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Effects of the U boson on the inner edge of neutron star crusts

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We explore effects of the light vector U boson, which is weakly coupled to nucleons, on the transition density ρ_t and pressure P_t at the inner edge separating the liquid core from the solid crust of neutron stars. Three methods, i.e., the thermodynamical approach, the curvature matrix approach, and the Vlasov equation approach, are used to determine the transition density ρ_t with the Skyrme effective nucleon-nucleon interactions. We find that the ρ_t and P_t depend on not only the ratio of coupling strength to mass squared of the U boson g^2/μ^2 but also its mass μ due to the finite-range interaction from the U-boson exchange. In particular, our results indicate that the ρ_t and P_t are sensitive to both g^2/μ^2 and μ if the U-boson mass μ is larger than about 2 MeV. Furthermore, we show that both g^2/μ^2 and μ can have significant influence on the mass-radius relation and the crustal fraction of total moment of inertia of neutron stars. In addition, we study the exchange term contribution of the U boson based on the density matrix expansion method, and demonstrate that the exchange term effects on the nuclear matter equation of state as well as the ρ_t and P_t are generally negligible.

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I. INTRODUCTION

The possible existence of a neutral weakly coupled light spin-1 gauge U boson [1], which is originated from supersymmetric extensions of the standard model with an extra $U(1)$ symmetry, has recently attracted much attention due to its multifaceted influences in particle physics, nuclear physics, astrophysics, and cosmology. For instance, the U boson can provide annihilation of light dark matter that can be responsible for the excess flux of 511 keV photons coming from the central region of our Galaxy observed by the SPI/INTEGRAL satellite [2–5]. It is also proposed that the U boson can be mediator of the putative "fifth force" providing a possible mechanism for non-Newtonian gravity, i.e., the violation of the inverse-square law of Newtonian gravitational force at short distance [6–23]. Thus far, various upper limits on the deviation from the inverse-square law have been put forward down to femtometer range [12,16,19]. Furthermore, the U boson can involve a rich phenomenology in particle physics and nuclear physics and may have observable effects in particle decays [24–27] and nucleon scattering processes [16,18,19,28,29], which can also put limits on the U-boson properties. Studying properties of the U boson is thus important for understanding the relevant new physics beyond the standard model.

Very recently, the effects of the U boson on the nuclear matter equation of state (EOS) and neutron star structure have been investigated [30–33] and it is shown that the vector U boson can significantly stiffen the nuclear matter EOS and thus enhance drastically the maximum mass of neutron stars. In particular, by considering the U boson, the stability and observed global properties of neutron stars can be reasonably explained by using the neutron-rich matter EOS with a supersoft nuclear symmetry energy at supersaturation densities consistent with the available terrestrial laboratory data on the π^-/π^+ radio in relativistic heavy-ion collisions from FOPI/GSI [34,35], while the supersoft nuclear symmetry energy at supersaturation densities generally cannot support a canonical mass ($1.4M_\odot$) neutron star if the U boson is not introduced [31]. The U boson has also been introduced to describe the recently discovered new holder of neutron star maximum mass of 1.97 ± 0.04 $M_\odot$ from PSR J1614-2230 [36] using soft EOS's consistent with existing terrestrial nuclear laboratory experiments for hybrid neutron stars containing a quark core described by MIT bag model using reasonable parameters [33,37], and it is found that the constraints on the U-boson properties are consistent with existing constraints from neutron-proton and neutron-lead scatterings [18,19] as well as the spectroscopy of antiproton atoms [16].

In the studies about the U-boson influences on neutron star structure [30–33], the exchange term contribution of the U boson to the nuclear matter EOS has been neglected and only the direct term contribution has been considered, leading to that the nuclear matter EOS depends only on the ratio the coupling strength to mass squared of the U boson, namely, g^2/μ^2. Physically, the exchange term contribution of the U boson to the nuclear matter EOS will depend on both the coupling constant g and the U-boson mass μ due to the finite-range interaction mediated by the U boson. It is thus interesting to see how the exchange term contribution of the U boson will influence the nuclear matter EOS. Furthermore, neutron stars are expected to have a solid inner crust surrounding a liquid core. Knowledge on

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properties of the crust plays an important role in understanding many astrophysical observations [38–48]. The inner crust spans the region from the neutron dripout point to the inner edge separating the solid crust from the homogeneous liquid core. While the neutron drip-out density $\rho_{\rm out}$ is relatively well determined to be about 4×10^{11} g/cm^3 [49], the transition density ρ_t at the inner edge is still largely uncertain mainly because of our very limited knowledge on the EOS of neutron-rich nucleonic matter, especially the density dependence of the symmetry energy [42,43]. The transition density ρ_t and the corresponding pressure P_t at the inner edge might be measurable indirectly from observations of pulsar glitches [43,45]. Since the U boson can have significant influence on the nuclear matter EOS, it is therefore very interesting to see how the U boson will affect the inner edge of neutron star crusts.

In the present work, we investigate effects of the light vector U boson on the transition density ρ_t and pressure P_t at the inner edge of neutron stars crust. The density matrix expansion approach [50,51] is used to describe the exchange term contribution of the finite-range interaction due to the U-boson exchange. Based on the Skyrme effective nucleon-nucleon interactions, we use three methods, i.e., the thermodynamical approach, the curvature matrix approach, and the Vlasov equation approach to determine the transition density ρ_t. As expected, our results indicate that the ρ_t and P_t depend on not only the ratio of coupling strength to mass squared of the U boson g^2/μ^2 but also its mass μ due to the finite-range interaction from the U-boson exchange. Furthermore, we find that the ρ_t and P_t are sensitive to both g^2/μ^2 and μ if the U-boson mass μ is larger than about 2 MeV and both g^2/μ^2 and μ can have significant influence on the mass-radius relation and the crustal fraction of total moment of inertia of neutron stars. We also demonstrate that the exchange term has minor influence on the nuclear matter EOS as well as the ρ_t and P_t for the parameter values of g^2/μ^2 and μ considered in this work.

II. THEORETICAL MODELS AND METHODS

A. Nuclear matter symmetry energy

The EOS of isospin asymmetric nuclear matter, defined by its binding energy per nucleon, can be expanded to 2nd order in isospin asymmetry δ as

$$E(\rho, \delta) = E_0(\rho) + E_{\rm sym}(\rho)\delta^2 + O(\delta^4), \tag{1}$$

where $\rho = \rho_n + \rho_p$ is the baryon density with ρ_n and ρ_p denoting the neutron and proton densities, respectively; $\delta = (\rho_n - \rho_p)/(\rho_p + \rho_n)$ is the isospin asymmetry; $E_0(\rho) = E(\rho, \delta = 0)$ is the binding energy per nucleon in symmetric nuclear matter, and the nuclear symmetry energy is expressed as

$$E_{\rm sym}(\rho) = \frac{1}{2!}\frac{\partial^2 E(\rho, \delta)}{\partial \delta^2}\bigg|_{\delta=0}. \tag{2}$$

In Eq. (1), the absence of odd-order terms in δ is due to the exchange symmetry between protons and neutrons in nuclear matter when one neglects the Coulomb interaction and assumes the charge symmetry of nuclear forces. The higher-order terms in δ are negligible, leading to the well-known empirical parabolic law for the EOS of asymmetric nuclear matter, which has been verified by all many-body theories to date, at least for densities up to moderate values (see, e.g., Ref. [52]). As a good approximation, the density-dependent symmetry energy $E_{\rm sym}(\rho)$ can thus be extracted from $E_{\rm sym}(\rho) \approx E(\rho, \delta = 1) - E(\rho, \delta = 0)$.

Around the nuclear matter saturation density ρ_0, the nuclear symmetry energy $E_{\rm sym}(\rho)$ can be expanded as

$$E_{\rm sym}(\rho) = E_{\rm sym}(\rho_0) + \frac{L}{3}\left(\frac{\rho - \rho_0}{\rho_0}\right) + O\left(\left(\frac{\rho - \rho_0}{\rho_0}\right)^2\right), \tag{3}$$

where L is the slope parameter of the nuclear symmetry energy at ρ_0, i.e.,

$$L = 3\rho_0 \frac{\partial E_{\rm sym}(\rho)}{\partial \rho}\bigg|_{\rho=\rho_0}. \tag{4}$$

The slope parameter L characterizes the density dependence of the nuclear symmetry energy around normal nuclear matter density, and thus carries important information on the properties of nuclear symmetry energy at both high and low densities.

The EOS of isospin asymmetric nuclear matter is a basic ingredient to determine the properties of neutron stars. For symmetric nuclear matter with equal fractions of neutrons and protons, its EOS $E_0(\rho)$ is relatively well determined. In particular, the incompressibility K_0 of symmetric nuclear matter at its saturation density ρ_0 has been determined to be 240 ± 20 MeV from analyses of the nuclear giant monopole resonances [53–62], and its EOS at densities of $2\rho_0 < \rho < 5\rho_0$ has also been constrained by measurements of collective flows [63] and subthreshold kaon production [64,65] in relativistic nucleus-nucleus collisions. On the other hand, the EOS of asymmetric nuclear matter, especially the density dependence of the nuclear symmetry energy, is largely unknown. Although the nuclear symmetry energy at ρ_0 is known to be around 30 MeV from the empirical liquid-drop mass formula [66,67], its values at other densities, especially at supra-saturation densities, are poorly known [52,68]. During the last decade, significant progress has been made both experimentally and theoretically on constraining the behavior of the symmetry energy at subsaturation density and the value of L constrained from different experimental data or methods has become consistently convergent to about 60 ± 30 MeV [69–71] (see, e.g., Refs. [72,73] for recent summary). Furthermore, the IBUU04 transport model analysis of the FOPI data on the π^-/π^+ ratio in central heavy-ion collisions at SIS/GSI [34] energies suggests a very soft symmetry energy at the supersaturation densities [35] (see also Refs. [74,75]). These studies have significantly improved our understanding for the EOS of asymmetric nuclear matter, which have important implications on the neutron star physics.

B. Skyrme-Hartree-Fock approach

For the nuclear effective interaction, we use in the present work the so-called standard Skyrme force (see, e.g., Ref. [76]), which has been shown to be very successful in describing the structure of finite nuclei. In the standard Skyrme Hartree-Fock (SHF) approach, the nuclear effective interaction is taken to have a zero-range, density- and momentum-dependent form [76], i.e.,

$$\begin{aligned} V_{12}(\mathbf{R},\mathbf{r}) = {} & t_0(1+x_0P_\sigma)\delta(\mathbf{r}) + \tfrac{1}{6}t_3(1+x_3P_\sigma)\rho^\sigma(\mathbf{R})\delta(\mathbf{r}) \\ & + \tfrac{1}{2}t_1(1+x_1P_\sigma)(K'^2\delta(\mathbf{r}) + \delta(\mathbf{r})K^2) \\ & + t_2(1+x_2P_\sigma)\mathbf{K}'\cdot\delta(\mathbf{r})\mathbf{K} \\ & + iW_0(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot[\mathbf{K}'\times\delta(\mathbf{r})\mathbf{K}], \end{aligned} \tag{5}$$

with $\mathbf{r} = \vec{r}_1 - \vec{r}_2$ and $\mathbf{R} = (\vec{r}_1 + \vec{r}_2)/2$. In the above expression, the relative momentum operators $\mathbf{K} = (\boldsymbol{\nabla}_1 - \boldsymbol{\nabla}_2)/2i$ and $\mathbf{K}' = -(\boldsymbol{\nabla}_1 - \boldsymbol{\nabla}_2)/2i$ act on the wave function on the right and left, respectively. The quantities P_σ and σ_i denote, respectively, the spin exchange operator and Pauli spin matrices. The σ, $t_0 - t_3$, $x_0 - x_3$ are the 9 Skyrme interaction parameters and W_0 is the spin-orbit coupling constant.

Within the standard form [see Eq. (5)], the total energy of the nuclear system can be written as

$$E = \int \mathcal{H}(\mathbf{r})d^3r, \tag{6}$$

with $\mathcal{H}$ the Skyrme energy density. In the standard SHF model, the total energy density of a spin-saturated nuclear system considered in this work is written as [76]

$$\mathcal{H} = \mathcal{K} + \mathcal{H}_0 + \mathcal{H}_3 + \mathcal{H}_{\text{eff}} + \mathcal{H}_{fin} + \mathcal{H}_{\text{Coul}}, \tag{7}$$

where $\mathcal{K} = \frac{\hbar^2}{2m}\tau$ is the kinetic-energy term and $\mathcal{H}_0$, $\mathcal{H}_3$, $\mathcal{H}_{\text{eff}}$, $\mathcal{H}_{fin}$ are given by

$$\mathcal{H}_0 = t_0[(2+x_0)\rho^2 - (2x_0+1)(\rho_p^2 + \rho_n^2)]/4, \tag{8}$$

$$\mathcal{H}_3 = t_3\rho^\sigma[(2+x_3)\rho^2 - (2x_3+1)(\rho_p^2+\rho_n^2)]/24, \tag{9}$$

$$\begin{aligned} \mathcal{H}_{\text{eff}} = {} & [t_2(2x_2+1) - t_1(2x_1+1)](\tau_n\rho_n + \tau_p\rho_p)/8 \\ & + [t_1(2+x_1) + t_2(2+x_2)]\tau\rho/8, \end{aligned} \tag{10}$$

$$\begin{aligned} \mathcal{H}_{fin} = {} & [3t_1(2+x_1) - t_2(2+x_2)](\nabla\rho)^2/32 - [3t_1(2x_1+1) \\ & + t_2(2x_2+1)][(\nabla\rho_n)^2 + (\nabla\rho_p)^2]/32 \end{aligned} \tag{11}$$

in terms of the 9 Skyrme interaction parameters σ, $t_0 - t_3$, $x_0 - x_3$. In the above equations, ρ_i and τ_i are, respectively, the local nucleon number and kinetic-energy densities, whereas ρ and τ are corresponding total densities. $\mathcal{H}_{\text{Coul}}$ is the Coulomb term given by

$$\mathcal{H}_{\text{Coul}} = \frac{1}{2}e^2\rho_p(\vec{r})\int\frac{\rho_p(\vec{r}')}{|\vec{r}-\vec{r}'|}d\vec{r}' - \frac{3}{4}e^2\left(\frac{3}{\pi}\right)^{1/3}\rho_p^{4/3}(\vec{r}). \tag{12}$$

The nucleon single-particle energy can be obtained from minimizing the total energy of the nuclear system with respect to its wave function as

$$\epsilon_q = \frac{p^2}{2m} + U_q = \frac{p^2}{2m_q^\star} + U_q^\star, \qquad q = n, p, \tag{13}$$

where U_q is the single-particle potential while $m_q^\star$ and $U_q^\star$ represent, respectively, the nucleon effective mass and effective single-particle potential, which can be expressed, respectively, as

$$\begin{aligned} \frac{\hbar^2}{2m_q^\star} = {} & \frac{\hbar^2}{2m_q} + \frac{1}{8}\rho[t_1(2+x_1) + t_2(2+x_2)] \\ & + \frac{1}{8}\rho_q[t_2(2x_2+1) - t_1(2x_1+1)], \end{aligned} \tag{14}$$

and

$$\begin{aligned} U_q^\star = {} & \frac{1}{2}t_0[(2+x_0)\rho - (2x_0+1)\rho_q] + \frac{1}{24}\sigma t_3\rho^{\sigma-1}[(2+x_3)\rho^2 - (2x_3+1)(\rho_p^2+\rho_n^2)] \\ & + \frac{1}{12}t_3\rho^\sigma[(2+x_3)\rho - (2x_3+1)\rho_q] + \frac{1}{8}[t_1(2+x_1) + t_2(2+x_2)]\tau + \frac{1}{8}[t_2(2x_2+1) - t_1(2x_1+1)]\tau_q \\ & + \frac{1}{16}[t_2(2+x_2) - 3t_1(2+x_1)]\nabla^2\rho + \frac{1}{16}[3t_1(2x_1+1) + t_2(2x_2+1)]\nabla^2\rho_q. \end{aligned} \tag{15}$$

For protons, the additional Coulomb potential is given by

$$U_{\text{Coul}} = e^2\int\frac{\rho_p(\vec{r}')}{|\vec{r}-\vec{r}'|}d\vec{r}' - e^2\left(\frac{3\rho_p(\vec{r})}{\pi}\right)^{1/3}. \tag{16}$$

For the kinetic-energy density, we use in this work the results from the extended Thomas-Fermi approximation [77], i.e.,

$$\tau_q = a\rho_q^{5/3} + b\frac{(\nabla\rho_q)^2}{\rho_q} + c\nabla^2\rho_q, \tag{17}$$

where $a = \frac{3}{5}(3\pi^2)^{2/3}$, $b = 1/36$, and $c = 1/3$. From the nucleon single-particle energy, the nucleon chemical potential in infinite nuclear matter, i.e., μ_q, can be obtained as the value of the single-particle energy without gradient terms at the Fermi surface $p_q^F = \hbar(3\pi^2\rho_q)^{1/3}$.

C. The weakly coupled light vector U boson

Fujii [6] first proposed that the non-Newtonian gravity can be described by adding a Yukawa term to the conventional gravitational potential between two objects of mass m_1 and m_2, i.e.,

$$V_{\rm gra}(r) = -\frac{Gm_1m_2}{r}(1 + \alpha e^{-r/\lambda}), \tag{18}$$

where α is a dimensionless strength parameter, λ is the length scale, and G is the gravitational constant. In the boson exchange picture, the light and weakly coupled vector U boson is a favorite candidate mediating the extra interaction for the non-Newtonian gravity [1], leading to the finite-range Yukawa potential between two nucleons, which can be expressed as

$$V_{\rm UB}(r) = \frac{g^2}{4\pi}\frac{e^{-\mu r}}{r}, \tag{19}$$

where g and μ represent the U-boson-nucleon coupling constant and the U-boson mass, respectively. Comparing Eq. (19) with the Yukawa term in Eq. (18), one can find the relations $\alpha = -g^2/(4\pi Gm^2)$ and $\lambda = 1/\mu$ (in natural units) where m is the nucleon mass. By adding the Yukawa potential of Eq. (19) to the standard Skyrme effective nucleon-nucleon interaction in Eq. (5), the extra binding energy of the nuclear system due to the U boson (UB) can be expressed as the integral of energy density $\mathcal{H}_{\rm UB}$ as follows:

$$E_{\rm UB} = \int \mathcal{H}_{\rm UB}(\mathbf{r})d^3r, \tag{20}$$

with

$$\mathcal{H}_{\rm UB} = \mathcal{H}_{\rm UB}^D + \mathcal{H}_{\rm UB}^E, \tag{21}$$

where $\mathcal{H}_{\rm UB}^D$ and $\mathcal{H}_{\rm UB}^E$ are the direct and exchange contribution to the energy density, respectively. For the finite-range Yukawa interaction of Eq. (19), the direct term contribution to the energy density can be easily obtained as [30–33]

$$\mathcal{H}_{\rm UB}^D = \frac{1}{2V}\int \rho(\vec{r}_1)\frac{g^2}{4\pi}\frac{e^{-\mu r}}{r}\rho(\vec{r}_2)d\vec{r}_1 d\vec{r}_2 = \frac{1}{2}\frac{g^2}{\mu^2}\rho^2, \tag{22}$$

where V is the normalization volume, $\rho = \rho_n + \rho_p$ is the baryon number density, and $r = |\vec{r}_1 - \vec{r}_2|$.

Although the direct term contribution of a finite-range interaction to the nuclear energy density can be treated exactly, it is numerically challenging to evaluate the exchange contribution. The latter can be, however, approximated by that from a Skyrme-like zero-range interaction using the density matrix expansion [50,51], and the results can be obtained as

$$\mathcal{H}_{\rm UB}^E = \sum_{q=p,n}\frac{g^2}{4}\left[2\rho_q\tau_q I_{1q} + \frac{1}{2}\left(I_{1q} + \rho_q\frac{\partial I_{1q}}{\partial\rho_q}\right)(\nabla\rho_q)^2\right] - \sum_{q=p,n}\frac{g^2}{4}\left[\frac{6}{5}(3\pi^2)^{2/3}\rho_q^{8/3}I_{1q} + \rho_q^2 I_{2q}\right]. \tag{23}$$

The integrations in Eq. (23) are defined as

$$I_{1q} = \int drr^4\rho_{SL}(k_q^F r)g(k_q^F r)\frac{e^{-\mu r}}{r}, \tag{24}$$

$$I_{2q} = \int drr^2\rho_{SL}^2(k_q^F r)\frac{e^{-\mu r}}{r}, \tag{25}$$

with

$$\rho_{SL}(k_q^F r) = \frac{3}{k_q^F r}j_1(k_q^F r), \tag{26}$$

$$g(k_q^F r) = \frac{35}{2(k_q^F r)^3}j_3(k_q^F r), \tag{27}$$

where j_1 and j_3 are, respectively, the first- and third-order spherical Bessel functions and $k_q^F = (3\pi^2\rho_q)^{1/3}$ is the Fermi momentum.

One can see from Eq. (22) that for the direct term, the U boson contributes to the nuclear energy density only through the combination g^2/μ^2. On the other hand, it is indicated from Eq. (23) that the exchange term contribution to the nuclear energy density depends on both the coupling constant g and the mass μ in a complicated way. Furthermore, the density gradient terms appear automatically in the exchange term contribution to the nuclear energy density due to the density matrix expansion of the finite-range Yukawa potential. It is interesting to see that the exchange term contribution to the nuclear matter EOS further depends on the isospin asymmetry although the direct term contribution to the nuclear matter EOS is isospin independent due to the fact that the U boson is an isoscalar boson. Therefore, the U boson will contribute to the nuclear symmetry energy through the exchange term. However, as will be shown later, the exchange term contribution to the nuclear matter EOS and the symmetry energy is quite small and can be neglected safely within the parameter value region of the coupling constant g and the U-boson mass μ considered in the present work.

The nucleon single-particle energy due to the U boson can be obtained from variation of the corresponding energy of Eq. (20) with respect to its wave function. The U-boson contribution to the nucleon effective mass can be expressed as

$$\frac{\hbar^2}{2m^{\star}_{q,\mathrm{UB}}} = \frac{g^2}{2}\rho_q I_{1q}, \tag{28}$$

which should be added to the right-hand side of Eq. (14) to obtain the total nucleon effective mass $m^{\star}_q$. It should be noted that the U-boson contribution to the nucleon effective mass is from the exchange term with the density matrix expansion of the finite-range Yukawa potential, which leads to the isospin-dependent contribution to the nucleon effective mass although the U boson is isoscalar. The U-boson contribution to the effective single-particle potential can be written as

$$\begin{aligned} U^{\star}_{q,\mathrm{UB}} &= \frac{g^2}{4\pi}\int \frac{e^{-\mu|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|}\rho(\vec{r}')d\vec{r}' \\ &\quad - \frac{g^2}{4}\left[\rho_q\left(2I_{2q} + \rho_q\frac{\partial I_{2q}}{\partial \rho_q}\right) + 2(3\pi^2)^{2/3}\rho_q^{5/3}I_{1q}\right] \\ &\quad + \frac{g^2}{4}\left(\frac{1}{18}\frac{I_{1q}}{\rho_q} - \frac{17}{18}\frac{\partial I_{1q}}{\partial\rho_q} - \frac{1}{2}\rho_q\frac{\partial^2 I_{1q}}{\partial\rho_q^2}\right)(\nabla\rho_q)^2 \\ &\quad - \frac{g^2}{12}\left(I_{1q} + \rho_q\frac{\partial I_{1q}}{\partial\rho_q}\right)\nabla^2\rho_q, \end{aligned} \tag{29}$$

where the first term in the right-hand side of Eq. (29) is from the direct term contribution while the other terms are from the exchange term contribution. As expected, the direct term contribution is isospin independent while the exchange term contribution depends on the isospin asymmetry as well as the density gradients. Furthermore, one can see that the U-boson contribution to the single-particle potential depends on both the coupling constant g and the mass μ in a complicated way.

D. The transition density in neutron stars

The transition density is the baryon number density that separates the liquid core from the inner crust in neutron stars and it plays an important role in determining the structural properties of neutron stars such as the crustal fraction of total moment of inertia and the mass-radius relations of static neutron stars. In principle, the transition density can be obtained from comparing relevant properties of the nonuniform solid crust and the uniform liquid core mainly consisting of neutrons, protons, and electrons (npe matter). However, this is practically very difficult since the inner crust may contain the so-called "nuclear pasta" with very complicated geometries [46,78–81]. In practice, a good approximation is to search for the density at which the uniform liquid first becomes unstable against small amplitude density fluctuations with clusterization. This approximation has been shown to produce a very small error for the actual core-crust transition density and it would yield the exact transition density for a second-order phase transition [41,82–84]. So far, several such methods including the thermodynamical method [43,85,86], the dynamical curvature matrix method [38–41,82,87,88], the Vlasov equation method [89–93], and the random phase approximation [84,91,94] have been applied extensively in the literature. Here, we briefly introduce the thermodynamical method, the dynamical curvature matrix method, and the Vlasov equation method, which will be used to calculate the transition density in this work.

1. The thermodynamical method for transition density in neutron stars

In the thermodynamical method, the system is required to obey the following intrinsic stability condition [43,85,95]:

$$-\left(\frac{\partial P}{\partial v}\right)_{\mu_{np}} > 0, \tag{30}$$

$$-\left(\frac{\partial \mu_{np}}{\partial q_c}\right)_{v} > 0, \tag{31}$$

where the $P = P_b + P_e$ is the total pressure of the npe matter system with P_b and P_e denoting the contributions from baryons and electrons, respectively, and the v and q_c are the volume and charge per baryon number. The μ_{np} is defined as the chemical potential difference between neutrons and protons, i.e.,

$$\mu_{np} = \mu_n - \mu_p. \tag{32}$$

The conditions of Eqs. (30) and (31) are equivalent to require the convexity of the energy per particle in the single phase [43,85] by ignoring the finite-size effects due to surface and Coulomb energies [96]. In fact, Eq. (30) is simply the well-known mechanical stability condition of the system at a fixed μ_{np}, which ensures that any local density fluctuation will not diverge. On the other hand, Eq. (31) is the charge or chemical stability condition of the system at a fixed density. It means that any local charge variation violating the charge neutrality condition will not diverge.

The pressure P_e is only a function of the chemical potential difference μ_{np} by assuming the β-equilibrium condition is satisfied, i.e., $\mu_{np} = \mu_e$. By using the relation $\frac{\partial E_b(\rho,x_p)}{\partial x_p} = -\mu_{np}$ with $E_b(\rho, x_p)$ being energy per baryon from the baryons in the β-equilibrium neutron star matter and $x_p = \rho_p/\rho$, and treating the electrons as free Fermi gas, one can show [96] that the thermodynamical relations

Eqs. (30) and (31) are actually equivalent to the following condition:

$$V_{\mathrm{ther}} = 2\rho \frac{\partial E_b(\rho, x_p)}{\partial \rho} + \rho^2 \frac{\partial^2 E_b(\rho, x_p)}{\partial \rho^2} - \left(\frac{\partial^2 E_b(\rho, x_p)}{\partial \rho \partial x_p}\rho\right)^2 / \frac{\partial^2 E_b(\rho, x_p)}{\partial x_p^2} > 0, \tag{33}$$

which determines the thermodynamical instability region of the β-equilibrium neutron star matter. The baryon number density that violates the condition Eq. (33) then corresponds to the core-crust transition density in neutron stars for the thermodynamical method.

2. The curvature matrix method for transition density in neutron stars

In the curvature matrix method, the instability region of homogeneous nuclear matters against clusterization is determined by introducing a finite-size spatially periodic density fluctuation $\delta\rho$ to the system and then examining how the system free energy varies with the fluctuation [88]. The fluctuation will affect the three components of homogeneous nuclear matter (neutrons, protons, and electrons) independently when assuming it occurs only on finite microscopic scale in the β-equilibrium nuclear matter as

$$\rho_q = \rho_q^0 + \delta\rho_q, \tag{34}$$

with $q = n, p, e$. Then, the free energy f at each point of density $\rho_q(\boldsymbol{r}) = \rho_q^0 + \delta\rho_q$ can be expressed as

$$f(\rho_q) = f(\rho_q^0) + \sum_{q=n,p,e}\left(\frac{\partial f}{\partial \rho_q}\right)_0 \delta\rho_q + \sum_{q,q'=n,p,e}\frac{1}{2} \times \left(\frac{\partial^2 f}{\partial \rho_q \partial \rho_{q'}}\right)_0 \delta\rho_q \delta\rho_{q'} + \cdots. \tag{35}$$

For a stable homogeneous npe matter system, the first-order term $(\frac{\partial f}{\partial \rho_q})_0$ in Eq. (35) must equal to zero and a density fluctuation $\delta\rho_q$ should lead to an increasing of the free energy, which is equivalent to requiring the second-order term in Eq. (35) to be positive for any density fluctuation $\delta\rho_q$. This can be ensured by the positive definiteness of the following curvature matrix:

$$C^f_{CM} = \begin{pmatrix} \frac{\partial^2 f}{\partial \rho_n^2} & \frac{\partial^2 f}{\partial \rho_n \partial \rho_p} & \frac{\partial^2 f}{\partial \rho_n \partial \rho_e} \\ \frac{\partial^2 f}{\partial \rho_p \partial \rho_n} & \frac{\partial^2 f}{\partial \rho_p^2} & \frac{\partial^2 f}{\partial \rho_p \partial \rho_e} \\ \frac{\partial^2 f}{\partial \rho_e \partial \rho_n} & \frac{\partial^2 f}{\partial \rho_e \partial \rho_p} & \frac{\partial^2 f}{\partial \rho_e^2} \end{pmatrix} = \begin{pmatrix} \frac{\partial \mu_n}{\partial \rho_n} & \frac{\partial \mu_n}{\partial \rho_p} & 0 \\ \frac{\partial \mu_p}{\partial \rho_n} & \frac{\partial \mu_p}{\partial \rho_p} & 0 \\ 0 & 0 & \frac{\partial \mu_e}{\partial \rho_e} \end{pmatrix} + k^2 \begin{pmatrix} D_{nn} & D_{np} & 0 \\ D_{pn} & D_{pp} & 0 \\ 0 & 0 & 0 \end{pmatrix} + \frac{g^2}{k^2 + \mu^2} \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} + \frac{4\pi e^2}{k^2} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{pmatrix}, \tag{36}$$

where k is the wave vector of the spatially periodic density fluctuations and the effective density gradient coefficients are defined as

$$D_{nn}(D_{pp}) = \frac{3}{16}[t_1(1 - x_1) - t_2(1 + x_2)] - \frac{1}{24}[t_1(1 - x_1) + 3t_2(1 + x_2)] + \frac{g^2}{12}\left(I_{1n(p)} + \rho_{n(p)} \frac{\partial I_{1n(p)}}{\partial \rho_{n(p)}}\right), \tag{37}$$

$$D_{np}(D_{pn}) = \frac{1}{16}[3t_1(2 + x_1) - t_2(2 + x_2)] - \frac{1}{24}[t_1(2 + x_1) + t_2(2 + x_2)]. \tag{38}$$

From Eq. (36), one can see that the matrix C^f_{CM} includes four parts. The bulk part, i.e., the first term in the right-hand side of Eq. (36), is k independent but the following 3 parts are all k dependent due to the density gradient terms in Eq. (15), the direct term contribution of the finite-range interaction from the U-boson exchange, and the Coulomb interaction, respectively.

The matrix C^f_{CM} is defined for each point (ρ_n, ρ_p, ρ_e), and the sign of its three eigenvalues determines the sign of the second-order term in Eq. (35), namely, only if all the eigenvalues of the matrix are positive, the free energy of the system will remain the minimum value and the nuclear system will be stable for all the density fluctuations. So, the baryon number density violating the positive definiteness of the matrix C^f_{CM} corresponds to the core-crust transition density in neutron stars for the curvature matrix method.

3. The Vlasov equation method for transition density in neutron stars

To determine the core-crust transition density in neutron stars within the Vlasov equation method, we include here for completeness a brief description for the method (see, e.g., Refs. [51,89] for the details). For a β-stable and electrically neutral npe matter, the Vlasov equation can be expressed as

$$\frac{\partial f_q(\vec{R},\vec{p},t)}{\partial t}+\vec{v}_q\cdot\nabla_{\vec{R}}f_q(\vec{R},\vec{p},t)-\nabla_{\vec{R}}U_q\cdot\nabla_{\vec{p}}f_q(\vec{R},\vec{p},t)=0 \tag{39}$$

in terms of the semiclassical Wigner function for particle type $q=n,p,e$, i.e.,

$$f_q(\vec{R},\vec{p},t)=\frac{1}{(2\pi)^3}\sum_i\int\phi_{qi}\left(\vec{R}-\frac{\vec{r}}{2},t\right)\phi^\star_{qi}\times\left(\vec{R}+\frac{\vec{r}}{2},t\right)e^{i\vec{p}\cdot\vec{r}}d^3r, \tag{40}$$

where ϕ_{qi} is the wave function of ith particle of type q, $\vec{R}$ and $\vec{r}$ are defined as the same as $\mathbf{R}$ and $\mathbf{r}$ in the previous section. In Eq. (39), $\vec{v}_q=\vec{p}/(m_q^{\star 2}+p^2)^{1/2}$ denotes the particle velocity and $m_e^\star=m_e$.

The density fluctuation due to a collective mode with frequency ω and wave vector $\vec{k}$ in nuclear matter can be studied through following the standard procedure [89] by writing

$$f_q(\vec{R},\vec{p},t)=f_q^0(\vec{p})+\delta f_q(\vec{R},\vec{p},t) \tag{41}$$

with

$$\delta f_q(\vec{R},\vec{p},t)=\delta\tilde{f}_q(\vec{p})e^{-i\omega t+i\vec{k}\cdot\vec{R}}. \tag{42}$$

By expressing $\rho_q(\vec{R},t)=\rho_q^0+\delta\rho_q(\vec{R},t)$ with

$$\delta\rho_q(\vec{R},t)=\frac{2}{(2\pi)^3}\int\delta f_q(\vec{R},\vec{p},t)d^3p, \tag{43}$$

we obtain the Vlasov equation

$$\delta\rho_q\approx X_qL_q\left(\sum_{q'}\frac{\delta U_q}{\delta\rho_{q'}}\delta\rho_{q'}\right), \tag{44}$$

where L_q is the usual Lindhard function

$$L_q=\int_{-1}^{1}\frac{\cos\theta d(\cos\theta)}{s_q-\cos\theta}=-2+s_q\ln\left(\frac{s_q+1}{s_q-1}\right), \tag{45}$$

with $s_q=\omega/kv_q^F$ and $v_q^F=p_q^F/(m_q^{\star 2}+p_q^{F2})^{1/2}$ being the Fermi velocity. The momentum integration can be evaluated approximately as

$$X_q=\frac{1}{2\pi^2}\int_0^{p_q^F}\left(-\frac{\partial f_q^0}{\partial\epsilon_q}\right)p^2dp\approx\frac{p_q^Fm_q^\star}{2\pi^2} \tag{46}$$

for $q=n,p$ with $\epsilon_q^F\approx p_q^{F2}/2m_q^\star$ and

$$X_e\approx\frac{\mu_e^2}{2\pi^2}, \tag{47}$$

for electrons, where $\mu_e\approx p_e^F$ is the electron chemical potential.

For protons, there are additional direct and exchange Coulomb contributions to the factor $\sum_{q'}\frac{\delta U_q}{\delta\rho_{q'}}\delta\rho_{q'}$ in Eq. (44) given, respectively, by

$$\delta U_p^{CD}=\frac{4\pi e^2}{k^2}(\delta\rho_p-\delta\rho_e), \tag{48}$$

$$\delta U_p^{CE}=-\frac{1}{3}e^2\left(\frac{3}{\pi}\right)^{1/3}\rho_p^{-2/3}\delta\rho_p. \tag{49}$$

For electrons, there are only direct and exchange Coulomb contributions to $\sum_{q'}\frac{\delta U_q}{\delta\rho_{q'}}\delta\rho_{q'}$.

After linearizing the Vlasov equation, we can reexpress Eq. (44) as a function of the collective density fluctuation

$$C_{VE}^f(\delta\rho_n,\delta\rho_p,\delta\rho_e)^T=0, \tag{50}$$

with

$$C_{VE}^f=\begin{pmatrix}X_nL_n\frac{\partial U_n}{\partial\rho_n}-1 & X_nL_n\frac{\partial U_n}{\partial\rho_p} & 0\\ X_pL_p\frac{\partial U_p}{\partial\rho_n} & X_pL_p\frac{\partial U_p}{\partial\rho_p}-1 & X_pL_p\frac{\partial U_p}{\partial\rho_e}\\ 0 & X_eL_e\frac{\partial U_e}{\partial\rho_p} & X_eL_e\frac{\partial U_e}{\partial\rho_e}-1\end{pmatrix}=\begin{pmatrix}X_nL_n & 0 & 0\\ 0 & X_pL_p & 0\\ 0 & 0 & X_eL_e\end{pmatrix}\begin{pmatrix}\frac{\partial U_n}{\partial\rho_n} & \frac{\partial U_n}{\partial\rho_p} & 0\\ \frac{\partial U_p}{\partial\rho_n} & \frac{\partial U_p}{\partial\rho_p} & \frac{\partial U_p}{\partial\rho_e}\\ 0 & \frac{\partial U_e}{\partial\rho_p} & \frac{\partial U_e}{\partial\rho_e}\end{pmatrix}-\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}. \tag{51}$$

In Eq. (51), the U_q is the single-particle potential in the npe system and the matrix

$$U=\begin{pmatrix}\frac{\partial U_n}{\partial\rho_n} & \frac{\partial U_n}{\partial\rho_p} & 0\\ \frac{\partial U_p}{\partial\rho_n} & \frac{\partial U_p}{\partial\rho_p} & \frac{\partial U_p}{\partial\rho_e}\\ 0 & \frac{\partial U_e}{\partial\rho_p} & \frac{\partial U_e}{\partial\rho_e}\end{pmatrix} \tag{52}$$

has the same k-dependent terms as in Eq. (36).

The determinant $|C_{VE}^f|=0$ determines the dispersion relation $\omega(k)$ of the collective density fluctuation that also determines the nontrivial solutions of Eq. (50). The transition density in neutron stars is the density at which the frequency ω becomes imaginary, leading to that the collective density fluctuation would grow exponentially and thus the instability of the neutron star matter occurs. To determine the condition for this to occur, we let $s_q=-i\nu_q$ ($\nu_q>0$) and rewrite the Lindhard function as $L_q=-2+2\nu_q\arctan(1/\nu_q)$. Since the values of L_q are in the range of $-2<L_q<0$, the critical values $L_n=L_p=L_e=-2$, corresponding to $\nu_q=0$, then determine the spinodal boundary of the system when they are substituted into $|C_{VE}^f|=0$. The baryon number density that makes $|C_{VE}^f|$ vanish then corresponds to the spinodal boundary in the neutron star matter or the core-crust transition density of neutron stars for the Vlasov equation method.

III. RESULTS

In the present work, for the Skyrme effective nucleon-nucleon interaction, we use the modified Skyrme-like (MSL) parameter [97,98] for which the 9 Skyrme interaction parameters σ, $t_0 - t_3$, $x_0 - x_3$ are obtained analytically in terms of 9 macroscopic quantities ρ_0, $E_0(\rho_0)$, the incompressibility K_0, the isoscalar effective mass $m^*_{s,0}$, the isovector effective mass $m^*_{v,0}$, $E_{\rm sym}(\rho_0)$, L, the gradient coefficient G_S, and the symmetry-gradient coefficient G_V. In particular, the MSL0 parameter set [97] is obtained by using the following empirical values for the 9 macroscopic quantities: $\rho_0 = 0.16\ \mathrm{fm}^{-3}$, $E_0(\rho_0) = -16$ MeV, $K_0 = 230$ MeV, $m^*_{s,0} = 0.8m$, $m^*_{v,0} = 0.7m$, $E_{\rm sym}(\rho_0) = 30$ MeV, $L = 60$ MeV, $G_V = 5\ \mathrm{MeV}\cdot\mathrm{fm}^5$, and $G_S = 132\ \mathrm{MeV}\cdot\mathrm{fm}^5$. And the spin-orbit coupling constant $W_0 = 133.3\ \mathrm{MeV}\cdot\mathrm{fm}^5$ is used to fit the neutron $p_{1/2} - p_{3/2}$ splitting in ^{16}O. It has been shown [97] that the MSL0 interaction can give a good description of the binding energies and charge rms radii for a number of closed-shell or semi-closed-shell nuclei.

For the U-boson-nucleon coupling constant g and the U-boson mass μ, their values are largely uncertain. As argued by Krivoruchenko *et al.* [30], in order to ensure that the U-boson effects on finite nuclei should be negligible, the Compton wavelength of the U boson is usually assumed to be greater than the radius of heavy nuclei, i.e., about 7 fm, leading to $\mu \lesssim 30$ MeV. At the same time, the g^2/μ^2 value of the U boson should be less than about 200 GeV^{-2}, which roughly corresponds to the value of the ordinary vector ω boson. Otherwise, the U boson is neither weakly coupled nor light. On the other hand, there also exist some constraints on properties of the U boson from cosmology and astrophysical observations. For example, the U-boson mass μ is required to exceed the mass of light cold dark matter, i.e., $\sim$ MeV, to explain the excess flux of 511 keV photons coming from the central region of our Galaxy observed by the SPI/INTEGRAL satellite [99]. Based on above discussions, in the present work we assume the U-boson mass is in the range of 2 MeV $\lesssim \mu \lesssim$ 30 MeV (the corresponding Compton wavelength of the U boson is thus between about 7 fm and 100 fm) and the g^2/μ^2 value of the U boson satisfies $g^2/\mu^2 \lesssim 150\ \mathrm{GeV}^{-2}$. The latter is further consistent with existing constraints from neutron-proton and neutron-lead scatterings, the spectroscopy of antiproton atoms as well as the recently discovered new holder of neutron star maximum mass of $1.97 \pm 0.04\ M_\odot$ from PSR J1614-2230 [16,18,19,33].

A. Nuclear matter symmetry energy from the U boson

As has been seen in the previous section, the direct term of the U boson contributes to the nuclear matter EOS only through the combination g^2/μ^2, and, in particular, its contribution to the energy per nucleon is given by $\frac{1}{2} g^2/\mu^2 \rho$. As it was emphasized by Fujii [100], for the direct term contribution, though both the coupling constant g and the mass μ are small for the light and weakly coupled bosons, the value of the ratio g^2/μ^2 can be large. Therefore, the light and weakly coupled bosons can significantly affect the nuclear matter EOS and thus the properties of neutron stars [30–33]. On the other hand, the exchange term contribution to the nuclear matter EOS depends on both the coupling constant g and the mass μ in a complicated way. Furthermore, it is interesting to see that the isoscalar U boson will contribute to the nuclear matter symmetry energy due to the exchange term contribution though the direct term does not have such contribution.

To see quantitatively how the exchange term of the U boson affects the nuclear matter EOS, we show in Fig. 1 the exchange term contribution of the U boson to the EOS of symmetric nuclear matter $E^E_{0,\rm UB}(\rho)$ [(a) and (b)] and the nuclear matter symmetry energy $E_{\rm sym,UB}(\rho)$ [(c) and (d)] as functions of density with several typical values of μ and g^2/μ^2. Here, the $E_{\rm sym,UB}(\rho)$ is extracted from the parabolic approximation $E_{\rm sym,UB}(\rho) \approx E_{\rm UB}(\rho, \delta = 1) - E_{\rm UB}(\rho, \delta = 0)$ with $E_{\rm UB}(\rho, \delta)$ being the energy per nucleon from the U-boson contribution. One can see from Fig. 1 that both $E^E_{0,\rm UB}(\rho)$ and $E_{\rm sym,UB}(\rho)$ increase with increasing values of both μ and g^2/μ^2. As expected, however, both the contributions of the exchange term to energy per nucleon of symmetric nuclear matter and the symmetry energy are quite small and can be safely neglected compared with the direct term contribution for the values of μ and g^2/μ^2 considered here. For example, even at very high baryon density such as $\rho = 1.0\ \mathrm{fm}^{-3}$ with $\mu = 30.0$ MeV and $g^2/\mu^2 = 150\ \mathrm{GeV}^{-2}$ (see the right

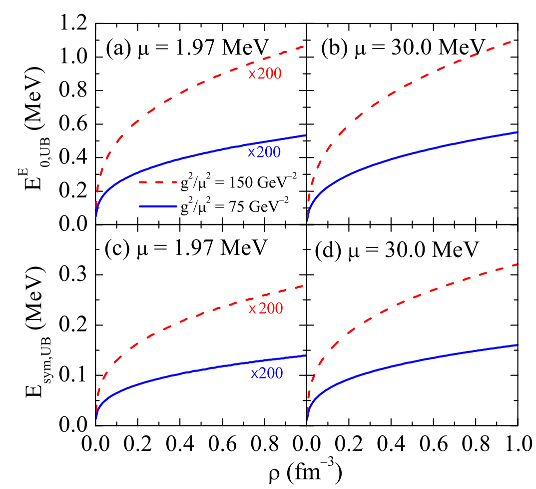

FIG. 1 (color online). The exchange term contribution of the U boson to the EOS of symmetric nuclear matter [(a) and (b)] and the symmetry energy [(c) and (d)] as functions of density with several typical values of μ and g^2/μ^2. Note: The results with $\mu = 1.97$ MeV [(a) and (c)] have been rescaled by multiplying a factor of 200 for convenience.

panels of Fig. 1), the $E^{E}_{0,\mathrm{UB}}(\rho)$ is only about 1.1 MeV and the $E_{\mathrm{sym,UB}}(\rho)$ is less than about 0.32 MeV while the direct term contribution to the energy per nucleon of symmetric nuclear matter reaches about 576 MeV. The $E^{E}_{0,\mathrm{UB}}(\rho)$ and $E_{\mathrm{sym,UB}}(\rho)$ will further decrease if smaller values of μ and g^2/μ^2 are used (see, e.g., the left panels of Fig. 1). These results verify the validity of neglecting the exchange term contribution of the U boson to nuclear matter EOS in the literature [30–33].

B. The core-crust transition density and pressure in neutron stars with the U boson

We now turn to the numerical results on the core-crust transition density and pressure in neutron stars with the thermodynamical approach, the curvature matrix approach, and the Vlasov equation approach. We note that both the matrices C^{f}_{CM} in Eq. (36) and C^{f}_{VE} in Eq. (51) are k dependent, and for the curvature matrix approach and the Vlasov equation approach, the core-crust transition density corresponds to the critical baryon number density above which the neutron star matter is always stable for all possible values of k while below which one can always find a k value to violate the stability conditions of the neutron star matter. On the other hand, for the thermodynamical method, the transition density can be directly obtained by solving the equation $V_{\mathrm{ther}} = 0$ [see Eq. (33) for the expression of V_{ther}].

Theoretically, it has been established that there exists a strong correlation between the transition density ρ_t and the nuclear symmetry energy. In particular, a strong linear correlation between the transition density ρ_t and the slope parameter L of the nuclear symmetry energy has been observed in many different theoretical calculations [96,97,101,102]. To see the symmetry energy dependence of the inner edge of neutron star crusts, we show in Fig. 2 the transition density ρ_t and pressure P_t in neutron stars as functions of the L parameter with MSL0 interaction by varying individually L using the thermodynamical method, the curvature matrix method, and the Vlasov equation method. When varying individually the L parameter, we keep all other macroscopic quantities ρ_0, $E_0(\rho_0)$, K_0, $m^*_{s,0}$, $m^*_{v,0}$, $E_{\mathrm{sym}}(\rho_0)$, G_S, G_V, and W_0 at their default values in MSL0. It should be noted that the original agreement of MSL0 with the experimental data of binding energies or charge radii of finite nuclei essentially still holds with the individual change of the L parameter.

For the results shown in Fig. 2, the U-boson contributions are not considered. It is seen that all the three methods give similar results for the L dependence of the transition density ρ_t and pressure P_t with the curvature matrix method giving slightly smaller values of ρ_t and P_t than the thermodynamical method while slightly higher values of ρ_t and P_t than the Vlasov equation method for a fixed value of L. The nonmonotonous variation of the L dependence of the transition pressure P_t in the thermodynamical method is due to the fact that the P_t is a complicated function of ρ_t, L, and the isospin asymmetry δ_t at the transition density (See, e.g., [96]). The smaller values of ρ_t from the curvature matrix method than from the thermodynamical method imply that the density gradient terms and Coulomb term considered in the former can make the neutron star matter more stable, which are consistent with the results in Refs. [96,97] (The curvature matrix method is called dynamical method there). On the other hand, the slightly smaller values predicted by the Vlasov equation method than the curvature matrix method are due to the quantum effects considered in the Vlasov equation method, indicating that the quantum effects will make the neutron star matter more stable as expected.

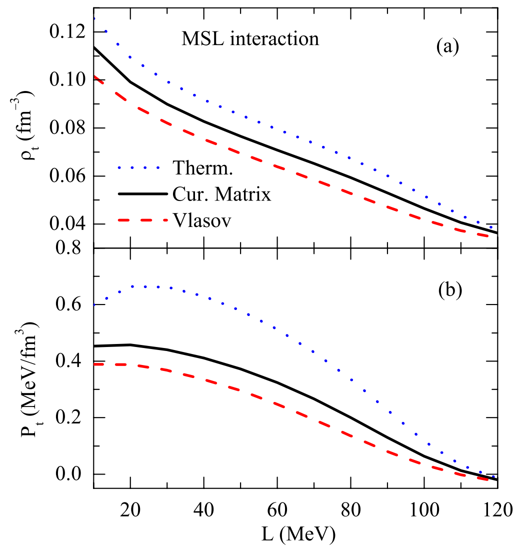

FIG. 2 (color online). Transition density ρ_t (a) and pressure P_t (b) in neutron stars as functions of the L parameter with the MSL interaction using the thermodynamical method, the curvature matrix method, and the Vlasov equation method.

To see how the light and weakly coupled vector U boson affects the inner edge of neutron star crusts, we show in Fig. 3 the g^2/μ^2 dependence of the transition density ρ_t and pressure P_t in neutron stars from the MSL0 interaction by including the U boson with $\mu = 1.97$ MeV and 30.0 MeV, respectively, using the thermodynamical method, the curvature matrix method, and the Vlasov equation method. One can see clearly that while the curvature matrix method and the Vlasov equation method predict very similar results for the g^2/μ^2 dependence of ρ_t and P_t, the thermodynamical method predicts very different results with ρ_t and P_t decreasing very quickly as the g^2/μ^2 increases. In particular, we find that for $g^2/\mu^2 > 20\ \mathrm{GeV}^{-2}$, the neutron star matter is always stable and the transition density does not exist in the thermodynamical method. This is due to the fact that the k-dependent terms in Eqs. (36) and (51) originating from

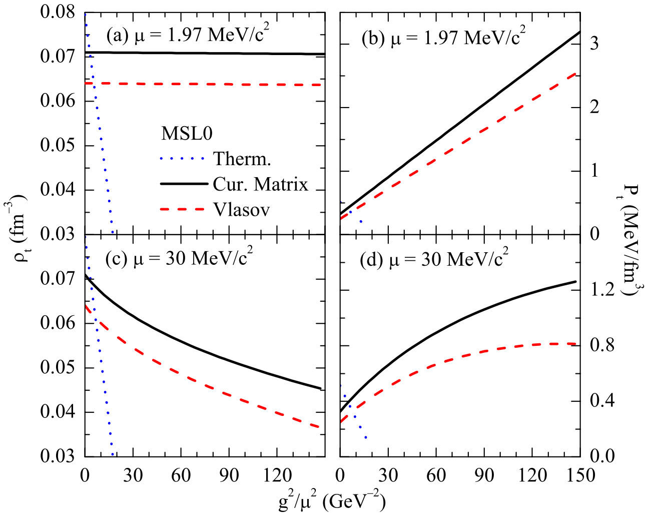

FIG. 3 (color online). The g^2/μ^2 dependence of the transition density ρ_t and pressure P_t in neutron stars from the thermodynamical method, the curvature matrix method, and the Vlasov equation method with the MSL0 interaction for $\mu = 1.97$ MeV and 30.0 MeV, respectively.

the finite-range interaction and density gradient contributions play an important role in determining the transition density when the U boson is considered. Therefore, the thermodynamical method that ignores the k-dependent terms would be no longer appropriate to determine the inner edge of neutron star crusts when the U boson is taken into account.

It is interesting to see from Fig. 3 that for the curvature matrix method and the Vlasov equation method, the effects of the U boson on the transition density ρ_t and pressure P_t depend on not only the ratio g^2/μ^2 but also the U-boson mass μ. In particular, for a heavier U boson (e.g., $\mu = 30.0$ MeV), the ρ_t decreases significantly with increment of g^2/μ^2 from 0 to 150 GeV^{-2}. On the other hand, for a very light U boson (e.g., $\mu = 1.97$ MeV), the transition density ρ_t exhibits very weak dependence on g^2/μ^2. These features imply that for a fixed value of g^2/μ^2, a heavier U boson can make the neutron star matter more stable while a very light U boson essentially has no influence on the transition density.

As shown in Fig. 3, for the curvature matrix method and the Vlasov equation method, although the transition density ρ_t displays a somewhat complicated relationship with the properties of the U boson, the transition pressure P_t simply increases with the ratio g^2/μ^2 whether the U-boson mass is light or heavy. Furthermore, it is interesting to see from Fig. 3(b) that the transition pressure P_t increases almost linearly with g^2/μ^2 for a very light U boson (i.e., $\mu = 1.97$ MeV), while it exhibits much slower increment with g^2/μ^2 for a heavier U boson (e.g., $\mu = 30.0$ MeV) as shown in Fig. 3(d). The linear correlation between P_t and g^2/μ^2 for $\mu = 1.97$ MeV observed in Fig. 3(b) is easily understood since the U-boson contribution to the pressure $P_{t,\mathrm{UB}}$ is dominated by the direct term contribution, i.e., $P_{t,\mathrm{UB}} \approx \frac{1}{2} g^2/\mu^2 \rho_t^2$, if μ is independent of the density as we assume in the present work, and the ρ_t remains approximately a constant value for $\mu = 1.97$ MeV when g^2/μ^2 varies from 0 to 150 GeV^{-2} as shown Fig. 3(a). In the case of $\mu = 30.0$ MeV, the ρ_t decreases significantly with the increment of g^2/μ^2 as shown Fig. 3(c), which leads to the transition pressure P_t displaying much slower increment with g^2/μ^2 as shown in Fig. 3(d).

In order to see the U-boson mass dependence of ρ_t and P_t at a fixed value of g^2/μ^2, we display in Fig. 4 the $1/\mu$ dependence of the transition density ρ_t and pressure P_t in neutron stars from the MSL0 interaction by including the U-boson contribution with $g^2/\mu^2 = 75$ GeV^{-2} using the curvature matrix method and the Vlasov equation method. It is seen that the two methods predict very similar $1/\mu$ dependence of the transition density ρ_t and pressure P_t with the Vlasov equation method giving smaller values. It is interesting to see that ρ_t becomes sensitive to μ when the U-boson mass μ is larger than about 2 MeV although the U boson almost has no influence on the transition density ρ_t if its mass μ is less than about 2 MeV. The transition pressure P_t displays similar $1/\mu$ dependence as the ρ_t due to the relation $P_{t,\mathrm{UB}} \approx \frac{1}{2} g^2/\mu^2 \rho_t^2$. Therefore, these results demonstrate that the transition density ρ_t in neutron stars can be sensitive to the value of both μ and g^2/μ^2, and any experimental or observational constraints on ρ_t may put important limits on μ and g^2/μ^2, or equivalently on μ and g.

As have been shown above, the exchange term contribution of the U boson to the nuclear matter EOS can be

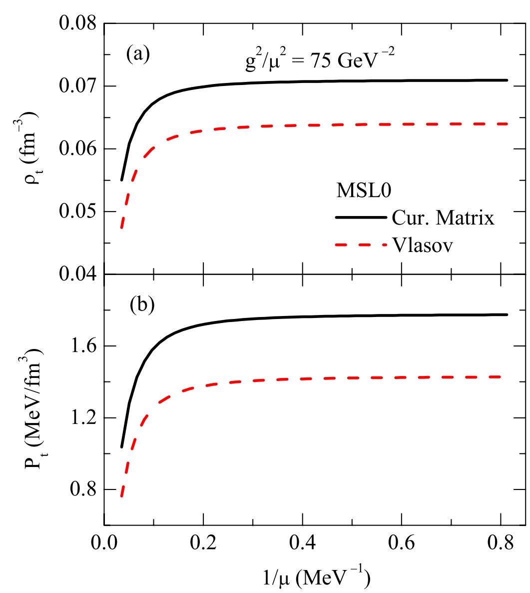

FIG. 4 (color online). The $1/\mu$ dependence of the transition density ρ_t and pressure P_t in neutron stars from the curvature matrix method and the Vlasov equation method with the MSL0 interaction for $g^2/\mu^2 = 75$ GeV^{-2}.

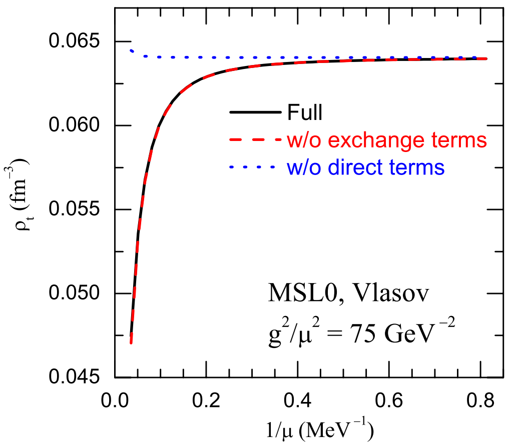

FIG. 5 (color online). The $1/\mu$ dependence of the transition density ρ_t in neutron stars from the Vlasov equation method with the MSL0 interaction for $g^2/\mu^2 = 75$ GeV^{-2}. The results of neglecting the U-boson direct term contribution or neglecting the U-boson exchange term contribution are included for comparison.

safely neglected. It is thus interesting to see how the exchange term affects the inner edge of neutron star crusts. To check this point, we show in Fig. 5 the $1/\mu$ dependence of the transition density ρ_t in neutron stars with the MSL0 interaction for $g^2/\mu^2 = 75$ GeV^{-2} from the Vlasov equation method together with that by neglecting the direct term contribution or neglecting the exchange term contribution. It is seen that the exchange term contribution of the U boson has very small influence on the transition density ρ_t, especially for the light mass U boson. On the other hand, the direct term significantly affects the $1/\mu$ dependence of the transition density ρ_t. Particularly, neglecting the direct term contribution of the U boson leads to very weak $1/\mu$ dependence of the transition density ρ_t, implying that the observed strong $1/\mu$ dependence of the transition density ρ_t is essentially due to the direct term contribution that produces a k-dependent term in the matrix (36) or (51), i.e., the third term in the right-hand side of Eq. (36). We note that using the curvature matrix method leads to the same conclusion.

C. The mass-radius relation and crustal fraction of moment of inertia for static neutron stars with the U boson

As we have showed above, the U boson may have significant influence on the nuclear matter EOS and the inner edge of neutron star crusts. Here, we investigate effects of the U boson on the global properties of static neutron stars. To calculate the global properties, such as the mass-radius relation and crustal fraction of moment of inertia, of static neutron stars, one needs the EOS of neutron star matter over a broad density region ranging from the center to the surface of neutron stars. Besides the possible appearance of nuclear pasta in the inner crust, various phase transitions and non-nucleonic degrees of freedom may appear in the core of neutron stars. In this work, we restrict ourselves to the simplest and traditional model, and make the minimum assumption that the core of neutron stars contains the uniform β-stable and electrically neutral $npe\mu$ matter only and there is no phase transition.

Generally, a typical neutron star contains the liquid core, inner crust, and outer crust from the center to surface. For the liquid core, we use the EOS of $npe\mu$ matter from SHF calculations including the U-boson contributions to the nuclear EOS. For the Skyrme effective nucleon-nucleon interaction, the MSL interaction with a soft symmetry energy of $L = 30$ MeV is used. We note that the MSL interaction with $L = 30$ MeV predicts a $npe\mu$ matter EOS very similar to the more sophisticated EOS containing nucleons, hyperons, and quark degrees of freedom [37]. In the present work, the U-boson contribution to the EOS of liquid core includes both the direct term contribution [i.e., Eq. (22)] and the exchange term contribution [i.e., Eq. (23) without the density gradient terms] although the latter is negligible. In particular, the fractions of neutrons, protons, electrons, and muons in the neutron star matter are obtained from self-consistently solving the set of equations for β-stable condition (i.e., $\mu_{np} = \mu_e = \mu_\mu$) and charge neutral condition (i.e., $\rho_p = \rho_e + \rho_\mu$) by considering the U-boson exchange term contribution to the chemical potential of neutrons and protons. It should be noted that the U-boson direct term does not change the neutron and proton chemical potential difference μ_{np} as it contributes equally to the single-particle potential of neutrons and protons, and thus the chemical compositions of the neutron star matter will not change if only the U-boson direct term contribution is considered as pointed out in previous work [30–33]. In this way, the contributions of U boson to the energy density, the pressure, the nucleon effective masses, and chemical potentials are considered self-consistently in the neutron star matter calculations for the liquid core.

In the inner crust with densities between $\rho_{\rm out}$ and ρ_t where the nuclear pastas may exist, because of our poor knowledge about its EOS from first principle, following Carriere *et al.* [84] (see also Ref. [96]) we construct its EOS according to

$$P = a + b\epsilon^{4/3}. \tag{53}$$

This polytropic form with an index of $4/3$ has been found to be a good approximation to the crust EOS [42,45]. The $\rho_{\rm out} = 2.46 \times 10^{-4}$ fm^{-3} is the density separating the inner from the outer crust. The constants a and b are then determined by

$$a = \frac{P_{\rm out}\epsilon_t^{4/3} - P_t\epsilon_{\rm out}^{4/3}}{\epsilon_t^{4/3} - \epsilon_{\rm out}^{4/3}}, \tag{54}$$

$$b = \frac{P_t - P_{\rm out}}{\epsilon_t^{4/3} - \epsilon_{\rm out}^{4/3}}, \tag{55}$$

where P_t, ϵ_t and $P_{\rm out}$, $\epsilon_{\rm out}$ are the pressure and energy density at ρ_t and $\rho_{\rm out}$, respectively. In the outer crust with $6.93 \times 10^{-13}\ {\rm fm}^{-3} < \rho < \rho_{\rm out}$, we use the EOS of BPS [38,103], and in the region of $4.73 \times 10^{-15}\ {\rm fm}^{-3} < \rho < 6.93 \times 10^{-13}\ {\rm fm}^{-3}$ we use the EOS of Feynman-Metropolis-Teller [38]. For the U-boson contribution to the EOS of neutron star crusts and surface, we add the energy density and pressure from the U-boson direct term contribution, i.e., $\epsilon_{\rm UB} = P_{\rm UB} \approx \frac{1}{2} g^2/\mu^2 \rho^2$, to the corresponding parts since the exchange term contribution is negligible as shown in the above.

As an example, we show in Fig. 6 the EOS for different parts of a neutron star. As we have discussed earlier, the transition density ρ_t is obtained by studying the onset of instabilities in the liquid core, namely, it is the critical density below which small density fluctuations will grow exponentially. Therefore, the transition density ρ_t and the EOS of the liquid core are obtained self-consistently from the same interaction and in this sense they are on the same footing. We use in Fig. 6 the ρ_t obtained within the Vlasov equation method using the full EOS with the MSL interaction of $L = 30$ MeV for different values of g^2/μ^2 with $\mu = 1.97$ MeV. Using the above EOS for different parts of the neutron star, the radial distribution of the total energy density and the pressure in neutron stars is continuous, but the derivative of the pressure is not continuous at ρ_t and $\rho_{\rm out}$. It is seen that the EOS's of the inner crust and the liquid core are quite different for different values of g^2/μ^2. Interestingly, one can see that because the transition density ρ_t displays very weak dependence on g^2/μ^2 for the very light U-boson mass (1.97 MeV here), the ρ_t (and thus the corresponding ε_t) has essentially the same value for different g^2/μ^2, and the difference of the EOS for different values of g^2/μ^2 observed in Fig. 6 is essentially due to the variation of P_t with g^2/μ^2 because of the relation $P_{t,{\rm UB}} \approx \frac{1}{2} g^2/\mu^2 \rho_t^2$.

Using the EOS constructed above, one can solve the Tolman-Oppenheimer-Volkoff equations to obtain the mass-radius relations and the results are shown in Fig. 7. Indicated by the shaded band in Fig. 7 is the latest new holder of the maximum mass of neutron stars of $1.97 \pm 0.04 M_\odot$ from PSR J1614-2230 [36]. For MSL interaction with a soft symmetry energy ($L = 30$ MeV) without considering the U-boson contribution, the neutron star mass M decreases quickly with increasing radius R and the maximum mass is about $1.47\ M_\odot$, which is significantly less than the observed maximum neutron star mass of $1.97 \pm 0.04 M_\odot$. On the other hand, the neutron star mass can be enhanced strongly if the effects of U boson are considered. In particular, a larger value of g^2/μ^2 leads to a larger neutron star mass at a fixed radius since the nuclear EOS is increasingly stiffened with increment of g^2/μ^2 as shown in Fig. 6. In particular, the neutron star maximum mass can reach $2.07 M_\odot$ with $g^2/\mu^2 = 75\ {\rm GeV}^{-2}$, and the corresponding radius of the maximum mass neutron star is about 14.8 km.

To see more clearly the U-boson effects on the properties of neutron stars, we include in Fig. 7 the mass-radius relations for different values of g^2/μ^2 with two values of the U-boson mass, i.e., $\mu = 1.97$ and 30.0 MeV, respectively. Furthermore, the results with ρ_t from both the Vlasov equation method and the curvature matrix method are included for comparison although the former is

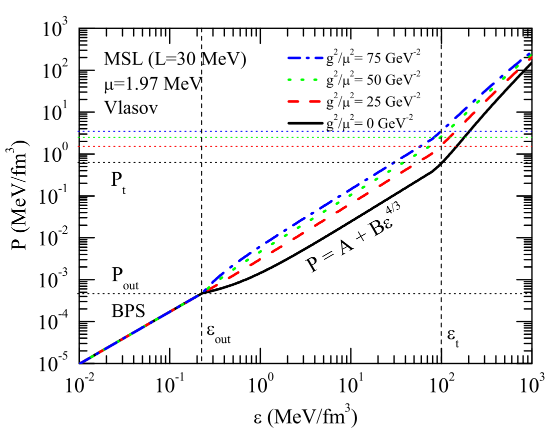

FIG. 6 (color online). The EOS's of different parts in neutron stars using the MSL interaction with $L = 30$ MeV for different values of g^2/μ^2 ranging from 0 to 75 GeV^{-2} with $\mu = 1.97$ MeV. The energy density (pressure) at ρ_t and $\rho_{\rm out}$ is indicated as ε_t (P_t) and $\varepsilon_{\rm out}$ ($P_{\rm out}$), respectively, and the ρ_t is obtained from the Vlasov equation method.

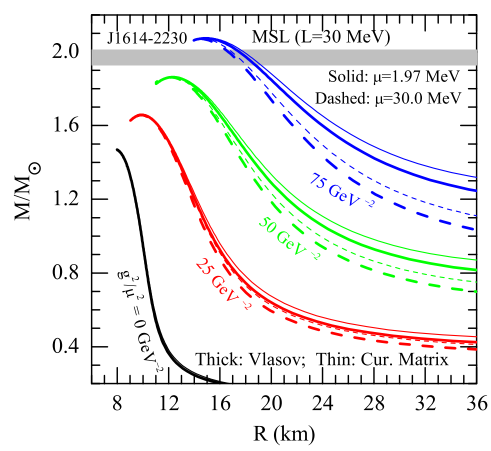

FIG. 7 (color online). The mass-radius relation of static neutron stars using the MSL interaction with $L = 30$ MeV and different values of g^2/μ^2 for $\mu = 1.97$ MeV (solid lines) and 30.0 MeV (dashed lines), respectively. The results with ρ_t from both the Vlasov equation method (thick lines) and the curvature matrix method (thin lines) are included for comparison. The shaded band represents the latest new holder of the maximum mass of neutron stars of $1.97 \pm 0.04 M_\odot$ from PSR J1614-2230 [36].

believed to be more realistic. As expected, for the larger value of the U-boson mass, a smaller value of the neutron star mass M at a fixed radius R is obtained due to the smaller transition density and pressure obtained for a larger μ as shown in the previous section. This U-boson mass effect on the neuron star mass-radius relation will become more pronounced for a larger value of g^2/μ^2 (e.g., $g^2/\mu^2 = 75\ \text{GeV}^{-2}$). In addition, one can see that using the ρ_t from the curvature matrix method predicts a little larger neutron star mass M at a fixed radius R than using the ρ_t from the Vlasov equation method. However, the maximum mass of the neutron stars is almost the same for different values of μ using the ρ_t from either the curvature matrix method or the Vlasov equation method. These results indicate that, essentially regardless of the values of the U-boson mass and the ρ_t, a value of $g^2/\mu^2 = 75\ \text{GeV}^{-2}$ can reasonably describe the latest new holder of the neutron star maximum mass of $1.97 \pm 0.04 M_\odot$ from PSR J1614-2230 [36].

The crustal fraction of total moment of inertia of a static neutron star, $\Delta I/I$, is a particularly interesting quantity as it can be inferred from observations of pulsar glitches, i.e., the occasional disruptions of the otherwise extremely regular pulsations from magnetized, rotating neutron stars. Furthermore, as was stressed by Lattimer and Prakash [42], the $\Delta I/I$ depends sensitively on the ρ_t and P_t, which are essentially determined by the EOS of asymmetric nuclear matter at subsaturation densities as shown above, but there is no explicit dependence upon the EOS of neutron star matter at higher densities. These features imply that the $\Delta I/I$ may provide a good probe for properties of the U boson. To illustrate the U-boson mass dependence of the $\Delta I/I$, we show in Fig. 8 the $\Delta I/I$ as a function of the neutron star mass for static neutron stars using the MSL interaction with $L = 30$ MeV for $g^2/\mu^2 = 75\ \text{GeV}^{-2}$ with $\mu = 1.97$ and 30.0 MeV, respectively. Furthermore, the results with ρ_t obtained from both the Vlasov equation method and the curvature matrix method are included for comparison. The $\Delta I/I$ is obtained here from direct numerical calculations as in Ref. [96]. As expected, one can see that the $\Delta I/I$ indeed exhibits a clear sensitivity to the U-boson mass with a heavier U-boson mass giving a smaller $\Delta I/I$ for a fixed neutron star mass. In addition, one can see that using the ρ_t obtained from the curvature matrix method gives a little larger value for $\Delta I/I$ at a fixed neutron star mass M than using the ρ_t from the Vlasov equation method. Empirically, the crustal fraction of total moment of inertia has been constrained as $\Delta I/I > 0.014$ from studying the glitches of the Vela pulsar [45] and the lower limit $\Delta I/I = 0.014$ of the constraint is also indicated in Fig. 8. It is seen from Fig. 8 that the calculated results of $\Delta I/I$ with the two values of $\mu = 1.97$ and 30.0 MeV using the ρ_t obtained from either the curvature matrix method or the Vlasov equation method are all consistent with the observation constraint of $\Delta I/I > 0.014$ for the Vela pulsar.

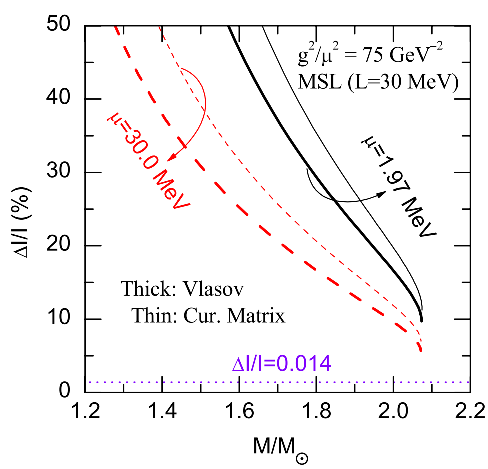

FIG. 8 (color online). The crustal fraction of total moment of inertia $\Delta I/I$ as a function of the neutron star mass for static neutron stars using the MSL interaction with $L = 30$ MeV for $g^2/\mu^2 = 75\ \text{GeV}^{-2}$ with $\mu = 1.97$ MeV (solid lines) and 30.0 MeV (dashed lines), respectively. The results with ρ_t from both the Vlasov equation method (thick lines) and the curvature matrix method (thin lines) are included for comparison. The lower limit $\Delta I/I = 0.014$ of the observation constraint for the Vela pulsar [45] is also indicated.

Based on the results above, we conclude that the vector U boson can significantly stiffen the nuclear matter EOS and thus enhance strongly the (maximum) mass of neutron stars. Furthermore, as the ρ_t and P_t are sensitive to both g^2/μ^2 and μ if the U-boson mass is larger than about 2 MeV, both g^2/μ^2 and μ can thus affect the mass-radius relation of neutron stars although the maximum mass of neutron stars is essentially independent of the U-boson mass μ. In addition, our results demonstrate that the crustal fraction of total moment of inertia $\Delta I/I$ may depend sensitively on the U-boson mass μ for a fixed value of g^2/μ^2.

IV. SUMMARY

Using the thermodynamical approach, the curvature matrix approach, and the Vlasov equation approach with the Skyrme effective nucleon-nucleon interaction, we have investigated effects of the light vector gauge U boson, that is weakly coupled to nucleons, on the core-crust transition density ρ_t and pressure P_t of neutron stars. For the exchange term contribution of the U boson, we have applied the density matrix expansion approach, which automatically leads to the density gradient terms in the single nucleon potential and nuclear energy density functional. Our results have demonstrated that the exchange term contribution to energy per nucleon of symmetric nuclear matter and the symmetry energy is quite small and can be safely neglected compared with the direct term contribution for the parameter range of μ and g^2/μ^2 considered in this work, verifying the validity of neglecting the U-boson exchange term contribution to the nuclear matter EOS as has been done in the literature. Furthermore, the exchange term has also been found to have negligible influence on the core-crust transition density ρ_t and pressure P_t, especially for very light U boson.

Interestingly, our results have shown that the ρ_t and P_t depend on not only the ratio of coupling strength to mass squared of the U boson g^2/μ^2 but also its mass μ. The U-boson mass dependence of ρ_t and P_t is due to the finite-range interaction from the U-boson exchange. In particular, we have found that the ρ_t and P_t will be sensitive to both g^2/μ^2 and μ if the U-boson mass μ is larger than about 2 MeV, and both g^2/μ^2 and μ can have significant influence on the mass-radius relation and the crustal fraction of total moment of inertia of neutron stars. Therefore, our results presented in this work have demonstrated that astrophysical observations on neutron star structures, such as the mass-radius relation and the crustal fraction of total moment of inertia from pulsar glitches, can be potentially useful to constrain properties of the U boson, e.g., its mass μ and the coupling constant g to nucleons.

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