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Dark matter and dark energy via nonperturbative (flavor) vacua

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A nonperturbative field theoretical approach to flavor physics (Blasone-Vitiello formalism) has been shown to imply a highly nontrivial vacuum state. Although still far from representing a satisfactory framework for a coherent and complete characterization of flavor states, in recent years the formalism has received attention for its possible implications at cosmological scales. In a previous work, we implemented the approach on a simple supersymmetric model (free Wess-Zumino), with flavor mixing, which was regarded as a model for free neutrinos and sneutrinos. The resulting effective vacuum (called *flavor vacuum*) was found to be characterized by a strong supersymmetry breaking. In this paper we explore the phenomenology of the model and we argue that the flavor vacuum is a consistent source for both dark energy (thanks to the bosonic sector of the model) and dark matter (via the fermionic one). Quite remarkably, besides the parameters connected with neutrino physics, in this model no other parameters have been introduced, possibly leading to a predictive theory of dark energy/matter. Despite its oversimplification, such a toy model already seems capable to shed some light on the observed energy hierarchy between neutrino physics, dark energy and dark matter. Furthermore, we move a step forth in the construction of a more realistic theory, by presenting a novel approach for calculating relevant quantities and hence extending some results to interactive theories, in a completely nonperturbative way.

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I. INTRODUCTION

Neutrino flavor oscillation is nowadays a fairly well-established fact, thanks to a wide range of experimental evidences [1]. A simple quantum mechanical model (based on the work of Pontecorvo, Maki, Nagawa, and Sakata [2–5]) is commonly considered as sufficient for accounting for experimental data. However, this hides nontrivial difficulties in the formulation of flavor oscillations in a quantum field-theoretical (QFT) framework [6]. flavor states indeed do not represent correct asymptotic states (by definition, since their oscillating behavior), which are required in the usual perturbative approach to QFT (in the Lehmann, Symanzik and Zimmermann scheme).

More than a decade ago, a nonperturbative approach for building flavor states was suggested by Blasone, Vitiello and coworkers (*BV formalism* for flavor physics) [7]. A first version was proposed in 1995 [7], but some inconsistencies in the derivation of oscillation formulae were noticed [8–11] shortly after; a revisited version, in which these discrepancies were clarified and removed, was suggested and developed [12–16] later on [6]. However, for some aspects, the formalism remains controversial and its physical relevance is still matter of debate [17–19].

The approach correctly reduces to the common quantum mechanical approach in the small (neutrinos) mass limit, but leads to corrections to those formulas that are currently beyond experimental sensitivity [6,20]. However, perhaps the most interesting feature of BV formalism is the nontrivial vacuum (called *flavor vacuum*) implied by the theory. Such a *flavor vacuum* (which can be regarded as a *vacuum condensate*) represents the physical state with no (flavor) particles in it. Despite being merely an ''empty'' state, the flavor vacuum is characterized by a rich structure revealed by the nonvanishing expectation value of the stress-energy tensor and the related equation of state. Within BV formalism one is able to fully describe it at a nonperturbative level, and its features depend on the specific model considered (spin, interactions, number of particles characterized by flavor mixing, etc.).

In a series of papers it was suggested that the *flavor vacuum* might behave as a source of dark energy [21–30]. Recently, it has been shown that in a simple supersymmetric (Wess-Zumino) model with flavor mixing, in which two Majorana fields, two scalars and two pseudoscalars were present (a simple model for neutrinos and sneutrinos), the *flavor vacuum* was actually characterized by a strong supersymmetry breaking [31,32]. In the present work we argue that this breaking is the origin of an interesting phenomenology, that might shed some lights both on the dark energy and the dark matter problem. More precisely, in the supersymmetric context of [31,32], the *flavor vacuum* can be thought as made of two different fluids that fill in all universe: a first one related to the bosonic sector of the model, and a second induced by the fermionic one. The former is characterized by negative pressure, equal in modulus to its energy density (acting as a source of dark energy). The latter is characterized by zero pressure, giving rise to a source of cold dark matter.

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The first part of the paper (Sec. I) will be dedicated to review BV formalism, complementing the original literature with a discussion on nonperturbative theories and Fock spaces.

In the second part of the paper (Sec. II) we shall explore in details the phenomenology of the model studied in [31,32]. We shall clarify why the *flavor vacuum* is a good dark energy and dark matter candidate, with emphasis on this latter. More importantly, we will explain how to relate all parameters of the model to observational data. This is a quite important aspect of the approach: the model introduces very few parameters which are all related to neutrino physics. Hopefully, more realistic models will not rely on uncontrolled free parameters, leading to a truly falsifiable theory for both dark energy and dark matter. A first encouraging result in this direction comes already from the simple model here studied: this is indeed fairly consistent with a choice of the parameters modeled on real world data, as we shall see in the relevant section.

A first step towards more realistic models will be moved in the last part of this work (Sec. III). We will present a novel method for calculating relevant quantities, specifically thought for analyzing the features of the *flavor vacuum*, which might enable us to study interactive theories completely at a nonperturbative level. In particular, we shall show how, under reasonable assumptions, the method can discriminate which interactions preserve the behavior of the condensate as dark energy/matter source.

II. BV FORMALISM

Neutrino oscillations can be described in a nonrelativistic quantum mechanical framework by constructing particle states, labeled by a *flavor number*, that are not eigenstates of the Hamiltonian. In its simplest formulation for two distinct flavors, the Pontecorvo model, flavor states are constructed as follows [33]:

$$\begin{aligned} |\nu_A\rangle &= \cos\theta|\nu_1\rangle + \sin\theta|\nu_2\rangle \\ |\nu_B\rangle &= -\sin\theta|\nu_1\rangle + \cos\theta|\nu_2\rangle, \end{aligned} \tag{1}$$

where $|\nu_1\rangle$ and $|\nu_2\rangle$ are massive eigenstate of the free Hamiltonian (particles with well-defined mass m_i, with $i = 1, 2$), and from which

$$\begin{aligned} \wp_{A\to B} &= |\langle\nu_B|\nu_A(t)\rangle|^2 = |\langle\nu_B|e^{-iHt}|\nu_A\rangle|^2 \\ &= \sin^2 2\theta \sin^2\left(\frac{\omega_1(k)t - \omega_2(k)t}{2}\right) \end{aligned} \tag{2}$$

with $\omega_i(k) \equiv \sqrt{\vec{k}^2 + m_i^2}$, describing the nonvanishing probability of a flavored particle with momentum $\vec{k}$ to be created with a certain flavor (A) and be detected later on with a different flavor (B).

The form of the transformation between flavored and massive particles (1) is reflected in the relativistic field formalism by the relation

$$\begin{aligned} \nu_A(x) &= \nu_1(x)\cos\theta + \nu_2(x)\sin\theta \\ \nu_B(x) &= -\nu_1(x)\sin\theta + \nu_2(x)\cos\theta \end{aligned} \tag{3}$$

that connects flavor fields ν_A, ν_B with massive ones ν_1, ν_2. Such a relation is connected with the linearization of the following Lagrangian for free spin-$\frac{1}{2}$ fields

$$\begin{aligned} \mathcal{L} = {} & i\bar{\nu}_A(x)\partial\!\!\!/\nu_A(x) + i\bar{\nu}_B(x)\partial\!\!\!/\nu_B(x) - m_A\bar{\nu}_A(x)\nu_A(x) \\ & - m_B\bar{\nu}_B(x)\nu_B(x) - m_{AB}(\bar{\nu}_A(x)\nu_B(x) + \bar{\nu}_B(x)\nu_A(x)), \end{aligned} \tag{4}$$

which becomes

$$\begin{aligned} \mathcal{L} = {} & i\bar{\nu}_1(x)\partial\!\!\!/\nu_1(x) + i\bar{\nu}_2(x)\partial\!\!\!/\nu_2(x) \\ & - m_1\bar{\nu}_1(x)\nu_1(x) - m_2\bar{\nu}_2(x)\nu_2(x), \end{aligned} \tag{5}$$

when

$$\begin{aligned} m_A = m_1\cos^2\theta + m_2\sin^2\theta \qquad m_B = m_1\sin^2\theta + m_2\cos^2\theta \\ m_{AB} = (m_2 - m_1)\sin\theta\cos\theta. \end{aligned} \tag{6}$$

However, in the field-theoretical framework the decomposition of the fields (3) into ladder operators associated with flavor particle states is highly nontrivial [6]. It has been shown, indeed, that states defined as the relativistic equivalent of (1), for which $|\nu_{1,2}\rangle$ belongs to the mass-$m_{1,2}$ irreducible representation of the Poincaré group, are *not* eigenstates of the flavor charge operators [34,35], which for the theory (4) read [34]

$$Q_A(t) = \int d\vec{x}\nu_A^\dagger(x)\nu_A(x), \quad Q_B(t) = \int d\vec{x}\nu_B^\dagger(x)\nu_B(x). \tag{7}$$

BV formalism compensate for this [7,16,36], by defining appropriate flavor eigenstates via the action of a certain operator G_θ on massive states:

$$|\vec{k}_1, f_1; \vec{k}_2, f_2; \vec{k}_3, f_3; \ldots\rangle \equiv G_\theta^\dagger|\vec{k}_1, m_{(1)}; \vec{k}_2, m_{(2)}; \vec{k}_3, m_{(3)}; \ldots\rangle, \tag{8}$$

where $|\vec{k}_1, f_1; \vec{k}_2, f_2; \vec{k}_3, f_3; \ldots\rangle$ denotes a state with different flavor particles described by their momenta $\vec{k}_i$ and their flavors $f_i = A, B$, whereas $|\vec{k}_1, m_{(1)}; \vec{k}_2, m_{(2)}; \vec{k}_3, m_{(3)}; \ldots\rangle$ denotes a state with different massive particles described by their momenta $\vec{k}_i$ and their masses $m_{(i)} = m_1, m_2$, defined from the linearized theory (5). The operator G_θ is defined by the equations:

$$\nu_A(x) = G_\theta^{-1}\nu_1(x)G_\theta \qquad \nu_B(x) = G_\theta^{-1}\nu_2(x)G_\theta \tag{9}$$

and its explicit form depends on the specific theory considered. For the theory (4), G_θ is written as [7]

$$G_\theta(t) = e^{(\theta/2)\int d\vec{x}(\nu_1^\dagger(x)\nu_1(x) - \nu_1^\dagger(x)\nu_2(x))}. \tag{10}$$

Among all flavor states defined in the BV approach, the one called *flavor vacuum* and defined by

$$|0\rangle_f \equiv G_\theta^\dagger |0\rangle \tag{11}$$

plays a special role, since it represents the physical vacuum. In this context, by *physical vacuum* we mean the state that represents the physical empty state, i.e. with no particle in it. Since only particles with well-defined flavor, rather than mass (i.e. Hamiltonian eigenstates), can be created/detected, one expects the physical vacuum to be represented by the state that counts no flavor particles in it. It can be shown that this is state is $|0\rangle_f$, rather than $|0\rangle$ [7].[1]

Furthermore, it has been proven that *all* flavor states $|\vec{k}_1, f_1; \vec{k}_2, f_2; \vec{k}_3, f_3; \ldots\rangle$ are *orthogonal* to each massive state $|\vec{k}_1, m_{(1)}; \vec{k}_2, m_{(2)}; \vec{k}_3, m_{(3)}; \ldots\rangle$, and therefore $\langle 0|0\rangle_f = 0$ follows as a particular case. This result enables us to talk of a *Fock space for flavor states*, in opposition of the usual Fock space, whose basis is given by $\{|\vec{k}_1, m_{(1)}; \vec{k}_2, m_{(2)}; \vec{k}_3, m_{(3)}; \ldots \vec{k}_n, m_{(n)}\rangle \mid \forall n \in \mathbb{N}\}$.

The orthogonality of the two spaces is not very surprising if one regards BV formalism as a nonperturbative approach to the interactive theory defined by (4).

In the second-quantization framework, the Hilbert space representing physical states is defined by vectors in the number occupation representation: assuming that a single-particle state can be classified by a discrete set of states labeled by the index $i = 1, 2, 3, \ldots$, a vector representing a many-particle state can be identified by the number n_i of particles occupying the i-th state and it is denoted with $|n_1, n_2, n_3, \ldots\rangle$; the Hilbert space of physical states $\mathfrak{H}$ is therefore defined as the vector space generated by the basis $\{|n_1, n_2, n_3, \ldots\rangle\}$. For bosons $n_i \in \mathbb{N}_0$, whereas for fermions $n_i = 0, 1$. It can be shown that in both cases the set $\{|n_1, n_2, n_3, \ldots\rangle\}$ is *uncountable* and therefore $\mathfrak{H}$ is nonseparable [37]. In particle physics a separable subset of $\mathfrak{H}$, a Fock space that we will denote with $\mathfrak{F}_0$, is usually considered [38]. $\mathfrak{F}_0$ carries an irreducible representation of the Poincaré group and particle states belonging to it have well-defined mass and spin. Such a subset is spanned by the *countable* basis of all states with an arbitrary, yet finite in total, number of free particles. Although this basis does not fully describe $\mathfrak{H}$ (for instance the vector $|1, 1, 1, \ldots\rangle$ which counts an infinite number of particles is not included), it is sufficient for accounting for scattering processes at a *perturbative* level. In the usual perturbation theory all *interactive* processes are indeed approximated by means of a superposition of a finite number of *free* particle states. This is quite clear in the functional formalism, when Feynman diagrams are considered. In a simplified picture of this framework, a scattering process is represented by a graph with a certain number of external legs (incoming and outgoing particles). The total number of internal lines is connected to the precision of the approximation used in the perturbative expansion: the higher the order of the perturbation, the higher the number of vertices, and therefore the higher the number of internal lines involved. Each line can be naïvely interpreted as a single free particle state, which is emitted in the starting vertex and then absorbed in the ending one. At each order in perturbation theory, a finite number of free particle states enters in the description of the scattering process. However, under the assumption that the perturbative series converges, its limit would be described by an *infinite* number of lines/one-free-particle-states. In bra-ket formalism such a limit state would therefore be represented by a vector of $\mathfrak{H}$, the space of *all* physical states, but not of $\mathfrak{F}_0$, the space of states with *finite* number of free particles. With this example we want to remark the nontrivial difference existing from a *free* theory and an *interactive* one: we can express interactive processes in terms of free states (which have no direct physical meaning or interpretation, being just a basis in which we choose to express our process) but only in a weakly-interactive/perturbed framework. A full nonperturbative treatment for interactive particle states requires subspaces of $\mathfrak{H}$, that are orthogonal to the Fock space of free states $\mathfrak{F}_0$ [39].

Coming back to our case, a Lagrangian with flavor mixing, such as (4), can be regarded as an interactive theory, thanks to its nondiagonal terms. Flavor particle states defined *à la* BV form a Fock space $\mathfrak{F}_\mathfrak{f}$ that is therefore orthogonal to $\mathfrak{F}_0$. In other words, we could express flavor states in a perturbative way by means of $\mathfrak{F}_0$; however, BV formalism enables us to construct flavor states in a completely nonperturbative manner, and therefore it requires states that are part of $\mathfrak{H}$ but not of $\mathfrak{F}_0$.

Different Fock spaces are ordinarily used in QFT on curved backgrounds and other contexts [40,41]. In the former, for instance, one identifies Fock spaces for physical states in flat regions. However, these Fock spaces do not coincide (they are different/orthogonal subset of $\mathfrak{H}$) if the regions are not connected and curved regions exist in between. As a consequence, the vacuum defined by an observer in a certain region is not necessarily described by the state with no particles by an observer in another region. The *particle creation* phenomenon may occur: a state that is empty for an observer can actually contain particles according with a different observer. This mechanism characterizes both of the two main results of QFT in curved spacetime: the Unruh effect [42] and the Hawking radiation [43].

In a formal analogy, BV formalism introduces a ground state, called *flavor vacuum*, which is not as trivial as the ground state for the free theory. As already said, since it is the state in which no flavor particles are present, it correctly represents the *physical vacuum*. Even though it is *empty*, it is characterized by a nonzero expectation value of the stress-energy tensor ${}_f\langle 0|T_{\mu\nu}|0\rangle_f$, whose effects must

[1]In short: flavor states can be built by means of specific creation/annihilation operators for flavor particles; it can be shown that the only state that is annihilated by all annihilation operators is the flavor vacuum defined via (11).

be gravitationally testable. This is true as long as we fix as zero-point of our theory the usual vacuum $|0\rangle$ for the free theory and belonging to $\mathfrak{F}_0$, or, in other words, we consider the usual *normal ordering* $f\langle 0|{:}T_{\mu\nu}{:}|0\rangle_f \equiv_f \langle 0|T_{\mu\nu}|0\rangle_f - \langle 0|T_{\mu\nu}|0\rangle$, which is valid in perturbation theory as well as in this nonperturbative approach. One commonly refers to the flavor vacuum as a *condensate* for the following reason: once expressed in terms of particles with well-defined mass (eigenstates of the Hamiltonian), the flavor vacuum contains an nonvanishing number of those particles, per unit of volume. In our example, they are characterized by the following distribution over the momentum space [20]

$$_f\langle 0|n(\vec{k})|0\rangle_f = \frac{\sin^2\theta}{4\pi^3}\frac{\omega_1(k)\omega_2(k) - m_1 m_2 - k^2}{\omega_1(k)\omega_2(k)} \tag{12}$$

with $n(\vec{k}) \equiv \sum_r (a_1^{r\dagger}(\vec{k})a_1^r(\vec{k}) + a_2^{r\dagger}(\vec{k})a_2^r(\vec{k}))$, $k \equiv |\vec{k}|$, and $a_i^{r(\dagger)}$ representing ladder operators for particles with well-defined mass. However, since the physical degrees of freedom of the theory are flavor particles (the only kind of particle that can be produced and detected), the interpretation as a *gas* or *collection of particles* remains at a mere mathematical level, the flavor vacuum being absolutely empty from a physical point of view (in the sense that no flavored particles are present in it), and only characterized by a nonvanishing stress-energy tensor expectation value which is detectable via gravitational effects.

The features of the flavor vacuum depend on the model considered and a preliminary investigation on a simple supersymmetric model [32] showed that it might behave very differently, according with the spin of the particles involved.

III. PHENOMENOLOGY OF A SUSY FLAVOR VACUUM

A. Free WZ *à la* BV

Our interest in BV formalism was firstly motivated by physics beyond the standard model (SM). The Wess-Zumino model here discussed has been considered in [32] after two works in which the *flavor vacuum* has been regarded as an effective vacuum arising in a string-theoretical framework [44,45]. Indeed, a specific model from the *braneworld* scenario, called *D-particle foam model* [46–50], seems to explain neutrino flavor oscillations in terms of flavor oscillation of fundamental strings, in presence of a "cloud" (or *foam*) of pointlike topological defects in the bulk space. In the spirit of weak coupling string theory, the interaction between the foam and strings/branes in the theory can be regarded as "vacuum defects" from the point of view of a macroscopical observer. Therefore it has been suggested that BV formalism, together with its "flavor vacuum" condensate, might provide a suitable description of the low energy limit of the model.

In [32] we presented the behavior of the *flavor vacuum*, in a simple supersymmetric theory. The model that was considered involves two free real scalars $S_A(x)$, $S_B(x)$ with mixing, two free real pseudoscalars $P_A(x)$, $P_B(x)$ with mixing and two free Majorana spinors $\psi_A(x)$, $\psi_B(x)$ with mixing:

$$\begin{aligned}\mathcal{L} = &\sum_{\iota=A,B}[\partial_\mu S_\iota(x)\partial^\mu S_\iota(x) + \partial_\mu P_\iota(x)\partial^\mu P_\iota(x)\\ &+ i\bar{\psi}_\iota(x)\partial\!\!\!/\psi_\iota(x)] - \sum_{\iota,\kappa=A,B}[m_{\iota\kappa}^2 S_\iota^2(x) + m_{\iota\kappa}^2 S_\iota^2(x)\\ &+ m_{\iota\kappa}\bar{\psi}_\iota(x)\psi_\kappa(x)]\end{aligned} \tag{13}$$

with $m_{AB} = m_{BA}$. Terms involving products of fields of different flavors disappear when one expresses the model in terms of new fields, obtained by appropriate rotations of the previous ones:

$$\begin{aligned}\phi_A(x) &= \cos\theta\phi_1(x) + \sin\theta\phi_2(x)\\ \phi_B(x) &= -\sin\theta\phi_1(x) + \cos\theta\phi_2(x)\end{aligned} \tag{14}$$

with $\phi = S, P, \psi$, leading to

$$\begin{aligned}\mathcal{L} = &\sum_{i=1,2}[\partial_\mu S_i(x)\partial^\mu S_i(x) - m_i^2 S_i^2(x) + \partial_\mu P_i(x)\partial^\mu P_i(x)\\ &- m_i^2 P_i^2(x) + \bar{\psi}_i(x)(i\partial\!\!\!/ - m_i)\psi_i(x)].\end{aligned} \tag{15}$$

From this latter it is possible to build the usual Fock space for massive particles, previously denoted as $\mathfrak{F}_0$, which has as ground state the *"massive"* vacuum $|0\rangle$. The *flavor* vacuum is hence defined as

$$|0\rangle_f \equiv e^{\theta\int d\vec{x}(X_{12}(x) - X_{21}(x))}|0\rangle \tag{16}$$

with

$$X_{12}(x) \equiv \frac{1}{2}\psi_1^\dagger(x)\psi_2(x) + i\dot{S}_2(x)S_1(x) + i\dot{P}_2(x)P_1(x). \tag{17}$$

Its features have been explored via its stress-energy tensor expectation value, being

$$\begin{aligned}T_{\mu\nu}(x) = &\sum_{i=1,2}[2\partial_{(\mu}S_i(x)\partial_{\nu)}S_i(x) + 2\partial_{(\mu}P_i(x)\partial_{\nu)}P_i(x)\\ &+ i\bar{\psi}_i(x)\gamma_{(\mu}\partial_{\nu)}\psi_i(x)] - \eta_{\mu\nu}\mathcal{L}.\end{aligned} \tag{18}$$

It has been shown that the flavor vacuum behaves as a perfect relativistic fluid, *i.e.*

$$_f\langle 0|T_{\mu\nu}|0\rangle_f = \mathrm{diag}\{\rho, \mathsf{P}, \mathsf{P}, \mathsf{P}\} \tag{19}$$

with

$$\begin{aligned}\rho &\equiv_f \langle 0|T_{00}(x)|0\rangle_f\\ &= \sin^2\theta\frac{(m_1 - m_2)^2}{2\pi^2}\int_0^K dk k^2\left(\frac{1}{\omega_1(k)} + \frac{1}{\omega_2(k)}\right)\end{aligned} \tag{20}$$

$$\mathsf{P} \equiv_f \langle 0|T_{jj}(x)|0\rangle_f = -\sin^2\theta \frac{(m_1^2 - m_2^2)}{2\pi^2} \int_0^K dk k^2 \left(\frac{1}{\omega_2(k)} + \frac{1}{\omega_1(k)}\right) \quad (21)$$

ρ representing its energy density, P its pressure and K a momentum cutoff (cf. [31,32]).[2]

In particular, disentangling the contribution of the bosonic sector from the fermionic one, one finds

$$\rho_b = \sin^2\theta \int_0^K dk \frac{k^2}{\pi^2}(\omega_1(k) + \omega_2(k)) \frac{(\omega_1(k) - \omega_2(k))^2}{2\omega_1(k)\omega_2(k)} \quad (22)$$

$$\rho_f = \sin^2\theta \int_0^K dk \frac{k^2}{\pi^2}(\omega_1(k) + \omega_2(k)) \times \left(\frac{\omega_1(k)\omega_2(k) - m_1 m_2 - k^2}{\omega_1(k)\omega_2(k)}\right) \quad (23)$$

$$\mathsf{P}_b = -\rho_b, \qquad \mathsf{P}_f = 0 \quad (24)$$

in which the standard normal order has being adopted.

B. Flavor vacuum as a source of dark matter

An important result emerges from the above outlined analysis: the equation of states $w \equiv \mathsf{P}/\rho$ for the bosonic and the fermionic sectors are different, $w_b = -1$ and $w_f = 0$ holding. The emphasis on the novelty of this supersymmetry (SUSY) breaking mechanism has already being remarked [32]. We are now aimed to explore the interesting phenomenology connected with such a result. Our simple model implies a physical vacuum that is a combination of two fluids which behave quite differently: both permeate the empty space uniformly and statically, but one has a cosmological-constant-like behavior ($w = -1$), while the other behaves as dust ($w = 0$). The role of the flavor vacuum as source of dark energy (which now is played only by the bosonic sector of the theory) has been extensively discussed in literature [21–30]. Here we present a new feature of the flavor vacuum: its contribution to dark matter.

Dark matter is the name given to unknown sources of gravitational effects, whose presence, primarily within and around galaxies, has been established by many astrophysical data [51,52]. Numerical simulations of structure formation have shown that "hot" (relativistic) particles cannot explain the observed structures at galactic scales, therefore dark matter is expected to be made out of fairly massive and "cold" (nonrelativistic) particles. Big-bang nucleosynthesis limits on the average baryonic content of the Universe exclude that (the majority of) dark matter is made out of ordinary baryonic matter (i.e. atoms). Furthermore, although "dark", in the sense that does not emit nor absorb light (i.e. electromagnetically neutral), dark matter might couple to ordinary matter in other ways (besides gravity); however, arguments on its density and thermal production at early times imply that such a coupling must be weak.

Both astrophysics and particle physics have been proposing suitable candidates for dark matter through the last three decades, giving rise to an enormous wealth of choice. However, because of the absence of direct detections and the lack of predictions by theoretical models, plagued by an undesirable abundance of free parameters, the nature of dark matter remains elusive.

The fermionic sector of the flavor vacuum in the model here presented clearly fulfills basic requests for a dark matter candidate: it contributes to the energy content of the universe; it is dark (i.e. it is an electromagnetically neutral object, since (s)neutrino fields do not couple with the electromagnetic field); furthermore, it does not interact with any other of the SM particles (excluding gravitational effects), being the *empty* state for the (s)neutrino sector; unlike its bosonic counterpart, it is purely *pressureless*.[3]

A possible concern about its uniform distribution in space, in contrast with the observed distribution of dark matter which is usually gathered in clusters around and inside galaxies, can be easily dispelled by recalling that we are actually modeling a simple empty universe. If a nonuniform matter distribution is considered in our toy universe in addition to the flavor vacuum, it would start to interact gravitationally with our vacuum condensate. Thanks to initial irregularities in the matter distribution, we expect them to form clusters via *gravitational instability* (gravity tends to enhance irregularities, pulling matter towards denser regions [53]), as the system evolves with time. It is known that such an effect, on the other hand, does not necessarily occurs for dark energy-like fluids [54], as the bosonic component of the flavor vacuum, which can persist in their state of spatial uniformity even in presence of clustered matter. The evolution of our flavor vacuum, considering both its bosonic and fermionic components, in presence of other matter and gravitational interaction, represents necessarily an object of future studies.

C. Testability

An interesting aspect of the model concerns the interplay between the two fluids. Supersymmetry imposes that

[2]From the perspective of considering BV formalism as an effective formalism for physics beyond the standard model [32,44], such a cutoff must be interpreted as the energy scale up to which the formalism provides the framework for a good effective theory.

[3]This is certainly true for the free above mentioned model; the possibility of extending this result to interactive models will be discussed later on.

the energy density of the bosonic component is tied up to the energy density of the fermionic component; in a more realistic theory, therefore, one should be able to reproduce the current experimental value of the ratio between the dark energy density and the dark matter energy density (~ 2.8), in the optimist belief that the flavor vacuum is the only responsible for both of them. The role of a curved background in the formulation of the theory might be crucial, since the energy density of a dustlike fluid gets diluted by the expansion of the universe, whereas such an effect does not occur for a cosmological-constant type, and therefore the ratio between those two quantities changes dramatically with time.

Within a momentum cutoff-regularization framework, as the one here presented, the two energy densities depend on such a cutoff, which is the *same* for both quantities. The ratio between them can in general be cutoff dependent, as it actually is in the case here presented. On one side, one might hope that in a more realistic theory (on a curved background, for instance) the ratio might be cutoff independent. On the other hand, one could consider the opposite situation, in which the ratio varies with the cutoff, as highly desirable: if the ratio is fixed from the cutoff, the same value for the cutoff would also fix the value of the energy. This implies that once the cutoff is decided on the basis of experimental data on the ratio, the model gives a precise prediction for the absolute values of the energy densities, which can be compared with their observational estimates.

In order to illustrate these ideas we will present a concrete example. Let us assume that our supersymmetric model is effective up to the energy scale K (which comes from deeper theories, as, for instance, in [32]). In the standard big-bang picture, this means that when the universe cools down to that energy, the flavor vacuum starts to be the effective description of the vacuum state of the (unknown) underlying theory. We call t_0 the time corresponding with this transition and a_0 the corresponding scale factor.

In our toy universe, we assume that at t_0, in absence of any other sources of energy or matter, the energy/matter content of our toy universe is only due to the flavor vacuum. Moreover, we assume that it can be describe, at a classic level (i.e. on sufficiently large scales), in terms of two fluids: a first one, due to the bosonic component of the flavor vacuum and described by ρ_b and $w = -1$, and a second one, due to its fermionic component and described by ρ_f and $w = 0$. We will regard the bosonic component as the only source of dark energy and the fermionic as the only source of dark matter. Both ρ_b and ρ_f are function of the following parameters: (s)neutrino masses, mixing angles, and the cutoff K. If we know (from observations) the neutrino masses and mixing angles, and we can constrain parameters induced by SUSY breaking, the cutoff is the only parameter left to determine.

As our toy universe expands, we assume that the two fluids obey Einstein equations and therefore they scale as

$$\rho_f(t) \quad a(t)^3 = \rho_f a_0^3 \qquad \rho_b(t) = \rho_b. \tag{25}$$

This means that *today* their value is

$$\rho_f(t_{\rm now}) = \rho_f a_0^3 \qquad \rho_b(t_{\rm now}) = \rho_b, \tag{26}$$

respectively, being $a(t_{\rm now}) = 1$ by convention. Those two quantities depends on the following parameters: (s)neutrino masses, mixing angles, cutoff energy, scale factor at t_0. Provided with these expressions, we can then test our model in two ways.

(1) If observational data enable us to constrain (s)neutrino masses, mixing angles, dark matter and dark energy densities, from (26) we can derive the other parameters left: the cutoff energy and the scale factor a_0. Well equipped with all the parameters of the theory, we will then be able to check if the model is in reasonable agreement with other standard models. For example, if the scale factor a_0 fitting all data corresponds to a time *in the future* (for $a_0 > 1$), the model has to be rejected, or corrected at least.

(2) On the other hand, theoretical reasons might suggest specific values for the cutoff (if for instance the flavor vacuum rises in the low energy limit of an underlying theory, and/or the scale factor a_0, being the temperature of the universe inversely proportional to its scale factor. In this case, we might be able to make *a prediction* on the value of the dark matter and dark energy density, via formulae (26), that might be compared with observational estimates. On this basis our model is therefore accepted or refused.

D. A preliminary test

The simple toy model discussed in Sec. III A is not realistic enough to hold the comparison with data already available: only two generations of neutrinos have been considered, neither matter or interactions are present, SUSY is unbroken, there is no prescription for the cutoff K.[4] However, some preliminary tests can be performed.

As just explained, in a realistic theory with three generations instead of two, it is possible to constrain the parameter space of mixing angles and masses thanks to observational data. In absence of such a theory, we will limit our selves to check if our simpler model admits a choice of parameters that gives rise to physically ‘‘plausible’’ estimations for dark energy and dark matter densities. More precisely, we shall check the compatibility of our model with the relation

[4]The assumption of treating neutrinos as Majorana particles might also be questioned.

$$\rho_\Lambda \sim \rho_{\rm DM} \sim (\Delta m_{ij}^2)^2, \qquad (27)$$

with $\rho_{\Lambda/{\rm DM}}$ the dark energy/matter density today and Δm_{ij} the difference of the squared masses of neutrinos,[5] which for our model becomes

$$\rho_b \sim \rho_f(t_{\rm now}) \sim (\Delta m^2)^2, \qquad (28)$$

in the assumption that all dark energy and dark matter of our toy universe is due to the flavor vacuum. In other words, is there a sensible region of the parameter space that gives rise to (28)?

Once provided with a realistic theory, the reasoning goes the other way around: given the space of parameters constrained by observational data, does (28) hold? However, the analysis on which we are embarking is neither irrelevant nor negligible: previous analyses hardly conciliate the very different scales of energies entering into the problem, such as the momentum cutoff, which presumably is greater then the TeV scale, and neutrino mass differences (cf. [28]). The aim of this section is therefore to show that even in our simple toy model, nonperturbative formulae describing the features of the flavor vacuum can accommodate very different scales in a natural way, giving rise to physically sensible values for dark energy and dark matter densities.

Recapitulating, in the following we shall assume that the physical vacuum is effectively described by the flavor vacuum defined by (16) for energies lower than K; over large distance scales, such a flavor vacuum behaves as a classical fluids, obeying Einstein equations (*i.e.* (25) and (26) hold). We will further assume that some radiation and matter are present in our toy universe, whose density is at least one order lower than the flavor vacuum density. Their presence justifies the notion of ‘‘temperature’’ and defines the profile of the time-evolution of our toy universe. Since we assume the neutrino sector not being coupled with any other fields, the flavor vacuum and the matter/radiation content of the universe interact only gravitationally. As mentioned, we expect this interaction to lead the fermionic flavor vacuum to cluster together with ordinary matter, leaving the bosonic flavor vacuum homogeneously distributed. These effects are reasonably expected as long as gravitational effects are relevant only on cosmological scales, at which the flavor vacuum is well approximated by a classical fluid. As a consequence of a possible nonuniform distribution in space of the fermionic fluid, we shall consider the value of (23) not as a local attribute but as a global, or ‘‘averaged’’ over sufficiently large scales, property.

In order to check the compatibility of our model with (28), we start by defining the quantity $\xi \equiv (1 - m_2^2/m_1^2)^2$, with m_2 the smaller of the two masses ($m_2 < m_1$). Since $0 < \xi < 1$, we can expand (22) and (23) in series around $\xi = 0$ and get to

$$\rho_b = \frac{\sin^2\theta}{\pi^2} m_1^4 f(K/m_1)\xi + \mathcal{O}(\xi^{3/2}) \approx \frac{\sin^2\theta}{\pi^2}(\Delta m^2)^2 f(K/m_1)$$
$$\rho_f = \frac{\sin^2\theta}{\pi^2} m_1^4 g(K/m_1)\xi + \mathcal{O}(\xi^{3/2}) \approx \frac{\sin^2\theta}{\pi^2}(\Delta m^2)^2 g(K/m_1) \qquad (29)$$

with

$$f(K/m_1) = \int_0^{K/m_1} dx \frac{x^2}{4(1+x^2)^{3/2}} \qquad (30)$$

$$g(K/m_1) = \int_0^{K/m_1} dx \frac{x^4}{4(1+x^2)^{3/2}} \qquad (31)$$

and $m_1^4\xi = (\Delta m^2)^2 \equiv (m_1^2 - m_2^2)^2$. Relations (29) are good approximations of the exact values, as long as the two masses are very similar $m_1 \sim m_2$. All divergencies connected with our problem are included in function $f(K/m_1)$ and $g(K/m_1)$, for their argument running to infinity. In the following analysis a physical cutoff of momenta (K, rescaled by the neutrino mass m_1) will be considered, in the belief that flavor physics *à la* BV must be regarded as an effective description at low energy scales of a deeper theory [44]. Clearly other renormalization tools might be required if this assumption is dropped, for instance, in a pure self consistent quantum field-theoretical approach. Despite this choice, it is remarkable, however, that the relation $\rho_{b/f} \propto (\Delta m^2)^2$ has been derived entirely analytically.

In the following, we would like to show that in a cutoff-regularization scheme the two functions in (30) can give rise to physically sensible values, i.e. the function $f(K/m_1)$ remains relatively small even when the cutoff is very high (giving rise to the hierarchy: high cutoff/low dark energy density), whereas $g(K/m_1)$ can be considerably greater then $f(K/m_1)$ for the same choice of cutoff, motivating the observed discrepancy between the dark energy and

[5] The energy scale of dark energy is far away from all natural scales provided by the SM via particle masses. Only one fundamental scale is known to be comparable with the dark energy one: the scale of neutrino physics. Boundary on total masses of neutrinos show that they are much lighter then all other particles: astrophysical data indicate that $\Sigma m_\nu < 058$ eV, with 95% of confidence [55] (the sum runs over all possible species—possibly more then three—that where present in the early Universe). Moreover, direct observations on solar and atmospheric neutrinos show that $\Delta m_{12}^2 \approx 8 \cdot 10^{-5}$ eV^2 and $\Delta m_{23}^2 \approx 2.5 \cdot 10^{-3}$ eV^2 (being $m_i^2 - m_j^2 \equiv \Delta m_{ij}^2$, cf. [33] and references therein). These mass scales $10^{-1} \div 10^{-2}$ eV have to be compared with the scale 10^{-3} eV, that one obtains from $\rho_\Lambda = 3 \times 10^{-11}$ eV^4. This ‘‘coincidence’’ gave rise to many works, besides the ones connected with BV formalism, aimed to provide a theoretical explanation for it (see [56] and references therein). The more famous *coincidence problem* regarding dark energy concerns the similar density of dark energy and dark matter ($\rho_\Lambda \approx 2.8\rho_{\rm DM}$) as measured today, which requires a notable fine-tuning of initial conditions considering their very different evolution in time. These two ‘‘coincidences’’ are combined together in formula (27).

dark matter densities at early times. We should stress once more that a strict comparison with available experimental data would be possible once a more realistic model will be given.

We will now focus on the former of (29). Being SUSY unbroken in our toy model, neutrinos and sneutrinos have the same masses. So the m_1 and m_2 appearing in (29) are the masses of two neutrinos, even though ρ_b encodes the contribution of the bosonic sector of the theory, which in a realistic case would be affected by the breaking of SUSY via effective masses greater than m_i. Recalling the observed relation (27), we wonder now if is there any region of the parameter space (θ, K) that might generate a similar situation (*i.e.* $\rho_b \sim (\Delta m^2)^2$) in our model. Quite interestingly, the condition

$$\frac{\sin^2\theta}{\pi^2} f(K/m_1) \sim 1 \tag{32}$$

is satisfied by a region of the plane (θ, K), which is not that far from the expected value for a realistic theory. As shown in Fig. 1, if the cutoff of the theory lies somewhere between the TeV(10^{12} eV) scale and the Plank scale (10^{28} eV), the value of $\sin^2\theta$ must be around the $0.5 \div 1$ region. Moreover, a complete overlap with one real mixing angle is obtained in a region with a very high cutoff, close to the Planck scale. It should be emphasized that the existence itself of such a region is highly nontrivial, since it appears from the combination of parameters spanning a wide range of energies, being $\Delta m^2 \sim 10^{-4}$ eV^2 and $K > 10^{12}$ eV.

Focusing now on the latter of (29), we shall proceed with a similar analysis in order to test the hypothesis that the fermionic sector of the model provides a sensible dark matter candidate. As explained in the previous section, under the assumption that the flavor vacuum behaves as a perfect classical fluid on large scales, the fermionic contribution would get diluted with time as the universe expands. Its value *today* would then be

$$\rho_f(t_{\rm now}) = \rho_f a_0^3 \tag{33}$$

with a_0 the scale factor corresponding to the time at which the model became effective. In order to reproduce the relation $(\Delta m^2) \sim \rho_f(t_{\rm now})$ we expect

$$a_0^3 \frac{\sin^2\theta}{\pi^2} g(K/m_1) \sim 1 \tag{34}$$

that can be obtained by combining (33), (28), and (29). Because of the constrain on the dark energy density (32) the above condition becomes

$$a_0^3 \frac{g(K/m_1)}{f(K/m_1)} \sim 1. \tag{35}$$

We already know that a_0 must be extremely small, but what is the ratio between $g(K/m_1)$ and $f(K/m_1)$ for large K/m_1 ($K/m_1 > 10^{14}$)? As shown in Fig. 2, Eq. (35) is indeed satisfied for large values of K and very small values of a_0, as one would expect from a realistic theory. If, for instance, the cutoff is set equal to the Planck scale, relation (35) is satisfied for $a_0 = 10^{-20}$, which sets the transition phase (when the flavor vacuum became effective) well far in the past, when our toy universe was 10^{20} times smaller than

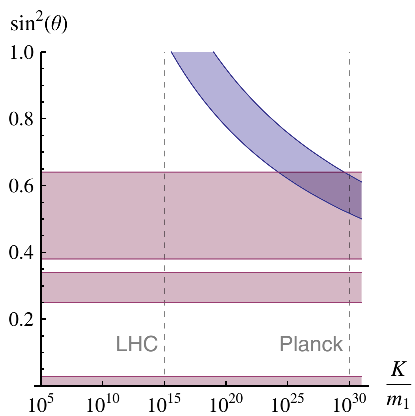

FIG. 1 (color online). The parameter space $(K/m_1, \sin^2\theta)$ is plotted. In blue, the acceptance region for the condition (32) (boundaries of the region correspond to $lhs = 0.9$ and $lhs = 1.1$). Physical mixing angles for three generation of neutrinos are: $\sin^2\theta_{13} = 0.000^{+0.028}$, $\sin^2\theta_{12} = 0.30^{+0.04}_{-0.05}$, $\sin^2\theta_{23} = 0.50^{+1.4}_{-1.2}$ (within 2σ) [61]. These values are also plotted in the graph (red regions). Dotted lines represents LHC energy scale (~ 1 TeV) and the Planck energy scale ($\sim 10^{28}$ eV) fixing $m_1 = 10^{-2}$ eV.

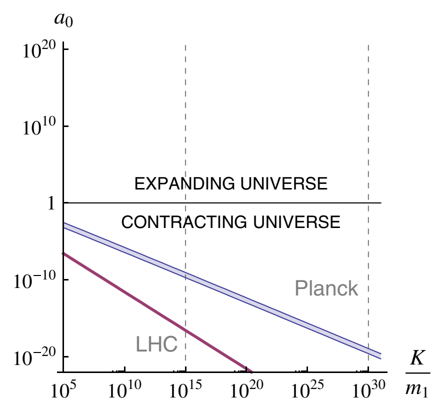

FIG. 2 (color online). The parameter space $(K/m_1, a_0)$ is plotted. In blue, the acceptance region for the condition (32) (boundaries of the region correspond to $lhs = 10$ and $lhs = 0.1$). The red line depicts the scale factor of the real universe as a function of its average temperature, fixing $m_1 = 10^{-2}$ eV. In a more realistic theory, the acceptance region (which may differ from the one here presented) is further reduced by comparison of the first of (29) with dark energy data, which gives a constraint on possible momentum cutoffs; the model is ruled out if the resulting region lies *below* the red line, since it would predict an amount of dark matter greater than what is observed; the model is consistent with data on dark matter if the region overlaps or lies *above* the red line; in this latter case the flavor vacuum contributes only to a *fraction* of total dark matter density.

"now". Once more, parameters characterized by very different values ($a_0 \sim 10^{-20}$, $K/m_1 \sim 10^{30}$) combine together to give rise to a third scale, which has a physical significance (nowadays dark matter density).

Despite the unrealistic nature of our toy model, these first quantitative tests go in the right direction and certainly motivate the study of more realistic models, which hopefully will share with our toy model all the good features here discussed.

IV. TOWARDS INTERACTIVE FLAVOR VACUA

A. A new method of calculation: Free WZ revised

Following the standard literature, the results in [32] discussed so far have been derived by the following approach:

(1) stress-energy tensor has been written in terms of massive fields;
(2) massive fields have been decomposed in terms of massive ladder operators;
(3) massive ladder operators have been written in terms of flavor ladder operators;
(4) hence, the stress-energy tensor has been written in terms of flavor ladder operators;
(5) flavor vacuum expectation value (vev) of the stress-energy tensor has been reduced by acting with the flavor ladder operators on the flavor vacuum.

However, such a long procedure can be avoided and the same exact result may be obtained in a much shorter way, following the steps:

(1) flavor vacuum is written as $|0\rangle_f = G_\theta^\dagger|0\rangle$, the flavor vev of $T_{\mu\nu}$ becoming a vev of $\mathcal{O} \equiv G_\theta T_{\mu\nu} G_\theta^\dagger$;
(2) the stress-energy operator is transformed under the action of the $G_\theta \blacksquare G_\theta^\dagger$, using (13); the transformed operator $\mathcal{O}$ will be expressed in terms of the flavor fields, rather then the massive ones;
(3) (3) is used to write the flavor fields in terms of the massive fields; $\mathcal{O}$ is again expressed in terms of the massive fields, but this time the operator G_θ, and its complicated exponential structure, is not present any more;
(4) a vev of $\mathcal{O}$, expressed as a simple combination of massive fields, is left, which can be reduced by decomposing the massive fields into massive ladder operators.

As a neat example of the above procedure, we will derive the results of Sec. III A via this new method. Recalling the discussion in Sec. III A, in the study of the free WZ model we can consider the bosonic and the fermionic component separately, by evaluating relevant quantities in two separated contexts (a bosonic theory and a fermionic one) and eventually combining together the results. Furthermore, the pseudoscalar and the scalar field are indistinguishable for our purposes, therefore we are allowed to consider just the scalar field, keeping in mind to sum its contribution to the relevant quantities twice.

In the real scalar case, we have

$$T^b_{00}(x) = \sum_i (\pi_i^2(x) + (\vec{\nabla}\phi_i(x))^2 + m_i^2\phi_i^2(x)) \tag{36}$$

with $\pi_i \equiv \dot{\phi}_i$, the conjugate momentum of ϕ_i. Since

$$G_\theta(t) = e^{i\theta \int d\vec{x}(\pi_2(x)\phi_1(x) - \pi_1(x)\phi_2(x))} \tag{37}$$

from which

$$\begin{aligned} G_\theta(t)\phi_1(x)G_\theta^\dagger(t) &= \phi_1(x)\cos\theta - \phi_2(x)\sin\theta \\ G_\theta(t)\phi_2(x)G_\theta^\dagger(t) &= \phi_1(x)\sin\theta + \phi_2(x)\cos\theta \end{aligned} \tag{38}$$

and

$$\begin{aligned} G_\theta(t)\pi_1(x)G_\theta^\dagger(t) &= \pi_1(x)\cos\theta - \pi_2(x)\sin\theta \\ G_\theta(t)\pi_2(x)G_\theta^\dagger(t) &= \pi_1(x)\sin\theta + \pi_2(x)\cos\theta \end{aligned} \tag{39}$$

via the Baker-Campbell-Hausdorff formula

$$\begin{aligned} e^Y X e^{-Y} = X + [Y, X] + \frac{1}{2}[Y, [Y, X]] \\ + \frac{1}{3!}[Y, [Y, [Y, X]]] + \ldots \end{aligned} \tag{40}$$

we can write

$$G_\theta(t)\Big(\sum_{i=1,2}\pi_i^2(x)\Big)G_\theta^\dagger(t) = \Big(\sum_{i=1,2}\pi_i^2(x)\Big), \tag{41}$$

$$G_\theta(t)\Big(\sum_{i=1,2}(\vec{\nabla}\phi_i(x))^2\Big)G_\theta^\dagger(t) = \Big(\sum_{i=1,2}(\vec{\nabla}\phi_i(x))^2\Big) \tag{42}$$

and

$$\begin{aligned} &\langle 0|G_\theta(t)(m_1^2\phi_1^2(x) + m_2^2\phi_2^2(x))G_\theta^\dagger(t)|0\rangle \\ &= \langle 0|(m_1^2\phi_1^2(x) + m_2^2\phi_2^2(x))|0\rangle \\ &\quad + \sin^2\theta(m_1^2 - m_2^2)\langle 0|(\phi_2^2(x) - \phi_1^2(x))|0\rangle. \end{aligned} \tag{43}$$

It follows that

$$\begin{aligned} {}_f\langle 0|T_{00}(x)|0\rangle_f = \langle 0|T_{00}(x)|0\rangle + \sin^2\theta(m_1^2 - m_2^2) \\ \times \langle 0|(\phi_2^2(x) - \phi_1^2(x))|0\rangle \end{aligned} \tag{44}$$

and therefore

$$\begin{aligned} \rho_b &= {}_f\langle 0|{:}T_{00}(x){:}|0\rangle_f \\ &= \sin^2\theta(m_1^2 - m_2^2)\langle 0|(\phi_2^2(x) - \phi_1^2(x))|0\rangle. \end{aligned} \tag{45}$$

Equivalently for $T_{jj}(x)$ we have

$$\begin{aligned} \mathsf{P}_b = {}_f\langle 0|{:}T_{jj}(x){:}|0\rangle_f = -\sin^2\theta(m_1^2 - m_2^2) \\ \times \langle 0|(\phi_2^2(x) - \phi_1^2(x))|0\rangle = -\rho_b. \end{aligned} \tag{46}$$

Once the fields in (45) and (46) are decomposed in terms of the ladder operators and the quantum algebra is simplified, expressions (22) and (23) are correctly reproduced.

In the fermionic case, a similar procedure leads to

$$\rho_f = {}_f\langle 0|{:}T^f_{00}(x){:}|0\rangle_f = \sin^2\theta(m_1 - m_2) \times \langle 0|(\bar{\psi}_2(x)\psi_2(x) - \bar{\psi}_1(x)\psi_1(x))|0\rangle \tag{47}$$

and

$$\mathsf{P}_f = {}_f\langle 0|{:}T^f_{jj}(x){:}|0\rangle_f = 0. \tag{48}$$

By comparing (47) and (45), the analogy between the fermionic and the bosonic condensate that earlier was hidden in formulae (22) and (23) is now more evident. Again, formula (23) is correctly reproduced, once the operatorial structure of the fields is simplified with respect to $\langle 0|\blacksquare|0\rangle$. The expression (47) dispels any doubts concerning formula (23) and its possible dependency on the specific form of the gamma matrices and spinors used to achieve the results of [32], being (47) independent of such a choice [57].

Furthermore, supersymmetry enables us to rewrite this result in terms of the bosonic fields only. For the *massive* vacuum $|0\rangle$ we know that

$$\langle 0|T_{\mu\nu}(x)|0\rangle = 0, \tag{49}$$

which leads to

$$\langle 0|\bar{\psi}_i(x)\psi_i(x)|0\rangle = -4m_i\langle 0|\phi_i^2(x)|0\rangle \tag{50}$$

and hence

$$\rho_{WZ} = 2\sin^2\theta(m_1 - m_2)^2\langle 0|(\phi_1^2(x) + \phi_2^2(x))|0\rangle \tag{51}$$

$$\mathsf{P}_{WZ} = -2\sin^2\theta(m_1^2 - m_2^2)\langle 0|(\phi_2^2(x) - \phi_1^2(x))|0\rangle \tag{52}$$

in accordance with (20) and (21).

The procedure exemplified in the previous section can be easily implemented for other fields: one might want to consider Dirac or two component Weyl spinors as well as complex scalar fields, getting to analogous results. For mere speculative reasons, applications to vector fields or even more complex objects might be thought: the method involves a manipulation of the stress-energy tensor, with the use of equation of motion of the field and its (anti-) commutation rules, regardless of the tensorial or spinorial structure of the field itself. Furthermore, extending these results to more then two flavors is rather straightforward.

Finally, the method enables us to distinguish among all the terms of the stress-energy tensors the ones that really contribute to the final result. This might be helpful in understanding the behavior of the flavor vacuum in more realistic theories, as we shall see in the forthcoming sections.[6]

B. Self-interactive bosons

An example of the applications just discussed is offered by a $\lambda\phi^4$ model. The theory

$$\mathcal{L} = \sum_{i=1,2}(\partial_\mu\phi_i\partial^\mu\phi_i - m_i^2\phi_i^2 - \lambda\phi_i^4) \tag{53}$$

can be regarded as derived from a model with flavor mixing:

$$\mathcal{L} = \partial_\mu\phi_A\partial^\mu\phi_A + \partial_\mu\phi_B\partial^\mu\phi_B - m_A^2\phi_A^2 - m_B^2\phi_B^2 - m_{AB}^2\phi_A\phi_B - \sum_{\iota,\kappa,\lambda,\rho=A,B} g_{\iota\kappa\lambda\rho}\phi_\iota\phi_\kappa\phi_\lambda\phi_\rho \tag{54}$$

with the usual rotation

$$\phi_A = \cos\theta\phi_1 - \sin\theta\phi_2 \quad \phi_B = \sin\theta\phi_1 + \cos\theta\phi_2 \tag{55}$$

and a specific choice of the coupling constants $g_\blacksquare$. Since the expression of G_θ in terms of the fields can be deduced from

$$G_\theta^\dagger\phi_1G_\theta = \cos\theta\phi_1 - \sin\theta\phi_2$$
$$G_\theta^\dagger\phi_2G_\theta = \sin\theta\phi_1 + \cos\theta\phi_2 \tag{56}$$

just using commutation relations between fields and conjugate momenta, which are not modified by the form of the Lagrangian [57], expression

$$G_\theta = e^{i\theta\int d\vec{x}(\dot{\phi}_2\phi_1 - \dot{\phi}_1\phi_2)} \tag{57}$$

that was found valid in the free case, holds also in the interactive one.

If we *assume* that the flavor vacuum is defined as

$$|0\rangle_f \equiv G_\theta^\dagger|0\rangle \tag{58}$$

naïvely generalizing the free case, with $|0\rangle$ the ground state of the theory described by (53), we can easily see that

$$\begin{aligned} {}_f\langle 0|T_{\mu\nu}|0\rangle_f = {}& \langle 0|T_{\mu\nu}|0\rangle + \eta_{\mu\nu}(\sin^2\theta(m_1^2 - m_2^2) \\ & \times \langle 0|\phi_2^2 - \phi_1^2|0\rangle) + \eta_{\mu\nu}\lambda\langle 0|((\phi_2\cos(\theta) \\ & - \phi_1\sin(\theta))^4 + (\phi_1\cos(\theta) \\ & + \phi_2\sin(\theta))^4 - \phi_1^4 - \phi_2^4)|0\rangle. \end{aligned} \tag{59}$$

[6] In addition, we would like to stress that the method does not require an explicit decomposition of the flavor fields in terms of flavor ladder operators. Such a decomposition has been object of a debate in literature, raised by the authors of [10]. Although the problem was exhaustively discussed in [9,11], not all the community was convinced by the arguments presented [18]. Without entering into the details of the dispute, here we would like to suggest that a different point of view on the formalism, such as the one offered by formulae (47) and (45), where an observable quantity concerning a flavor state has been calculated without the explicit use of the controversial decomposition, might help in a deeper understanding of the problem and the formalism itself.

We can therefore state that the equation of state is given by

$$w = \frac{{}_f\langle 0|{:}T_{jj}{:}|0\rangle_f}{{}_f\langle 0|{:}T_{00}{:}|0\rangle_f} = \frac{-\langle 0|\because \sum_{i=1,2}(m_i^2\phi_i^2 + \lambda\phi_i^4)\because|0\rangle}{\langle 0|\because \sum_{i=1,2}(m_i^2\phi_i^2 + \lambda\phi_i^4)\because|0\rangle} = -1 \tag{60}$$

in which

$$\because f(\varphi_1, \varphi_2)\because \equiv f(\cos\theta\varphi_1 - \sin\theta\varphi_2, \sin\theta\varphi_1 + \cos\theta\varphi_2) - f(\varphi_1, \varphi_2). \tag{61}$$

Quite notably, this result generalizes the analogous result for the free theory, in a completely *nonperturbative* way: Eq. (60) is independent of the explicit form of the fields and the ground state $|0\rangle$, which we might be able to recover just in a perturbative treatment of the model.

In fact, it is possible the further generalize the above result for *any* interactive theory for two scalar fields with flavor mixing in the following form:

$$\mathcal{L} = \partial_\mu\phi_A\partial^\mu\phi_A + \partial_\mu\phi_B\partial^\mu\phi_B - m_A^2\phi_A^2 - m_B^2\phi_B^2 + -m_{AB}^2\phi_A\phi_B + \mathcal{L}_{int}(\phi_A, \phi_B) \tag{62}$$

with $\mathcal{L}(\phi_A, \phi_B)$ any polynomial function of ϕ_A and ϕ_B. It is easy to show that

$${}_f\langle 0|{:}T_{\mu\nu}{:}|0\rangle_f = \eta_{\mu\nu}\langle 0|\because \sum_{i=1,2} m_i^2\phi_i^2 - \mathcal{L}_{int}\because|0\rangle \tag{63}$$

leading *always* to the equation of state $w = -1$.

C. Self-interactive fermions

Analogously, we can generalize the result presented in Sec. III A for fermionic fields (namely, $w = 0$) for a certain class of self-interactive theories. We start by considering a theory written in terms of the massive fields ψ_1 and ψ_2:

$$\mathcal{L} = \sum_i \bar\psi_i(i\partial\!\!\!/ - m_i)\psi_i + \mathcal{L}_{\rm int} \tag{64}$$

with $\mathcal{L}_{\rm int}$ a suitable polynomial function of ψ_i and $\bar\psi_i$. Again, we regard (64) as the *diagonalized* Lagrangian: in case of flavor mixing, ψ_1 and ψ_2 come from a rotation of the flavored fields ψ_A and ψ_B.

Combining our previous discussion on the bosonic case and results of Sec. IVA, we can write

$$\begin{aligned}{}_f\langle 0|{:}T_{00}{:}|0\rangle_f &= \langle 0|\because T_{00}\because|0\rangle \\ &= \langle 0|\because\sum_i m_i\bar\psi_i\psi_i + \mathcal{L}_{int}\because|0\rangle\end{aligned} \tag{65}$$

in which we used

$$\because \bar\psi_i\vec\gamma\cdot\vec\partial\psi_i\because = 0. \tag{66}$$

Analogously, the $jj \neq 00$ component of the stress-energy tensor is given by

$$T_{jj} = \sum_i(\bar\psi_i\gamma_j\partial_j\psi_i) - \eta_{jj}\mathcal{L} \tag{67}$$

that on shell can be written as

$$T_{jj} = \sum_i(\bar\psi_i\gamma_j\partial_j\psi_i + \psi_i^\dagger[L_{\rm int}, \psi_i]) + \mathcal{L}_{\rm int} \tag{68}$$

with $L_{\rm int} = \int d^3x\mathcal{L}_{\rm int}$, leading to

$$\begin{aligned}{}_f\langle 0|{:}T_{jj}{:}|0\rangle_f &= \langle 0|\because T_{jj}\because|0\rangle \\ &= \langle 0|\because\sum_i\bar\psi_i[L_{\rm int}\psi_i] + \mathcal{L}_{\rm int}\because|0\rangle\end{aligned} \tag{69}$$

We can now distinguish two cases:

(1) If the interactive term of the Lagrangian $\mathcal{L}_{\rm int}$ (and consequently $L_{\rm int}$) is invariant under the transformation

$$\begin{aligned}\psi_1 &\to \cos\theta\psi_1 - \sin\theta\psi_2 \\ \psi_2 &\to \sin\theta\psi_1 + \cos\theta\psi_2,\end{aligned} \tag{70}$$

we can then write

$$\because\mathcal{L}_{\rm int}\because = \because L_{\rm int}\because = \because\sum_i\bar\psi_i[L_{\rm int}, \psi_i]\because = 0 \tag{71}$$

leading to $w = 0$.

(2) If $\mathcal{L}_{\rm int}$ is not invariant under (70), we cannot push our analysis farther and we are unable to decide whether the pressure is zero or not, provided just with the tools here presented. It should be emphasized that other cancellation mechanisms might occur, leading to a full generalization of $w = 0$ for *all* self-interactive cases, just like in the bosonic case. However, these mechanisms are not reproduced within our method.

D. Remarks on interactive theories

To conclude the discussion of these examples, a few remarks are in order. Throughout our analysis we assumed that the flavor vacuum was *defined* by

$$|0\rangle_f \equiv G_\theta^\dagger|0\rangle \tag{72}$$

with G_θ the operator mapping flavor fields into massive fields, and vice versa, and $|0\rangle$ being the massive ground state of the interactive theory. The derivation of our results was purely formal and did not require any other knowledge of the theory. Nonetheless, although it might look reasonable, the assumption (72) remains a mere guess in absence of a complete (either perturbative or nonperturbative) interactive theory.

An interactive theory is a rather different object than a free one, from a nonperturbative level. In Sec. II, we already mentioned that the usual Fock space $\mathfrak{F}_0$ is not sufficient for fully describing the theory. More generally we can say that in the framework of second quantization few progresses on a coherent definition of the theory have been made so far, and the explicit construction of physical states in interactive theories still represents an open issue (cf. [58] and references therein).

Moreover, the familiar Perturbation Theory scheme, in the formulation of Lehmann, Symanzik and Zimmermann [59], is thought specifically for scattering processes and it might be unfit for describing the *flavor vacuum*. Since it relies on the assumption that particles are free at early and late times, all relevant quantities (scattering probabilities) are expressed in terms of time ordered products of field acting on the vacuum of the free theory $|0\rangle$, which is suppose to coincide with the true vacuum of theory at early and late times. However, the features of the flavor vacuum are not expressed in these terms, i.e. as probabilities of having certain states at late times, given some initial conditions.

It follows that implementing BV formalism on interactive theories is not a trivial task and requires very much care. Such a generalization is not among the aims of the present work. However, the purpose of this Sec. was to indicate a possible path for further developments of the formalism, taking advantage of the method of calculation discussed so far. Although an interactive theory might suffer from serious problems when it comes to construct particle states, as above mentioned, we believe that certain quantities, such as the equation of state of the flavor vacuum, might not require an explicit expression of such a state. Our analysis is valid under the assumption that (72) holds, irrespective of a detailed knowledge of $|0\rangle$ or any particle states in the interactive theory. Therefore, we might expect to be able to get some features of the phenomenology of the flavor vacuum, even though the underlying theory is not understood in full detail. However, a dedicated analysis is in order to fully justify the use of (72).

V. CONCLUSIONS

Neutrino physics required in the last years a dedicated theoretical effort, beyond the usual quantum field-theoretical framework of scattering processes [6]. Among other approaches, BV formalism is aimed to describe flavor states in a completely nonperturbative way [7]. The approach fostered an intense discussion in last years [9,17–19,60], and it has not being accepted by the community as a whole. However, we believe it represents a valid starting point towards a complete and coherent treatment of nonperturbative aspects of flavor physics.

Besides providing a consistent relativistic generalization for well-established nonrelativistic neutrino oscillations formulae, the formalism implies a nontrivial vacuum (the so called *flavor vacuum*), which has been regarded as a source of dark energy in various works [29]. Recently, the formalism has been used to describe the low energy limit of a quantum gravity model [44], leading to an implementation on a supersymmetric model [32]. Preliminary works have shown that BV formalism gives rise to a novel mechanism of SUSY breaking [31,32]. This work moves a step forth in the analysis of the supersymmetric flavor vacuum.

On one hand, we analyzed the phenomenology of the model of [31,32] (a free Wess-Zumino with flavor mixing), arguing that the supersymmetric flavor vacuum might consistently provide a source for both dark energy (thanks to the bosonic sector of the theory) and dark matter (via the fermionic one). At the moment, a quantitative comparison with available data is not yet possible, due to the oversimplifications of the model (just free fields labeled by only two flavors have been considered). However, encouraging results come from a preliminary analysis here performed: despite the huge difference of magnitudes of the involved parameters (ranging from the Plank energy scale to neutrino masses, from the density of dark matter today and its density at very early times), the model seems to be capable of reproducing the hierarchy $(\Delta m^2)^2 \sim \rho_\Lambda$, in the assumption that all dark energy density ρ_Λ is due to the bosonic sector of the flavor vacuum, and compatible with the hypothesis that the flavor vacuum contributes to (at least a fraction of) dark matter.

On the other hand, we started developing new tools in order to test the behavior of the flavor vacuum in interactive theories. We presented a novel method for evaluating relevant quantities connected with our problem, which, not only facilitate the nontrivial calculations involved in the free model, but also opens the way towards nonperturbative analyses of interactive theories. As a concrete example of the advantages of the method, we first showed how to reproduce in a few lines results of [31,32], getting a deeper insight of known formulae. Furthermore, in order to test the potential of the method, we used it to analyze the equation of state of the flavor vacuum in self-interactive theories, generalizing results of the free theory for a wide class of interactions, under reasonable assumptions.

The possibility here discussed that a source for both dark matter and dark energy might arise from neutrino physics, whether it derives from new physics beyond the Standard Model or from nonperturbative aspects of QFT, is quite attractive. The model here presented needs to be understood and developed much further. In particular, the following developments are in order:

(i) more realistic theories need to be constructed and compared with observational data: three flavors, SM/minimal supersymmetric standard model interactions, evolution in time (possibly on a curved background) are essential ingredients;

(ii) provided with a more realistic theory, the behavior of the flavor vacuum on large distance scales must be examined in presence of matter, in order to be compared with phenomenological models of dark matter and dark energy;

(iii) if we regard BV formalism as an effective description at low energy scales of the stringy model of [44], the gap between the macroscopic and microscopic description needs be reduced[7];

(iv) other ways to prove experimentally the existence of the flavor vacuum must be found; besides its gravitational effects, it might play an active role in interactive theories and hence in scattering processes, which require a dedicated analysis;

[7] An upcoming work of the authors of [44] will start filling this gap.

Despite these and many other questions that remain open, the models presented in this work suggest an intriguing possibility for a deeper understanding of fundamental problems in cosmology. The promising results here discussed certainly motivate, we believe, further developments of the approach.

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[1] A. Strumia and F. Vissani, arXiv:hep-ph/0606054v3.
[2] V. N. Gribov and B. Pontecorvo, Phys. Lett. **28B**, 493 (1969).
[3] Z. Maki, M. Nakagawa, and S. Sakata, Prog. Theor. Phys. **28**, 870 (1962).
[4] B. Pontecorvo, Sov. Phys. JETP **7**, 172 (1958).
[5] B. Pontecorvo, Sov. Phys. JETP **26**, 984 (1968).
[6] M. Beuthe, Phys. Rep. **375**, 105 (2003).
[7] M. Blasone and G. Vitiello, Ann. Phys. (N.Y.) **244**, 283 (1995).
[8] M. Blasone, A. Capolupo, and G. Vitiello, arXiv:hep-ph/0107183v1
[9] M. Blasone and G. Vitiello, Phys. Rev. D **60**, 111302 (1999).
[10] K. Fujii, C. Habe, and T. Yabuki, Phys. Rev. D **59**, 113003 (1999).
[11] K. Fujii, C. Habe, and T. Yabuki, Phys. Rev. D **64**, 013011 (2001).
[12] M. Blasone, A. Capolupo, O. Romei, and G. Vitiello, Phys. Rev. D **63**, 125015 (2001).
[13] M. Blasone, A. Capolupo, and G. Vitiello, Phys. Rev. D **66**, 025033 (2002).
[14] M. Blasone, P. A. Henning, and G. Vitiello, Phys. Lett. B **451**, 140 (1999).
[15] C-R Ji and Y. Mishchenko, Phys. Rev. D **64**, 076004 (2001).
[16] C-R Ji and Y. Mishchenko, Phys. Rev. D **65**, 096015 (2002).
[17] M. Blasone, A. Capolupo, F. Terranova, and G. Vitiello, Phys. Rev. D **72**, 013003 (2005).
[18] C. Giunti, Eur. Phys. J. C **39**, 377 (2005).
[19] C. Giunti, J. Phys. G **34**, R93 (2007).
[20] A. Capolupo, arXiv:hep-th/0408228v2.
[21] M. Blasone, A. Capolupo, S. Capozziello, S. Carloni, and G. Vitiello, Phys. Lett. A **323**, 182 (2004).
[22] M. Blasone, A. Capolupo, S. Capozziello, and G. Vitiello, AIP Conf. Proc. **841**, 406 (2006).
[23] M. Blasone, A. Capolupo, S. Capozziello, and G. Vitiello, AIP Conf. Proc. **957**, 185 (2007).
[24] M. Blasone, A. Capolupo, S. Capozziello, and G. Vitiello, arXiv:0809.0082v1.
[25] M. Blasone, A. Capolupo, S. Capozziello, and G. Vitiello, Nucl. Instrum. Methods Phys. Res., Sect. A **588**, 272 (2008).
[26] Massimo Blasone, A. Capolupo, S. Capozziello, S. Carloni, and G. Vitiello, Braz. J. Phys. **35**, 455 (2005).
[27] A. Capolupo, S. Capozziello, and G. Vitiello, Int. J. Mod. Phys. A **23**, 4979 (2008).
[28] A. Capolupo, S. Capozziello, and G. Vitiello, Phys. Lett. A **373**, 601 (2009).
[29] A. Capolupo, S. Capozziello, and G. Vitiello, Phys. Lett. A **363**, 53 (2007).
[30] A. Capolupo, S. Capozziello, and G. Vitiello, J. Phys. Conf. Ser. **67**, 012032 (2007).
[31] A. Capolupo, M. Di Mauro, and A. Iorio, Phys. Lett. A **375**, 3415 (2011).
[32] N. E. Mavromatos, S. Sarkar, and W. Tarantino, Phys. Rev. D **84**, 044050 (2011).
[33] S. Bilenky, Lect. Notes Phys. **817**, 1 (2010).
[34] M. Blasone, A. Capolupo, C. R. Ji, and G. Vitiello, Nucl. Phys. B, Proc. Suppl. **188**, 37 (2009).
[35] M. Blasone, A. Capolupo, C-R Ji, and G. Vitiello, Int. J. Mod. Phys. A **25**, 4179 (2010).
[36] K. C. Hannabuss and D. C. Latimer, J. Phys. A **33**, 1369 (2000).
[37] H. Umezawa, H. Matsumoto, and M. Tachiki, *Thermo Field Dynamics and Condensed States* (North-Holland, Amsterdam, 1982), p. 591.
[38] R. F. Streater and A. S. Wightman, *PCT, Spin and Statistics, and All That (Advanced Book Classics)*, (Addison-Wesley, Redwood City, CA, 1989), p. 207.
[39] R. Haag, Kong. Dan. Vid. Sel. Mat. Fys. Med. **29N12**, 1 (1955).
[40] N. D. Birrell and P. C. W. Davies, *Quantum Fields in Curved Space* (Cambridge University Press, Cambridge, England, 1982) p. 340.

[41] E. A. Calzetta and B. L. Hu, *Cambridge Monographs on Mathematical Physics* (Cambridge University Press, Cambridge, England, 2008).

[42] W. G. Unruh, Phys. Rev. D **14**, 870 (1976).

[43] S. W. Hawking, Commun. Math. Phys. **43**, 199 (1975).

[44] N. E. Mavromatos and S. Sarkar, New J. Phys. **10**, 073009 (2008).

[45] N E. Mavromatos, S. Sarkar, and W. Tarantino, Phys. Rev. D **80**, 084046 (2009).

[46] J. R. Ellis, N. E. Mavromatos, and M. Westmuckett, Phys. Rev. D **70**, 044036 (2004).

[47] J. R. Ellis, N. E. Mavromatos, and M. Westmuckett, Phys. Rev. D **71**, 106006 (2005).

[48] N. E. Mavromatos and S. Sarkar, Phys. Rev. D **74**, 036007 (2006).

[49] N. Mavromatos and S. Sarkar, Phys. Rev. D **72**, 065016 (2005).

[50] N. E. Mavromatos and S. Sarkar, Phys. Rev. D **79**, 104015 (2009).

[51] G. Bertone, D. Hooper, and J. Silk, Phys. Rep. **405**, 279 (2005).

[52] G. Jungman, M. Kamionkowski, and K. Griest, Phys. Rep. **267**, 195 (1996).

[53] A. Liddle, *An Introduction to Modern Cosmology* (Wiley-VCH, Weinheim, Germany, 2009), p. 201.

[54] L. Amendola and S. Tsujikawa, *Dark Energy: Theory and Observations* (Cambridge University Press, Cambridge, England, 2010).

[55] E. Komatsu *et al.*, Astrophys. J. Suppl. Ser. **192**, 18 (2011).

[56] I. Gurwich, Natural Neutrino Dark Energy. 2010.

[57] M. E. Peskin and D. V. Schroeder, *An Introduction to Quantum Field Theory. Reading* (Addison-Wesley, Reading, MA, 1995), p. 842.

[58] T. Borne, G. Lochak, and H. Stumpf, *Nonperturbative Quantum Field Theory and the Structure of Matter: Fundamental Theories of Physics*. Kluwer Academic Publishers, Berlin, 2001.

[59] H. Lehmann, K. Symanzik, and W. Zimmermann, Nuovo Cimento **1**, 205 (1955).

[60] Y. F. Li and Q. Y. Liu, J. High Energy Phys. 10 (2006) 048.

[61] C. Giunti and C. W. Kim. *Fundamentals of Neutrino Physics and Astrophysics* (Oxford University Press, Oxford, UK, 2007).