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# $CP$ asymmetries in singly-Cabibbo-suppressed $D$ decays to two pseudoscalar mesons

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The LHCb Collaboration has recently reported evidence for a $CP$ asymmetry approaching the percent level in the difference between $D^0 \to \pi^+\pi^-$ and $D^0 \to K^+K^-$. We analyze this effect as if it is due to a penguin amplitude with the weak phase of the standard model $c \to b \to u$ loop diagram, but with a $CP$-conserving enhancement as if due to the strong interactions. In such a case the magnitude and strong phase of this amplitude $P_b$ are correlated in order to fit the observed $CP$ asymmetry, and one may predict $CP$ asymmetries for a number of other singly-Cabibbo-suppressed decays of charmed mesons to a pair of pseudoscalar mesons. Nonzero $CP$ asymmetries are expected for $D^+ \to K^+\bar{K}^0$ (the most promising channel for which a nonzero $CP$ asymmetry has not yet been reported), as well as $D^0 \to \pi^0\pi^0$, $D_s^+ \to \pi^+K^0$, and $D_s \to \pi^0K^+$. No $CP$ asymmetry is predicted for $D^+ \to \pi^+\pi^0$ or $D^0 \to K^0\bar{K}^0$ in this framework.

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## I. INTRODUCTION

Although $CP$ violation was first observed in neutral kaon decays and $CP$ asymmetries have been seen at the tens of percent level in $B$ meson decays, the standard model describing these decays predicts naturally very small $CP$ asymmetries in decays of charmed particles, of order $10^{-3}$ or less [1–3]. These decays are dominated by physics of the first two quark families, with the contribution of the third family suppressed both by smallness of elements of the Cabibbo-Kobayashi-Maskawa (CKM) matrix and by the relatively small $b$ quark mass in the $c \to b \to u$ penguin diagram. This is in contrast to the $b \to t \to s$ penguin amplitude, which can profit from both a larger CKM factor and a much larger top quark mass. Following an early suggestion [4] that the penguin amplitude in $D$ decays may be enhanced by nonperturbative effects in analogy to the $s \to d$ penguin amplitude in $K \to \pi\pi$, recent studies [2,3,5] indicate that an order of magnitude enhancement is not impossible.

The LHCb Collaboration has reported $3.5\sigma$ evidence for $CP$-violating charm decays in the difference between $CP$ asymmetries in $D^0 \to K^-K^+$ and $D^0 \to \pi^-\pi^+$ [6]:

$$\Delta A_{CP} \equiv A_{CP}(K^+K^-) - A_{CP}(\pi^+\pi^-) = [-0.82 \pm 0.21(\mathrm{stat}) \pm 0.11(\mathrm{syst})]\%. \quad (1)$$

For the decay of a charmed meson $D$ to any final state $f$ we are defining

$$A_{CP}(f) \equiv \frac{\Gamma(D \to f) - \Gamma(\bar{D} \to \bar{f})}{\Gamma(D \to f) + \Gamma(\bar{D} \to \bar{f})}. \quad (2)$$

Although the CDF II Collaboration at the Fermilab Tevatron does not see statistically compelling evidence for $CP$ violation in either of these two decays, their results are consistent with those of LHCb [7]:

$$A_{CP}(D^0 \to K^+K^-) = (-0.24 \pm 0.22 \pm 0.09)\%,$$
$$A_{CP}(D^0 \to \pi^+\pi^-) = (0.22 \pm 0.24 \pm 0.11)\%. \quad (3)$$

We calculate the corresponding 90% confidence level limits to be

$$-0.63\% \le A_{CP}(D^0 \to K^+K^-) \le 0.15\%,$$
$$-0.21\% \le A_{CP}(D^0 \to \pi^+\pi^-) \le 0.65\%. \quad (4)$$

The LHCb results have led to numerous hypotheses of physics beyond the standard model (e.g., [2,8–10]) some of which had been studied earlier [11]. A more conservative approach, studying the above $CP$ asymmetries within the standard model under relaxed assumptions about nonperturbative hadronic weak matrix elements, has been adopted recently in two papers applying flavor SU(3). Reference [12] extended the hypothesis of triplet operator enhancement introduced in Ref. [4] by including, in the effective weak Hamiltonian, SU(3) breaking terms which are first order in the strange quark mass. A second work [13] (appearing while we were writing up our results), applying a diagrammatic flavor SU(3) approach similar to the one discussed by us below, associated $W$-exchange and annihilation amplitudes with final state resonant effects [14]. While these two papers have some overlap

 

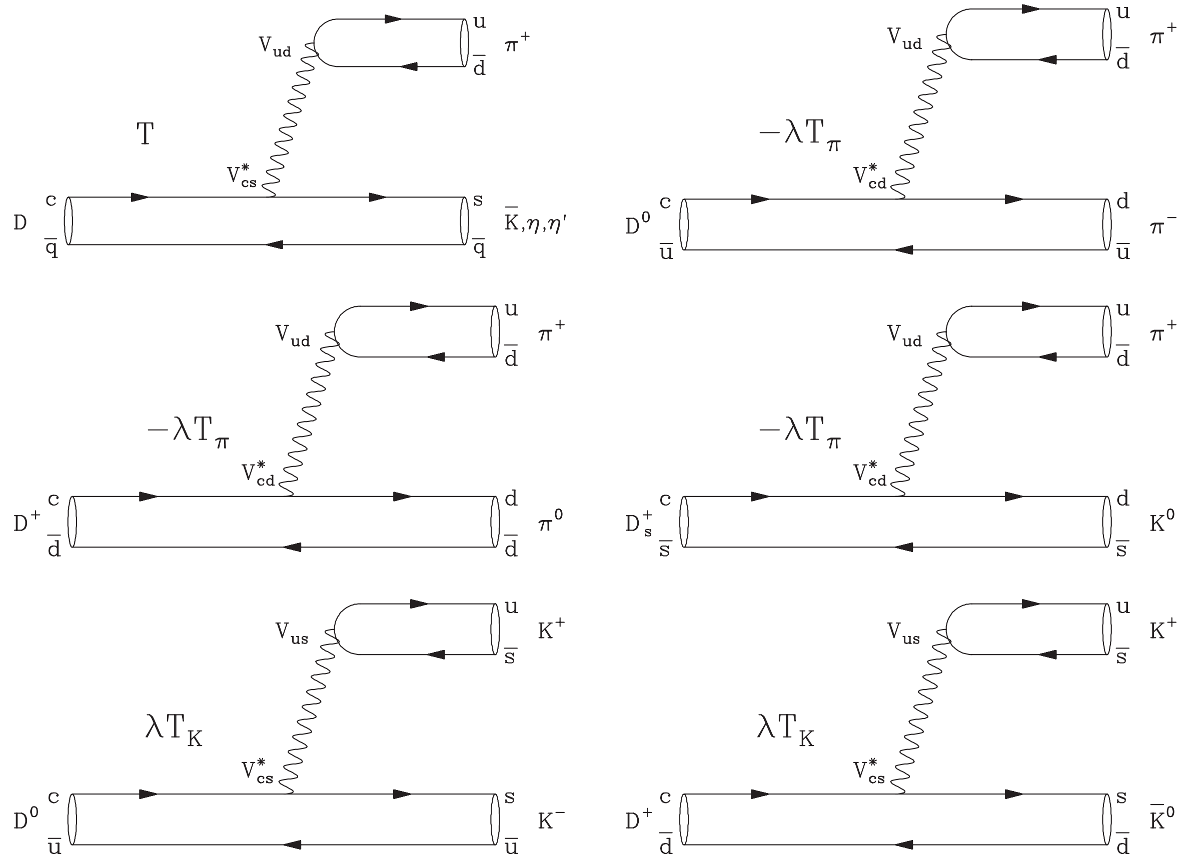


FIG. 1. Relation between CF and SCS color-favored tree amplitudes.

with ours, the specific assumptions and detailed predictions of the three studies, all based on flavor SU(3) analyses, are different.

In the present paper we explore a scenario in which the standard model $c \to b \to u$ penguin amplitude receives a sufficient enhancement from strong interaction physics to account for the effect observed by LHCb. This amplitude then must contribute to other direct $CP$ asymmetries in decays of $D^0$, $D^+$, and $D_s^+$ to pairs of pseudoscalar mesons. Nonzero $CP$ asymmetries are expected for $D^+ \to K^+\bar{K}^0$, as well as $D^0 \to \pi^0\pi^0$, $D_s^+ \to \pi^+K^0$, and $D_s \to \pi^0K^+$. No $CP$ asymmetry is predicted for $D^+ \to \pi^+\pi^0$ or $D^0 \to K^0\bar{K}^0$ in this model. The former receives no penguin contribution, being a $\Delta I = 3/2$ process, while the latter involves a different penguin amplitude than the one we are considering.

We perform the analysis in the context of a flavor-SU(3) model of charm decays presented previously [15,16]. We introduce notation in Sec. II and fit decay rates for singly-Cabibbo-suppressed (SCS) processes (including SU(3) breaking) in Sec. III. We then introduce a phenomenological penguin amplitude $P_b$ in Sec. IV to account for the $CP$ violation observed by LHCb, and predict other $CP$ asymmetries for SCS charmed meson decays. We summarize in Sec. V.

## II. FORMALISM AND NOTATION

Cabibbo-favored (CF) charm decays may be characterized by amplitudes $T$, $C$, $E$, and $A$, corresponding to color-favored tree, color-suppressed tree, exchange, and annihilation flavor topologies [15,16]. A fit to CF decays of $D$ mesons to two pseudoscalar mesons leads to the following ($|T| > |C|$) solution:

$$T = 2.927, \tag{5}$$

$$C = 2.337e^{-i151.66^\circ} = -2.057 - 1.109i, \tag{6}$$

$$E = 1.573e^{i120.56^\circ} = -0.800 + 1.355i, \tag{7}$$

$$A = 0.33e^{i70.47^\circ} = 0.110 + 0.311i, \tag{8}$$

quoted in units of $10^{-6}$ GeV. We note that complex conjugates of the amplitudes (6)–(8) give identical decay rates. To describe amplitudes corresponding to SCS processes the above amplitudes are multiplied by $\pm\lambda$, where $\lambda = \tan\theta_{\text{Cabibbo}} = 0.2317$. (Tiny phases of $V_{ud}$, $V_{cs}$, $V_{cd}$, and $V_{us}$ are neglected. These contribute to negligible direct $CP$ asymmetries in $D^0 \to \pi^+\pi^-$ and $D^0 \to K^+K^-$ at a level below one per thousand by interference of the above amplitudes with SU(3)-breaking penguin amplitudes

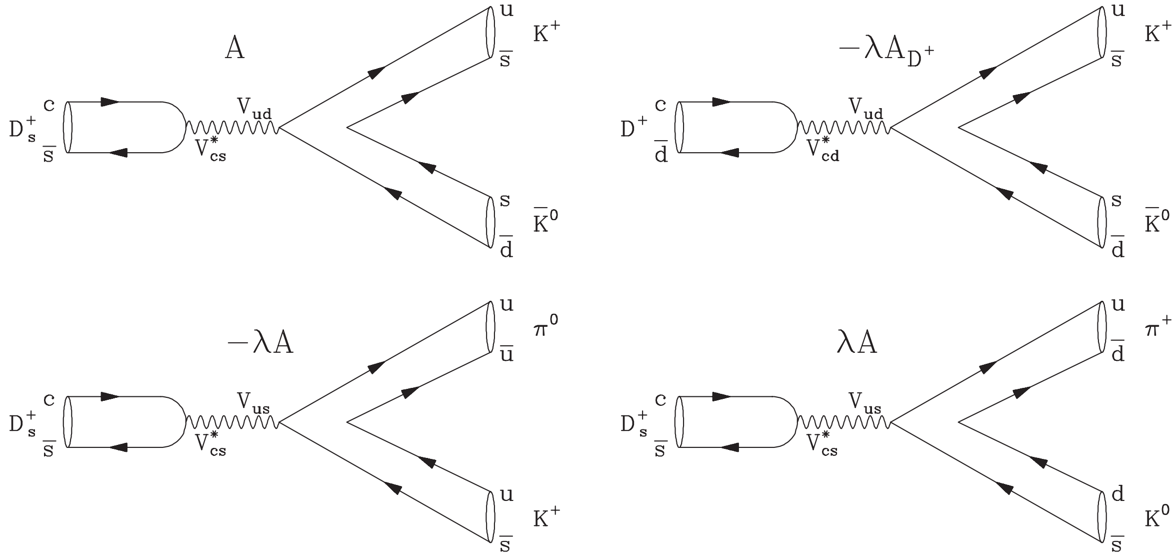


FIG. 2. Relation between CF and SCS $A$ amplitudes.

discussed in Sec. III.) The relation between CF and SCS color-favored tree amplitudes is illustrated in Fig. 1.

In order to account for SU(3) breaking in the SCS $T$ amplitude we may use the following expressions:

$$T_{D^0\to\pi^+\pi^-} = T_{D^+\to\pi^+\pi^0} = T_{D_s^+\to\pi^+K^0} = T_\pi, \quad (9)$$

$$T_{D^0\to K^+K^-} = T_{D^+\to K^+\bar{K}^0} = T_K, \quad (10)$$

where, neglecting the contribution of $f_-(q^2)$ at $q^2 = m^2_{\pi,K}$,

$$T_\pi = T\cdot\frac{|f_{+(D^0\to\pi^-)}(m_\pi^2)|}{|f_{+(D^0\to K^-)}(m_\pi^2)|}\cdot\frac{m_D^2 - m_\pi^2}{m_D^2 - m_K^2}, \quad (11)$$

$$T_K = T\cdot\frac{|f_{+(D^0\to K^-)}(m_K^2)|}{|f_{+(D^0\to K^-)}(m_\pi^2)|}\cdot\frac{f_K}{f_\pi}. \quad (12)$$

Similarly, the relation between CF and SCS $A$ amplitudes is illustrated in Fig. 2. Here, one may introduce SU(3) breaking as follows:

$$A_{D^+\to K^+\bar{K}^0} = A\cdot\frac{f_{D^+}}{f_{D_s^+}} = A_{D^+}, \quad (13)$$

$$A_{D_s^+\to\pi^+K^0} \simeq A_{D_s^+\to K^+\pi^0} = A. \quad (14)$$

We know the relevant decay constants [17] and meson masses [18] (in GeV):

$$f_\pi = 0.130\,41; \qquad f_K = 0.1561; \qquad f_{D^+} = 0.2067; \qquad f_{D_s^+} = 0.2575; \quad (15)$$

$$m_{D^0} = 1.8648; \qquad m_\pi = 0.139\,570\,18; \qquad m_K = 0.493\,677. \quad (16)$$

The following approximate values are also known for the form factors from semileptonic $D^0$ decays [19,20]:

$$|f_{+(D^0\to\pi^-)}(m_\pi^2)| \simeq 0.705, \quad (17)$$

$$|f_{+(D^0\to K^-)}(m_\pi^2)| \simeq 0.768, \quad (18)$$

$$|f_{+(D^0\to K^-)}(m_K^2)| \simeq 0.811. \quad (19)$$

After using relevant form factors and decay constants, we find, in units of $10^{-6}$ GeV,

$$T_\pi = 2.87, \quad (20)$$

$$T_K = 3.70, \quad (21)$$

$$A_{D^+} = (0.89 + 2.50i)\times 10^{-1}. \quad (22)$$

The tree-level amplitudes for $D^0\to\pi^+\pi^-$, $D^0\to K^+K^-$, and $D^0\to\pi^0\pi^0$ involve the following respective combinations (in units of $10^{-7}$ GeV):

$$-\lambda(T_\pi + E) = -4.80 - 3.14i, \quad (23)$$

$$\lambda(T_K + E) = 6.72 + 3.14i, \quad (24)$$

$$\lambda(C - E) = -2.91 - 5.71i, \quad (25)$$

as well as SU(3)-breaking terms which we shall now introduce.

## III. FITS TO DECAY RATES INCLUDING SU(3) VIOLATION

The penguin amplitude $P$ for $c\to u$ transitions is normally thought to be very small because the contributions of $d$ and $s$ quarks in the intermediate state cancel one another [21]. If we regard this cancellation as inexact due

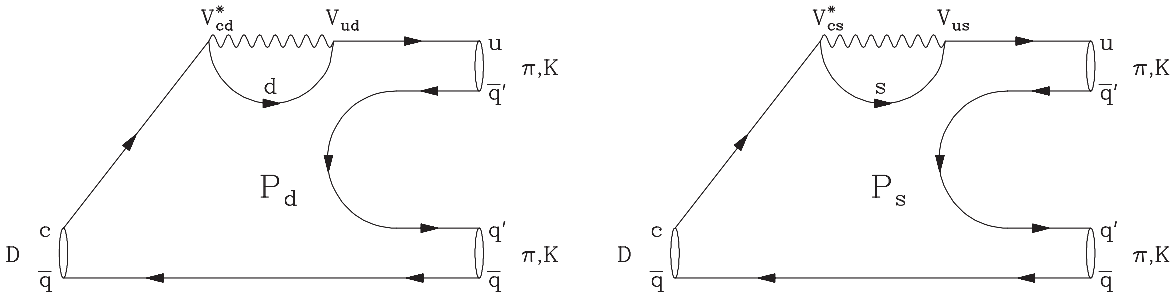


FIG. 3. Penguin diagrams leading to a nonzero amplitude $P = P_d + P_s$ in the presence of imperfect cancellation between intermediate $d$ and $s$ quarks.

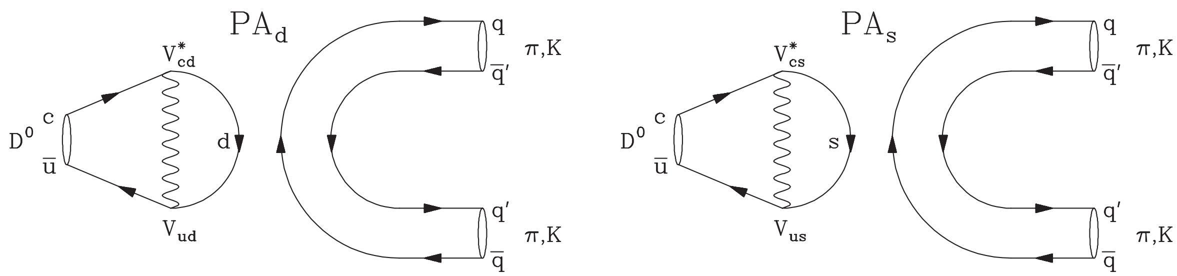


FIG. 4. Penguin annihilation diagrams leading to a nonzero amplitude $PA = PA_d + PA_s$ in the presence of imperfect cancellation between intermediate $d$ and $s$ quarks.

to SU(3)-violating masses of intermediate-state particles, we can regard the penguin amplitude $P$ as a proxy for SU(3) violation [see Fig. 3]. It will then have the same weak phase as other standard model contributions to $D$ decays. The same can be said for a penguin annihilation ($PA$) amplitude, contributing only to $D^0$ decays. It corresponds to the exchange processes $c\bar{u} \to s\bar{s}$ and $c\bar{u} \to d\bar{d}$ followed by $s\bar{s}$ or $d\bar{d}$ annihilation into a pair of charge-conjugate pseudoscalar mesons [Fig. 4].

The amplitudes for $D^0 \to \pi^+\pi^-$, $D^0 \to K^+K^-$, and $D^0 \to \pi^0\pi^0$ then may be expressed as shown in the first three lines of Table I. Given the magnitudes of the relevant amplitudes determined from the decay rates [16,22], in units of $10^{-7}$ GeV,

$$|\mathcal{A}(D^0 \to \pi^+\pi^-)| = 4.70 \pm 0.08, \tag{26}$$

$$|\mathcal{A}(D^0 \to K^+K^-)| = 8.49 \pm 0.10, \tag{27}$$

$$\sqrt{2}|\mathcal{A}(D^0 \to \pi^0\pi^0)| = 4.96 \pm 0.16, \tag{28}$$

one may then plot circles with these radii and centers defined by Eqs. (23)–(25) to solve for a common value of $P + PA$. The existence of a self-consistent solution for $P + PA$ is supported by a $\chi^2$-minimization fit, which leads to

TABLE I. Representations and comparison of experimental and fit amplitudes for SCS decays of charmed mesons to two pseudoscalar mesons.

| Decay mode | Amplitude representation | $\lvert\mathcal{A}\rvert$ ($10^{-7}$ GeV) Experiment | Theory fit | $\chi^2$ |
|---|---|---|---|---|
| $D^0 \to \pi^+\pi^-$ | $-\lambda(T_\pi + E) + (P + PA)$ | $4.70 \pm 0.08$ | 4.70 | 0 |
| $D^0 \to K^+K^-$ | $\lambda(T_K + E) + (P + PA)$ | $8.49 \pm 0.10$ | 8.48 | 0.01 |
| $D^0 \to \pi^0\pi^0$ | $-\lambda(C - E)/\sqrt{2} - (P + PA)/\sqrt{2}$ | $3.51 \pm 0.11$ | 3.51 | 0 |
| $D^+ \to \pi^+\pi^0$ | $-\lambda(T_\pi + C)/\sqrt{2}$ | $2.66 \pm 0.07$ | 2.26 | 33 |
| $D^0 \to K^0\bar{K}^0$ | $-(P + PA) + P$ | $2.39 \pm 0.14$ | 2.37 | 0.02 |
| $D^+ \to K^+\bar{K}^0$ | $\lambda(T_K - A_{D^+}) + P$ | $6.55 \pm 0.12$ | 6.87 | 7 |
| $D_s^+ \to \pi^+K^0$ | $-\lambda(T_\pi - A) + P$ | $5.94 \pm 0.32$ | 7.96 | 40 |
| $D_s^+ \to \pi^0K^+$ | $-\lambda(C + A)/\sqrt{2} - P/\sqrt{2}$ | $2.94 \pm 0.55$ | 4.44 | 7 |

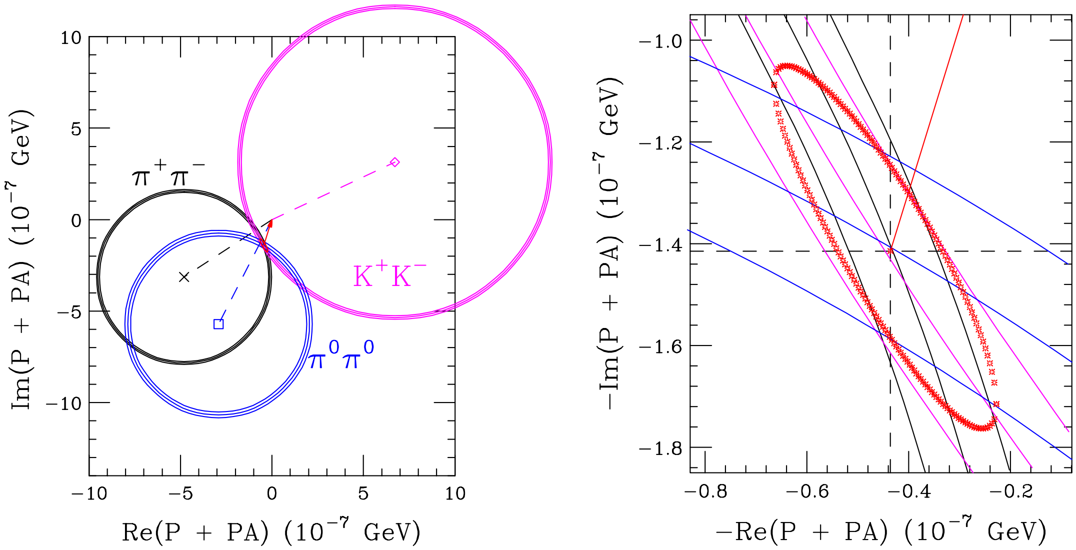


FIG. 5 (color online). Construction to determine $P + PA$. The relative sign between the left-hand and (magnified) right-hand panels is due to the fact that the vector $P + PA$ points *toward* the origin in the left-hand figure.

$$P + PA = [(0.44 \pm 0.23) + (1.41 \pm 0.36)i] \times 10^{-7}\ \text{GeV};$$
$$\chi^2/\text{d.o.f.} = 0.012/1 = 0.012. \tag{29}$$

The construction and the corresponding $\Delta\chi^2 = 2.3$ error ellipse (corresponding to 68% probability) are shown in Fig. 5.

Using the extracted value of $P + PA$ we apply a similar construction technique to extract $P$. The relevant parts of the tree-level amplitudes that determine centers of the circles are as follows (in units of $10^{-7}$ GeV):

$$-(P + PA) = -0.44 - 1.41i, \tag{30}$$

$$\lambda(T_K - A_{D^+}) = 8.37 - 0.58i, \tag{31}$$

$$-\lambda(T_\pi - A) = -6.40 + 0.72i, \tag{32}$$

$$\lambda(C + A) = -4.51 - 1.85i. \tag{33}$$

The relevant experimental rates [16,22] lead to amplitudes once again determining the radii of the circles as follows (in units of $10^{-7}$ GeV):

$$|\mathcal{A}(D^0 \to K^0\bar{K}^0)| = 2.39 \pm 0.14, \tag{34}$$

$$|\mathcal{A}(D^+ \to K^+\bar{K}^0)| = 6.55 \pm 0.12, \tag{35}$$

$$|\mathcal{A}(D_s^+ \to K^0\pi^+)| = 5.94 \pm 0.32, \tag{36}$$

$$\sqrt{2}|\mathcal{A}(D_s^+ \to K^+\pi^0)| = 2.94 \pm 0.55. \tag{37}$$

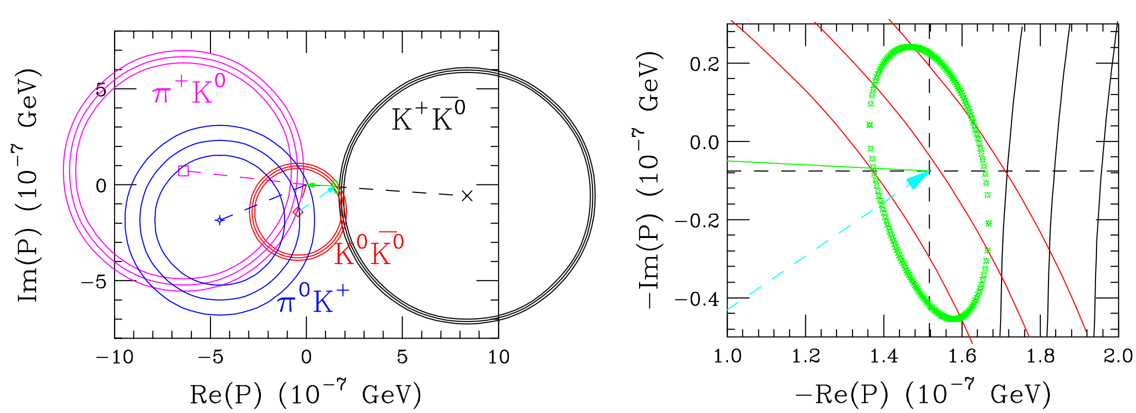


FIG. 6 (color online). Construction to determine $P$. The relative sign between the left-hand and (magnified) right-hand panels is due to the fact that the vector $P$ points *toward* the origin in the left-hand figure.

$\chi^2$-minimization gives us

$$P = [(-1.52 \pm 0.15) + (0.08^{+0.38}_{-0.32})i] \times 10^{-7}\ \text{GeV};$$
$$\chi^2/\text{d.o.f.} = 54/2 = 27. \qquad (38)$$

The construction and the corresponding 68% error ellipse ($\Delta\chi^2 = 2.3$) are shown in Fig. 6. In Table I we quote the representations and compare the experimental and fit amplitudes. Using the extracted values of $P$ and $P + PA$ we find

$$PA = [(1.95 \pm 0.38) + (1.34 \pm 0.71)i] \times 10^{-7}\ \text{GeV}. \qquad (39)$$

We recall the two-fold ambiguity permitting amplitudes which are complex conjugates of (30)–(33), (38), and (39).

The poor $\chi^2$ in this fit is driven primarily by the large contribution from the $D_s^+ \to \pi^+ K^0$ amplitude. It is quite possible that our description of SU(3) breaking in this quantity is imperfect. In any case, the large experimental errors on the SCS decays of $D_s$ to two pseudoscalar mesons will hinder the study of $CP$-violating asymmetries in their decays for some time to come, so we shall not be greatly concerned with such decays for the present.

## IV. DESCRIPTION AND PREDICTION OF OBSERVED DIRECT *CP* ASYMMETRIES

We now consider the effects of an additional phenomenological-penguin amplitude $P_b$, the weak phase of which differs from the weak phase of $P$ and $PA$ by the CKM angle $\gamma$. (The subscript $b$ refers to a $b$ quark in the intermediate quark loop in Fig. 3.) In Table II we summarize the amplitudes for SCS processes obtained in the previous section, and extend the amplitude representations to include $P_b$. The quantities $\phi_T^f = \text{Arg}[T_f]$ denote the strong phases of the non-$P_b$ amplitudes with respect to $T$. (The amplitudes $T_f$ include factors $\pm\lambda$.)

In general, the amplitude for $D \to f$ may be written as follows:

$$\mathcal{A}(D \to f) = |T_f| e^{i\phi_T^f}(1 + r_f e^{i(-\gamma+\phi^f)}), \qquad (40)$$

where $T_f$ represents terms that have the same weak phase as the tree-level terms contributing to that amplitude, $\phi_T^f$ represents the strong phase of $T_f$, $r_f$ represents the ratio of the magnitude of the $CP$-violating penguin contribution to that of $T_f$, $-\gamma$ represents the weak phase of the $CP$-violating penguin ($\gamma$ is the same as the CKM angle), and $\phi^f$ is the strong phase of the $CP$-violating penguin relative to $T_f$. Let us take the example of the process $D^0 \to \pi^+\pi^-$ for clarity. Then

$$T_{\pi^+\pi^-} = -\lambda(T_\pi + E) + (P + PA), \qquad (41)$$

$$\phi_T^{\pi^+\pi^-} = \text{Arg}[T_{\pi^+\pi^-}], \qquad (42)$$

$$r_{\pi^+\pi^-} = \frac{|P_b|}{|T_{\pi^+\pi^-}|}, \qquad (43)$$

$$\phi^{\pi^+\pi^-} = \text{Arg}[P_b] - \phi_T^{\pi^+\pi^-} + \gamma. \qquad (44)$$

The amplitude for $\bar{D} \to \bar{f}$ may be written as follows:

$$\mathcal{A}(\bar{D} \to \bar{f}) = |T_f| e^{i\phi_T^f}(1 + r_f e^{i(\gamma+\phi^f)}). \qquad (45)$$

For a two-body decay, the rate is proportional to the absolute square of the amplitude. Thus, one may now define a $CP$ asymmetry as follows:

$$A_{CP}(f) = \frac{\Gamma(D \to f) - \Gamma(\bar{D} \to \bar{f})}{\Gamma(D \to f) + \Gamma(\bar{D} \to \bar{f})} = \frac{2r_f \sin\gamma \sin\phi^f}{1 + r_f^2 + 2r_f \cos\gamma \cos\phi^f} = \frac{2p|T_f| \sin\gamma \sin(\delta - \phi_T^f)}{|T_f|^2 + p^2 + 2p|T_f| \cos\gamma \cos(\delta - \phi_T^f)}, \qquad (46)$$

where in the final step we have used $P_b = pe^{i(\delta-\gamma)}$.

TABLE II. Fit amplitudes for SCS charmed meson decays including $P$ and $PA$, and their representations including $P_b$.

| Decay mode | Amplitude representation | $\phi_T^f = \text{Arg}[T_f]$ (degrees) |
|---|---|---|
| $D^0 \to \pi^+\pi^-$ | $-\lambda(T_\pi + E) + (P + PA) + P_b$ | $-158.5$ |
| $D^0 \to K^+K^-$ | $\lambda(T_K + E) + (P + PA) + P_b$ | 32.5 |
| $D^0 \to \pi^0\pi^0$ | $-\lambda(C - E)/\sqrt{2} - (P + PA)/\sqrt{2} - P_b/\sqrt{2}$ | 60.0 |
| $D^+ \to \pi^+\pi^0$ | $-\lambda(T_\pi + C)/\sqrt{2}$ | 126.3 |
| $D^0 \to K^0\bar{K}^0$ | $-(P + PA) + P$ | $-145.6$ |
| $D^+ \to K^+\bar{K}^0$ | $\lambda(T_K - A_{D^+}) + P + P_b$ | $-4.2$ |
| $D_s^+ \to \pi^+K^0$ | $-\lambda(T_\pi - A) + P + P_b$ | 174.3 |
| $D_s^+ \to \pi^0K^+$ | $-\lambda(C + A)/\sqrt{2} - P/\sqrt{2} - P_b/\sqrt{2}$ | 16.4 |

The LHCb result (1) [6] may be used as a constraint on the magnitude and strong phase of the *CP*-violating penguin $P_b$. [*Note added in proof*—To lowest order in $p$, all asymmetries $A_{CP}(f)$ depend on the combination $p\sin\gamma$. Thus, if we impose the $\Delta A_{CP}$ constraint, our predictions for other asymmetries are the same for any weak phase of $P_b$ as long as effects of higher order in $p$ are negligible. We thank N. Deshpande for a question leading to this result.] We use the following relationships:

$$A_{CP}(K^+K^-) = \frac{2p|T_{K^+K^-}|\sin\gamma\sin(\delta - \phi_T^{K^+K^-})}{|T_{K^+K^-}|^2 + p^2 + 2p|T_{K^+K^-}|\cos\gamma\cos(\delta - \phi_T^{K^+K^-})}, \tag{47}$$

$$A_{CP}(\pi^+\pi^-) = \frac{2p|T_{\pi^+\pi^-}|\sin\gamma\sin(\delta - \phi_T^{\pi^+\pi^-})}{|T_{\pi^+\pi^-}|^2 + p^2 + 2p|T_{\pi^+\pi^-}|\cos\gamma\cos(\delta - \phi_T^{\pi^+\pi^-})}. \tag{48}$$

We may use the theory fit results quoted in Table I for $|T_f|$. The strong phase $\phi_T^f$ can be taken from the results quoted in Table II. The CKM angle $\gamma$ may be taken to be 77° [18]. Corresponding to each value of $\delta$ allowed by the $\Delta A_{CP}$ constraint, one may extract the allowed values of $p$. In addition we expect $|P_b| < |T_f|$ ($r_f < 1$), which in turn restricts us to small values of $p$. In Fig. 7 we plot the allowed values of $p$ as a function of $\delta$ using Eqs. (47) and (48). For a wide range of $\delta$, a penguin amplitude of magnitude $0.01 \times 10^{-7}$ GeV, or $\mathcal{O}(0.1\%)$ of the $D^0 \to K^+K^-$ amplitude, is sufficient to account for the observed value of $\Delta A_{CP}$. This is in accord with a conclusion reached in Ref. [3].

The constraint on $p$ as a function of $\delta$ now allows us to predict *CP* asymmetries in other channels such as $D^+ \to K^+\bar{K}^0$ as a function of the angle $\delta$. In Fig. 8 we plot $A_{CP}(K^+\bar{K}^0)$ as a function of $\delta$. Values of $p$ are plotted only for the range of $\delta$ consistent with the limits (4), which will be specified shortly. In Fig. 9 we plot $A_{CP}$ for the final states $K^+K^-$, $\pi^+\pi^-$, and $\pi^0\pi^0$. The limits (4) imply the following allowed ranges of $\delta$:

$$A_{CP}(K^+K^-) \Rightarrow -2.64 \le \delta \le 0.41,$$
$$A_{CP}(\pi^+\pi^-) \Rightarrow -2.65 \le \delta \le 0.41. \tag{49}$$

Figures 7 and 8 are plotted only for values of $\delta$ consistent with both these limits. Note the correlation between the *CP* asymmetries in the channels $D^0 \to \pi^0\pi^0$ and $D^+ \to K^+\bar{K}^0$. More precise measurements of the individual asymmetries in $D^0 \to \pi^+\pi^-$ and $D^0 \to K^+K^-$ can help to pin down the unknown strong phase $\delta$.

As mentioned in the preceding section, all the contributions to $T_f$ listed in Table II involve an ambiguity due to complex conjugation. Thus, the phase $\phi_T^f$ has a sign ambiguity, $\phi_T^f \to -\phi_T^f$, which is common to all final states $f$. The *CP* asymmetry (46) is approximately invariant under a joint transformation,

$$\phi_T^f \to -\phi_T^f, \qquad \delta \to \pi - \delta, \tag{50}$$

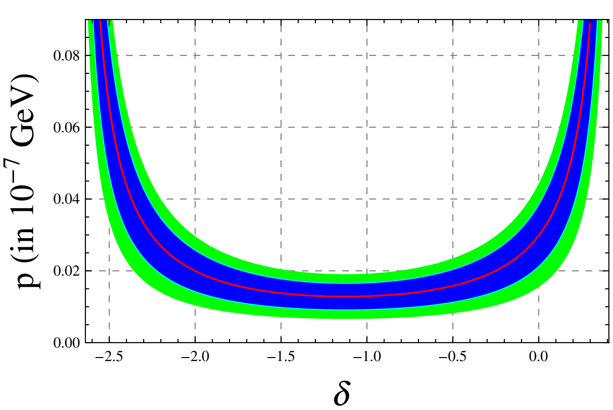


FIG. 7 (color online). $p$ and $\delta$ allowed by the measured range of $\Delta A_{CP}$. The (red) line represents the central value, while inner (blue) and outer (green) bands, respectively, represent 68% confidence level ($1\sigma$) and 90% confidence level ($1.64\sigma$) regions based on error in $\Delta A_{CP}$.

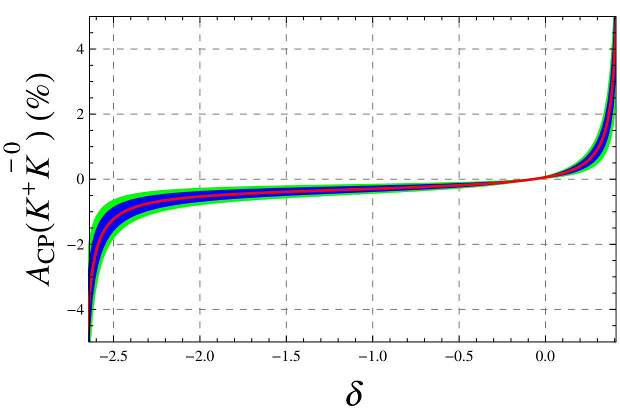


FIG. 8 (color online). $A_{CP}(K^+\bar{K}^0)$ as a function of the allowed values of $\delta$. The (red) line represents the central value, while inner (blue) and outer (green) bands, respectively, represent 68% confidence level ($1\sigma$) and 90% confidence level ($1.64\sigma$) regions based on error in $\Delta A_{CP}$.

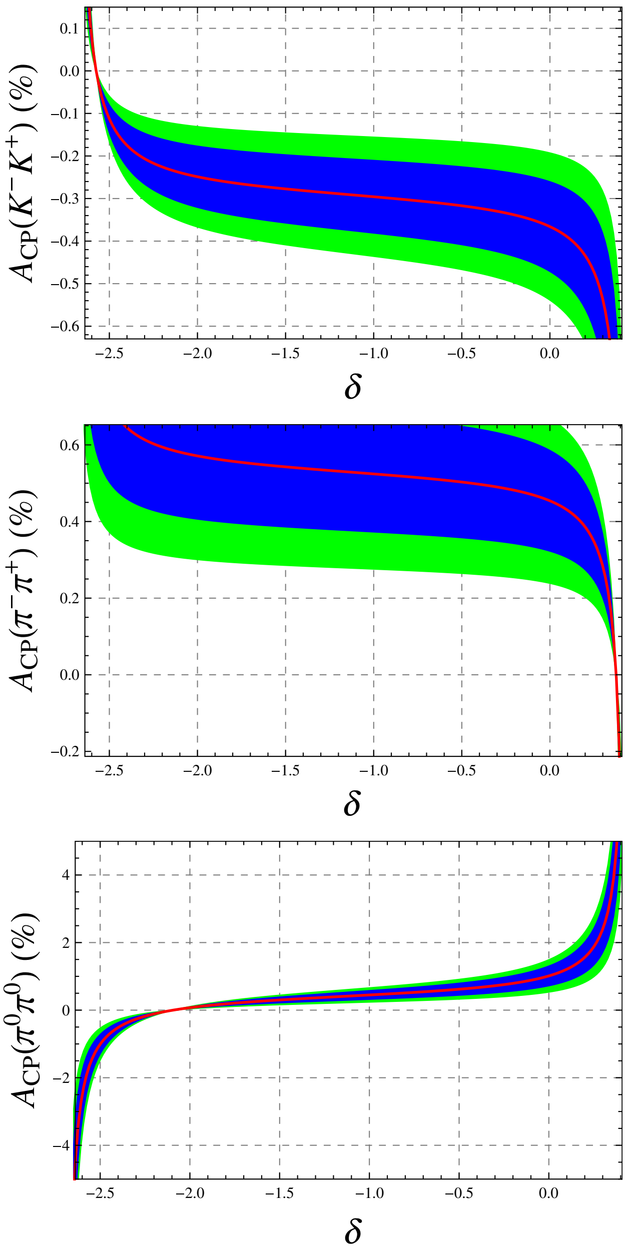


FIG. 9 (color online). $A_{CP}$ as a function of the allowed values of $\delta$. The (red) line represents the central value, while inner (blue) and outer (green) bands, respectively, represent 68% confidence level ($1\sigma$) and 90% confidence level ($1.64\sigma$) regions based on error in $\Delta A_{CP}$.

neglecting a very small contribution to the asymmetry quadratic in $p/|T_f|$. Thus, while plots similar to Figs. 7–9 may be plotted with $\delta \to \pi - \delta$, the correlations between asymmetries in different decay modes are invariant under this redefinition.

We have left out $D_s^+$ decay asymmetries since the corresponding branching ratios have large fractional errors. The process $D^+ \to \pi^+\pi^0$ does not depend on $P_b$ in the isospin symmetry limit, and therefore its $CP$ asymmetry is zero at this high level of approximation. The $CP$ asymmetry in $D^0 \to K^0\bar{K}^0$ depends only on a penguin annihilation diagram as there are no $u$ quarks in the final state. If it is found to be nonzero, our discussion must be expanded to include the possibility of $CP$ violation due to interference between a $(PA)_b$ amplitude involving a $b$ quark in the loop and an SU(3) breaking term in $E$.

## V. DISCUSSION AND SUMMARY

The observation by the LHCb Collaboration of a difference between the $CP$-violating asymmetries in $D^0 \to K^+K^-$ and $D^0 \to \pi^+\pi^-$ likely implies observable asymmetries in other decays of charmed mesons to pairs of pseudoscalar mesons. The present description of that difference assumes that a penguin amplitude with an intermediate $b$ quark, normally thought to provide a contribution below the observed effect, is amplified by $CP$—conserving physics (e.g., unforeseen QCD effects) to an extent which can account for the asymmetry. In that case several direct $CP$ asymmetries are predicted as functions of a single strong phase difference $\delta$. These include asymmetries in the individual decays $D^0 \to K^+K^-$ and $D^0 \to \pi^+\pi^-$, as well as $D^0 \to \pi^0\pi^0$ and $D^+ \to K^+\bar{K}^0$. These asymmetries are typically of order (a few) $\times 10^{-3}$, and the latter two are correlated with one another. Experimental limits (4) on the direct $CP$ asymmetries in $D^0 \to K^+K^-$ and $D^0 \to \pi^+\pi^-$ [7] provide constraints on $\delta$. The observed asymmetry [23] $A_{CP}(D^+ \to K^+\bar{K}^0) = (7.1 \pm 6.1 \pm 1.2)\%$ carries far too large an uncertainty at present to test its prediction. [*Note added in proof*—(1) The CDF Collaboration has now reported a value of $\Delta A_{CP} = (-0.62 \pm 0.21 \pm 0.10)$ [24]. (2) We thank Anze Zupanc for reminding us that the Belle Collaboration has reported the much more precise value $A_{CP}(D^+ \to K^+\bar{K}^0) = (-0.16 \pm 0.58 \pm 0.25)$ [25].]

In Fig. 9, while $A_{CP}(K^+K^-)$ and $A_{CP}(\pi^+\pi^-)$ are predicted to have opposite signs for a wide range of $\delta$, their relative *magnitudes* provide information about $\delta$, with the ratio $|A_{CP}(\pi^+\pi^-)/A_{CP}(K^+K^-)|$ exceeding 1 for the mid-range of $\delta$ and behaving as a decreasing function of $\delta$. Thus, better measurements of these individual asymmetries will enable improved predictions of asymmetries such as $A_{CP}(K^+\bar{K}^0)$ and $A_{CP}(\pi^0\pi^0)$. We look forward to improvement of many of these determinations.

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