Charged-lepton mass matrix and nonzero θ_{13} with TeV scale new physics

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We provide an explicit structure of the charged-lepton mass matrix which is 2-3 symmetric except for a single breaking of this symmetry by the muon mass. We identify a flavor symmetric limit for the mass matrices where the first generation is decoupled from the other two in the charged-lepton sector while in the neutrino sector the third generation is decoupled from the first two generations. The leptonic mixing in the symmetric limit can be, among other structures, the bimaximal or the tri-bimaximal mixing. Symmetry breaking effects are included both in the charged-lepton and the neutrino sector to produce corrections to the leptonic mixing and explain the recent θ_{13} measurements. A model that extends the standard model by three right-handed neutrinos, an extra Higgs doublet, and two singlet scalars is introduced to generate the leptonic mixing.

I. INTRODUCTION

We now know that neutrinos have masses and just like the quark mixing matrix there is a leptonic mixing matrix. This fact has been firmly established through a variety of solar, atmospheric, and terrestrial neutrino oscillation experiments [1]. The Pontecorvo-Maki-Nakagawa-Sakata lepton-mixing matrix, U_{PMNS}, is parametrized as follows [2]:

$$U_{\text{PMNS}} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} K, \qquad (1)$$

where $s_{13} \equiv \sin\theta_{13}$, $c_{13} \equiv \cos\theta_{13}$ with θ_{13} being the reactor angle, $s_{12} \equiv \sin\theta_{12}$, $c_{12} \equiv \cos\theta_{12}$ with θ_{12} being the solar angle, $s_{23} \equiv \sin\theta_{23}$, $c_{23} \equiv \cos\theta_{23}$ with θ_{23} being the atmospheric angle, δ is the Dirac CP-violating phase, and $K = \text{diag}(1, e^{i\phi_1}, e^{i\phi_2})$ contains additional (Majorana) CP-violating phases ϕ_1, ϕ_2. We ignore the Majorana CP-violating phases in this work.

Unlike the Cabibbo-Kobayashi-Maskawa quark-mixing matrix that can be thought of as a perturbation about the identity matrix, the leading term in the leptonic mixing contains large mixing angles. Some examples of the leading-order mixing matrix are the bimaximal mixing [3] and the tri-bimaximal mixing [4]. However, current experiments indicate deviations from these standard zeroth-order forms.

For instance, recent results from the T2K [5] and MINOS [6,7] experiments have indicated a large reactor angle θ_{13} for neutrino mixing. At the 90% C.L., T2K gives $0.03(0.04) < \sin^2 2\theta_{13} < 0.28(0.34)$, with zero Dirac CP phase, δ_D, for normal (inverted) hierarchy. The MINOS group gives $0.01(0.026) < \sin^2 2\theta_{13} < 0.088(0.150)$. There are already several papers that have attempted to explain the recent θ_{13} results [8,9].

The leptonic mixing arises from the overlap of matrices that diagonalize the charged-lepton and the neutrino mass matrices. Many approaches to studying the leptonic mixing start in the basis where the charged-lepton mass is diagonal. Our approach to obtaining the leading-order leptonic mixing as well as deviations from it starts from the charged-lepton sector. A recent attempt to understand θ_{13} from the charged-lepton sector can be found in Ref. [8] and in the past corrections to the leptonic mixing from the charged-lepton sector were considered in Ref. [10]. An approach to suppress flavor-changing neutral current effects in the quark sector, based on shared flavor symmetry, was proposed in Ref. [11]. As an example of this shared symmetry, the decoupled 2-3 symmetry was used for the down-quark sector to suppress flavor-changing neutral current effects and explain anomalies [12] observed in the B meson system. In the decoupled limit, the first generation is decoupled from the other two generations. We extend this decoupled 2-3 symmetry to the charged-lepton sector. This is a reasonable extension given the fact that the down quark and charged leptons exhibit similar hierarchical structure and they may be combined in representations of grand unified theory groups.

One of the central ideas of this approach is the requirement that the mass matrices, in a symmetric limit, be diagonalized by unitary matrices composed of pure

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numbers independent of the parameters of the mass matrices. This is similar to the idea of form diagonalizable matrices discussed in Ref. [13]. If one starts with a 2-3-symmetric mass matrix for the charged-lepton sector and requires it to be diagonalized by unitary matrices of pure numbers, one recovers the decoupled 2-3 symmetry. In the neutrino sector, we assume the third generation to be decoupled from the first two generations. With real entries in the neutrino mass matrix, it is diagonalized by a rotation matrix and the resulting leptonic mixing has a μ-τ symmetry. Requiring the mass matrix to be diagonalized by pure numbers can lead to, among other structures, the bimaximal (BM) and the tri-bimaximal (TBM) leptonic mixing.

To generate the mixing matrices in the charged-lepton and the neutrino sector, we present a Lagrangian that extends the standard model (SM) by three right-handed neutrinos, an additional Higgs doublet, and two singlet scalar fields.[1] The Lagrangian uses the same class of Z_2 symmetries as has been used in Ref. [15]. However, the structure as well as the phenomenology of our model is very different from the above-mentioned papers. The Lagrangian is constructed to have a 2-3 symmetry, Z_2^{23}, along with two additional Z_2 symmetries Z_2^e and Z_2^D. The neutrino masses and mixing are generated through the usual seesaw mechanism. The presence of the $Z_2^{23} \times Z_2^e$ symmetries leads to the decoupled 2-3 symmetry in the charged-lepton sector and fixes the interactions of the right-handed neutrinos with the singlet scalar fields. The presence of the Z_2^D symmetry forces the neutrinos to acquire Dirac masses by coupling to a second Higgs doublet which has a different Z_2^D transformation than the usual SM Higgs doublet that give masses to the charged leptons. The full Lagrangian is symmetric under the product of the Z_2 symmetries, $Z_2^{23} \times Z_2^e \times Z_2^D$.

The neutrino masses and mixing arise when the Higgs doublets and the singlet scalars acquire vacuum expectation values (VEVs) and break the symmetries of the Lagrangian. The leptonic mixing is predicted to be of the bimaximal type when both the singlet scalars acquire the same VEV. If the VEV of the second Higgs doublet is small enough ~MeV, then the seesaw scale as well as the masses of the singlet scalars can be in the TeV range. To obtain the TBM mixing, one has to use different flavor symmetries.

Symmetry breaking is introduced in the charged-lepton sector by higher-dimensional operators that break the decoupled 2-3 symmetry but generate a 2-3-symmetric mass matrix except for a single breaking generated by the muon mass. In the neutrino sector, symmetry breaking is introduced by breaking the alignment of the VEVs of the singlet scalars by terms in the effective potential. The corrections to leptonic mixing go as $\sim \frac{v^2}{\omega^2}$ where v is the VEV of the SM

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[1]Recent motivations for considering two Higgs doublet models can be found in Ref. [14].

Higgs and ω the scale of the singlet scalar VEVs. If $\omega \sim$ TeV, then the corrections to the leptonic mixing are enough to explain the experimental observations.

The paper is organized in the following manner: We begin in Sec. II with a discussion of the flavor symmetric limit that leads to among other structures the BM and TBM mixing. In Sec. III, we present the Lagrangian to generate the mixing matrices in the symmetric limit. In Sec. IV, we study the effect of symmetry breaking in the charged-lepton and neutrino sector to generate the realistic leptonic mixing matrix. In Sec. V, we show the numerical results due to the symmetry breaking, and, finally, in Sec. VI we conclude with a summary of the results reported in this work.

II. THE LEPTONIC MIXING IN THE SYMMETRIC LIMIT

We start with the charged-lepton sector, and assume that the Yukawa matrix is 2-3 symmetric [16]. The Yukawa couplings of the charged leptons are given by

$$Y^L = \begin{pmatrix} l_{11} & l_{12} & -l_{12} \\ l_{12} & l_{22} & l_{23} \\ -l_{12} & l_{23} & l_{22} \end{pmatrix}. \quad (2)$$

The above Yukawa matrix can be diagonalized as

$U^\dagger Y^L U = Y^L_{\text{diag}},$

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \cdot \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad (3)$$

where the mixing angle θ is determined by the positive solution to

$$\tan\theta = \frac{2\sqrt{2}l_{12}}{l_{22} - l_{23} - l_{11} \pm \sqrt{(l_{22} - l_{23} - l_{11})^2 + 8l_{12}^2}}. \quad (4)$$

The eigenvalues of Y^L are $\frac{1}{2}[l_{11} + l_{22} - l_{23} \pm \sqrt{(l_{11} - l_{22} + l_{23})^2 + 8l_{12}^2}]$ and $l_{22} + l_{23}$. According to our assumption, the elements of the matrix that diagonalizes Y^L must be pure numbers in the symmetric limit. It is clear that we can achieve that by setting $l_{12} = 0$ ($\theta = 0$) in Eq. (4). This generates the decoupled 2-3 symmetry [11], as the flavor symmetry in the charged-lepton sector in which the first generation is decoupled from the second and third generations.

One can represent the Yukawa matrix with the decoupled 2-3 symmetry by Y^L_{23} as

$$Y_{23}^L = \begin{pmatrix} l_{11} & 0 & 0 \\ 0 & \frac{1}{2}l_{22} & \frac{1}{2}l_{23} \\ 0 & \frac{1}{2}l_{23} & \frac{1}{2}l_{22} \end{pmatrix}. \tag{5}$$

This Yukawa matrix Y_{23}^L is diagonalized by the unitary matrix W_{23}^l given by

$$W_{23}^l = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}. \tag{6}$$

Note that this matrix differs from the one in Eq. (3) in the limit $\theta = 0$ by an irrelevant diagonal phase matrix. Writing the diagonalized Yukawa matrix as Y_{23d}^L, we have

$$Y_{23d}^L = W_{23}^{l\dagger} Y_{23}^L W_{23}^l = \begin{pmatrix} l_{11} & 0 & 0 \\ 0 & \frac{1}{2}(l_{22} - l_{23}) & 0 \\ 0 & 0 & \frac{1}{2}(l_{22} + l_{23}) \end{pmatrix}. \tag{7}$$

The charged-lepton masses are given by

$$\begin{aligned} m_e &= \pm \frac{v_1}{\sqrt{2}} l_{11}, \\ m_\mu &= \pm \frac{v_1}{\sqrt{2}} \frac{(l_{22} - l_{23})}{2}, \\ m_\tau &= \pm \frac{v_1}{\sqrt{2}} \frac{(l_{22} + l_{23})}{2}. \end{aligned} \tag{8}$$

Since $m_\mu \ll m_\tau$, there has to be a fine-tuned cancellation between l_{22} and l_{23} to produce the muon mass. Hence, it is more natural to consider the symmetry limit $l_{22} = l_{23}$ which leads to $m_\mu = 0$. The Yukawa matrix which leads to the zero muon mass within the decoupled 2-3 symmetry is

$$Y_{23}^L = \begin{pmatrix} l_{11} & 0 & 0 \\ 0 & \frac{1}{2}l_T & \frac{1}{2}l_T \\ 0 & \frac{1}{2}l_T & \frac{1}{2}l_T \end{pmatrix}. \tag{9}$$

In the neutrino sector, we assume that, in the symmetric limit, $\mathcal{M}_\nu$ has the general structure

$$\mathcal{M}_\nu = \begin{pmatrix} a & d & 0 \\ d & b & 0 \\ 0 & 0 & c \end{pmatrix}, \tag{10}$$

where all the parameters are real. This can be diagonalized by the matrix

$$W_{12}^\nu = \begin{pmatrix} c_{12} & s_{12} & 0 \\ s_{12} & -c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \qquad s_{12} \equiv \sin\theta_{12}, \quad c_{12} \equiv \cos\theta_{12}, \tag{11}$$

where

$$\tan 2\theta_{12} = \frac{2d}{(a - b)}. \tag{12}$$

We can then calculate U_{PMNS}^s as

$$U_{\text{PMNS}}^s = U_\ell^\dagger U_\nu, \tag{13}$$

with

$$U_\ell = W_{23}^l, \qquad U_\nu = W_{12}^\nu, \tag{14}$$

where W_{23}^l and W_{23}^ν are given in Eq. (6) and in Eq. (11).

This gives

$$U_{\text{PMNS}}^s = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -\frac{1}{\sqrt{2}}s_{12} & \frac{1}{\sqrt{2}}c_{12} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}}s_{12} & -\frac{1}{\sqrt{2}}c_{12} & \frac{1}{\sqrt{2}} \end{pmatrix}, \tag{15}$$

which is just the μ-τ-symmetric leptonic mixing. If we require θ_{12} in Eq. (12) to be independent of the parameters a, b, and d, then we either have $a = b$ which leads to $\theta_{12} = \pi/4$ and generates the BM mixing or $d = k(a - b)$ and, in particular, we obtain the tri-bimaximal mixing with $k = \sqrt{2}$. Hence, by choosing $a = b$, the neutrino mass matrix is given as

$$\mathcal{M}_\nu = \begin{pmatrix} a & d & 0 \\ d & a & 0 \\ 0 & 0 & c \end{pmatrix}. \tag{16}$$

III. THE LAGRANGIAN IN THE SYMMETRIC LIMIT

In this section, we present a simple Lagrangian that generates the mixing matrices considered in the previous section. We find that the model naturally generates the BM mixing though the TBM mixing can also be obtained but with introducing different flavor symmetries. Our phenomenology will be done in the scenario in which the leptonic mixing is BM in the symmetric limit.

We will use the seesaw mechanism to obtain the neutrino masses. Our model extends the SM by an additional Higgs doublet and two singlet scalars. The particle content of the model is given as
  (i) three left-handed lepton doublets D_{α_L}, where α denotes e, μ, and τ,
  (ii) three right-handed charged-lepton singlets α_R, and
  (iii) three right-handed neutrino singlets $\nu_{\alpha R}$.

In the scalar sector, we employ
  (i) two Higgs doublets ϕ_j with VEVs $\langle 0|\phi_j^0|0\rangle = \frac{v_j}{\sqrt{2}}$ and
  (ii) two real singlet scalar fields ϵ_1 and ϵ_2, with VEVs $\langle 0|\epsilon_k^0|0\rangle = w_k$.

The symmetries of the Lagrangian are introduced as

$$Z_2^{23}: D_{\mu_L} \leftrightarrow -D_{\tau_L}, \qquad \mu_R \leftrightarrow -\tau_R, \qquad \nu_{\mu R} \leftrightarrow -\nu_{\tau R},$$

$$D_{e_L} \rightarrow D_{e_L}, \qquad e_R \rightarrow e_R, \qquad \nu_{eR} \rightarrow \nu_{eR},$$

$$\epsilon_1 \rightarrow -\epsilon_1, \qquad \epsilon_2 \rightarrow \epsilon_2, \qquad \phi_1 \rightarrow \phi_1, \qquad \phi_2 \rightarrow \phi_2,$$

$$Z_2^e: \nu_{eR}, \qquad e_R, \qquad D_{e_L}, \qquad \epsilon_1, \qquad \epsilon_2, \qquad \text{(Change sign, and the rest of the fields remain same)}$$

$$Z_2^D: \nu_{eR}, \qquad \nu_{\mu R}, \qquad \nu_{\tau R}, \qquad \phi_2, \qquad \text{(Change sign, and the rest of the fields remain same).} \tag{17}$$

The most general Lagrangian consistent with the symmetries is

$$\mathcal{L}_Y = [y_1 \bar{D}_{e_L} e_R + y_2 (\bar{D}_{\mu_L} \mu_R + \bar{D}_{\tau_L} \tau_R)$$

$$+ y_3 (\bar{D}_{\mu_L} \tau_R + \bar{D}_{\tau_L} \mu_R)] \phi_1$$

$$+ [y_4 \bar{D}_{e_L} \nu_{eR} + y_5 (\bar{D}_{\mu_L} \nu_{\mu R} + \bar{D}_{\tau_L} \nu_{\tau R})] \tilde{\phi}_2 + \text{H.c.}, \tag{18}$$

$$\mathcal{L}_M = \frac{1}{2} [M \nu_{eR}^T C^{-1} \nu_{eR} + M_P \nu_{\mu R}^T C^{-1} \nu_{\mu R} + M_P \nu_{\tau R}^T C^{-1} \nu_{\tau R}]$$

$$- \frac{1}{2} y \nu_{eR}^T C^{-1} \left(\nu_{\mu R} \frac{(a\epsilon_1 + b\epsilon_2)}{\sqrt{2}} + \nu_{\tau R} \frac{(a\epsilon_1 - b\epsilon_2)}{\sqrt{2}} \right)$$

$$+ \text{H.c.} \tag{19}$$

Here, $\tilde{\phi}_i \equiv i\sigma_2 \phi_i^*$ is the conjugate Higgs doublet and we have chosen to work in a basis where the Dirac mass matrix for the neutrinos is diagonal. We can simplify the Lagrangian in several ways. First, we can redefine $a\epsilon_1 \rightarrow \epsilon_1$ and $b\epsilon_2 \rightarrow \epsilon_2$. Second, to reduce the number of parameters we can impose an approximate symmetry of the Lagrangian. A $SU(3)$ symmetry where the right-handed singlet fields and the left-handed doublet fields transform as the $SU(3)$ triplets leads to $y_4 = y_5 = y_D$. The $SU(3)$ symmetry is only satisfied by the Dirac mass term for the neutrinos and is broken by the other terms in the Lagrangian. Third, we will require the Lagrangian to be invariant under the transformation of the right-handed charged leptons $(\mu_R \leftrightarrow -\tau_R, e_R \rightarrow -e_R, \phi_1 \rightarrow -\phi_1)$, with all other fields remaining unchanged. This symmetry requires $y_2 = y_3$ leading to vanishing μ mass. The μ mass is introduced later as a symmetry breaking term. Finally, we will set the Majorana mass terms $M = M_P$. We can then rewrite the Lagrangian as

$$\mathcal{L}_Y = [y_1 \bar{D}_{e_L} e_R + y_2 (\bar{D}_{\mu_L} \mu_R + \bar{D}_{\tau_L} \tau_R)$$

$$+ y_2 (\bar{D}_{\mu_L} \tau_R + \bar{D}_{\tau_L} \mu_R)] \phi_1$$

$$+ y_D [\bar{D}_{e_L} \nu_{eR} + \bar{D}_{\mu_L} \nu_{\mu R} + \bar{D}_{\tau_L} \nu_{\tau R}] \tilde{\phi}_2 + \text{H.c.}, \tag{20}$$

$$\mathcal{L}_M = \frac{1}{2} M [\nu_{eR}^T C^{-1} \nu_{eR} + \nu_{\mu R}^T C^{-1} \nu_{\mu R} + \nu_{\tau R}^T C^{-1} \nu_{\tau R}]$$

$$- \frac{1}{2} y \nu_{eR}^T C^{-1} \left(\nu_{\mu R} \frac{(\epsilon_1 + \epsilon_2)}{\sqrt{2}} + \nu_{\tau R} \frac{(\epsilon_1 - \epsilon_2)}{\sqrt{2}} \right) + \text{H.c.} \tag{21}$$

The most general scalar potential V that is invariant under $Z_2^{23} \times Z_2^e \times Z_2^D$ is given by

$$V = -\mu_1^2 \epsilon_1^2 - \mu_2^2 \epsilon_2^2 + \lambda_1 \epsilon_1^4 + \lambda_2 \epsilon_2^4 + \lambda_1' \epsilon_1^2 \epsilon_2^2$$

$$+ \sigma_1 \epsilon_1^2 |\phi_1|^2 + \sigma_2 \epsilon_1^2 |\phi_2|^2 + \sigma_3 \epsilon_2^2 |\phi_1|^2$$

$$+ \sigma_4 \epsilon_2^2 |\phi_2|^2 + V_{2HD}(\phi_1, \phi_2), \tag{22}$$

where $V_{2HD}(\phi_1, \phi_2)$ is the potential of the two Higgs doublets,

$$V_{2HD}(\phi_1, \phi_2) = -\mu_{\phi_1}^2 \phi_1^\dagger \phi_1 - \mu_{\phi_2}^2 \phi_2^\dagger \phi_2$$

$$+ \lambda_{\phi_1}(\phi_1^\dagger \phi_1)^2 + \lambda_{\phi_2}(\phi_2^\dagger \phi_2)^2$$

$$+ \lambda_{\phi_{12}}(\phi_1^\dagger \phi_1 + \phi_2^\dagger \phi_2)^2$$

$$+ \lambda_{\phi_{12}}'(\phi_1^\dagger \phi_1 - \phi_2^\dagger \phi_2)^2$$

$$+ \lambda_{\phi_{21}}((\phi_1^\dagger \phi_1)(\phi_2^\dagger \phi_2) - (\phi_1^\dagger \phi_2)(\phi_2^\dagger \phi_1))$$

$$+ \lambda_{\phi_{21}}'((\phi_1^\dagger \phi_1)(\phi_2^\dagger \phi_2) + (\phi_1^\dagger \phi_2)(\phi_2^\dagger \phi_1)). \tag{23}$$

If we impose an additional symmetry to the above potential such as $\epsilon_1 \leftrightarrow \epsilon_2$, then the potential takes the form

$$V = -\mu^2(\epsilon_1^2 + \epsilon_2^2) + (\epsilon_1^2 + \epsilon_2^2) \sum_{i=1}^2 \sigma_i \phi_i^\dagger \phi_i$$

$$+ \lambda(\epsilon_1^2 + \epsilon_2^2)^2 + \lambda'(\epsilon_1^2 - \epsilon_2^2)^2 + V_{2HD}(\phi_1, \phi_2). \tag{24}$$

We can parametrize the VEVs of the singlet scalars as follows:

$$\langle 0|\epsilon_1|0\rangle = w\cos\gamma \quad \text{and} \quad \langle 0|\epsilon_2|0\rangle = w\sin\gamma. \tag{25}$$

Thus, the only term that depends on γ is

$$f(\gamma) \equiv \lambda' w^4 \cos^2 2\gamma. \tag{26}$$

By minimizing $f(\gamma)$, one gets

$$\cos 2\gamma = 0. \tag{27}$$

Thus,

$$\langle 0|\epsilon_1|0\rangle = \langle 0|\epsilon_2|0\rangle = \frac{w}{\sqrt{2}}. \tag{28}$$

By minimizing the above potential, one can find the parameter w and the VEVs of the two Higgs doublets which are nonzero and different in the symmetric limit

$$v_1 = \sqrt{\frac{\alpha_1}{2\beta_1}}, \qquad v_2 = \sqrt{\frac{\alpha_2}{2\beta_2}}, \tag{29}$$

where

$$\begin{aligned}
\alpha_1 = {} & 4\lambda(\lambda_{\phi_{12}}\mu_{\phi_1}^2 + \lambda_{\phi_2}\mu_{\phi_1}^2 - \lambda'_{\phi_{21}}\mu_{\phi_2}^2 - \lambda_{\phi_{12}}\mu_{\phi_2}^2 \\
& + \lambda'_{\phi_{12}}(\mu_{\phi_1}^2 + \mu_{\phi_2}^2)) - 2\lambda_{\phi_{12}}\mu^2\sigma_1 - 2\lambda_{\phi_2}\mu^2\sigma_1 \\
& + 2\lambda'_{\phi_{21}}\mu^2\sigma_2 + 2\lambda_{\phi_{12}}\mu^2\sigma_2 + \mu_{\phi_2}^2\sigma_1\sigma_2 \\
& - \mu_{\phi_1}^2\sigma_2^2 - 2\lambda'_{\phi_{12}}\mu^2(\sigma_1 + \sigma_2), \\
\beta_1 = {} & 4\lambda(-\lambda'^2_{\phi_{21}} - 2\lambda'_{\phi_{21}}\lambda_{\phi_{12}} + \lambda_{\phi_1}\lambda_{\phi_{12}} + \lambda_{\phi_1}\lambda_{\phi_2} \\
& + \lambda_{\phi_{12}}\lambda_{\phi_2} + \lambda'_{\phi_{12}}(2\lambda'_{\phi_{21}} + \lambda_{\phi_1} + 4\lambda_{\phi_{12}} + \lambda_{\phi_2})) \\
& - \lambda_{\Phi_{12}}\sigma_1^2 - \lambda_{\phi_2}\sigma_1^2 + 2\lambda'_{\phi_{21}}\sigma_1\sigma_2 + 2\lambda_{\phi_{12}}\sigma_1\sigma_2 \\
& - \lambda_{\phi_1}\sigma_2^2 - \lambda_{\phi_{12}}\sigma_2^2 - \lambda'_{\phi_{12}}(\sigma_1 + \sigma_2)^2, \\
\alpha_2 = {} & 4\lambda(\lambda_{\phi_{12}}\mu_{\phi_2}^2 + \lambda_{\phi_1}\mu_{\phi_2}^2 - \lambda'_{\phi_{21}}\mu_{\phi_1}^2 - \lambda_{\phi_{12}}\mu_{\phi_1}^2 \\
& + \lambda'_{\phi_{12}}(\mu_{\phi_2}^2 + \mu_{\phi_1}^2)) - 2\lambda_{\phi_{12}}\mu^2\sigma_2 - 2\lambda_{\phi_1}\mu^2\sigma_2 \\
& + 2\lambda'_{\phi_{21}}\mu^2\sigma_1 + 2\lambda_{\phi_{12}}\mu^2\sigma_1 + \mu_{\phi_1}^2\sigma_2\sigma_1 \\
& - \mu_{\phi_2}^2\sigma_1^2 - 2\lambda'_{\phi_{12}}\mu^2(\sigma_2 + \sigma_1), \\
\beta_2 = {} & 4\lambda(-\lambda'^2_{\phi_{21}} - 2\lambda'_{\phi_{21}}\lambda_{\phi_{12}} + \lambda_{\phi_2}\lambda_{\phi_{12}} + \lambda_{\phi_2}\lambda_{\phi_1} \\
& + \lambda_{\phi_{12}}\lambda_{\phi_1} + \lambda'_{\phi_{12}}(2\lambda'_{\phi_{21}} + \lambda_{\phi_2} + 4\lambda_{\phi_{12}} + \lambda_{\phi_1})) \\
& - \lambda_{\Phi_{12}}\sigma_2^2 - \lambda_{\phi_1}\sigma_2^2 + 2\lambda'_{\phi_{21}}\sigma_2\sigma_1 + 2\lambda_{\phi_{12}}\sigma_2\sigma_1 \\
& - \lambda_{\phi_2}\sigma_1^2 - \lambda_{\phi_{12}}\sigma_1^2 - \lambda'_{\phi_{12}}(\sigma_2 + \sigma_1)^2. \tag{30}
\end{aligned}$$

Also, the parameter w can simply be written as follows:

$$w^2 = \frac{\mu^2 - (\sigma_1|v_1|^2 + \sigma_2|v_2|^2)}{2\lambda}, \tag{31}$$

which shows that the VEV of the singlet scalars is independent of (v_1, v_2) when $\sigma_1 = \sigma_2 = 0$.

The explicit form of the Yukawa matrix, Y_{23}^L, and the Dirac neutrino mass matrix can be written from the Lagrangian (18) as follows:

$$Y_{23}^L = \frac{v_1}{\sqrt{2}}\begin{pmatrix} y_1 & 0 & 0 \\ 0 & y_2 & y_2 \\ 0 & y_2 & y_2 \end{pmatrix}, \tag{32}$$

$$M_D = \mathrm{diag}(A, A, A), \quad \text{with} \quad A = y\frac{v_2}{\sqrt{2}}. \tag{33}$$

Also, the Majorana mass matrix can be obtained from Eq. (19) as follows:

$$M_R = \begin{pmatrix} M & -v_w & 0 \\ -v_w & M & 0 \\ 0 & 0 & M \end{pmatrix}, \tag{34}$$

with $v_w = yw$. Using the seesaw formula [17], the neutrino mass matrix is given as

$$\mathcal{M}_\nu = -M_D^T M_R^{-1} M_D. \tag{35}$$

Then, $\mathcal{M}_\nu$ has the structure

$$\mathcal{M}_\nu = \begin{pmatrix} X & G & 0 \\ G & X & 0 \\ 0 & 0 & Z \end{pmatrix}, \tag{36}$$

where

$$X = -\frac{A^2 M}{M^2 - v_w^2}, \qquad G = -\frac{A^2 v_w}{M^2 - v_w^2}, \qquad Z = -\frac{A^2}{M}. \tag{37}$$

By diagonalizing Eq. (36), we obtain the neutrino masses as

$$m_1 = -\frac{A^2}{M + v_w},$$

$$m_2 = -\frac{A^2}{M - v_w}, \tag{38}$$

$$m_3 = -\frac{A^2}{M}.$$

Note that from the above equations one can estimate the scale of the VEV, v_2, of the second Higgs doublet ϕ_2. As the absolute neutrino masses are in the eV scale, therefore, v_2 has to be in the MeV scale if the seesaw scale (M) is in the TeV range. The mass relations satisfy the relation

$$\frac{1}{m_1} + \frac{1}{m_2} = \frac{2}{m_3}. \tag{39}$$

Similar relations among the masses are discussed in Ref. [18]. We can use the above sum rule to obtain an upper limit for the heaviest mass, $|m_3| \leq \frac{2|m_1||m_2|}{||m_1| + |m_2||}$ for the normal hierarchy or $|m_2| \leq \frac{|m_1||m_3|}{|2|m_1| - |m_3||}$ for the inverted hierarchy.

IV. SYMMETRY BREAKING

The breaking of the flavor symmetries in the charged-lepton and the neutrino sectors will cause deviation from the BM form, and we study these deviations in this section.

A. Charged-lepton sector

In the charged-lepton sector, we break the decoupled 2-3 symmetry by adding the following higher-dimensional terms:

$$O_1 = cy_2 \bar{D}_{\mu_L} \mu_R \phi_1 \frac{\phi_1^\dagger \phi_1}{\Lambda^2}, \tag{40}$$

and

$$O_2 = y'(\bar{D}_{e_L}\mu_R - \bar{D}_{e_L}\tau_R + \bar{D}_{\mu_L}e_R - \bar{D}_{\tau_L}e_R)\phi_1 \frac{\phi_1^\dagger \phi_1}{\Lambda^2}. \tag{41}$$

The operator O_2 breaks the decoupled 2-3 symmetry, $Z_2^{23} \times Z_2^e$, but is still 2-3 symmetric. The operator O_1 explicitly breaks the 2-3 symmetry, Z_2^{23}, and generates the muon mass. To generate explicit 2-3 breaking, we have introduced the higher-dimensional operator in the position of the muon field, 2-2 element, in the Yukawa matrix which is the most straightforward way to generate the muon mass. Introducing this operator in the 3-3 position generates the same numerical solutions for the correction angles. But, introducing it in the 2-3 or 3-2 positions does not generate physical values for the mixing angles. Even introducing 2-3 symmetric terms in (2-2, 3-3) or (2-3, 3-2) generates either unphysical mixing angles or gives very large correction mixing angles that do not lead to successful phenomenology.

In the presence of the higher-dimensional terms, the charged-lepton Yukawa matrix has the following form:

$$Y^L = \begin{pmatrix} l_{11} & l_{12} & -l_{12} \\ l_{12} & \frac{1}{2}l_T(1 + 2\kappa_l) & \frac{1}{2}l_T \\ -l_{12} & \frac{1}{2}l_T & \frac{1}{2}l_T \end{pmatrix}, \tag{42}$$

with $\kappa_l = cv_1^2/2\Lambda^2$ and $l_{12} = y'v_1^3/2\sqrt{2}\Lambda^2$ after the Higgs field gets its VEV. Three relations can be obtained among the Y^L matrix elements,

$$Y_{12}^L = -Y_{13}^L, \qquad Y_{23}^L = Y_{33}^L, \qquad Y_{22}^L = (1 + 2\kappa_l)Y_{23}^L. \tag{43}$$

We can solve for the unitary matrix, U_l, that diagonalizes Y^L in Eq. (42). We write,

$$U_l = W_{23}^l R_{23}^l R_{13}^l R_{12}^l, \tag{44}$$

where

$$R_{12}^l = \begin{pmatrix} c_{12l} & s_{12l} & 0 \\ -s_{12l} & c_{12l} & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

$$c_{12l} = \cos\theta_{12l}; \qquad s_{12l} = \sin\theta_{12l}, \tag{45}$$

$$R_{13}^l = \begin{pmatrix} c_{13l} & 0 & s_{13l}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13l}e^{i\delta} & 0 & c_{13l} \end{pmatrix},$$

$$c_{13l} = \cos\theta_{13l}; \qquad s_{13l} = \sin\theta_{13l}, \tag{46}$$

$$R_{23}^l = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23l} & s_{23l} \\ 0 & -s_{23l} & c_{23l} \end{pmatrix},$$

$$c_{23l} = \cos\theta_{23l}; \qquad s_{23l} = \sin\theta_{23l}. \tag{47}$$

The Yukawa matrix, Y^L, can be written as

$$Y^L = U_l Y_d^L U_l^\dagger, \tag{48}$$

with

$$Y_d^L = \begin{pmatrix} l_e & 0 & 0 \\ 0 & l_\mu & 0 \\ 0 & 0 & l_\tau \end{pmatrix}. \tag{49}$$

Applying the relations in Eq. (43) to the Y^L matrix elements in Eq. (48) using Eq. (44), one can solve for the corrections of the mixing angles. Two ways can be used to find the angles, analytically or numerically. Solving for the mixing angles analytically, see details in the Appendix, can determine the size of the Yukawa matrix parameters in Eq. (42),

$$\begin{aligned} z_\mu &\equiv \frac{m_\mu}{m_\tau}, \\ \kappa_l &= zz_\mu, \\ l_{12} &\approx \sqrt{\frac{z_\mu}{2}}(l_e - l_\mu), \\ l_T &\approx (l_\tau - l_\mu)\left(1 - \frac{1}{2}(zz_\mu)^2\right). \end{aligned} \tag{50}$$

with $z \sim 2$. It is interesting to note that

$$\kappa = \frac{cv_1^2}{2\Lambda^2} = zz_\mu \tag{51}$$

which fixes $\Lambda \sim$ TeV. We assume that the charged-lepton corrections are "Cabibbo–Kobayashi–Maskawa-quark-mixing-matrix-like," i.e.

$$\sin\theta_{12l} \approx \lambda, \qquad \sin\theta_{23l} \approx A\lambda^2, \qquad \sin\theta_{13l} \approx B\lambda^3, \tag{52}$$

where A and B are real and of order one and λ is the sine of the Cabibbo angle, $\lambda = \sin\theta_C \simeq 0.227$. We present numerical solutions to the relations in Eq. (43) for various z values that produce the pattern in Eq. (52). In our calculations, we assume $\delta = \pi$,

(i) For $z = 2.0$: $s_{12l} \approx \pm 0.34$, $s_{13l} \approx \pm 0.0011$, $s_{23l} \approx -0.059$,

(ii) For $z = 2.06$: $s_{12l} \approx \pm 0.3$, $s_{13l} \approx \pm 0.001$, $s_{23l} \approx -0.061$,

(iii) For $z = 2.2$: $s_{12l} \approx \pm 0.2$, $s_{13l} \approx \pm 0.00075$, $s_{23l} \approx -0.065$.

We expand the angles in Eq. (1) as

$$s_{13} = \frac{r}{\sqrt{2}}, \qquad s_{12} = \frac{1}{\sqrt{2}}(1 + s), \qquad s_{23} = \frac{1}{\sqrt{2}}(1 + a),$$

(53)

where the three real parameters r, s, a describe the deviations of the reactor, solar, and atmospheric angles from their bimaximal values. We use global fits of the conventional mixing parameters (s, a) [19] that can be translated

into 3σ ranges and the mixing parameter r with 2.5σ significance (90% C.L.) [5]

$$0.12 < r < 0.39, \quad -0.29 < s < -0.14, \quad -0.15 < a < 0.16.$$

(54)

To first order in r, s, a, the lepton mixing matrix can be written as

$$U \approx \begin{pmatrix} \frac{1}{\sqrt{2}}(1-s) & \frac{1}{\sqrt{2}}(1+s) & \frac{1}{\sqrt{2}}re^{-i\delta} \\ -\frac{1}{2}\left(1+s-a+\frac{r}{\sqrt{2}}e^{i\delta}\right) & \frac{1}{2}\left(1-s-a-\frac{r}{\sqrt{2}}e^{i\delta}\right) & \frac{1}{\sqrt{2}}(1+a) \\ \frac{1}{2}\left(1+s+a-\frac{r}{\sqrt{2}}e^{i\delta}\right) & -\frac{1}{2}\left(1-s+a+\frac{r}{\sqrt{2}}e^{i\delta}\right) & \frac{1}{\sqrt{2}}(1-a) \end{pmatrix},$$

(55)

which is similar to the parametrization in Ref. [20] with the TBM mixing. We have assumed that $\delta = \pi$ where the present data prefers a negative value for s [20] and r is positive, in our discussion we do not consider *CP* violation. Now, we can write the parameters (r, s, a) in terms of the elements of the mixing matrix

$$\begin{aligned} s &= -1 + \sqrt{2}U_{12}, \\ r &= \sqrt{2}(1 + s - a + 2U_{21}), \\ a &= -1 + \sqrt{2}U_{23}. \end{aligned}$$

(56)

From the details in the Appendix, one obtains

$$\begin{aligned} s &\approx -\frac{1}{\sqrt{2}}(s_{12l} + s_{13l}), \\ r &\approx s_{12l} - s_{13l}, \\ a &\approx -s_{23l}. \end{aligned}$$

(57)

From the above equations, one can get the deviation parameters as follows:
  (i) For $z = 2.0$: $s \approx -0.24$, $r \approx 0.34$, $a \approx 0.059$,
  (ii) For $z = 2.06$: $s \approx -0.21$, $r \approx 0.30$, $a \approx 0.061$,
  (iii) For $z = 2.2$: $s \approx -0.14$, $r \approx 0.20$, $a \approx 0.065$.

The above results demonstrate that the contributions from the charged-lepton sector can accommodate the T2K data of θ_{13} as well as the other mixing angles.

B. Neutrino sector

In this section, we consider deviations of the BM mixing from the neutrino sector. We maintain the invariance of the Majorana Lagrangian under the symmetry group in Eq. (17) and generate the deviation from the BM matrix by breaking the $\epsilon_1 \leftrightarrow \epsilon_2$ symmetry in Eq. (24) by introducing the most general dimension-four symmetry breaking terms in the potential

$$(\epsilon_1^2 - \epsilon_2^2)\sum_{i=1}^{2} \sigma_i' \phi_i^\dagger \phi_i + \varrho(\epsilon_1^2 - \epsilon_2^2)(\epsilon_1^2 + \epsilon_2^2).$$

(58)

[2]We require that all terms in the symmetry breaking potential are of the same size which results in $\rho \sim \frac{v^2}{w^2}\sigma_i'$ where v is the electroweak VEV with $v^2 = v_1^2 + v_2^2$ and ω is the scale of the VEVs of the singlet scalars. Thus, the potential is

$$\begin{aligned} V &= -\mu^2(\epsilon_1^2 + \epsilon_2^2) + (\epsilon_1^2 + \epsilon_2^2)\sum_{i=1}^{2} \sigma_i \phi_i^\dagger \phi_i \\ &\quad + \lambda(\epsilon_1^2 + \epsilon_2^2)^2 + \lambda'(\epsilon_1^2 - \epsilon_2^2)^2 \\ &\quad + (\epsilon_1^2 - \epsilon_2^2)\sum_{i=1}^{2} \sigma_i' \phi_i^\dagger \phi_i + \varrho(\epsilon_1^2 - \epsilon_2^2)(\epsilon_1^2 + \epsilon_2^2) \\ &\quad + V_{2HD}(\phi_1, \phi_2). \end{aligned}$$

(59)

Now, parameterizing the VEVs as in Eq. (25) and minimizing the potential leads to

$$\cos 2\gamma = -\frac{\varrho w^2 + (\sigma_1'|v_1|^2 + \sigma_2'|v_2|^2)}{2\lambda' w^2},$$

$$w^2 = \frac{2\lambda'(\mu^2 - (\sigma_1|v_1|^2 + \sigma_2|v_2|^2)) + \varrho(\sigma_1'|v_1|^2 + \sigma_2'|v_2|^2)}{4\lambda\lambda' - \varrho^2}.$$

(60)

Keeping in mind the size of the various coefficients in the symmetry breaking potential discussed above, we find that $\cos 2\gamma \approx 0$ up to corrections of order $\frac{v^2}{\omega^2}$. We assume that w is in the TeV scale and with v in the electroweak scale the symmetry breaking corrections are of the right size to explain the experimental numbers.

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[2]The most general symmetry breaking terms can be expressed in terms of the form in Eq. (58) and symmetry conserving terms that can be absorbed in the symmetric potential.

We shift the VEVs of the two singlet scalars ($w_1 \neq w_2$) up to the first order of the symmetry breaking parameter. Then, the Majorana neutrino mass matrix in Eq. (34) takes the form

$$M_R = \begin{pmatrix} M & -v_{wp} & -v_{wn} \\ -v_{wp} & M & 0 \\ -v_{wn} & 0 & M \end{pmatrix}, \qquad (61)$$

where

$$v_{wp} = \frac{y}{\sqrt{2}}(w_1 + w_2), \qquad v_{wn} = \frac{y}{\sqrt{2}}(w_1 - w_2). \qquad (62)$$

We write the VEVs of the singlet scalars after symmetry breaking as

$$w_1 = \frac{w + \rho_1}{\sqrt{2}}, \qquad w_2 = \frac{w + \rho_2}{\sqrt{2}}, \qquad (63)$$

where ρ_1 and ρ_2 are small quantities and

$$\rho_1 = -\rho_2 = \frac{w\tau}{2}. \qquad (64)$$

Up to the first order of the symmetry breaking parameter τ,

$$\tau \equiv -\frac{\varrho w^2 + (\sigma_1'|v_1|^2 + \sigma_2'|v_2|^2)}{2\lambda' w^2}, \qquad (65)$$

one gets

$$v_{wp} = v_w, \qquad v_{wn} = \frac{\tau}{2} v_w. \qquad (66)$$

It turns out that breaking the $\epsilon_1 \leftrightarrow \epsilon_2$ symmetry to generate different VEVs for the singlet scalars is not sufficient to break the almost degeneracy of (m_1, m_2) to satisfy the squared-mass-difference measurements. Therefore, we introduce an additional term in the Lagrangian which is consistent with the symmetries of the Lagrangian,

$$M_1[\nu_{\mu R}^T C^{-1} \nu_{\mu R} + \nu_{\tau R}^T C^{-1} \nu_{\tau R}]. \qquad (67)$$

Thus,

$$M_R = \begin{pmatrix} M & -v_w & -\frac{\tau}{2} v_w \\ -v_w & M' & 0 \\ -\frac{\tau}{2} v_w & 0 & M' \end{pmatrix}, \qquad (68)$$

where $M' = M + M_1$.

The neutrino mass matrix in Eq. (36) changes to be

$$\mathcal{M}_\nu = \begin{pmatrix} X' & G' & P' \\ G' & Y' & W' \\ P' & W' & Z' \end{pmatrix}, \qquad (69)$$

where

$$X' = -\frac{4A^2 M'}{4MM' - v_w^2(4 + \tau^2)},$$

$$Y' = -\frac{A^2(4MM' - v_w^2 \tau^2)}{M'(4MM' - v_w^2(4 + \tau^2))},$$

$$Z' = -\frac{4A^2(MM' - v_w^2)}{M'(4MM' - v_w^2(4 + \tau^2))}, \qquad (70)$$

$$G' = -\frac{4A^2 v_w}{4MM' - v_w^2(4 + \tau^2)},$$

$$P' = -\frac{2A^2 v_w \tau}{4MM' - v_w^2(4 + \tau^2)},$$

$$W' = -\frac{2A^2 v_w^2 \tau}{M'(4MM' - v_w^2(4 + \tau^2))}.$$

By diagonalizing Eq. (69), one gets the mass eigenvalues

$$m_1 = -\frac{2A^2((M + M') - \sqrt{M^2 - 2MM' + M'^2 + v_w^2(4 + \tau^2)})}{4MM' - v_w^2(4 + \tau^2)},$$

$$m_2 = -\frac{2A^2((M + M') + \sqrt{M^2 - 2MM' + M'^2 + v_w^2(4 + \tau^2)})}{4MM' - v_w^2(4 + \tau^2)},$$

$$m_3 = -\frac{A^2}{M'}. \qquad (71)$$

Now, we can diagonalize the mass matrix in Eq. (69) using the unitary matrix $U_\nu = W_{12}^\nu R_{23}^\nu R_{12}^\nu$ with

$$R_{12}^\nu = \begin{pmatrix} c_{12\nu} & s_{12\nu} & 0 \\ -s_{12\nu} & c_{12\nu} & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

$$c_{12\nu} = \cos\theta_{12\nu}; s_{12\nu} = \sin\theta_{12\nu}, \qquad (72)$$

$$R_{23}^\nu = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23\nu} & s_{23\nu} \\ 0 & -s_{23\nu} & c_{23\nu} \end{pmatrix},$$

$$c_{23\nu} = \cos\theta_{23\nu}; s_{23\nu} = \sin\theta_{23\nu}.$$

The mass matrix elements in Eq. (70) satisfy the two relations

$$X'(Z' - Y') = P'^2 - G'^2, \qquad (73)$$
$$G'P'(Z' - Y') = W'(P'^2 - G'^2).$$

By applying the above relations to the matrix elements of

$$\mathcal{M}_\nu = U_\nu \mathcal{M}_\nu^d U_\nu^\dagger, \qquad (74)$$

one can obtain the mixing angles

$$s_{23\nu} = \sqrt{\frac{2m_1(m_2 - m_3)}{m_2(m_1 - m_3)}},$$

$$s_{12\nu} = \sqrt{\frac{-m_1 m_2 + 2m_1 m_3 - m_2 m_3}{2m_3(m_1 - m_2)}}. \qquad (75)$$

Eventually, we obtain the elements of the lepton mixing matrix $U_{\text{PMNS}} = U_l^\dagger U_\nu$ with $U_\ell = W_{23}^l R_{23}^l R_{13}^l R_{12}^l$ and $U_\nu = W_{12}^\nu R_{23}^\nu R_{12}^\nu$. The deviation parameters (s, r, a) can be obtained from Eq. (56) as follows:

$$s \approx -\frac{1}{\sqrt{2}}(s_{12l} + s_{13l}) + s_{12\nu},$$

$$r \approx s_{12l} - s_{13l} - s_{23\nu}, \qquad (76)$$

$$a \approx -s_{23l} + \frac{1}{\sqrt{2}}s_{23\nu}.$$

V. NUMERICAL RESULTS

From the neutrino mass matrix (16), one observes that in the degenerate case, when $m_1 \approx m_2 \approx m_3$, $a \approx c$, $d \approx 0$ which means that the neutrino mass matrix is already diagonalized as $\mathcal{M}_\nu \approx \text{diag}(a, a, a)$. That means the lepton mixing matrix does not include a contribution from the neutrino sector, and the resultant leptonic mixing is inconsistent with the experimental data. Thus, in the symmetric limit our model excludes the case of the degenerate neutrino masses. Even, after symmetry breaking, the degenerate case in Eq. (71) leads to vanishing the VEVs of the singlet scalar fields which does not lead to successful phenomenology.

The numerics goes as follows: we choose masses (m_1, m_2, m_3) which satisfy the experimental values of the squared-mass differences

$$\Delta m_{21}^2 = m_2^2 - m_1^2 = (7.59 \pm 0.20) \times 10^{-5} \text{ eV}^2,$$
$$\Delta m_{32}^2 = |m_3^2 - m_2^2| = (2.43 \pm 0.13) \times 10^{-3} \text{ eV}^2. \qquad (77)$$

We substitute those mass values in (r, s, a) in Eq. (56), using $(s_{12\nu}, s_{23\nu})$ given in Eq. (75) and $(s_{12l}, s_{23l}, s_{13l})$ in Sec. IV A. If the results satisfy the experimental constraints in Eq. (54), we plot the possible values of the absolute masses and the mixing angles. By using Eq. (71), we calculate values for the Lagrangian parameters (v_w, A, M, M') which generate the values of the absolute masses obtained from the graphs. From the graphs, one finds that (v_w, M, M') are obtained in the TeV scale and A in the MeV range.

Three mass-dependent neutrino observables are probed in different types of experiments. The sum of absolute neutrino masses $m_{\text{cosm}} \equiv \Sigma m_i$ is probed in cosmology, the kinetic electron neutrino mass in beta decay (M_β) is probed in direct search for neutrino masses, and the effective mass (M_{ee}) is probed in neutrinoless double-beta decay experiments with the decay rate for the process $\Gamma \propto M_{ee}^2$. In terms of the "bare" physical parameters m_i and $U_{\alpha i}$, the observables are given by [18]

$$\Sigma m_i = |m_1| + |m_2| + |m_3|,$$
$$M_{ee} = ||m_1||U_{e1}|^2 + |m_2||U_{e2}|^2 e^{i\phi_1} + |m_3||U_{e3}|^2 e^{i\phi_2}|,$$
$$M_\beta = \sqrt{|m_1|^2|U_{e1}|^2 + |m_2|^2|U_{e2}|^2 + |m_3|^2|U_{e3}|^2}. \qquad (78)$$

In our analysis, we ignore the Majorana phases (ϕ_1, ϕ_2) and plot M_β versus Σm_i and M_{ee} versus m_{light}, where m_{light} is the lightest neutrino mass.

In Figs. 1–3, we assume specific values of z with the corresponding correction mixing angles $(s_{13l}, s_{12l}, s_{23l})$ and plot the absolute masses and the mixing angles which satisfy the neutrino mixing constraints. By choosing a value for the symmetry breaking term τ, we plot the parameters (v_w, A, M, M') that satisfy the squared-mass-difference measurements. This model supports the normal mass hierarchy as shown in the graphs with the scale of the neutrino masses in the few meV to ~50 meV range. The results agree with the recent T2K data which find a relatively large θ_{13}. The graphs show that the seesaw scale (M, M') are in the TeV range, and the second Higgs that couples to the right-handed neutrinos has VEV v_2, included in A, in the MeV scale. Also, they indicate that the VEV of the singlet scalar fields v_w is in the TeV scale. The graphs show that $\Sigma m_i \approx 0.06$ eV and $M_{ee} < M_\beta$ and $M_{ee} < 0.35$ eV [21]. Various other mechanisms to generate the neutrino masses with TeV scale new physics are mentioned in Ref. [22].

VI. CONCLUSION

In this paper, we presented a model for leptonic mixing which accommodates the sizable neutrino mixing angle θ_{13}, recently measured by the T2K and MINOS experiments. We worked in a basis where the charged-lepton mass matrix is not diagonal and proposed an explicit structure for the charged-lepton mass matrix which is 2-3 symmetric except for a single breaking of this symmetry by the muon mass. We identified a flavor symmetric limit for the mass matrices where the first generation is decoupled from the other two in the charged-lepton sector while in the neutrino sector the third generation is decoupled from the first two generations. The leptonic mixing in the symmetric limit was shown to have, among other structures, the BM and the TBM mixing.

A model that extended the SM by three right-handed neutrinos, an extra Higgs doublet, and two singlet scalars was introduced to generate the leptonic mixing. In the symmetric limit, the model had two Z_2 symmetries in addition to the μ-τ symmetry and the BM leptonic mixing was obtained when the two singlet scalars got equal VEVs.

Symmetry breaking effects were included in the charged-lepton sector via higher-dimensional operators that generated a μ-τ symmetric mass matrix except for a single breaking due to the finite muon mass. In the neutrino sector, symmetry breaking was included via slightly different VEVs for the two singlet scalars. To explain the Δm^2 data, two different Majorana mass terms, one for ν_e and one for ν_μ and ν_τ, were used keeping in mind that the μ-τ symmetry fixes the Majorana mass terms for the ν_μ and ν_τ to be the same.
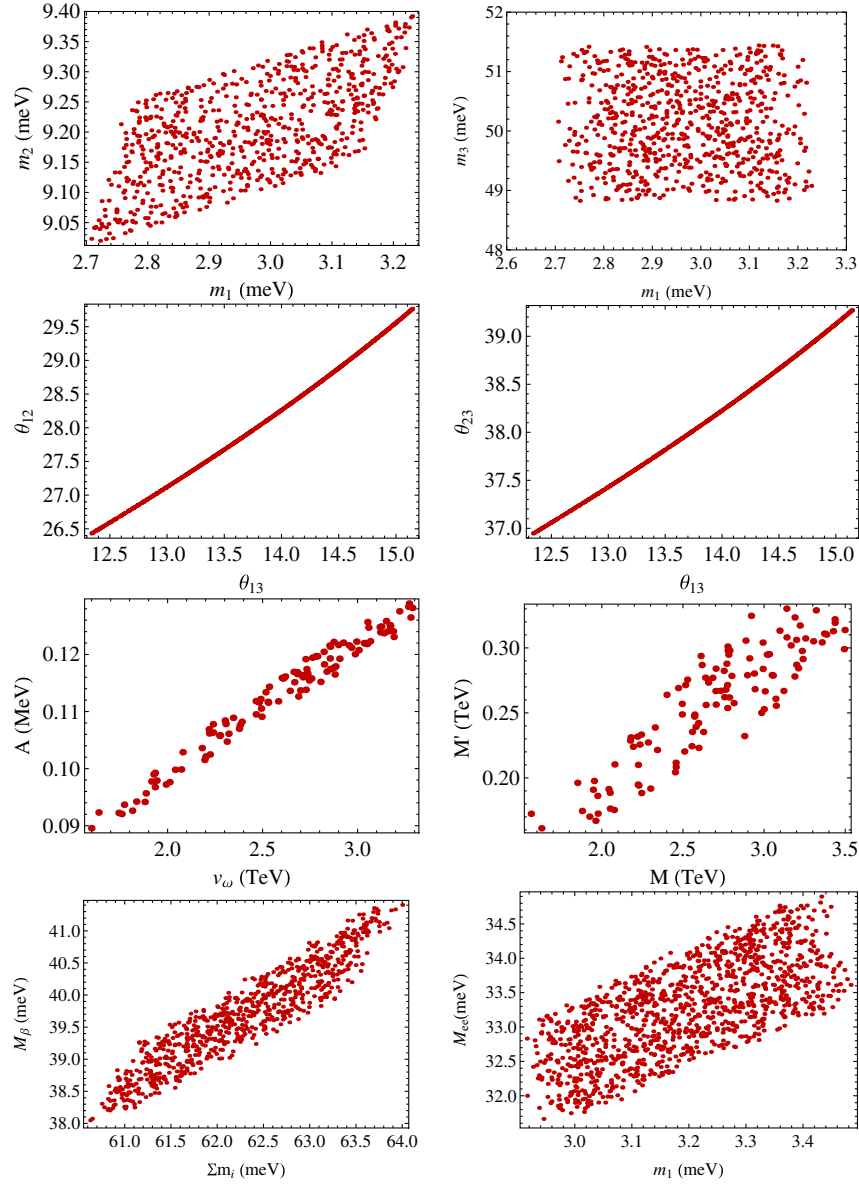
FIG. 1 (color online). Scatter plots for $z = 2.0$ with $s_{12l} \approx -0.34$, $s_{13l} \approx -0.0011$, and $s_{23l} \approx -0.059$. In the neutrino sector, we assume that $\tau = 0.1$ (meV $\equiv 10^{-3}$ eV).

A fit to the experimental measurements showed that our model predicted normal hierarchy for the neutrino masses with the masses being in the few meV to ∼50 meV range. The Majorana mass terms as well as the VEVs of the singlet scalar fields were predicted to be in the TeV scale and consequently the VEV of the second Higgs doublet was shown to be in the MeV range. We calculated predictions for the mass-dependent observables (Σm_i), (M_β), and (M_{ee}). We found that $\Sigma m_i \approx 0.06$ eV, $M_{ee} < M_\beta$, and $M_{ee} < 0.35$ eV.

ACKNOWLEDGMENTS

APPENDIX A: CHARGED-LEPTON SECTOR

We analytically calculate the deviation of the leptonic mixing from the symmetric limit due to corrections from the charged-lepton sector. We, here, are going to determine the sizes for the Yukawa matrix elements in Y^L in Eq. (42). We first consider the breaking of the 2-3 symmetry in the
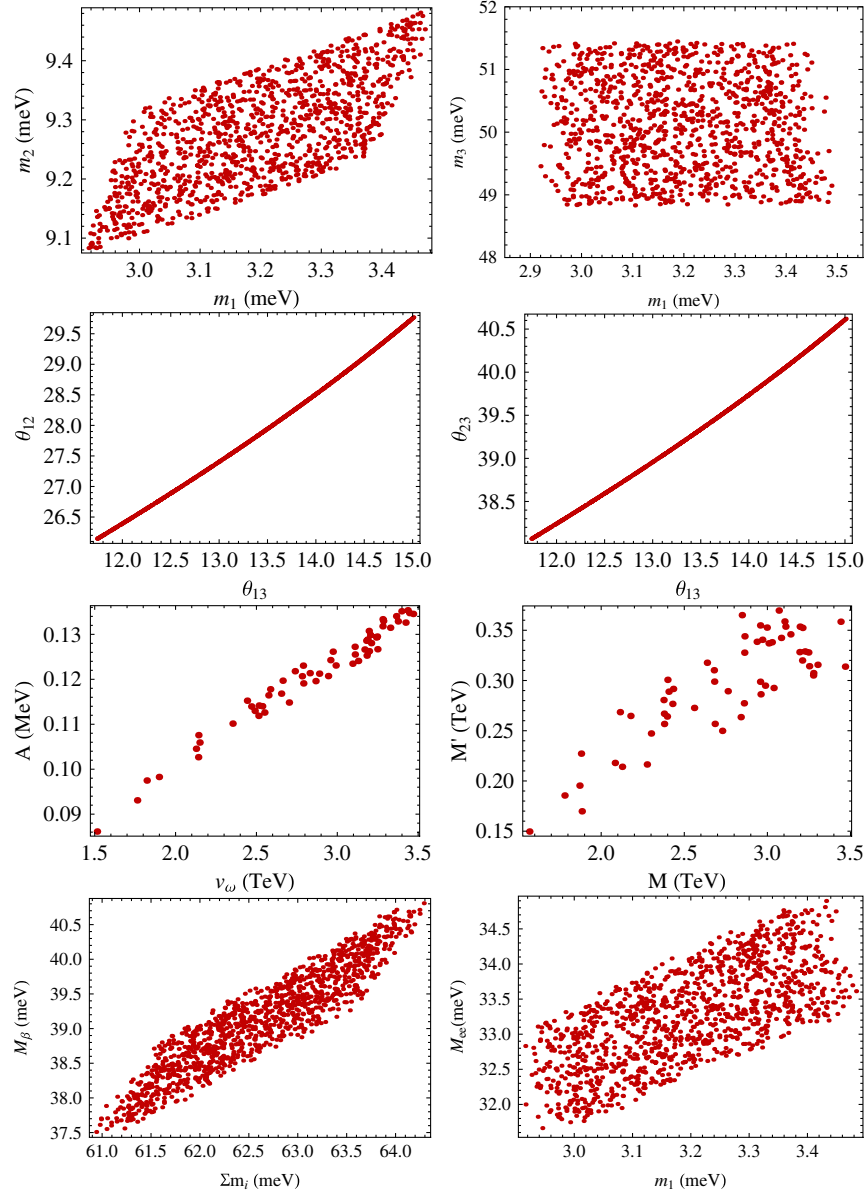
FIG. 2 (color online).   Scatter plots for $z = 2.06$ with $s_{12l} \approx -0.3$, $s_{13l} \approx -0.001$, and $s_{23l} \approx -0.061$. In the neutrino sector, we assume that $\tau = 0.05$ (meV $\equiv 10^{-3}$ eV).

charged-lepton sector via the introduction of a higher-dimensional operator that generates the muon mass

$$O_1 = cy_2 \bar{D}_{\mu_L} \mu_R \phi_1 \frac{\phi_1 \phi_1^\dagger}{\Lambda^2}. \qquad (A1)$$

Thus, we consider the Yukawa matrix,

$$Y_{23}^L = \begin{pmatrix} l_e & 0 & 0 \\ 0 & \frac{1}{2}l_T(1 + 2\kappa_l) & \frac{1}{2}l_T \\ 0 & \frac{1}{2}l_T & \frac{1}{2}l_T \end{pmatrix}. \qquad (A2)$$

The structure above breaks the 2-3 symmetry because of the correction to the 22 element. Note that we do not break the 2-3 symmetry in the 23 element so that the Yukawa matrix remains symmetric. The matrix Y_{23}^L is now diago-

nalized by the unitary matrix, $U_l = W_{23}^l R_{23}^l$. Applying the relation $(Y_{23}^L)_{23} = (Y_{23}^L)_{33}$ in Eq. (A2) to $Y_{23}^L = U_l Y_{\mathrm{diag}}^L U_l^\dagger$ leads to

$$t_{23l} = \frac{1}{2}[z_\mu - 1 + \sqrt{z_\mu^2 - 6z_\mu + 1}], \qquad (A3)$$

where $t_{23l} \equiv \tan\theta_{23l}$ and we have chosen the solution that leads to small angle θ_{23l} and to small flavor symmetry breaking. Keeping terms up to first order in z_μ, we get

$$t_{23l} \approx -z_\mu. \qquad (A4)$$

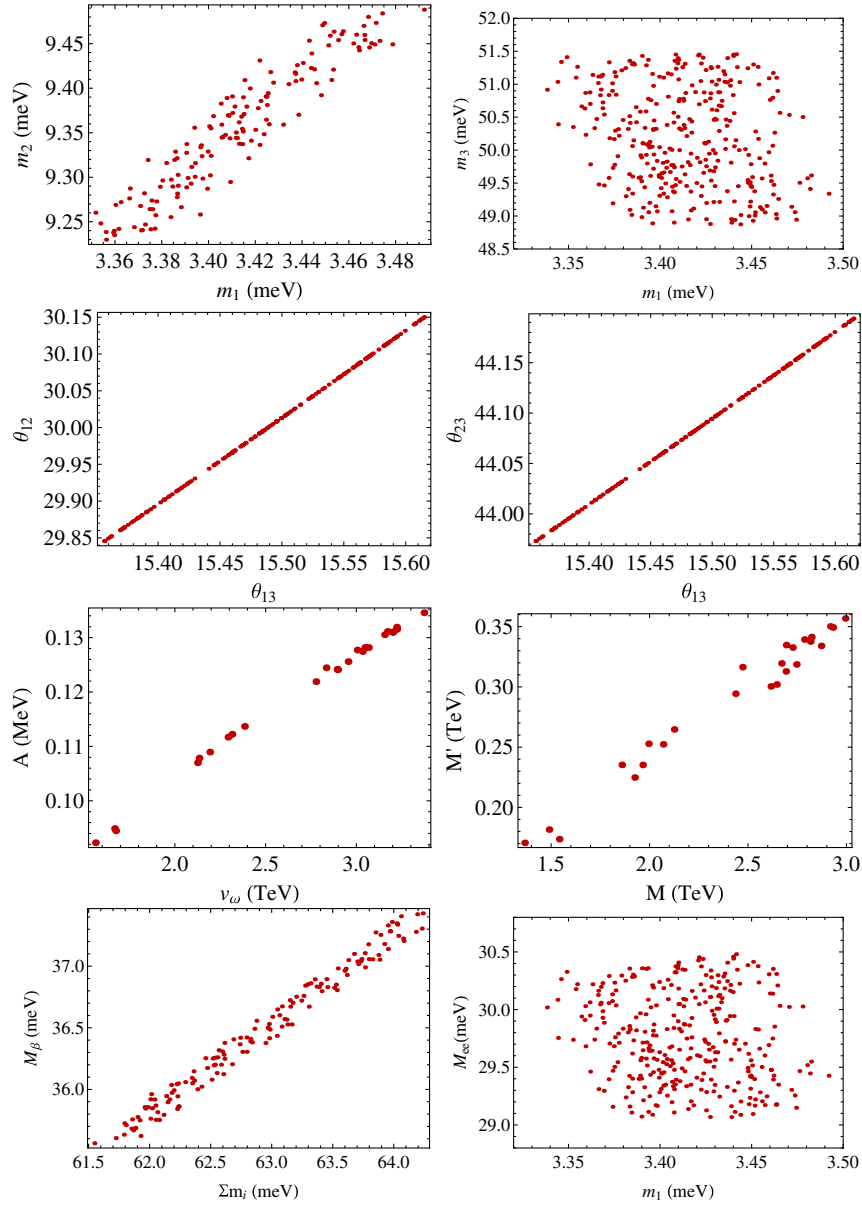We further obtain for κ_l and l_T in Eq. (A2),

FIG. 3 (color online). Scatter plots for $z = 2.2$ with $s_{12l} \approx -0.2$, $s_{13l} \approx -0.00075$, and $s_{23l} \approx -0.065$. In the neutrino sector, we assume that $\tau = 0.1$ (meV $\equiv 10^{-3}$ eV).

$$\kappa_l = -\tan 2\theta_{23l} \approx 2z_\mu, \qquad l_T = (l_\tau - l_\mu)\cos 2\theta_{23l}. \tag{A5}$$

Comparing the above equation with Eq. (A1), the size of the higher-dimensional operator can be estimated as

$$\frac{cv_1^2}{2\Lambda^2} \approx 2z_\mu. \tag{A6}$$

Since $v_1 \approx 250$ GeV, the scale of Λ is in the TeV range.

To obtain a realistic charged-lepton matrix, we take into account the mixing involving the second and the third generations in the full Yukawa matrix

$$O_2 = y'(\bar{D}_{e_L}\mu_R - \bar{D}_{e_L}\tau_R + \bar{D}_{\mu_L}e_R - \bar{D}_{\tau_L}e_R)\phi_1 \frac{\phi_1 \phi_1^\dagger}{\Lambda^2}. \tag{A7}$$

Thus, the full mixing matrix will be given by

$$Y^L = \begin{pmatrix} l_{11} & l_{12} & -l_{12} \\ l_{12} & \frac{1}{2}l_T(1 + 2\kappa_l) & \frac{1}{2}l_T \\ -l_{12} & \frac{1}{2}l_T & \frac{1}{2}l_T \end{pmatrix}. \tag{A8}$$

We will assume that the Yukawa matrix Y^L is now diagonalized by the unitary matrix U_l given by

$$U_l = W_{23}^l R_{23}^l R_{13}^l R_{12}^l. \tag{A9}$$

From the Yukawa matrix (A8), one can find the two relations

$$Y_{12} = -Y_{13}, \qquad Y_{22} = \tfrac{1}{2}(Y_{23} + Y_{33})(1 + 2\kappa_l). \tag{A10}$$

Applying the above two relations to

$$Y^L = U_l Y^L_{\text{diag}} U_l^\dagger \tag{A11}$$

using Eq. (A9), one can obtain the solutions

$$s_{12l} \approx \pm c_{23l} \sqrt{\frac{z_\mu - 2\kappa_l + (-3 + 3z_\mu - 2\kappa_l)c_{23l}s_{23l} + 2z_\mu\kappa_l c_{23l}s_{23l}}{z_\mu - z_\mu^2(3 + 2\kappa_l)c_{23l}s_{23l}}},$$

$$s_{13l} \approx \pm \sqrt{z_\mu} c_{12l}s_{23l} \sqrt{\frac{z_\mu - 2\kappa_l + (-3 + 3z_\mu - 2\kappa_l)c_{23l}s_{23l} + 2z_\mu\kappa_l c_{23l}s_{23l}}{1 - z_\mu(3 + 2\kappa_l)c_{23l}s_{23l}}}. \tag{A12}$$

By comparing Eqs. (A8) and (A11), one can get the matrix element l_{12} after substituting Eqs. (A4), (A5), and (A12) up to the first order in z_μ,

$$l_{12} \approx \sqrt{\frac{z_\mu}{2}}(l_e - l_\mu). \tag{A13}$$

The leptonic mixing matrix is now given by

$$U_{\text{PMNS}} = U_\ell^\dagger U_\nu, \tag{A14}$$

where $U_\ell = W_{23}^l R_{23}^l R_{13}^l R_{12}^l$ and $U_\nu = W_{12}^\nu$.

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