(Y_d^\dagger Y_d) + \text{Tr}(Y_e^\dagger Y_e)$, and $G_e = (\frac{9}{5}g_1^2 + 3g_2^2)S(t)$. Further, the evolution of the square of the Yukawa coupling matrices becomes

$$16\pi^2 \frac{d(Y_d^\dagger Y_d)}{dt} = 2(T_d \pi S^2 - G_d)(Y_d^\dagger Y_d) + 6(Y_d^\dagger Y_d)^2 \pi S^2$$
$$+ [(Y_u^\dagger Y_u)(Y_d^\dagger Y_d) + (Y_d^\dagger Y_d)(Y_u^\dagger Y_u)]\pi S^2,$$

$$16\pi^2 \frac{d(Y_u^\dagger Y_u)}{dt} = 2(T_u \pi S^2 - G_u)(Y_u^\dagger Y_u) + 6(Y_u^\dagger Y_u)^2 \pi S^2$$
$$+ [(Y_u^\dagger Y_u)(Y_d^\dagger Y_d) + (Y_d^\dagger Y_d)(Y_u^\dagger Y_u)]\pi S^2,$$

$$16\pi^2 \frac{d(Y_e^\dagger Y_e)}{dt} = 2(T_e \pi S^2 - G_e)(Y_e^\dagger Y_e) + 6(Y_e^\dagger Y_e)^2 \pi S^2.$$

$$(17)$$

The square of the quark Yukawa coupling matrices can be diagonalized by using two unitary matrices U and V,

$$\text{diag}(f_u^2, f_c^2, f_t^2) = UY_u^\dagger Y_u U^\dagger,$$
$$\text{diag}(h_d^2, h_s^2, h_b^2) = VY_d^\dagger Y_d V^\dagger, \quad (18)$$

in which f_u^2, f_c^2, f_t^2, and h_d^2, h_s^2, h_b^2 are the eigenvalues of $Y_u^\dagger Y_u$ and $Y_d^\dagger Y_d$, respectively. It follows that the CKM matrix appears as a result of the transition from the quark flavor eigenstates to the quark mass eigenstates upon this diagonalization of the quark mass matrices:

$$V_{\text{CKM}} = UV^\dagger. \quad (19)$$

By imposing the unitary transformation Eq. (18), on both sides of the evolution equations of $Y_u^\dagger Y_u$ and $Y_d^\dagger Y_d$, and taking the diagonal elements, we obtain the following two relations:

$$16\pi^2 \frac{df_i^2}{dt} = f_i^2 \left[2(T_u \pi S^2 - G_u) + 6\pi S^2 f_i^2 \right.$$
$$\left. + 2\pi S^2 \sum_j h_j^2 |V_{ij}|^2 \right],$$

$$16\pi^2 \frac{dh_j^2}{dt} = h_j^2 \left[2(T_d \pi S^2 - G_d) + 6\pi S^2 h_j^2 \right.$$
$$\left. + 2\pi S^2 \sum_i f_i^2 |V_{ij}|^2 \right], \tag{20}$$

where V_{ij} are the elements of V_{CKM}. These, along with the equation

$$16\pi^2 \frac{dy_e^2}{dt} = y_e^2 [2(T_e \pi S^2 - G_e) + 6\pi S^2 y_e^2], \tag{21}$$

for the lepton sector, constitute a complete set of coupled differential equations for the three families, in which $y_e^2 = \text{diag}(y_e^2, y_\mu^2, y_\tau^2) = Y_e^\dagger Y_e$.

Considering the variation of the square of the quark Yukawa couplings, we may impose two new unitary matrices to make them diagonal. Thus, by applying Eq. (19), we are led to the variation of the CKM matrix, and thus its evolution equation when the energy scale is beyond the threshold R^{-1}:

$$16\pi^2 \frac{dV_{ik}}{dt} = \pi S^2 \left[\sum_{m, j \neq i} \frac{f_i^2 + f_j^2}{f_i^2 - f_j^2} h_m^2 V_{im} V_{jm}^* V_{jk} \right.$$
$$\left. + \sum_{j, m \neq k} \frac{h_k^2 + h_m^2}{h_k^2 - h_m^2} f_j^2 V_{jm}^* V_{jk} V_{im} \right]. \tag{22}$$

Likewise, for the case where the matter fields are localized to the brane, Eq. (9) gives us the following:

$$16\pi^2 \frac{dY_d}{dt} = Y_d \{ T_d - G_d + (6Y_d^\dagger Y_d + 2Y_u^\dagger Y_u)S \},$$

$$16\pi^2 \frac{dY_u}{dt} = Y_u \{ T_u - G_u + (6Y_u^\dagger Y_u + 2Y_d^\dagger Y_d)S \}, \tag{23}$$

$$16\pi^2 \frac{dY_e}{dt} = Y_e \{ T_e - G_e + (6Y_e^\dagger Y_e)S \},$$

where $T_d = 3\text{Tr}(Y_d^\dagger Y_d) + \text{Tr}(Y_e^\dagger Y_e)$, $G_d = (\frac{19}{30}g_1^2 + \frac{9}{2}g_2^2 + \frac{32}{3}g_3^2)S(t)$, $T_u = 3\text{Tr}Y_u^\dagger Y_u$, $G_u = (\frac{43}{30}g_1^2 + \frac{9}{2}g_2^2 + \frac{32}{3}g_3^2)S(t)$, $T_e = 3\text{Tr}(Y_d^\dagger Y_d) + \text{Tr}(Y_e^\dagger Y_e)$, and $G_e = (\frac{33}{10}g_1^2 + \frac{9}{2}g_2^2)S(t)$. As was done earlier, these can be manipulated to

$$16\pi^2 \frac{d(Y_d^\dagger Y_d)}{dt} = 2(T_d - G_d)(Y_d^\dagger Y_d) + 12(Y_d^\dagger Y_d)^2 S$$
$$+ 2[(Y_u^\dagger Y_u)(Y_d^\dagger Y_d) + (Y_d^\dagger Y_d)(Y_u^\dagger Y_u)]S,$$

$$16\pi^2 \frac{d(Y_u^\dagger Y_u)}{dt} = 2(T_u - G_u)(Y_u^\dagger Y_u) + 12(Y_u^\dagger Y_u)^2 S$$
$$+ 2[(Y_u^\dagger Y_u)(Y_d^\dagger Y_d) + (Y_d^\dagger Y_d)(Y_u^\dagger Y_u)]S,$$

$$16\pi^2 \frac{d(Y_e^\dagger Y_e)}{dt} = 2(T_e - G_e)(Y_e^\dagger Y_e) + 12(Y_e^\dagger Y_e)^2 S. \tag{24}$$

This gives us

$$16\pi^2 \frac{df_i^2}{dt} = f_i^2 \left[2(T_u - G_u) + 12Sf_i^2 + 4S \sum_j h_j^2 |V_{ij}|^2 \right],$$

$$16\pi^2 \frac{dh_j^2}{dt} = h_j^2 \left[2(T_d - G_d) + 12Sh_j^2 + 4S \sum_i f_i^2 |V_{ij}|^2 \right],$$

$$16\pi^2 \frac{dy_e^2}{dt} = y_e^2 [2(T_e - G_e) + 12Sy_e^2], \tag{25}$$

as well as the evolution equation of the CKM matrix elements

$$16\pi^2 \frac{dV_{ik}}{dt} = 2S \left[\sum_{m, j \neq i} \frac{f_i^2 + f_j^2}{f_i^2 - f_j^2} h_m^2 V_{im} V_{jm}^* V_{jk} \right.$$
$$\left. + \sum_{j, m \neq k} \frac{h_k^2 + h_m^2}{h_k^2 - h_m^2} f_j^2 V_{jm}^* V_{jk} V_{im} \right], \tag{26}$$

in which the f_i^2, h_j^2, y_e^2 are the eigenvalues of $Y_u^\dagger Y_u$, $Y_d^\dagger Y_d$, $Y_e^\dagger Y_e$ as defined earlier.

However, when the energy is below the compactification scale, we have the usual 4D MSSM evolution equations for the Yukawa couplings and the CKM matrix:

$$16\pi^2 \frac{df_i^2}{dt} = f_i^2 \left[2(T_u - G_u) + 6f_i^2 + 2\sum_j h_j^2 |V_{ij}|^2 \right],$$

$$16\pi^2 \frac{dh_j^2}{dt} = h_j^2 \left[2(T_d - G_d) + 6h_j^2 + 2\sum_i f_i^2 |V_{ij}|^2 \right], \tag{27}$$

$$16\pi^2 \frac{dy_e^2}{dt} = y_e^2 [2(T_e - G_e) + 6y_e^2],$$

and

$$16\pi^2 \frac{dV_{ik}}{dt} = \left[\sum_{m, j \neq i} \frac{f_i^2 + f_j^2}{f_i^2 - f_j^2} h_m^2 V_{im} V_{jm}^* V_{jk} \right.$$
$$\left. + \sum_{j, m \neq k} \frac{h_k^2 + h_m^2}{h_k^2 - h_m^2} f_j^2 V_{jm}^* V_{jk} V_{im} \right], \tag{28}$$

where $T_u = 3\text{Tr}(Y_u^\dagger Y_u)$, $G_u = \frac{13}{15}g_1^2 + 3g_2^2 + \frac{16}{3}g_3^2$, $T_d = 3\text{Tr}(Y_d^\dagger Y_d) + \text{Tr}(Y_e^\dagger Y_e)$, $G_d = \frac{7}{15}g_1^2 + 3g_2^2 + \frac{16}{3}g_3^2$, $T_e = 3\text{Tr}(Y_d^\dagger Y_d) + \text{Tr}(Y_e^\dagger Y_e)$, and $G_e = \frac{9}{5}g_1^2 + 3g_2^2$.

Before proceeding to numerically solve these systems of equations, we choose the initial input values for the gauge couplings, fermion masses, and CKM elements at the M_Z scale (as in Ref. [5]). The 4D MSSM contains the particle spectrum of a two-Higgs doublet model extension of the SM and the corresponding supersymmetric partners. After the spontaneous breaking of the electroweak symmetry, five physical Higgs particles are left in the spectrum. The two-Higgs doublets H_u and H_d, with opposite hypercharges, are responsible for the generation of the up-type and down-type quarks, respectively. The vacuum expectation values of the neutral components of the two-Higgs fields satisfy the relation $v_u^2 + v_d^2 = (\frac{246}{\sqrt{2}})^2 = (174 \text{ GeV})^2$. The fermion mass matrices appear after the spontaneous symmetry breaking from the fermion-Higgs Yukawa couplings. As a result, the initial Yukawa couplings are given by the ratios of the fermion masses to the appropriate Higgs vacuum expectation values as follows:

$$f_{u,c,t} = \frac{m_{u,c,t}}{v_u}, \quad h_{d,s,b} = \frac{m_{d,s,b}}{v_d}, \quad y_{e,\mu,\tau} = \frac{m_{e,\mu,\tau}}{v_d}, \quad (29)$$

where we define $\tan\beta = v_u/v_d$, which is the ratio of vacuum expectation values of the two-Higgs fields H_u and H_d.

IV. DISCUSSION OF THE RESULTS

In the previous section, we derived the full set of one-loop coupled RGEs for the Yukawa and gauge couplings, together with the CKM matrix elements for both the universal 5D MSSM and brane-localized matter field scenarios. From these complete sets of the RGEs, we can obtain the renormalization group flow of all observables related to up- and down-quark masses and the quark flavor mixings. For our numerical analysis, we assume the fundamental

scale is not far from the range of LHC scale and set the compactification scale to be $R^{-1} = 2$ TeV, 8 TeV, and 15 TeV, respectively.

Actually, below the supersymmetric breaking scale, the Yukawa and gauge couplings run in the usual logarithmic fashion, giving a rather slow change for their values. Therefore, for supersymmetric breaking theories around TeV scales, for simplicity, we take the supersymmetric breaking scale $M_{\text{SUSY}} = M_Z$ in the present numerical study and run the RGEs from M_Z up to the high-energy scales for our three different compactification scales. In Fig. 2, we find the evolution of the gauge couplings have different properties for these two 5D MSSM scenarios.

Additionally, as illustrated in Fig. 3, for the case of universal 5D MSSM, once the first KK threshold is crossed at $\mu = R^{-1}$, the power-law running of the various beta functions causes the Yukawa coupling to rapidly increase following the rapid increase in the gauge coupling constants in the left panel of Fig. 2. From Eq. (20), we can find the quadratic term of $S(t)$, providing a positive contribution to the Yukawa beta functions, which is in contrast to beta functions of the gauge couplings (which include terms only linear in $S(t)$). Therefore, from Eq. (20), the positive contribution from $S(t)$ terms will dominate the negative contributions from the gauge couplings and cause the Yukawa couplings to increase rapidly. This behavior can be observed for both small and large $\tan\beta$ cases. However, as illustrated in the first graph of Fig. 3, for small $\tan\beta$, the Yukawa coupling has a large initial value; therefore, it blows up at a relatively low energy as compared with the case for large $\tan\beta$. As a result, as one evolves upward in the scale, the top Yukawa coupling is rising with a fast rate and is pushed up against the Landau pole, where it becomes divergent and blows up. In the vicinity of this singular point, the perturbative calculation becomes
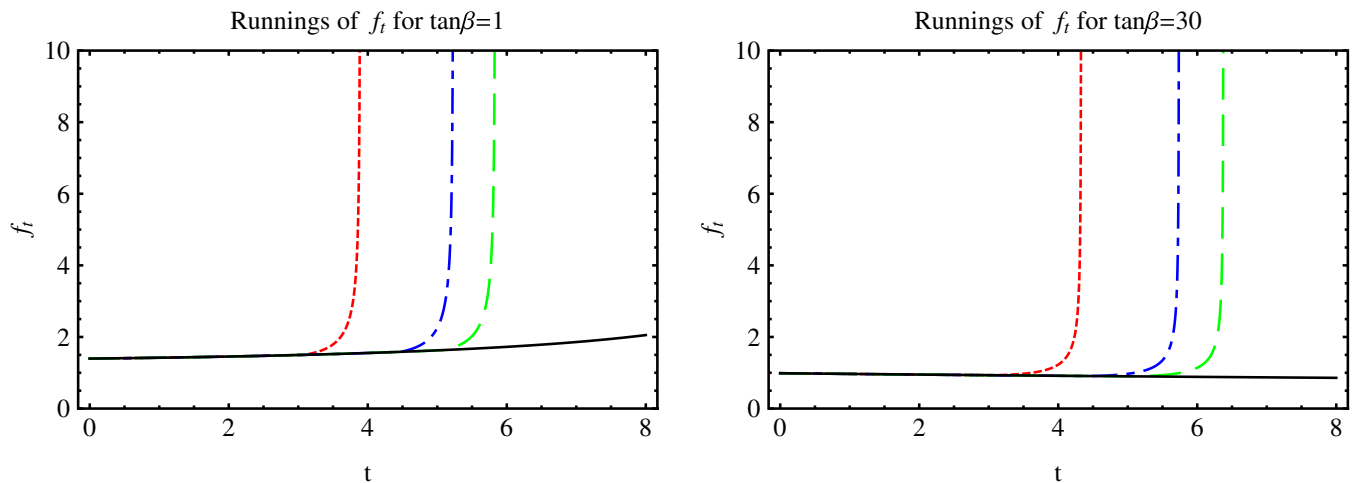

FIG. 3 (color online). The Yukawa coupling f_t for the top quark (in the bulk) as a function of the scale parameter t, for (left panel) $\tan\beta = 1$ and (right panel) $\tan\beta = 30$ for different compactification scales: $R^{-1} = 2$ TeV (red, dotted line), 8 TeV (blue, dotted-dashed line), and 15 TeV (green, dashed line).
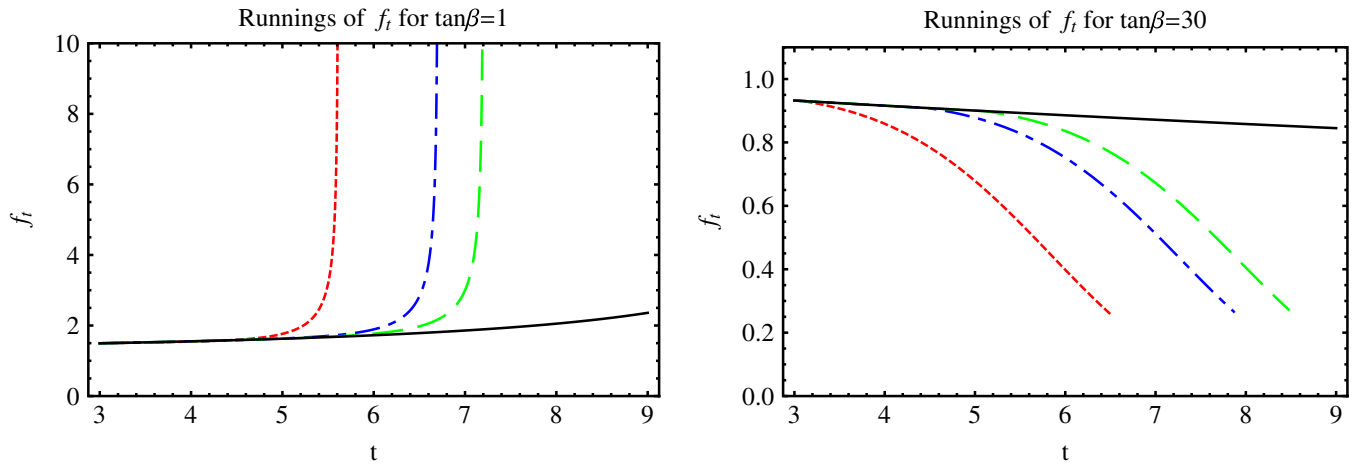
FIG. 4 (color online).   The Yukawa coupling f_t for the top quark (on the brane) as a function of the scale parameter t, for (left panel) $\tan\beta = 1$ and (right panel) $\tan\beta = 30$ for different compactification scales: $R^{-1} = 2$ TeV (red, dotted line), 8 TeV (blue, dotted-dashed line), and 15 TeV (green, dashed line).

invalid, and the higher-order corrections become significant. The Landau pole also indicates that there is an upper limit on the value of the gauge couplings where new physics must emerge before the Yukawa couplings diverge.

In the brane-localized matter field scenario, the beta function has only linear terms in $S(t)$, which is comparable with the $S(t)$ term in the beta function for the gauge couplings. As depicted in Fig. 4, for a small value of
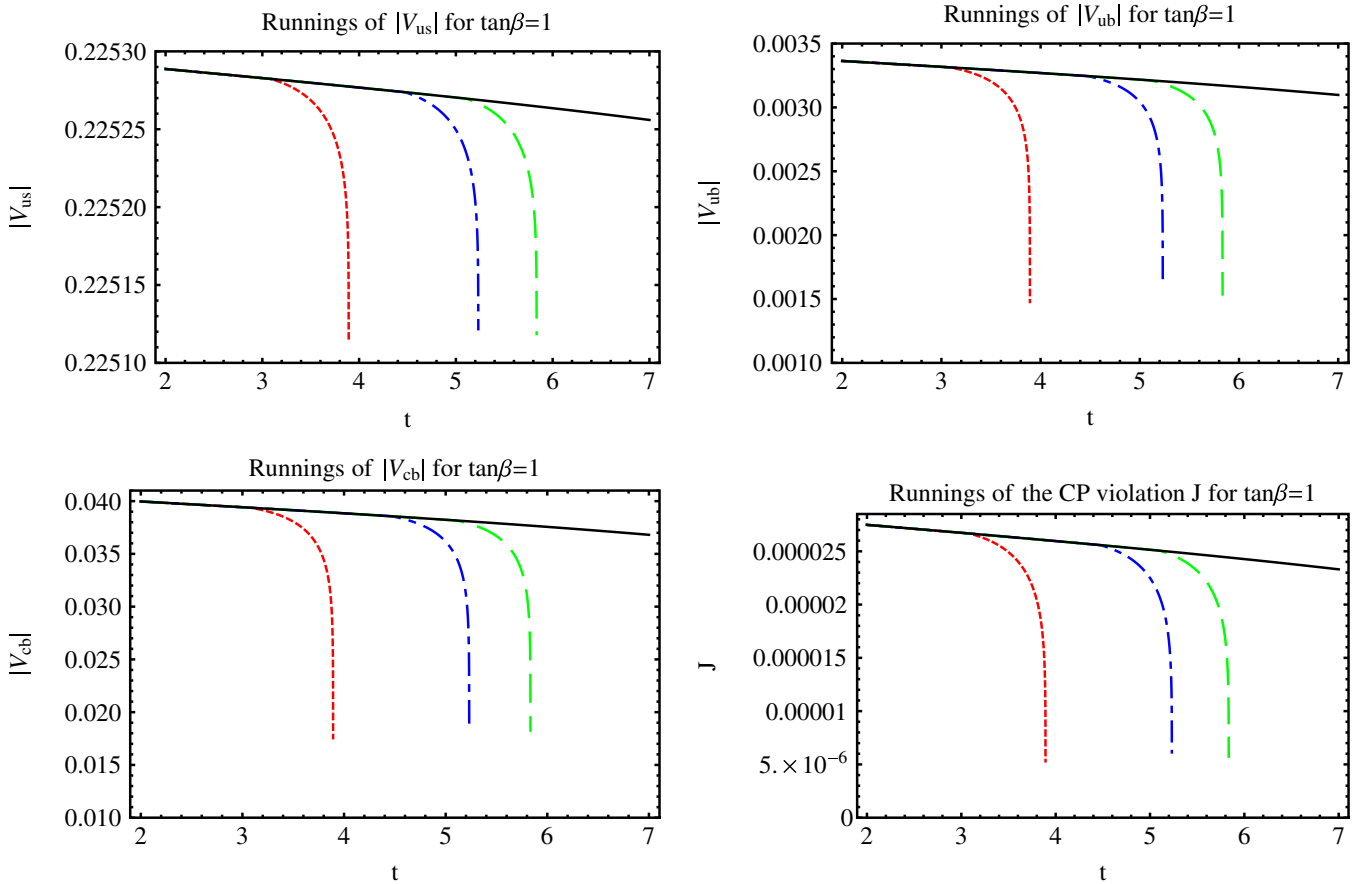

FIG. 5 (color online).   The CMK matrix elements $|V_{us}|$ (top left), $|V_{ub}|$ (top right), $|V_{cb}|$ (bottom left), and the Jarlskog parameter J (bottom right) as functions of the scale parameter t for $\tan\beta = 1$. All matter fields are in the bulk for a variety of compactification scales: $R^{-1} = 2$ TeV (red, dotted line), 8 TeV (blue, dotted-dashed line), and 15 TeV (green, dashed line).
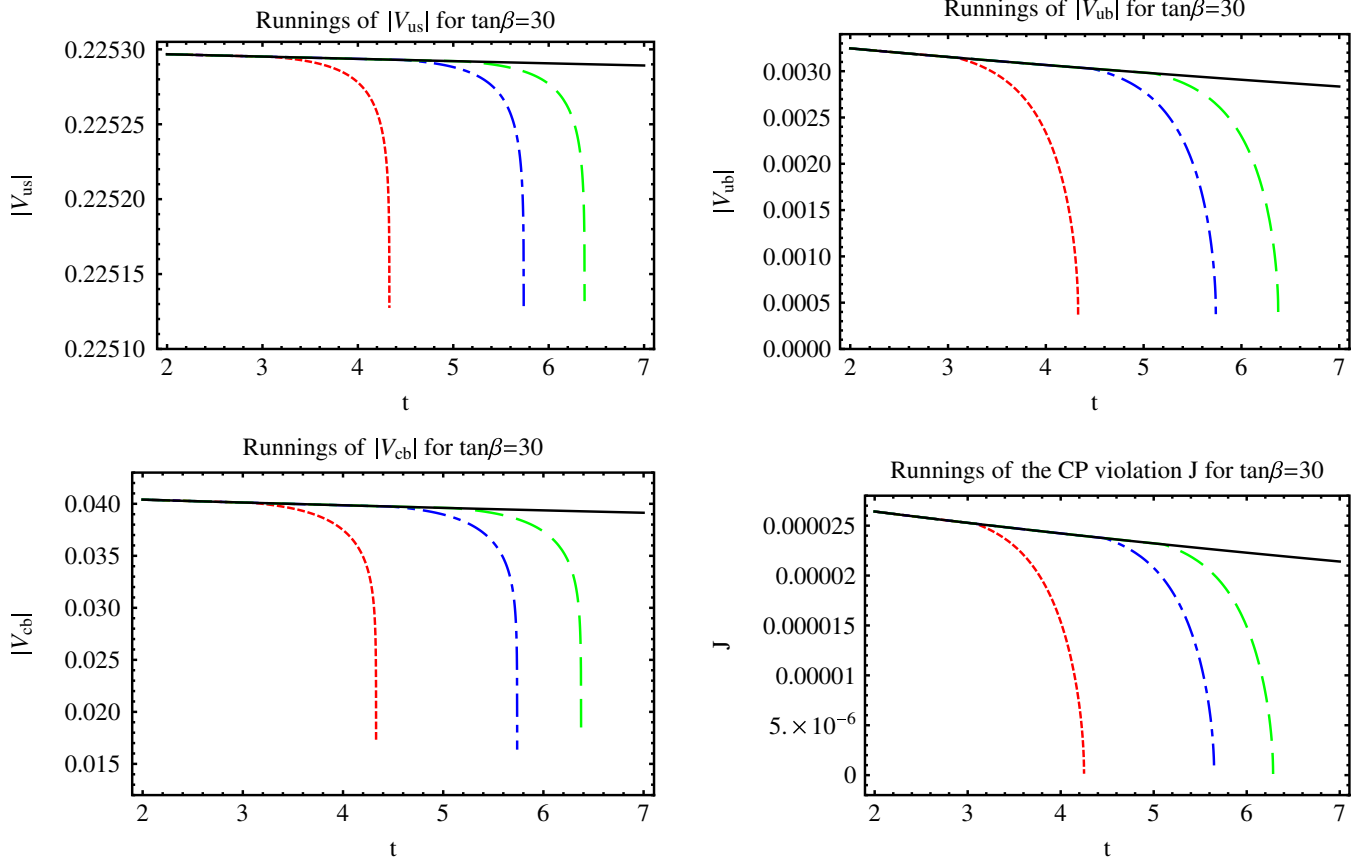
FIG. 6 (color online).   The CMK matrix elements $|V_{us}|$ (top left), $|V_{ub}|$ (top right), $|V_{cb}|$ (bottom left), and the Jarlskog parameter J (bottom right) as functions of the scale parameter t for $\tan\beta = 30$. All matter fields are in the bulk for a variety of compactification scales: $R^{-1} = 2$ TeV (red, dotted line), 8 TeV (blue, dotted-dashed line), and 15 TeV (green, dashed line).

$\tan\beta$, we have a large initial value of f_t, and the gauge coupling contribution to the Yukawa beta function is subdominant only. Therefore, as discussed previously, the Yukawa coupling f_t increases rapidly as one crosses the KK threshold at $\mu = R^{-1}$, resulting in a rapid approach of the singularity before the unification scale is reached. However, for an intermediate value of $\tan\beta$, we have a relative smaller initial condition for the top Yukawa coupling, and the Yukawa terms in the beta function become less important. The contributions from the gauge couplings may then become significant, which leads to a net negative contribution to the beta functions. Therefore, the curvature of the trajectory of the top Yukawa evolution might change direction, and the Yukawa evolution will decrease instead of increasing. This behavior would become more obvious for a large value of $\tan\beta$. As observed in Fig. 4, for $\tan\beta = 30$, we indeed observe the decreasing behavior of the top Yukawa couplings. This behavior provides a very clear phenomenological signature, especially for scenarios with a larger $\tan\beta$ and that are valid up to the unification scale where the gauge couplings converge.

We next turn our attention to the quark flavor mixing matrix, especially the complex phase of the CKM matrix

which characterizes CP-violating phenomena. This phenomena has been unambiguously verified in a number of $K - \bar{K}$ and $B - \bar{B}$ systems. Because of the arbitrariness in choice of phases of the quark fields, the phases of individual matrix elements of the V_{CKM} are not themselves directly observable. Among these, we therefore use the absolute values of the matrix element $|V_{ij}|$ as the independent set of rephasing invariant variables. Of the nine elements of the CKM matrix, only four of them are independent, which is consistent with the four independent variables of the standard parametrization of the CKM matrix. For definiteness, we choose the $|V_{ub}|$, $|V_{cb}|$, $|V_{us}|$, and the Jarlskog rephasing invariant parameter $J = \text{Im} V_{ud} V_{cs} V_{us}^* V_{cd}^*$ as the four independent parameters of V_{CKM}. In Figs. 5 and 6, we plot the energy dependence of these four variables from the weak scale all the way up to the high-energy scales for different values of compactification radii R^{-1} for the universal 5D MSSM case, and in Figs. 7 and 8, for the brane-localized matter fields case. In these sets of pictures, we consider two indicative choices of $\tan\beta$, that of $\tan\beta = 1$ and $\tan\beta = 30$.

The running of the CKM matrix is governed by the terms related to the Yukawa couplings. These Yukawa couplings
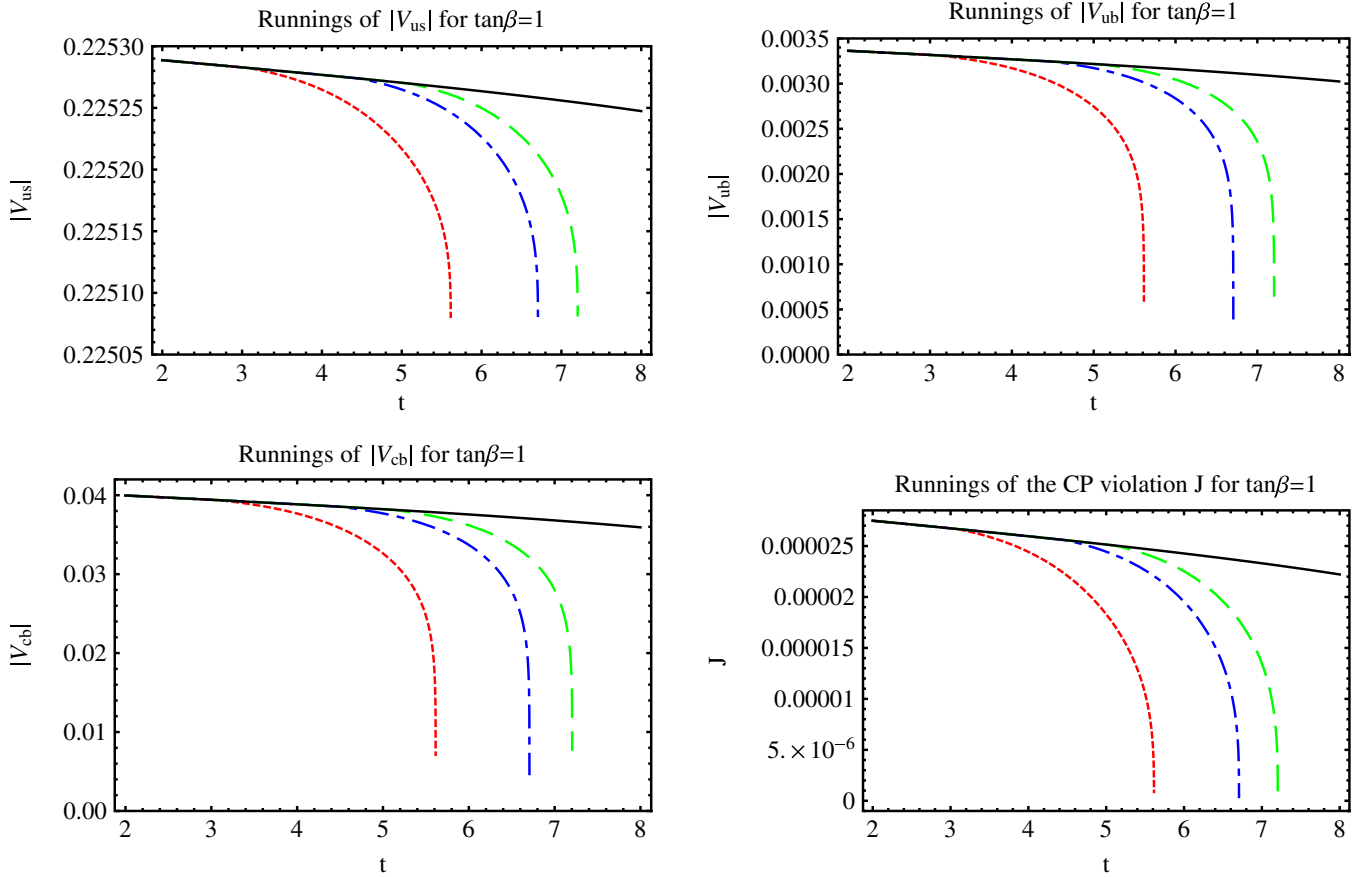
FIG. 7 (color online). The CMK matrix elements $|V_{us}|$ (top left), $|V_{ub}|$ (top right), $|V_{cb}|$ (bottom left), and the Jarlskog parameter J (bottom right) as functions of the scale parameter t for $\tan\beta = 1$. All matter fields are on the brane for a variety of compactification scales: $R^{-1} = 2$ TeV (red, dotted line), 8 TeV (blue, dotted-dashed line), and 15 TeV (green, dashed line).

are usually very small, except for the top Yukawa coupling (which could give a sizeable contribution). The CKM matrix elements $V_{us} \simeq \theta_{12}$, $V_{ub} \simeq \theta_{13}e^{-i\delta}$, $V_{cb} \simeq \theta_{23}$, can be used to observe the Cabibbo angle, θ_{13} and θ_{23}. For the universal 5D MSSM scenario, we plot their evolution in Figs. 5 and 6 and find that they decrease with the energy scale in a similar manner regardless of whether $\tan\beta$ is small or large. However, for a large initial value of f_t (small $\tan\beta$), the mixing angles have a more rapid evolution and end in the regime where the top Yukawa diverges and develops a singularity. Similarly, the Jarlskog parameter also decreases quite rapidly once the initial KK threshold is passed. However, when $\tan\beta$ is large, we have a relatively longer distance between the initial and terminating energy track, and the evolution of J can be driven towards zero or even further. Besides, as can be seen explicitly in Ref. [7], the beta functions of the evolution equations of the CKM elements are up to the third order in the CKM elements, which are comparably smaller than that of Jarlskog parameter's quadratic dependence on the CKM elements. This fact then leads to the relatively large variation of J with the increase of energy. Quantitatively,

we observe from these plots the following: the decrease in the values of V_{us} is not sizeable, while for V_{ub} and V_{cb}, their variations change by more than 50%. Furthermore, for $\tan\beta = 30$, the Jarlskog parameter drops almost to zero, which sets the effect of the SM CP violation to being very small. Note, however, that in a supersymmetric theory, other sources of CP violation beyond the SM ones are typically present. Therefore, only a complete and detailed study of a specific model would allow us to establish the strength of the CP-violating effects.

For the case of the matter fields being constrained to the brane, in Figs. 7 and 8, we observe that the evolutions of these mixing angles and CP-violation parameter are decreasing irrespective of whether the top Yukawa coupling grows or not. For small $\tan\beta$, we see similar evolution behaviors for these parameters as in the bulk case. The decreases in the values of V_{us}, V_{ub}, V_{cb}, and J are much steeper, due to rapid growth of the top Yukawa coupling near the singular point. However, as $\tan\beta$ becomes larger, e.g. $\tan\beta = 30$, the top Yukawa coupling evolves downward instead of upward. The decreases in these CKM parameters then become much milder towards the
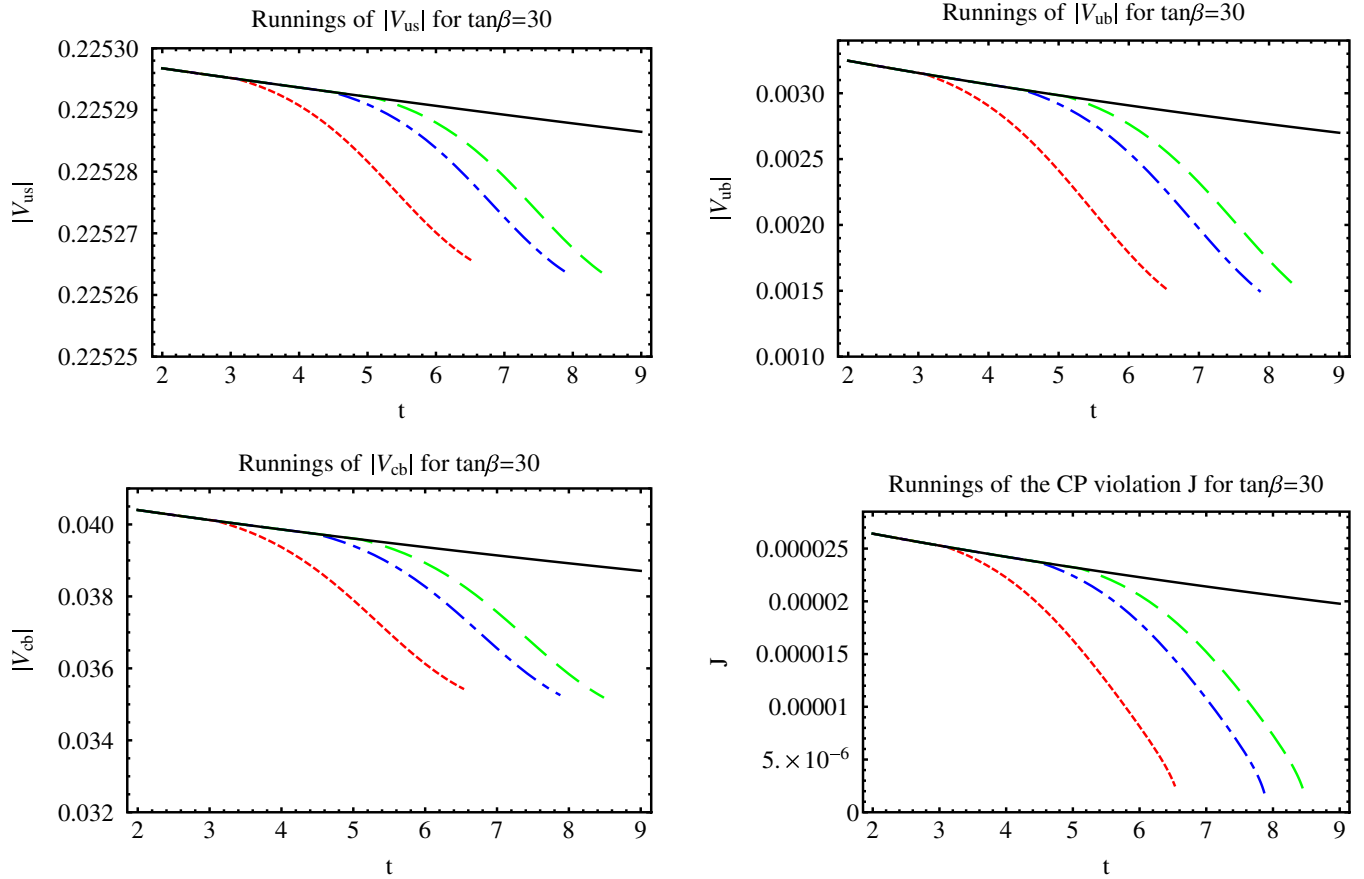
FIG. 8 (color online). The CMK matrix elements $|V_{us}|$ (top left), $|V_{ub}|$ (top right), $|V_{cb}|$ (bottom left), and the Jarlskog parameter J (bottom right) as functions of the scale parameter t for $\tan\beta = 30$. All matter fields are on the brane for a variety of compactification scales: $R^{-1} = 2$ TeV (red, dotted line), 8 TeV (blue, dotted-dashed line), and 15 TeV (green, dashed line).

unification scale; though the reduction to effectively zero in the Jarlskog parameter persists. As a result, for the brane-localized matter field scenario, it is more desirable to have a large $\tan\beta$ for theories that are valid up to the gauge coupling unification scale.

V. CONCLUSIONS

In summary, for the two 5D MSSM scenarios with matter fields in the bulk or on the brane, we have performed the numerical analysis of the evolution of the various parameters of the CKM matrix, and both cases give us a scenario with small or no quark flavor mixings at high energies, especially for the mixings with the heavy generation. The evolution equations which relate various observables at different energies, and also allow the study of their asymptotic behaviors, are particularly important in view of testing the evolution of the Yukawa couplings. In the universal 5D MSSM model, the evolution of these CKM parameters have a rapid variation prior to reaching a cutoff scale where the top Yukawa coupling develops a singularity point and the model breaks down. For the brane-localized matter fields model, we can only observe similar behaviors for small values of $\tan\beta$, while for large $\tan\beta$, the initial top Yukawa coupling becomes smaller, the gauge couplings then play a dominant role during the evolution of the Yukawa couplings, which cause the Yukawa couplings to decrease instead of increasing. As such, the variations of these CKM parameters have a relatively milder behavior, and the theory is valid up the gauge coupling unification scale.

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