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Linearly polarized gluon distributions in the color dipole model

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We show that the linearly polarized gluon distributions appear in the color dipole model as we derive the full cross sections of the DIS dijet production and the Drell-Yan dijet (γ^* jet correlation) process. Together with the normal Weizsäcker-Williams gluon distribution, the linearly polarized one will contribute to the DIS dijet production cross section as the coefficient of the $\cos(2\Delta\phi)$ term in the correlation limit. We also derive the exact results for the cross section of the Drell-Yan dijet process, and find that the linearly polarized dipole gluon distribution which is identical to the normal dipole gluon distribution involves in the cross section. The results obtained in this paper agree with the previous transverse momentum dependent factorization study. We further derive the small-x evolution of these linearly polarized gluon distributions and find that they rise as x gets small at high energy.

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I. INTRODUCTION

In small-x physics, two different unintegrated gluon distributions [1,2] (also known as transverse momentum dependent gluon distributions), namely, the Weizsäcker-Williams gluon distribution $xG^{(1)}$ [3,4] and the dipole gluon distribution $xG^{(2)}$, have been widely used in the literature. The Weizsäcker-Williams gluon distribution can be interpreted as the number density of gluons inside dense hadrons in light-cone gauge. The dipole gluon distribution, despite of lacking the probabilistic interpretation, has been thoroughly studied since it appears in many physical processes [5,6] and it is defined via the Fourier transform of the simple color dipole amplitude. This dipole gluon distribution can be probed directly in photon-jet correlations and Drell-Yan dijet measurement in pA collisions. Recent studies [7,8] on the Weizsäcker-Williams gluon distribution indicate that it can be directly measured in DIS dijet production and its operator definition is related to color quadrupoles instead of normal color dipoles. Other more complicated dijet processes in pA collisions (e.g., qg or gg dijets) involve both of these gluon distributions through convolution in transverse momentum space. The complete calculations were performed in Ref. [7,8] in both the transverse momentum dependent (TMD) factorization formalism and the color dipole model. The results demonstrate the complete agreement between these two formalisms in the kinematical region where they are both valid.

Linearly polarized gluon distributions, denoted as $h_\perp^{(i)}(x, q_\perp)$,[1] where x and $q_\perp$ are the active gluon's longitudinal momentum fraction and its transverse momentum, respectively, were first introduced in Ref. [9]. This new gluon distribution effectively measures an averaged quantum interference between a scattering amplitude with an active gluon polarized along the x(or y)-axis and a complex conjugate amplitude with an active gluon polarized along the y(or x)-axis inside an unpolarized hadron. Because of the unique transverse spin correlation between the two-gluon fields of the distribution, the linearly polarized gluon distribution can contribute to a physical observable with $\cos(2\Delta\phi)$-type azimuthal angular dependence, or the azimuthally symmetric observables if they come in pairs. As proposed in Ref. [10], the linearly polarized gluon distributions can be directly probed in dijet and heavy quark pair production processes in electron-hadron collisions. As expected, this distribution also contributes to the cross section in photon pair productions [11,12] and the standard model Higgs boson production [13–15] in hadron-hadron collisions. Since the integrated parton distributions for incoming protons were used in the calculations of pA collisions in Ref. [7,8], the linearly polarized gluon distribution does not enter the cross section except for the Drell-Yan dijet processes as we show in the later discussion.

[1]Normally it is denoted as $h_1^{\perp g}(x, q_\perp)$. Here throughout the paper, in order to avoid confusion on the notation, we use $h_\perp^{(i)}(x, q_\perp)$ with $i = 1, 2$ to represent the linearly polarized gluon distributions.

In Ref. [16], the linearly polarized partner of both Weizsäcker-Williams and dipole gluon distributions inside an unpolarized nucleus target is studied in Color Glass Condensate (CGC) formalism. The corresponding cross sections of deep inelastic scattering (DIS) dijet production and the Drell-Yan processes in pA collisions are computed in terms of the TMD formalism. In both processes, the linearly polarized gluon distributions appear as the coefficients of the $\cos(2\Delta\phi)$ term in the cross section, where they were found to be consistent with the small-x formalism as well [16].

Inspired by Ref. [16], we perform the detailed calculation in the color dipole model for the DIS dijet production and the Drell-Yan dijet processes in pA collisions and we find identical results as those with the TMD formalism for the cross sections in the correlation limit, which is defined as a limit when the final state dijets are almost back-to-back. For the DIS dijet production, the complete analysis of the quadrupole amplitude shows that the linearly polarized gluon distribution of the Weizsäcker-Williams type comes from the off-diagonal expansion of the quadrupole amplitude. Using a hybrid factorization, we obtain the exact results for the cross section of the Drell-Yan processes in pA collisions. In the correlation limit, this exact result reduces to the TMD cross section obtained in Ref. [16].

Another objective of this paper is to study the small-x evolution of the linearly polarized gluon distributions. The small-x evolution of the dipole type linearly polarized gluon distribution is essentially the evolution of the dipole amplitude, which is governed by the Balitsky-Kovchegov equation [17,18]. Derived from the evolution of quadrupoles, the evolution of the linearly polarized Weizsäcker-Williams gluon distribution is quite complicated. Nevertheless, in the dilute regime, we find that both linearly polarized gluon distributions receive the exponential enhancement in terms of rapidity at high energy as the normal unpolarized gluon distributions do due to the small-x evolution.

The rest of the paper is organized as follows. In Sec. II, we calculate the cross sections of the DIS dijet production and the Drell-Yan processes in pA collisions and demonstrate that the linearly polarized gluon distributions naturally arise in the dipole model. We discuss the small-x evolution equations of the linearly polarized gluon distributions in Sec. III. The summary and further discussions are given in Sec. IV.

II. THE LINEARLY POLARIZED GLUON DISTRIBUTION IN DIPOLE MODEL

In this section, following Ref. [8], we show that the cross section of the DIS dijet production and the Drell-Yan dijet process in the color dipole model, namely, the CGC approach, involves the linearly polarized gluon distribution as well. The reason why this does not appear in the original work in [8] is that there the azimuthal orientation of the outgoing partons was averaged over.

A. DIS dijet production

After averaging over the photon's polarization and summing over the quark and antiquark helicities and colors, the cross section of the DIS dijet production in the color dipole model can be cast into

$$\begin{aligned}\frac{d\sigma^{\gamma^*_{T,L}A\to q\bar{q}X}}{d^3k_1d^3k_2} &= N_c\alpha_{\rm em}e_q^2\delta(p^+-k_1^+-k_2^+)\int\frac{\mathrm{d}^2x_1}{(2\pi)^2}\frac{\mathrm{d}^2x_1'}{(2\pi)^2}\frac{\mathrm{d}^2x_2}{(2\pi)^2}\\&\quad\times\frac{\mathrm{d}^2x_2'}{(2\pi)^2}e^{-ik_{1\perp}\cdot(x_1-x_1')}e^{-ik_{2\perp}\cdot(x_2-x_2')}\sum_{\lambda\alpha\beta}\psi^{T,L\lambda}_{\alpha\beta}(x_1-x_2)\\&\quad\times\psi^{T,L\lambda*}_{\alpha\beta}(x_1'-x_2')[1+S^{(4)}_{x_g}(x_1,x_2;x_2',x_1')\\&\quad-S^{(2)}_{x_g}(x_1,x_2)-S^{(2)}_{x_g}(x_2',x_1')],\end{aligned}\tag{1}$$

where the two- and four-point functions, which are characterized by the Wilson lines, take care of the multiple scatterings between the $q\bar{q}$-pair and the target. They are defined as

$$S^{(2)}_{x_g}(x_1,x_2)=\frac{1}{N_c}\langle\mathrm{Tr}U(x_1)U^\dagger(x_2)\rangle_{x_g},\tag{2}$$

$$S^{(4)}_{x_g}(x_1,x_2;x_2',x_1')=\frac{1}{N_c}\langle\mathrm{Tr}U(x_1)U^\dagger(x_1')U(x_2')U^\dagger(x_2)\rangle_{x_g},\tag{3}$$

with

$$U(x)=\mathcal{P}\exp\left\{ig_S\int_{-\infty}^{+\infty}\mathrm{d}x^+T^cA_c^-(x^+,x)\right\}.\tag{4}$$

The notation $\langle\ldots\rangle_{x_g}$ is used for the CGC average of the color charges over the nuclear wave function where x_g is the smallest fraction of longitudinal momentum probed, and is determined by the kinematics. The splitting wave function of the virtual photon with longitudinal momentum p^+ and virtuality Q^2 in transverse coordinate space takes the form,

$$\psi^{T\lambda}_{\alpha\beta}(p^+,z,r)=2\pi\sqrt{\frac{2}{p^+}}\begin{cases}i\epsilon_fK_1(\epsilon_f|r|)\frac{r\cdot\epsilon_\perp^{(1)}}{|r|}[\delta_{\alpha+}\delta_{\beta+}(1-z)+\delta_{\alpha-}\delta_{\beta-}z], & \lambda=1,\\ i\epsilon_fK_1(\epsilon_f|r|)\frac{r\cdot\epsilon_\perp^{(2)}}{|r|}[\delta_{\alpha-}\delta_{\beta-}(1-z)+\delta_{\alpha+}\delta_{\beta+}z], & \lambda=2,\end{cases}\tag{5}$$

$$\psi^L_{\alpha\beta}(p^+, z, r) = 2\pi\sqrt{\frac{4}{p^+}} z(1-z)QK_0(\epsilon_f|r|)\delta_{\alpha\beta} \tag{6}$$

where z is the momentum fraction of the photon carried by the quark, λ is the photon polarization, α and β are the quark and antiquark helicities, r the transverse separation of the pair, $\epsilon_f^2 = z(1-z)Q^2$, and the quarks are assumed to be massless.

In order to take the correlation limit, we introduce the transverse coordinate variables: $u = x_1 - x_2$ and $v = zx_1 + (1-z)x_2$, and similarly for the primed coordinates, with respective conjugate momenta $\tilde{P}_\perp = (1-z)k_{1\perp} - zk_{2\perp}$[2] and $q_\perp$. The correlation limit ($\tilde{P}_\perp \simeq k_{1\perp} \simeq k_{2\perp} \gg q_\perp$) is therefore enforced by assuming u and u' are small as compared to v and v' and then expanding the integrand with respect to these two variables before performing the Fourier transform. Following the derivation in Ref. [8], one can find that the lowest order expansion in u and u' of the last line of Eq. (1) gives

$$-u_i u'_j \frac{1}{N_c}\langle \mathrm{Tr}[\partial^i U(v)]U^\dagger(v')[\partial^j U(v')]U^\dagger(v)\rangle_{x_g}. \tag{7}$$

With the help of the identities

$$\int \frac{d^2u}{(2\pi)^2}\frac{d^2u'}{(2\pi)^2} e^{-i\tilde{P}_\perp\cdot(u-u')} u_i u'_j \nabla_u K_0(\epsilon_f u)\cdot\nabla_{u'}K_0(\epsilon_f u') = \frac{1}{(2\pi)^2}\left[\frac{\delta_{ij}}{(\tilde{P}_\perp^2+\epsilon_f^2)^2} - \frac{4\epsilon_f^2\tilde{P}_{\perp i}\tilde{P}_{\perp j}}{(\tilde{P}_\perp^2+\epsilon_f^2)^4}\right], \tag{8}$$

$$\int \frac{d^2u}{(2\pi)^2}\frac{d^2u'}{(2\pi)^2} e^{-i\tilde{P}_\perp\cdot(u-u')} u_i u'_j K_0(\epsilon_f u)K_0(\epsilon_f u') = \frac{1}{(2\pi)^2}\frac{4\tilde{P}_{\perp i}\tilde{P}_{\perp j}}{(\tilde{P}_\perp^2+\epsilon_f^2)^4}, \tag{9}$$

one can integrate over u and u' and obtain the complete differential cross section in the correlation limit,

$$\frac{d\sigma^{\gamma_T^* A\to q\bar{q}X}}{d\mathcal{P}.\mathcal{S}.} = \alpha_{\mathrm{em}} e_q^2 \alpha_s \delta(x_{\gamma^*}-1) z(1-z)(z^2+(1-z)^2) \times\left[\frac{\delta_{ij}}{(\tilde{P}_\perp^2+\epsilon_f^2)^2} - \frac{4\epsilon_f^2\tilde{P}_{\perp i}\tilde{P}_{\perp j}}{(\tilde{P}_\perp^2+\epsilon_f^2)^4}\right](16\pi^3) \times\int\frac{d^3v d^3v'}{(2\pi)^6}e^{-iq_\perp\cdot(v-v')}2\langle\mathrm{Tr}[F^{i-}(v)\mathcal{U}^{[+]\dagger} \times F^{j-}(v')\mathcal{U}^{[+]}]\rangle_{x_g}, \tag{10}$$

$$\frac{d\sigma^{\gamma_L^* A\to q\bar{q}X}}{d\mathcal{P}.\mathcal{S}.} = \alpha_{\mathrm{em}} e_q^2 \alpha_s \delta(x_{\gamma^*}-1)4z^2(1-z)^2 \times\frac{4\epsilon_f^2\tilde{P}_{\perp i}\tilde{P}_{\perp j}}{(\tilde{P}_\perp^2+\epsilon_f^2)^4}(16\pi^3)\int\frac{d^3v d^3v'}{(2\pi)^6}e^{-iq_\perp\cdot(v-v')} \times 2\langle\mathrm{Tr}[F^{i-}(v)\mathcal{U}^{[+]\dagger}F^{j-}(v')\mathcal{U}^{[+]}]\rangle_{x_g}. \tag{11}$$

Here we have used the identity

$$-\langle\mathrm{Tr}[\partial_i U(v)]U^\dagger(v')[\partial_j U(v')]U^\dagger(v)\rangle_{x_g} = g_S^2\int_{-\infty}^{\infty} dv^+ dv'^+ \langle\mathrm{Tr}[F^{i-}(v)\mathcal{U}^{[+]\dagger}F^{j-}(v')\mathcal{U}^{[+]}]\rangle_{x_g}, \tag{12}$$

where the gauge link $\mathcal{U}^{[+]}$ connects the two coordinate points by means of longitudinal gauge links going to $+\infty$ and a transverse link at infinity which does not contribute when the appropriate boundary conditions are taken.

If one integrates over the orientation of $\tilde{P}_\perp$, one can replace $\tilde{P}_{\perp i}\tilde{P}_{\perp j}$ by $\frac{1}{2}\delta_{ij}\tilde{P}_\perp^2$.[3] This replacement allows us to reduce the above expressions into Eqs. (30) and (31) in Ref. [8] which only involve the conventional Weizsäcker-Williams gluon distribution.

Now we are ready to show that the linearly polarized Weizsäcker-Williams gluon distribution can also arise naturally in the color dipole model. Since the indices i, j are symmetric, we can decompose the operator expression appearing in Eqs. (10) and (11) into two parts with one part involving only δ_{ij} and the other part being traceless,

$$4\int\frac{d^3v d^3v'}{(2\pi)^3}e^{-iq_\perp\cdot(v-v')}\langle\mathrm{Tr}[F^{i-}(v)\mathcal{U}^{[+]\dagger}F^{j-}(v')\mathcal{U}^{[+]}]\rangle_{x_g} = \frac{1}{2}\delta^{ij}xG^{(1)}(x,q_\perp) + \frac{1}{2}\left(\frac{2q_\perp^i q_\perp^j}{q_\perp^2} - \delta^{ij}\right)xh_\perp^{(1)}(x,q_\perp). \tag{13}$$

Here $xG^{(1)}(x, q_\perp)$ is the conventional Weizsäcker-Williams gluon distribution while the coefficient of the traceless tensor $xh_\perp^{(1)}(x, q_\perp)$ is the so-called linearly polarized partner of the conventional Weizsäcker-Williams gluon distribution.

The physical meaning or interpretation of these two gluon distributions can be better represented in a frame in which the two components of the transverse momentum $q_\perp^j$ with $j = 1, 2$ or $j = x, y$ are the same. With $q_\perp^x = q_\perp^y$ in this frame, the two symmetric projection operators in Eq. (13) can be written as,

[2] One could also define $v = \frac{1}{2}(x_1 + x_2)$ in this process since the virtual photon does not have initial interactions with the nucleus target, then the respective conjugate momentum is $P_\perp = \frac{1}{2}(k_{1\perp} - k_{2\perp}) \simeq \tilde{P}_\perp$. $P_\perp$ is the relative momentum of outgoing partons respect to the center of mass frame. Nevertheless, the following derivation remains the same in this case.

[3] In the derivation of Ref. [8], we have employed this as an underlying assumption.

$$\frac{1}{2}\delta^{ij} = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \frac{1}{2}(e_x^i e_x^j + e_y^i e_y^j) = \frac{1}{2}[\varepsilon_+^{*i}\varepsilon_+^j + \varepsilon_-^{*i}\varepsilon_-^j], \tag{14}$$

$$\frac{1}{2}\left(\frac{2q_\perp^i q_\perp^j}{q_\perp^2} - \delta^{ij}\right) = \frac{1}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \frac{1}{2}(e_x^i e_y^j + e_y^i e_x^j) = \frac{1}{2i}[\varepsilon_+^{*i}\varepsilon_-^j - \varepsilon_-^{*i}\varepsilon_+^j], \tag{15}$$

where $e_x^i = (1, 0)$ and $e_y^i = (0, 1)$ are 2-dimensional unit vectors along x-axis and y-axis, respectively, which could be interpreted as two orthogonal *linear* polarization vectors for transversely polarized gluons. As shown in Eqs. (14) and (15), these two symmetric projection operators can also be expressed in terms of the two orthogonal *circular* polarization vectors for transversely polarized gluons, $\varepsilon_\pm^j \equiv [\mp e_x^j - ie_y^j]/\sqrt{2}$. For the comparison, we also list here the antisymmetric projection operator for the polarized gluon helicity distribution,

$$\frac{1}{2}(i\epsilon_\perp^{ij}) = \frac{1}{2}\begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix} = \frac{1}{2}i(e_x^i e_y^j - e_y^i e_x^j) = \frac{1}{2}[\varepsilon_+^{*i}\varepsilon_+^j - \varepsilon_-^{*i}\varepsilon_-^j]. \tag{16}$$

From Eqs. (14) and (16), it is natural to interpret $G^{(1)}$ as a probability distribution to find unpolarized gluons, while the polarized gluon helicity distribution could be interpreted as a *difference* of two probability distributions to find positive helicity gluons and negative helicity gluons, respectively. From Eq. (15), it appears that $h_\perp^{(1)}$ does not have a probability interpretation in terms of the base polarization vectors $\varepsilon_\pm^j$, which are the eigenstates of angular momentum operators.[4] Instead, it could be interpreted as a transverse spin correlation function to find the gluons in the amplitude and complex conjugate amplitude to be in two orthogonal polarization states. In a general frame, $q_\perp^j = (q_\perp^x, q_\perp^y) = q_\perp(\cos\phi, \sin\phi)$, the projection operator for $h_\perp^{(1)}$ can be written as,

$$\frac{1}{2}\left(\frac{2q_\perp^i q_\perp^j}{q_\perp^2} - \delta^{ij}\right) = \frac{1}{2}\begin{pmatrix} \cos(2\phi) & \sin(2\phi) \\ \sin(2\phi) & -\cos(2\phi) \end{pmatrix}, \tag{17}$$

[4]However, if one chooses different base polarization vectors as $e_1^i = \frac{1}{\sqrt{2}}(1, 1)$ and $e_2^i = \frac{1}{\sqrt{2}}(1, -1)$, which are not the eigenstates of angular momentum operators, one can find that Eq. (15) becomes $\frac{1}{2}(e_1^i e_1^j - e_2^i e_2^j)$ which would allow us to interpret $h_\perp^{(1)}$ as the linearly polarized gluon density along the direction of the linear polarization. In a general frame, the polarization vectors are found to be $e_1^i = (\cos\phi, \sin\phi)$ and $e_2^i = (\sin\phi, -\cos\phi)$ which convert Eq. (17) into $\frac{1}{2}(e_1^i e_1^j - e_2^i e_2^j)$ as well. This indicates that the interpretation of the linearly polarized gluon distributions depends on the choice of the polarization vectors.

which includes the special case in Eq. (15) when $\phi = \pi/4$. Since the projection operator in Eq. (17) is proportional to a rotation matrix of the azimuthal angle, the $h_\perp^{(1)}$ could also be interpreted as "azimuthal correlated" gluon distributions [12,13]. Because the gluons in the amplitude and complex conjugate amplitude are in different transverse spin states, this kind of gluon distributions could contribute to the observables with $\cos(2\Delta\phi)$-type azimuthal dependence, or azimuthal symmetric observables if they come in pairs.

Substitute Eq. (13) into Eqs. (10) and (11), we obtain

$$\frac{d\sigma^{\gamma_T^* A\to q\bar{q}X}}{d\mathcal{P}.\mathcal{S}.} = \alpha_{\rm em} e_q^2 \alpha_s \delta(x_{\gamma^*} - 1) z(1-z) \times (z^2 + (1-z)^2)\frac{\epsilon_f^4 + \tilde{P}_\perp^4}{(\tilde{P}_\perp^2 + \epsilon_f^2)^4} \times \left[xG^{(1)}(x, q_\perp) - \frac{2\epsilon_f^2 \tilde{P}_\perp^2}{\epsilon_f^4 + \tilde{P}_\perp^4} \times \cos(2\Delta\phi) x h_\perp^{(1)}(x, q_\perp)\right], \tag{18}$$

$$\frac{d\sigma^{\gamma_L^* A\to q\bar{q}X}}{d\mathcal{P}.\mathcal{S}.} = \alpha_{\rm em} e_q^2 \alpha_s \delta(x_{\gamma^*} - 1) z^2(1-z)^2 \frac{8\epsilon_f^2 \tilde{P}_\perp^2}{(\tilde{P}_\perp^2 + \epsilon_f^2)^4} \times [xG^{(1)}(x, q_\perp) + \cos(2\Delta\phi) x h_\perp^{(1)}(x, q_\perp)], \tag{19}$$

where $\Delta\phi = \phi_{\tilde{P}_\perp} - \phi_{q_\perp}$ with $\phi_{\tilde{P}_\perp}$ and $\phi_{q_\perp}$ being the azimuthal angle of $\tilde{P}_\perp$ and $q_\perp$, respectively. This result is in complete agreement with the one obtained in Ref. [16] by using the TMD approach. The coefficient of the $\cos(2\Delta\phi)$ term in the above cross section can provide us the direct information of the linearly polarized Weizsäcker-Williams gluon distribution $xh_\perp^{(1)}(x, q_\perp)$. It is also easy to see that the $xh_\perp^{(1)}(x, q_\perp)$ term vanishes if one averages the cross section over the orientation of either $\tilde{P}_\perp$ or $q_\perp$ due to the factor $\cos(2\Delta\phi)$. This is transparent when one uses the variables $P_\perp$ and $q_\perp$ since they can be interpreted as the relative transverse momentum with respect to the center of mass frame of these two outgoing partons and the total transverse momentum of the CM frame, respectively.

Last but not least, one can see that the contribution from the linearly polarized gluon distribution vanishes if $Q = 0$, i.e., the real photon nucleus scattering only involves the conventional Weizsäcker-Williams gluon distribution. This is because the real photon cannot generate a $\cos(2\Delta\phi)$-type transverse spin correlation that matches the transverse spin correlation generated by $h_\perp^{(1)}$.

Let us now study the behavior of $xh_\perp^{(1)}(x, q_\perp)$ in the McLerran-Venugopalan (MV) model [19] for a large nucleus with A nucleons inside. Using the quadrupole results calculated in Ref. [8], one can cast the analytical form of $xh_\perp^{(1)}(x, q_\perp)$ into [16]

$$xh_\perp^{(1)}(x, q_\perp) = \frac{2}{\alpha_s}\left(\delta^{ij} - \frac{2q_\perp^i q_\perp^j}{q_\perp^2}\right)\int \frac{d^2 v d^2 v'}{(2\pi)^2(2\pi)^2} \times e^{-iq_\perp\cdot(v-v')}\langle \mathrm{Tr}[\partial_i U(v)]U^\dagger(v') \times [\partial_j U(v')]U^\dagger(v)\rangle_{x_g}$$
$$= \frac{S_\perp}{2\pi^3\alpha_s}\frac{N_c^2-1}{N_c}\int_0^\infty dr_\perp r_\perp \frac{J_2(q_\perp r_\perp)}{r_\perp^2 \ln\frac{1}{r_\perp^2\Lambda^2}} \times \left[1 - \exp\left(-\frac{1}{4}r_\perp^2 Q_{sg}^2\right)\right], \tag{20}$$

where $J_2(q_\perp r_\perp)$ is the Bessel function of the first kind and $Q_{sg}^2 = \alpha_s g^2 N_c \mu^2 \ln\frac{1}{r_\perp^2\Lambda^2}$ with $\mu^2 = \frac{A}{2S_\perp}$. For $q_\perp^2 \gg Q_{sg}^2$, we find that $xh_\perp^{(1)}(x, q_\perp) \simeq \frac{\alpha_s A C_F N_c}{\pi^2 q_\perp^2}$, which is identical to $xG^{(1)}(x, q_\perp)$ and agrees with the perturbative QCD results. It is important to notice that it scales like A since each nucleon contributes additively in the dilute regime. In this regime, the dominant contribution to the gluon distribution comes from a single two-gluon exchange with a transverse momentum transfer $q_\perp$ in the color dipole picture. For the case $\Lambda^2 \ll q_\perp^2 \ll Q_{sg}^2$ one absorbs the $\ln\frac{1}{r_\perp^2\Lambda^2}$ factor into the definition of the saturation momentum and finds $xh_\perp^{(1)}(x, q_\perp) \simeq \frac{\alpha_s A C_F N_c}{\pi^2 Q_{sg}^2}$ which is an approximate constant. It scales like $A^{2/3}$ since $Q_{sg}^2 \sim A^{1/3}$ as a result of strong nuclear shadowing. It is interesting to note that, in the low $q_\perp^2$ region, the effect of multiple scatterings between probes and target nuclei can be viewed as or attributed to a single scattering with the momentum transfer of order Q_{sg}^2. As compared to the small $q_\perp^2$ behavior of the conventional Weizsäcker-Williams gluon distribution $xG^{(1)}(x, q_\perp) \simeq \frac{S_\perp}{4\pi^3\alpha_s}\frac{N_c^2-1}{N_c}\ln\frac{Q_{sg}^2}{q_\perp^2}$, we find that $\frac{xG^{(1)}(x,q_\perp)}{xh_\perp^{(1)}(x,q_\perp)} \simeq \ln\frac{q_\perp^2}{\Lambda^2}\ln\frac{Q_{sg}^2}{q_\perp^2} \gg 1$ where we have replaced $r_\perp$ by $\frac{1}{q_\perp}$. These gluon distributions obtained in the MV model can be viewed as an initial condition for the small-x evolution. In addition, we can also find that $xG^{(1)}(x, q_\perp) \geq xh_\perp^{(1)}(x, q_\perp)$ for any value of $q_\perp$ which ensures the positivity of the total cross section.

B. Drell-Yan dijet process

Following the prompt photon-jet correlation calculation in Ref. [8], it is straightforward to calculate the cross section of dijet ($q\gamma^*$) production in Drell-Yan processes in pA collisions. The calculation is essentially the same, except for the slightly different splitting function since the final state virtual photon, which eventually decays into a di-lepton pair, has a finite invariant mass M. By taking into account the photon invariant mass, the splitting wave functions of a quark with longitudinal momentum p^+ splitting into a quark and virtual photon pair in transverse coordinate space become

$$\psi_{\alpha\beta}^{T\lambda}(p^+, k_1^+, r) = 2\pi\sqrt{\frac{2}{k_1^+}}\begin{cases} i\epsilon_M K_1(\epsilon_M|r|)\frac{r\cdot\epsilon_\perp^{(1)}}{|r|}(\delta_{\alpha-}\delta_{\beta-} + (1-z)\delta_{\alpha+}\delta_{\beta+}), & \lambda = 1, \\ i\epsilon_M K_1(\epsilon_M|r|)\frac{r\cdot\epsilon_\perp^{(2)}}{|r|}(\delta_{\alpha+}\delta_{\beta+} + (1-z)\delta_{\alpha-}\delta_{\beta-}), & \lambda = 2. \end{cases}, \tag{21}$$

$$\psi_{\alpha\beta}^{L}(p^+, k_1^+, r) = 2\pi\sqrt{\frac{2}{k_1^+}}(1-z)MK_0(\epsilon_M|r|)\delta_{\alpha\beta}, \tag{22}$$

where $\epsilon_M^2 = (1-z)M^2$, λ is the photon polarization, α, β are helicities for the incoming and outgoing quarks, and $z = \frac{k_1^+}{p^+}$ is the momentum fraction of the incoming quark carried by the photon.

At the end of the day, for the correlation between the final state virtual photon and quark in pA collisions, we have

$$\frac{d\sigma_{\mathrm{DP}}^{pA\to\gamma^* q+X}}{dy_1 dy_2 d^2k_{1\perp}d^2k_{2\perp}} = \sum_f x_p q_f(x_p, \mu)\frac{\alpha_{\mathrm{e.m.}}e_f^2}{2\pi^2}(1-z)z^2 S_\perp F_{x_g}(q_\perp)\left\{[1+(1-z)^2]\frac{q_\perp^2}{[\tilde{P}_\perp^2 + \epsilon_M^2][(\tilde{P}_\perp + zq_\perp)^2 + \epsilon_M^2]} - \epsilon_M^2\left[\frac{1}{\tilde{P}_\perp^2 + \epsilon_M^2} - \frac{1}{(\tilde{P}_\perp + zq_\perp)^2 + \epsilon_M^2}\right]^2\right\}, \tag{23}$$

with $F_{x_g}(q_\perp) = \int\frac{d^2r_\perp}{(2\pi)^2}e^{-iq_\perp\cdot r_\perp}\frac{1}{N_c}\langle \mathrm{Tr}U(0)U^\dagger(r_\perp)\rangle_{x_g}$, $q_\perp = k_{1\perp} + k_{2\perp}$ and $\tilde{P}_\perp = (1-z)k_{1\perp} - zk_{2\perp}$. In the MV model, $F_{x_g}(q_\perp) \simeq \frac{1}{\pi Q_{sq}^2}\exp(-\frac{q_\perp^2}{Q_{sq}^2})$ with $Q_{sq}^2 = \frac{C_F}{N_c}Q_{sg}^2$ being the quark saturation momentum. $q_f(x_p, \mu)$ is the integrated quark distribution with flavor f in the proton projectile. Here we used the hybrid factorization which allows us to use integrated parton distributions since the proton projectile is considered to be dilute as compared to the nucleus target. The first term in the curly brackets arises solely from the transverse splitting function in Eq. (21) while the second term is the sum of contributions from both the transverse and longitudinal splitting functions. We would like to emphasize that the above cross section in Eq. (23) is an exact result regardless of the relative size between $q_\perp$ and $\tilde{P}_\perp$. By taking the correlation limit, namely $q_\perp \ll \tilde{P}_\perp$, we arrive at the result which is identical to the one obtained from TMD factorization [16][5]

$$\frac{d\sigma_{\mathrm{DP}}^{pA\to\gamma^* q+X}}{dy_1 dy_2 d^2k_{1\perp} d^2k_{2\perp}}\bigg|_{q_\perp\ll\tilde{P}_\perp} = \sum_f x_p q_f(x_p,\mu) xG^{(2)}(x_g,q_\perp)[H_{qg\to q\gamma^*} - \cos(2\Delta\phi)H^\perp_{qg\to q\gamma^*}], \tag{24}$$

with $\Delta\phi = \phi_{\tilde{P}_\perp} - \phi_{q_\perp}$, $xG^{(2)}(x,q_\perp) = \frac{q_\perp^2 N_c}{2\pi^2\alpha_s} S_\perp F_{x_g}(q_\perp)$ and

$$H_{qg\to q\gamma^*} = \frac{\alpha_s\alpha_{\mathrm{e.m.}}e_f^2(1-z)z^2}{N_c}\left\{\frac{1+(1-z)^2}{[\tilde{P}_\perp^2+\epsilon_M^2]^2} - \frac{2z^2\epsilon_M^2\tilde{P}_\perp^2}{[\tilde{P}_\perp^2+\epsilon_M^2]^4}\right\}, \tag{25}$$

$$H^\perp_{qg\to q\gamma^*} = \frac{\alpha_s\alpha_{\mathrm{e.m.}}e_f^2(1-z)z^2}{N_c}\frac{2z^2\epsilon_M^2\tilde{P}_\perp^2}{[\tilde{P}_\perp^2+\epsilon_M^2]^4}. \tag{26}$$

In this case, the relevant gluon distribution is the so-called dipole gluon distribution as demonstrated in Refs. [7,8,20]. As discussed in Ref. [16], according to the operator definition of dipole type gluon distributions [7,8,20],

$$xG_{\mathrm{DP}}^{ij}(x,q_\perp) = 2\int\frac{d\xi^- d\xi_\perp}{(2\pi)^3P^+}e^{ixP^+\xi^- - iq_\perp\cdot\xi_\perp} \times\langle P|\mathrm{Tr}[F^{+i}(\xi^-,\xi_\perp)\mathcal{U}^{[-]\dagger} \times F^{+j}(0)\mathcal{U}^{[+]}]|P\rangle, \tag{27}$$

$$= \frac{q_\perp^i q_\perp^j N_c}{2\pi^2\alpha_s}S_\perp F_{x_g}(q_\perp), \tag{28}$$

$$= \frac{1}{2}\delta^{ij}xG^{(2)}(x,q_\perp) + \frac{1}{2}\left(\frac{2q_\perp^i q_\perp^j}{q_\perp^2} - \delta^{ij}\right)xh_\perp^{(2)}(x,q_\perp), \tag{29}$$

where the gauge link $\mathcal{U}^{[-]}$ is composed by longitudinal gauge links going to $-\infty$. This shows that the linearly polarized partner of the dipole gluon distribution is exactly the same as the dipole gluon distribution.[6] From Eq. (29), with the proper normalization, we can also find that the linearly polarized gluon distribution $xh_\perp^{(2)}(x,q_\perp) = xG^{(2)}(x,q_\perp)$.

Furthermore, one can see that for the prompt photon-jet correlation, the linearly polarized gluon distribution does not contribute since $H^\perp_{qg\to q\gamma^*}$ vanishes when $M=0$. This is also due to the fact that the real photon in the final state cannot generate the transverse spin correlation that matches the transverse spin correlation of the incoming gluon in the $qg\to q\gamma$ subprocess. It takes two matched transverse spin correlations to get a nonvanish observable effect.

C. Resummation

For the purpose of the Collins-Soper-Sterman resummation [21] discussed in Ref. [14], it is also useful to define the coordinate expression of the linearly polarized Weizsäcker-Williams gluon distribution as follows

$$x\tilde{h}_\perp^{(1)ij}(x,b_\perp) = \frac{1}{2}\int \mathrm{d}^2q_\perp e^{-iq_\perp\cdot b_\perp} \times\left(\frac{2q_\perp^i q_\perp^j}{q_\perp^2} - \delta^{ij}\right)xh_\perp^{(1)}(x,q_\perp), \tag{30}$$

and it is straightforward to find that in the MV model

$$x\tilde{h}_\perp^{(1)ij}(x,b_\perp) = \frac{1}{2}\left(\delta^{ij} - \frac{2b_\perp^i b_\perp^j}{b_\perp^2}\right)\frac{S_\perp}{\pi^2\alpha_s}\frac{N_c^2-1}{N_c}\frac{1}{b_\perp^2\ln\frac{1}{b_\perp^2\Lambda^2}} \times\left[1-\exp\left(-\frac{1}{4}b_\perp^2Q_{sg}^2\right)\right]. \tag{31}$$

This can be compared to the normal Weizsäcker-Williams gluon distribution in $b_\perp$ space defined as $x\tilde{G}^{(1)}(x,b_\perp) = \int \mathrm{d}^2q_\perp e^{-iq_\perp\cdot b_\perp}xG^{(1)}(x,q_\perp)$,

$$x\tilde{G}^{(1)}(x,b_\perp) = \frac{S_\perp}{\pi^2\alpha_s}\frac{N_c^2-1}{N_c}\frac{\ln\frac{1}{b_\perp^2\Lambda^2}-2}{b_\perp^2\ln\frac{1}{b_\perp^2\Lambda^2}} \times\left[1-\exp\left(-\frac{1}{4}b_\perp^2Q_{sg}^2\right)\right]. \tag{32}$$

At small $b_\perp$, $x\tilde{h}_\perp^{(1)ij}(x,b_\perp)$ is proportional to $(\delta^{ij} - 2b_\perp^i b_\perp^j/b_\perp^2)$ times a constant, whereas $xG^{(1)}(x,b_\perp)$ behaves as $\ln\frac{1}{\Lambda^2 b_\perp^2}$ due to the logarithmic term in Q_{sg}^2. These properties are consistent with their perturbative behaviors at large transverse momentum [14].

[5]To compare with Ref. [16], one can compute the Mandelstam variables and find that $\hat{s} = (k_1+k_2)^2 = M^2 + \frac{(1-z)(M^2+k_{1\perp}^2)}{z} + \frac{zk_{2\perp}^2}{(1-z)} - 2k_{1\perp}\cdot k_{2\perp} = \frac{\tilde{P}_\perp^2+\epsilon_M^2}{z(1-z)}$, $\hat{u} = \frac{\tilde{P}_\perp^2+\epsilon_M^2}{z}$ and $\hat{t} = \frac{\tilde{P}_\perp^2}{1-z}$.

[6]There is a factor of 2 between these two distributions in Ref. [16] due to different normalization.

Similarly for the dipole gluon counterparts, one gets

$$x\tilde{h}_\perp^{(2)ij}(x,b_\perp)=\frac{1}{2}\left(\delta_\perp^{ij}-\frac{2b_\perp^i b_\perp^j}{b_\perp^2}\right)\frac{N_c S_\perp}{2\pi^2\alpha_s}\times\exp\left[-\frac{1}{4}Q_{sq}^2 b_\perp^2\right]Q_{sq}^2\times\left[\frac{1}{\ln\frac{1}{\Lambda^2 b_\perp^2}}+\frac{b_\perp^2 Q_{sq}^2}{4}\left(1-\frac{1}{\ln\frac{1}{\Lambda^2 b_\perp^2}}\right)^2\right], \quad (33)$$

and

$$x\tilde{G}^{(2)}(x,b_\perp)=\frac{N_c S_\perp}{2\pi^2\alpha_s}\exp\left[-\frac{1}{4}Q_{sq}^2 b_\perp^2\right]Q_{sq}^2\times\left[1-\frac{2}{\ln\frac{1}{\Lambda^2 b_\perp^2}}-\frac{b_\perp^2 Q_{sq}^2}{4}\left(1-\frac{1}{\ln\frac{1}{\Lambda^2 b_\perp^2}}\right)^2\right]. \quad (34)$$

Again, in the small $b_\perp$ limit, they behave the same as those Weizsäcker-Williams gluon distributions, respectively. It is interesting to notice that their large $b_\perp$ behaviors are different. For the dipole gluon distributions, they decrease exponentially whereas the Weizsäcker-Williams ones have power behaviors. These expressions can be viewed as the initial conditions of the resummation discussed in Ref. [14].

III. SMALL-x EVOLUTION OF THE LINEARLY POLARIZED GLUON DISTRIBUTIONS

In this section, we discuss the small-x evolution of the linearly polarized gluon distributions. We separate the discussions into two parts: the first part is on the evolution of the linearly polarized dipole gluon distribution since it is trivial and it only involves the dipole amplitude; then we derive the evolution equation for the linearly polarized Weizsäcker-Williams gluon distribution from the small-x evolution equation of quadrupoles.

A. The evolution of the linearly polarized dipole gluon distribution

According to the definition of the linearly polarized dipole gluon distribution, and the above calculation of the cross section of dijet ($q\gamma^*$) production in Drell-Yan processes in pA collisions, we know that the linearly polarized partner of the dipole gluon distribution is identical to the normal dipole gluon distribution, i.e., $xh_\perp^{(2)}(x,q_\perp)=xG^{(2)}(x,q_\perp)$. In general, one can write these distributions in terms of the dipole amplitude, namely, the two point function of Wilson lines $\frac{1}{N_c}\langle \mathrm{Tr}(U(x_\perp)U^\dagger(y_\perp))\rangle$ as follows

$$xh_\perp^{(2)}(x,q_\perp)=xG^{(2)}(x,q_\perp)=\frac{q_\perp^2 N_c}{2\pi^2\alpha_s}\int d^2x_\perp\int\frac{d^2y_\perp}{(2\pi)^2}e^{-iq_\perp\cdot(x_\perp-y_\perp)}\times\frac{1}{N_c}\langle \mathrm{Tr}U(x_\perp)U^\dagger(y_\perp)\rangle_Y. \quad (35)$$

The small-x evolution of the dipole amplitude follows the well-known Balitsky-Kovchegov equation [17,18] which reads

$$\frac{\partial}{\partial Y}\langle \mathrm{Tr}[U(x)U^\dagger(y)]\rangle_Y=-\frac{\alpha_s N_c}{2\pi^2}\int d^2z_\perp\frac{(x_\perp-y_\perp)^2}{(x_\perp-z_\perp)^2(z_\perp-y_\perp)^2}\times\left\{\langle \mathrm{Tr}[U(x)U^\dagger(y)]\rangle_Y-\frac{1}{N_c}\langle \mathrm{Tr}[U(x)U^\dagger(z)]\mathrm{Tr}[U(z)U^\dagger(y)]\rangle_Y\right\}. \quad (36)$$

In the dilute regime, the Balitsky-Kovchegov equation reduces to the famous BFKL equation which leads to the exponential growth in terms of the rapidity $Y\simeq\ln\frac{1}{x}$.

B. The evolution of the linearly polarized Weizsäcker-Williams gluon distribution

The operator definition of the Weizsäcker-Williams gluon distribution can be obtained from the quadrupole correlator whose initial condition can be throughly calculated in the MV model. In Refs. [22–25], the small-x evolution equation of the quadrupole has been derived and studied analytically. Similarly to the Balitsky-Kochegov equation for dipoles, quadrupoles follow BFKL evolution in the dilute regime and reach the saturation regime as a stable fixed point. In addition, one expects that quadrupoles should also exhibit the same geometrical scaling behavior as dipoles. Recently, using the JIMWLK renormalization equation [26,27], the first numerical studies [28] of the small-x evolution of quadrupoles indeed observe evidence of traveling wave solutions and geometric scaling for the quadrupole. According to [16] and Refs. [7,8,20], the Weizsäcker-Williams gluon distribution can be written as

$$xG_{\mathrm{WW}}^{ij}(x,k_\perp)=-\frac{2}{\alpha_S}\int\frac{d^2v}{(2\pi)^2}\frac{d^2v'}{(2\pi)^2}e^{-ik_\perp\cdot(v-v')}\times\langle \mathrm{Tr}[\partial^i U(v)]U^\dagger(v')[\partial^j U(v')]U^\dagger(v)\rangle_Y \quad (37)$$

$$=\frac{\delta^{ij}}{2}xG^{(1)}(x,q_\perp)+\frac{1}{2}\left(2\frac{q_\perp^i q_\perp^j}{q_\perp^2}-\delta^{ij}\right)\times xh_\perp^{(1)}(x,q_\perp). \quad (38)$$

The evolution equation for the correlator $\langle \mathrm{Tr}[\partial^i U(v)]U^\dagger(v')[\partial^j U(v')]U^\dagger(v)\rangle_Y$ can be obtained from the evolution equation of the quadrupole $\frac{1}{N_c}\langle \mathrm{Tr}(U(x_1)U^\dagger(x_1')U(x_2)U^\dagger(x_2'))\rangle_Y$ by differentiating with respect to x_1^i and x_2^j, and then setting $x_1^i = x_2'^i = v^i$ and $x_2^j = x_1'^j = v'^j$. Then the resulting evolution equation becomes[7]

$$\begin{aligned}
&\frac{\partial}{\partial Y}\langle \mathrm{Tr}[\partial^i U(v)]U^\dagger(v')[\partial^j U(v')]U^\dagger(v)\rangle_Y \\
&= -\frac{\alpha_s N_c}{2\pi^2}\int d^2 z_\perp \frac{(v-v')^2}{(v-z)^2(z-v')^2}\langle \mathrm{Tr}[\partial^i U(v)]U^\dagger(v')[\partial^j U(v')]U^\dagger(v)\rangle_Y \\
&\quad -\frac{\alpha_s N_c}{2\pi^2}\int d^2 z_\perp \frac{1}{N_c}\frac{(v-v')^2}{(v-z)^2(z-v')^2}\left[\frac{(v-v')^i}{(v-v')^2}-\frac{(v-z)^i}{(v-z)^2}\right] \\
&\quad \times\{\langle \mathrm{Tr}[U(v)U^\dagger(v')[\partial^j U(v')]U^\dagger(z)]\mathrm{Tr}[U(z)U^\dagger(v)]\rangle_Y - \langle \mathrm{Tr}[U(z)U^\dagger(v')[\partial^j U(v')]U^\dagger(v)]\mathrm{Tr}[U(v)U^\dagger(z)]\rangle_Y\} \\
&\quad -\frac{\alpha_s N_c}{2\pi^2}\int d^2 z_\perp \frac{1}{N_c}\frac{(v-v')^2}{(v-z)^2(z-v')^2}\left[\frac{(v'-v)^j}{(v'-v)^2}-\frac{(v'-z)^j}{(v'-z)^2}\right] \\
&\quad \times\{\langle \mathrm{Tr}[[\partial^i U(v)]U^\dagger(z)U(v')U^\dagger(v)]\mathrm{Tr}[U(z)U^\dagger(v')]\rangle_Y - \langle \mathrm{Tr}[[\partial^i U(v)]U^\dagger(v')U(z)U^\dagger(v)]\mathrm{Tr}[U(v')U^\dagger(z)]\rangle_Y\} \\
&\quad -\frac{\alpha_s N_c}{4\pi^2}\int d^2 z_\perp \frac{1}{N_c}\left[\partial_v^i\partial_{v'}^j\frac{(v-v')^2}{(v-z)^2(z-v')^2}\right] \\
&\quad \times\{\langle \mathrm{Tr}[U(v')U^\dagger(z)]\mathrm{Tr}[U(z)U^\dagger(v')]\rangle_Y + \langle \mathrm{Tr}[U(v)U^\dagger(z)]\mathrm{Tr}[U(z)U^\dagger(v)]\rangle_Y - \langle \mathrm{Tr}[U(v')U^\dagger(v)]\mathrm{Tr}[U(v)U^\dagger(v')]\rangle_Y - N_c^2\}.
\end{aligned} \tag{39}$$

The evolution equation of the Weizsäcker-Williams gluon distributions can be obtained by contracting the above correlator with δ_{ij} and the one for the linearly polarized partner by contracting with $(2\frac{q_\perp^i q_\perp^j}{q_\perp^2} - \delta^{ij})$. Although the expression is quite complicated in general, the result gets simplified in the dilute regime as in Ref. [23]. In the dilute regime, the correlator which yields the Weizsäcker-Williams gluon distribution can be reduced to a simple form in terms of $\Gamma(v, v')$

$$\langle \mathrm{Tr}[\partial^i U(v)]U^\dagger(v')[\partial^j U(v')]U^\dagger(v)\rangle_Y = \frac{C_F}{2}\partial_v^i\partial_{v'}^j\Gamma(v, v')_Y, \tag{40}$$

where $\frac{C_F}{2}\Gamma(v, v')$ is the leading order dipole amplitude which satisfies the BFKL equation

$$\frac{\partial}{\partial Y}\Gamma(x_1, x_2)_Y = \frac{N_c\alpha_s}{2\pi^2}\int d^2 z\frac{(x_1-x_2)^2}{(x_1-z)^2(x_2-z)^2}\times[\Gamma(x_1, z)_Y + \Gamma(z, x_2)_Y - \Gamma(x_1, x_2)_Y]. \tag{41}$$

In the dilute regime where the gluon density is low, we know that the Weizsäcker-Williams gluon distributions, $xG^{(1)}(x, q_\perp)$ and $xh_\perp^{(1)}(x, q_\perp)$, as well as the dipole gluon distributions all reduce to the same leading twist result. Therefore, despite of the distinct behavior in the saturation regime, we find that all these four types of gluon distributions follow the BFKL equation in the dilute regime where the gluon density is low. The physical consequence of this results is that the linearly polarized gluon distributions should be as important as the normal gluon distributions in the low-x region since they also receive the exponential rise in rapidity Y due to the BFKL evolution. Furthermore, according to the discussion in Refs. [29,30], the BFKL evolution together with a saturation boundary can give rise to the geometrical scaling behavior [31–33] of the dipole gluon distribution. Since the quadrupole evolution equation also contains the same property as discussed in Refs. [23,28], the Weizsäcker-Williams gluon distribution and its linearly polarized partner should exhibit geometrical scaling behavior as well, although their evolution equations are much more complicated in the saturation regime. In terms of the traveling wave picture [29,34] for the evolution of dipoles and quadrupoles, the velocities of the traveling waves for dipoles and quadruples are identical, since the velocity is determined by BFKL evolution. This implies that the energy dependence of the saturation momentum $Q_s^2 \simeq Q_0^2(x_0/x)^\lambda$ with $Q_0 = 1$ GeV, $x_0 = 3\times 10^{-3}$ and $\lambda = 0.29$, should be universal for all these four different gluon distributions.

[7]This evolution equation involves derivatives of the Wilson lines and complicated kernels which make it very hard to solve directly. However, one can extract the evolution information by numerically solving the evolution equation for quadrupoles first and then making numerical differentiation and identification of coordinates.

IV. CONCLUSION

We perform the color dipole model calculation of the cross section of DIS dijet and Drell-Yan dijet processes, and demonstrate that the linearly polarized partners of the Weizsäcker-Williams and dipole gluon distributions naturally arise in these processes. This result is in complete agreement with Ref. [16] and implies that the measurement of the $\cos(2\Delta\phi)$ asymmetries in these dijet processes can be a direct probe of these two different linearly polarized gluon distributions. In addition, the small-x evolution studies of the linearly polarized gluon distributions reveals that they also rise exponentially as function of the rapidity at high energy and they should also exhibit the geometrical scaling behavior as the normal unpolarized gluon distributions do.

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