

Testing the Kerr nature of stellar-mass black hole candidates by combining the continuum-fitting method and the power estimate of transient ballistic jets

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Astrophysical black hole candidates are thought to be the Kerr black holes predicted by General Relativity, as these objects cannot be explained otherwise without introducing new physics. However, there is no observational evidence that the space-time around them is really described by the Kerr solution. The Kerr black hole hypothesis can be tested with the already available X-ray data by extending the continuum-fitting method, a technique currently used by astronomers to estimate the spins of stellar-mass black hole candidates. In general, we cannot put a constraint on possible deviations from the Kerr geometry, but only on some combination between these deviations and the spin. The measurement of the radio power of transient jets in black hole binaries can potentially break this degeneracy, thus allowing for testing the Kerr-nature of these objects.

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I. INTRODUCTION

The $5\text{--}20M_{\odot}$ compact objects in X-ray binary systems and the $10^5\text{--}10^9M_{\odot}$ dark bodies at the center of every normal galaxy are thought to be the Kerr black holes (BHs) predicted by General Relativity [1]. There is no evidence that the space-time around these objects is really described by the Kerr metric, but, at the same time, there is no other explanation in the framework of conventional physics. A Kerr BH is completely specified by two parameters: its mass, M , and its spin angular momentum, J . A fundamental limit for a BH in 4-dimensional General Relativity is the bound $|a_*| \leq 1$, where $a_* = J/M^2$ is the dimensionless spin parameter.¹ This is just the condition for the existence of the event horizon: For $|a_*| > 1$, there is no horizon and the Kerr metric describes a naked singularity, which is forbidden by the weak cosmic censorship conjecture [2].

In the case of the stellar-mass BH candidates in X-ray binary systems, the mass M can be deduced by studying the orbital motion of the stellar companion. This measurement is reliable, because the system can be described in the framework of Newtonian mechanics, with no assumptions about the nature of the compact object. The situation changes when we want to get an estimate of the spin parameter a_* . The most reliable approach is currently the continuum-fitting method [3–8]. Basically, one fits the X-ray continuum spectrum of the BH candidate using the standard accretion disk model of Novikov and Thorne [9]. Under the assumption that the background geometry is described by the Kerr metric, it is possible to infer the spin parameter, a_* , and the mass accretion rate, \dot{M} , if the mass of the BH candidate, its distance from us, and the inclination angle of the disk are known independently.

The possibility of testing the Kerr nature of astrophysical BH candidates with present and near future experiments is becoming an active research field [10–16]. In particular, one can extend the continuum-fitting method to constrain possible deviations from the Kerr geometry [12]. That can be achieved by considering a more general background, which includes the Kerr solution as special case. The compact object will be thus characterized by M , a_* , and at least one “deformation parameter,” measuring deviations from the Kerr geometry. If observational data require a vanishing deformation parameter, the Kerr BH hypothesis is verified. However, the fit of the X-ray spectrum cannot be used to measure a_* and the deformation parameter at the same time, but it is only possible to constrain a combination of them. This is not a problem of the continuum-fitting method, but of any approach (see e.g. Ref. [13] for the case of the analysis of the $K\alpha$ iron line).

In what follows, I will apply the recent finding of Ref. [17] to show that one can potentially break the degeneracy between a_* and the deformation parameter by combining the continuum-fitting method with the power estimate of transient ballistic jets.

II. TRANSIENT BALLISTIC JETS

Observationally, BH binaries can emit two kinds of jets [18]. *Steady jets* occur in the hard spectral state, over a wide range of luminosity of the source, and they seem to be not very relativistic. *Transient ballistic jets* are instead launched when a BH binary with a low-mass companion undergoes a transient outburst: The jet appears when the source switches from the hard to soft state and its luminosity is close to the Eddington limit. Transient jets are observed as blobs of plasma moving ballistically outward at relativistic velocities. The common interpretation is that steady jets are produced relatively far from the compact

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¹Throughout the paper, I use units in which $G_N = c = 1$.

TABLE I. The four stellar-mass BH candidates of which the spin parameter a_* has been estimated with the continuum-fitting method and we have radio data of their outbursts. The accretion efficiency η in the third column has been deduced from the corresponding a_* for a Kerr background. The mass-normalized jet power P_{jet} in the fourth column has been inferred from the data reported in Ref. [17], using Eq. (1).

BH Binary	a_*	η	P_{jet} (kpc ² GHz Jy/ M_\odot)	Reference
GRS 1915 + 105	0.975, $a_* > 0.95$	0.224, $\eta > 0.190$	39.4	[5]
GRO J1655-40	0.7 ± 0.1	$0.104^{+0.018}_{-0.013}$	19.7	[6]
XTE J1550-564	0.34 ± 0.24	$0.072^{+0.017}_{-0.011}$	2.79	[7]
A0620-00	0.12 ± 0.19	$0.061^{+0.009}_{-0.007}$	0.173	[8]

object, say at about 10 to 100 gravitational radii [19], while transient jets are launched within a few gravitational radii [20]. As discussed in Ref. [17], it is therefore plausible that transient jets are powered by the rotational energy of the BH and, since they occur at a well defined luminosity, they may be used as “standard candles.”

In Ref. [17], the authors show there is a correlation between the spin parameter a_* , as inferred by the continuum-fitting method, and the radio power of transient ballistic jets. Moreover, the behavior is close to what should be expected if these jets were powered by the BH spin via the Blandford-Znajek mechanism [21].

So far, the continuum-fitting method has provided the estimate of the spin parameter of nine stellar-mass BH candidates [4]. Five of these objects have a low-mass companion and undergo mass transfer via Roche lobe outflow: During their outbursts, they produce ballistic jets. For three of them (GRS 1915 + 105, GRO J1655-40, and XTE J1550-564), we have good radio data during at least one of their outbursts. For A0620-00, the data are not so good. 4U 1543-47 has never been monitored well at radio wavelength during any of its outbursts. For GRS 1915 + 105, GRO J1655-40, XTE J1550-564, and A0620-00, the authors of Ref. [17] compute the mass-normalized jet radio power:

$$P_{\text{jet}} = \frac{D^2(\nu S_\nu)_{\text{max},5 \text{ GHz}}}{M}, \quad (1)$$

where D is the distance of the binary system from us and $(\nu S_\nu)_{\text{max},5 \text{ GHz}}$ is the estimate of the maximum of the radio power at 5 GHz (see Table I). Then, they plot the jet power P_{jet} against the BH spin parameter a_* , as inferred from the continuum-fitting method, and against the corresponding BH angular frequency

$$\Omega_H = -\left. \frac{g_{t\phi}}{g_{\phi\phi}} \right|_{r=r_H} = \frac{a}{r_H^2 + a^2}, \quad (2)$$

where r_H is the radius of the BH outer event horizon and $a = a_*M$. The scaling $P_{\text{jet}} \sim a_*^2$ was derived in Ref. [21], under the assumption $|a_*| \ll 1$. $P_{\text{jet}} \sim \Omega_H^2$ was instead obtained in Ref. [22] and works even for spin parameters quite close to 1. The top left panel of Fig. 1 shows the plot P_{jet} vs. Ω_H , which is basically the plot in Fig. 3 of Ref. [17]. The dashed line has a slope of 2, as expected from

the theoretical scaling. The uncertainty in P_{jet} is the somehow arbitrary uncertainty of 0.3 in the log adopted in Ref. [17]. Despite there being only four objects, there is evidence for a correlation between jet power and Ω_H , and one finds the behavior expected in the case of a jet powered by the rotational energy of the BH. For more details about the systematics, the interpretation of the finding, and the comparison with previous results, see Ref. [17]. The conclusions of the authors are therefore that: *i*) they have provided the first evidence that some jets may be powered by the BH spin energy, and *ii*) the observed correlation also provides an additional confirmation of the continuum-fitting method.

III. NON-KERR SPACE-TIMES

In this section, I will show that the jet power of a BH candidate can provide additional information about the nature of the compact object and potentially can be used to break the degeneracy between the spin and the deformation parameter. I will outline the basic idea without following a rigorous study: The latter would require a complete reanalysis of the X-ray continuum spectrum of the four objects and new general relativistic magnetohydrodynamic simulations in a particular non-Kerr background, both beyond the purpose of this work, as well as more observational data, which we do not have yet. I will consider two specific non-Kerr space-times: the braneworld-inspired BHs of Ref. [23] and the Johannsen-Psaltis (JP) BHs of Ref. [24]. These space-times can be seen as the two prototypes of non-Kerr background, or at least of the ones proposed in the literature [25].

A. Example 1: braneworld black holes

A braneworld-inspired BH solution was found in Ref. [23]. In Boyer-Lindquist coordinates, the nonzero components of the induced 4D metric are

$$\begin{aligned} g_{tt} &= -\left(1 - \frac{2Mr - \beta}{\rho^2}\right), & g_{t\phi} &= -\frac{2a(2Mr - \beta)}{\rho^2} \sin^2\theta, \\ g_{\phi\phi} &= \left[r^2 + a^2 + \frac{2Mr - \beta}{\rho^2} a^2 \sin^2\theta\right] \sin^2\theta, \\ g_{rr} &= \frac{\rho^2}{\Delta}, & g_{\theta\theta} &= \rho^2, \end{aligned} \quad (3)$$

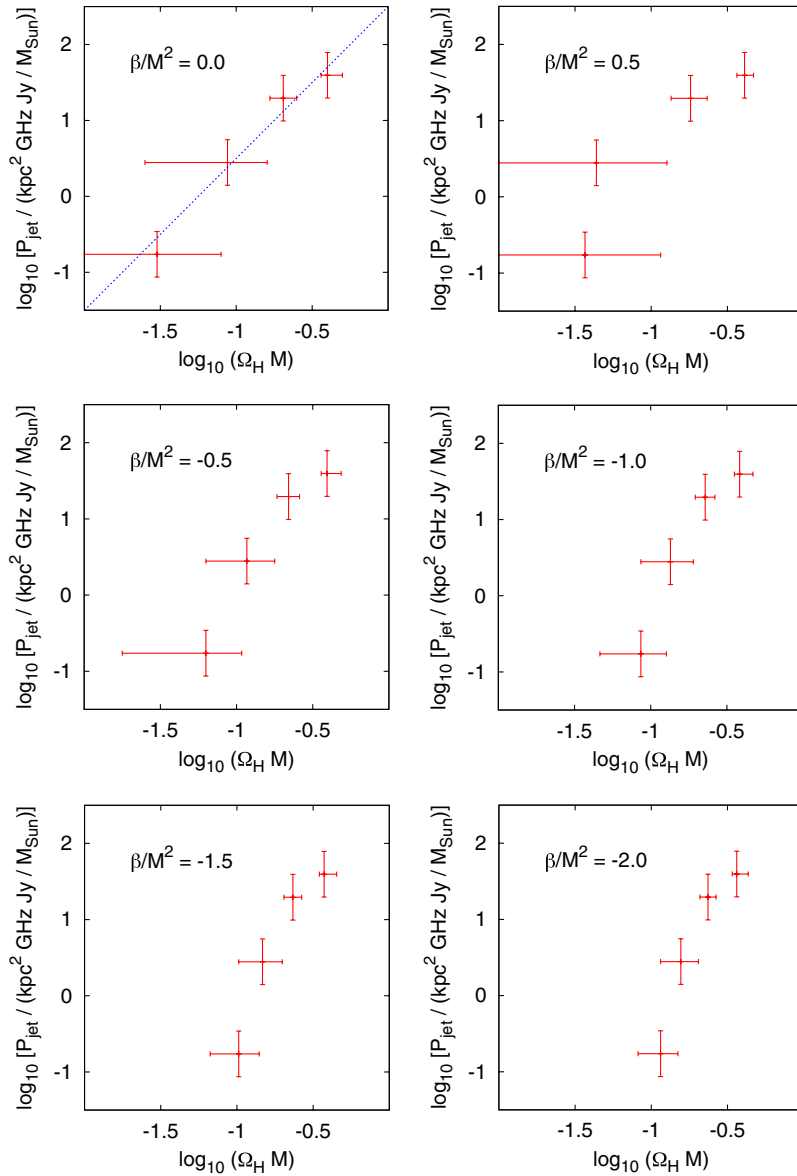


FIG. 1 (color online). Braneworld-inspired black holes of Eq. (3). Plots of the jet power P_{jet} against the BH angular frequency Ω_H . The top left panel shows the data in the case of the familiar Kerr background and the dotted line corresponds to $P_{\text{jet}} \sim \Omega_H^2$, the theoretical scaling derived in Ref. [22].

where

$$\rho^2 = r^2 + a^2 \cos^2 \theta, \quad \Delta = r^2 - 2Mr + a^2 + \beta, \quad (4)$$

and β is the tidal charge parameter, encoding the imprints of the nonlocal effects from the extra dimension. The metric looks like the usual Kerr-Newman solution of General Relativity, which describes a rotating BH with electric charge Q , with $\beta = Q^2$. However, here β can be either positive or negative. The event horizon is defined by $\Delta = 0$; the radius of the outer event horizon is

$$r_H = M + \sqrt{M^2 - a^2 - \beta}. \quad (5)$$

The event horizon exists only for $M \geq \sqrt{a^2 - \beta}$. When $M < \sqrt{a^2 - \beta}$, there is no horizon and the space-time has a naked singularity.² For the metric in Eq. (3) it is

²Let us notice that these braneworld BHs may violate the familiar bound $|a_*| \leq 1$, without violating the weak cosmic censorship conjecture. It is also possible to check that there exist astrophysical processes capable of producing such fast-rotating objects [26].

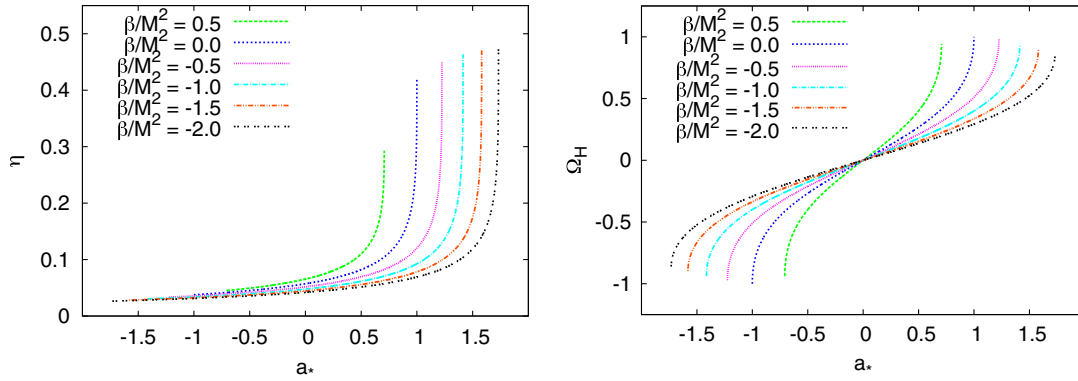


FIG. 2 (color online). Braneworld-inspired black holes of Eq. (3). Accretion efficiency $\eta = 1 - E_{\text{ISCO}}$ (left panel) and BH angular frequency Ω_H (right panel) as a function of the spin parameter a_* for different values of β/M^2 .

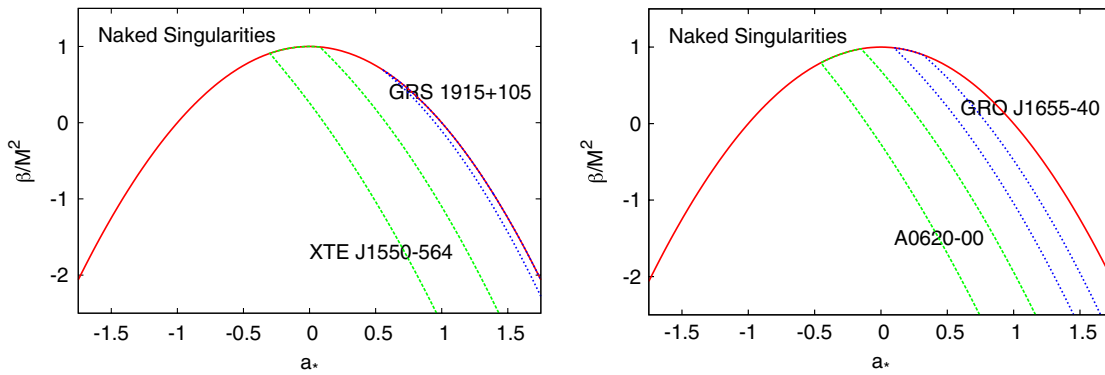


FIG. 3 (color online). Braneworld-inspired black holes of Eq. (3). Allowed regions in the parameter space ($a_*, \beta/M^2$) for the BH candidates GRS 1915+105 and XTE J1550-564 (left panel) and GRO J1655-40 and A0620-00 (right panel). The solid curve separates BHs from naked singularities. See text for details.

straightforward to repeat the analytical derivation of the jet power (see Ref. [27] and Appendix A of [22]) and one still finds $P_{\text{jet}} \sim \Omega_H^2$, as in Kerr.

The analysis of the X-ray continuum spectra of the four objects in Table I would provide a constraint on a_* and β/M^2 .³ The correct procedure would be to reanalyze the X-ray data of these objects in the background (3); however, that would take a long time and is beyond the purpose of the present paper. A simple estimate can be obtained from the following consideration. In the standard case of Kerr background, the continuum-fitting method provides the BH spin parameter a_* and its mass accretion rate \dot{M} , when the BH mass, its distance from us, and the inclination angle of the disk are known. Actually, the low frequency region of the spectrum constrains \dot{M} [28],

while the position of the peak constrains the accretion efficiency $\eta = 1 - E_{\text{ISCO}}$ [12], where E_{ISCO} is the specific energy of the gas at the innermost stable circular orbit (ISCO), which is supposed to be the inner edge of the accretion disk. The common statement in the literature that the continuum-fitting method measures the inner radius of the disk, r_{in} , is correct because in the Kerr metric there is a one-to-one correspondence between η and r_{in} . However, in a non-Kerr background one can see that the actual key-parameter is η . We can then write the present estimates of a_* of the four objects in terms of the accretion efficiency η (see the third column in Table I), and then get the allowed regions in the space ($a_*, \beta/M^2$) for every BH candidate (see the Appendix for more details). The accretion efficiency and the BH angular frequency as a function of the spin parameter are shown in Fig. 2. The final results are reported in Fig. 3, where the solid curve separates BHs from naked singularities. The region with naked singularities can be excluded for at least two reasons: These spacetimes have equatorial stable circular orbits with negative energy, which would imply $\eta > 1$, and they are presumably unstable due to the ergoregion instability [29]. As we

³If the Birkhoff's Theorem holds, Solar System experiments would require $|\beta/M^2| < 4.6 \times 10^{-4}$. While it is not clear if this is the case in braneworld models, the aim of this paper is not to constrain these theories, but to show how two independent measurements can break the degeneracy between the spin and the deformation parameter.

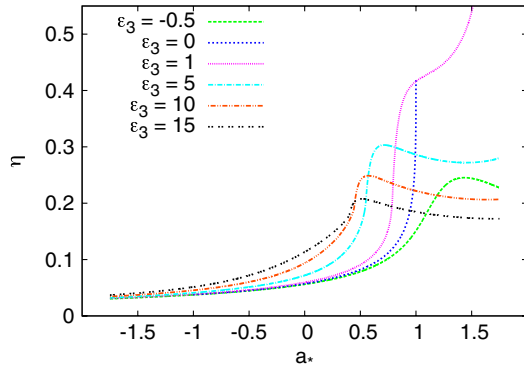


FIG. 4 (color online). JP black holes of Eq. (6) with deformation parameter ϵ_3 and $\epsilon_i = 0$ for $i \neq 3$. Accretion efficiency $\eta = 1 - E_{\text{ISCO}}$ as a function of the spin parameter a_* for different values of ϵ_3 .

can see in Fig. 3, we cannot estimate a_* and β/M^2 independently, but we can only constrain a combination of these two parameters. This is the usual situation we find when we want to test the Kerr BH hypothesis.

For braneworld BHs, Ω_H is still given by Eq. (2). It is also important to notice that P_{jet} is proportional to the second power of Ω_H ; that is, P_{jet} does not depend on the sense of BH rotation with respect to the one of the disk. In Fig. 1, I plot the power jet against Ω_H for some values of β/M^2 . Here I assume that all the BH candidates have the same value of β/M^2 . This assumption can be relaxed and tested when more data will be available.

B. Example 2: JP black holes

The JP BHs have been proposed in [24] explicitly to be used to test the Kerr BH hypothesis. The nonvanishing metric coefficients in Boyer-Lindquist coordinates are:

$$\begin{aligned} g_{tt} &= -\left(1 - \frac{2Mr}{\rho^2}\right)(1+h), & g_{t\phi} &= -\frac{2aMr\sin^2\theta}{\rho^2}(1+h), \\ g_{\phi\phi} &= \sin^2\theta\left[r^2 + a^2 + \frac{2a^2Mr\sin^2\theta}{\rho^2}\right] + \frac{a^2(\rho^2 + 2Mr)\sin^4\theta}{\rho^2}h, \\ g_{rr} &= \frac{\rho^2(1+h)}{\Delta + a^2h\sin^2\theta}, & g_{\theta\theta} &= \rho^2, \end{aligned} \quad (6)$$

where

$$\begin{aligned} \rho^2 &= r^2 + a^2\cos^2\theta, & \Delta &= r^2 - 2Mr + a^2, \\ h &= \sum_{k=0}^{\infty} \left(\epsilon_{2k} + \frac{Mr}{\rho^2}\epsilon_{2k+1}\right) \left(\frac{M^2}{\rho^2}\right)^k. \end{aligned} \quad (7)$$

The metric has an infinite number of free parameters ϵ_i and the Kerr solution is recovered when all these parameters are set to zero. However, in order to recover the correct Newtonian limit we have to impose $\epsilon_0 = \epsilon_1 = 0$, while ϵ_2 is constrained at the level of 10^{-4} from current tests in the Solar System [24]. For the sake of simplicity, in what follows I will consider only the case with the deformation parameter ϵ_3 and $\epsilon_i = 0$ for $i \neq 3$.

For some values of the deformation parameters, the JP BHs have a few properties common to other non-Kerr metrics, but absent in the Kerr solution (existence of vertically unstable circular orbits on the equatorial plane, topologically nontrivial event horizons, etc.). In particular, here we cannot define the BH angular frequency, at least in the usual way, as from Eq. (2) we would obtain something that depends on the polar angle θ . Anyway, if we want to check the Kerr-nature of astrophysical BH candidates, we can still plot P_{jet} against the spin parameter a_* and see if the correlation is the one expected for Kerr BHs.

The accretion efficiency $\eta = 1 - E_{\text{ISCO}}$ as a function of the spin parameter a_* for some values of the deformation parameter ϵ_3 is shown in Fig. 4. To get the constraints on a_* and ϵ_3 for the four objects in Table I, we can still apply the

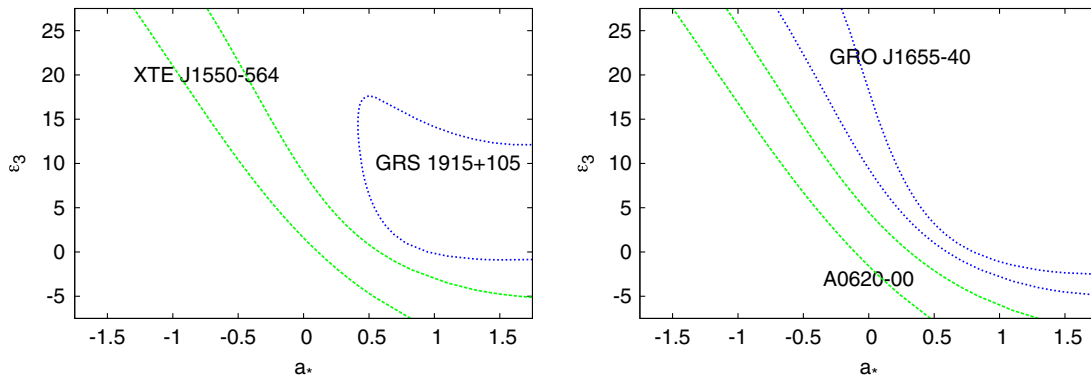


FIG. 5 (color online). JP black holes of Eq. (6) with deformation parameter ϵ_3 and $\epsilon_i = 0$ for $i \neq 3$. Allowed regions in the parameter space (a_*, ϵ_3) for the BH candidates GRS 1915+105 and XTE J1550-564 (left panel) and GRO J1655-40 and A0620-00 (right panel). See text for details.

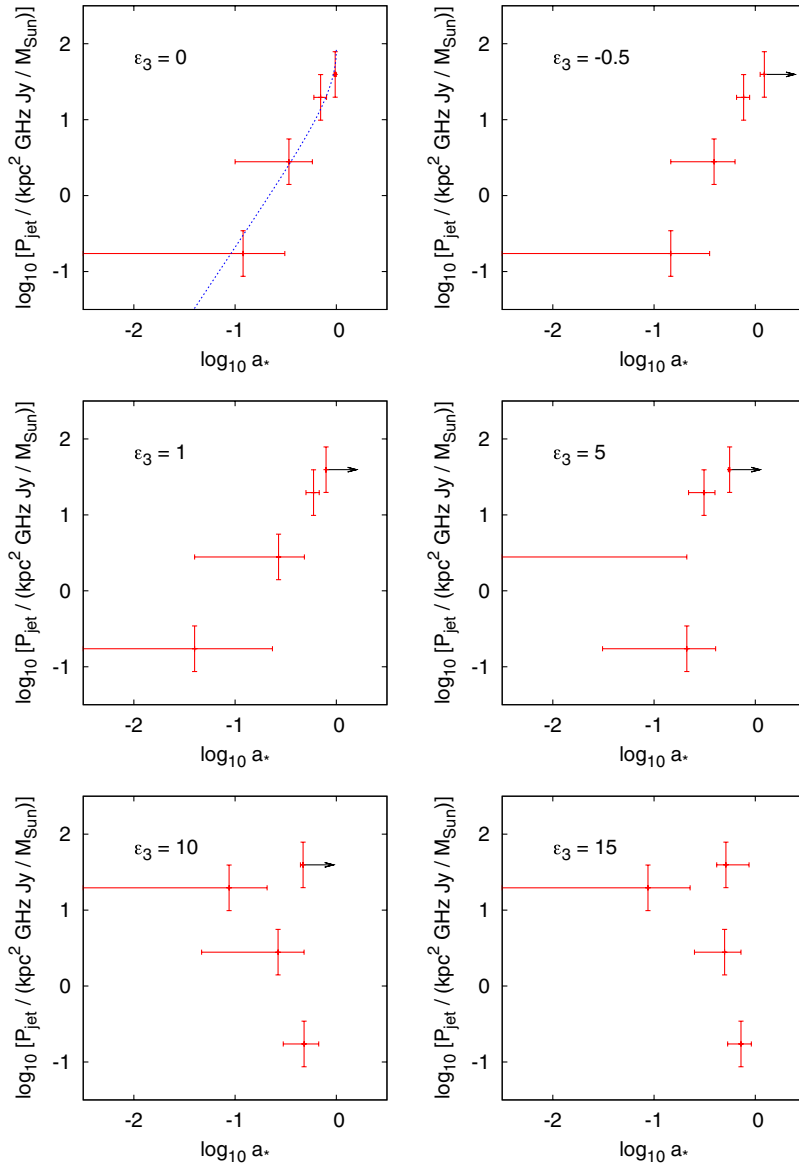


FIG. 6 (color online). JP black holes of Eq. (6) with deformation parameter ϵ_3 and $\epsilon_i = 0$ for $i \neq 3$. Plots of the jet power P_{jet} against the BH spin parameter a_* . The top left panel shows the data in the case of the familiar Kerr background and the dotted curve corresponds to $P_{\text{jet}} \sim \Omega_H^2$, the theoretical scaling derived in Ref. [22].

simplified analysis of the previous subsection. The results are shown in Fig. 5. Fig. 6 shows the plots P_{jet} vs. a_* in the JP space-time with ϵ_3 . The dotted curve in the top left panel is the theoretical scaling $P_{\text{jet}} \sim \Omega_H^2$ in Kerr background. Let us notice that the cases with $\epsilon_3 = 10$ and 15 are allowed with the sole use of the continuum-fitting method, while they seem to be at least strongly disfavored when we add the information coming from the jet power. Indeed, when $\epsilon_3 = 10$ and 15 , the continuum-fitting method would predict a counter-rotating disk (i.e. $a_* < 0$) for some sources, while the jet power should be independent of the sense of BH rotation with respect to the accreting matter.

IV. CONCLUSIONS

Astrophysical BH candidates are thought to be the Kerr BHs predicted in General Relativity, but direct observational evidence for this identification is still lacking. In order to test and verify the Kerr BH hypothesis, we have to probe the geometry of the space-time around these objects. The current most robust approach to do that with already available data seems to be the continuum-fitting method, a technique used by astronomers to measure the spin of the stellar-mass BH candidates. The physics involved are relatively simple and there are both astrophysical observations and numerical calculations supporting the crucial ingredients of this approach. However, the continuum-fitting

method cannot provide at the same time an estimate of the spin and of some deformation parameter measuring the deviations from the Kerr geometry. The problem is that there is a degeneracy between these two parameters and therefore it is only possible to get a constraint on some combination of them. The reason is that the continuum-fitting method is sensitive to the accretion efficiency, which depends on the spin and on the deformation parameter.

In this paper, I explored a way to break this degeneracy and get an estimate of the spin and on the deformation parameter separately. If transient ballistic jets in BH binaries are powered by the BH spin via the Blandford-Znajek mechanism, the jet power and the BH spin should be correlated in a specific way. In Ref. [17], the authors showed for the first time evidence for such a correlation. Here, I showed that, if this interpretation is correct, the estimate of jet power provides an additional information about the nature of the stellar-mass BH candidates and, when combined with the continuum-fitting method, it can potentially be used to constrain the deformation parameter. As it is particularly clear in Fig. 6, where ϵ_3 is the deformation parameter and $\epsilon_3 = 0$ corresponds to the Kerr metric, the expected correlation (the dotted curve in the top left panel of Fig. 6) is not consistent with observations when the space-time has large deviations from the Kerr solution (the cases $\epsilon_3 = 10$ and 15 in Fig. 6). The interpretation of the authors of Ref. [17] needs to be confirmed and the study of a larger number of objects is compulsory. However, as shown in this work through a simplified analysis, the combination of the continuum-fitting method and the estimate of jet power may be able to test the Kerr-nature of stellar-mass BH candidates in the near future.

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APPENDIX: ACCRETION EFFICIENCY IN THE NOVIKOV-THORNE MODEL

The Novikov-Thorne model is the standard model for accretion disks [9]. It describes geometrically thin and optically thick disks and it is the relativistic generalization of the Shakura-Sunyaev model [30]. Accretion is possible because viscous magnetic/turbulent stresses and radiation transport energy and angular momentum outwards. The model assumes that the disk is on the equatorial plane and that the disk's gas moves on nearly geodesic circular

orbits. The model can be applied for a generic stationary, axisymmetric, and asymptotically space-time. Here, the line element can always be written as

$$ds^2 = g_{tt}dt^2 + 2g_{t\phi}dtd\phi + g_{rr}dr^2 + g_{\theta\theta}d\theta^2 + g_{\phi\phi}d\phi^2. \quad (\text{A1})$$

Since the metric is independent of the t and ϕ coordinates, we have the conserved specific energy at infinity, E , and the conserved axial-component of the specific angular momentum at infinity, L . From the conservation of the rest-mass, $g_{\mu\nu}u^\mu u^\nu = -1$, we can write

$$g_{rr}\dot{r}^2 + g_{\theta\theta}\dot{\theta}^2 = V_{\text{eff}}(r, \theta), \quad (\text{A2})$$

where the effective potential V_{eff} is given by

$$V_{\text{eff}} = \frac{E^2 g_{\phi\phi} + 2ELg_{t\phi} + L^2 g_{tt}}{g_{t\phi}^2 - g_{tt}g_{\phi\phi}} - 1. \quad (\text{A3})$$

Circular orbits in the equatorial plane are located at the zeros and the turning points of the effective potential: $\dot{r} = \dot{\theta} = 0$, which implies $V_{\text{eff}} = 0$, and $\ddot{r} = \ddot{\theta} = 0$, requiring respectively $\partial_r V_{\text{eff}} = 0$ and $\partial_\theta V_{\text{eff}} = 0$. From these conditions, one can obtain the angular velocity, E , and L :

$$\Omega = \frac{-\partial_r g_{t\phi} + \sqrt{(\partial_r g_{t\phi})^2 - (\partial_r g_{tt})(\partial_r g_{\phi\phi})}}{\partial_r g_{\phi\phi}}, \quad (\text{A4})$$

$$E = -\frac{g_{tt} + g_{t\phi}\Omega}{\sqrt{-g_{tt} - 2g_{t\phi}\Omega - g_{\phi\phi}\Omega^2}}, \quad (\text{A5})$$

$$L = \frac{g_{t\phi} + g_{\phi\phi}\Omega}{\sqrt{-g_{tt} - 2g_{t\phi}\Omega - g_{\phi\phi}\Omega^2}}. \quad (\text{A6})$$

The orbits are stable under small perturbations if $\partial_r^2 V_{\text{eff}} \leq 0$ and $\partial_\theta^2 V_{\text{eff}} \leq 0$. In Kerr space-time, the second condition is always satisfied, so one can deduce the radius of the ISCO from $\partial_r^2 V_{\text{eff}} = 0$. In general, however, that may not be true. For instance, in the JP space-times, the ISCO radius may be determined by the orbital stability along the vertical direction. When we know the ISCO radius, we can compute the corresponding specific energy E_{ISCO} and then the accretion efficiency:

$$\eta = 1 - E_{\text{ISCO}}. \quad (\text{A7})$$

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