Wigner solution of the quark gap equation at nonzero current quark mass and partial restoration of chiral symmetry at finite chemical potential

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According to the generally accepted phase diagram of QCD, at low temperature and high baryon number density the chiral phase transition of QCD is of first order and the coexistence of the Nambu-Goldstone phase and the Wigner phase should appear. This is in conflict with the usual claim that the quark gap equation has no Wigner solution in the case of nonzero current quark mass. In this paper we analyze the reason why the Wigner solution does not exist in the usual treatment and try to propose a new approach to discuss this question. As a first step, we adopt a modified Nambu-Jona-Lasinio (NJL) model to study the Wigner solution at finite current quark mass. We then generalize this approach to the case of finite chemical potential and discuss partial restoration of chiral symmetry at finite chemical potential and compare our results with those in the normal NJL model.

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I. INTRODUCTION

Dynamical chiral symmetry breaking (DCSB) and confinement are two fundamental features of quantum chromodynamics (QCD). It is generally believed that with increasing temperature and baryon number density strongly interacting matter will undergo a phase transition from the hadronic matter to the quark-gluon plasma (QGP) which is expected to appear in the ultrarelativistic heavy ion collisions. These two phases are generally referred to as the Nambu-Goldstone phase which is characterized by DCSB and confinement of dressed quarks and the Wigner phase corresponding to QGP in which chiral symmetry is partially restored and quarks are not confined. Theoretically, these two phases are described by two different solutions, the Nambu-Goldstone solution and the Wigner solution of the quark gap equation. The existence of these two solutions in the chiral limit (the current quark mass $m = 0$) has been shown in the framework of Dyson-Schwinger equation (DSE) approach of QCD (see, for example, [1,2]). However, it is a general conclusion in the previous literature that, when the current quark mass m is nonzero, the quark gap equation has only one solution which corresponds to the Nambu-Goldstone phase and the solution corresponding to the Wigner phase does not exist [3,4]. This conclusion is hard to understand and one will naturally ask why the Wigner solution of the quark gap equation only exists in the chiral limit while it does not exist at finite current quark mass. Furthermore, this conclusion is in fact not compatible with the current study of chiral phase transition of QCD. In order to see this more clearly, let us have a look at the generally accepted QCD phase diagram (see, for example, Fig. 3 of Ref. [5]). As is shown in the QCD phase diagram, it is generally believed that at low temperature and high baryon number density the chiral phase transition of QCD is of first order and the coexistence of the Nambu-Goldstone phase and the Wigner phase should appear. It is well known that in the real world the current quark mass is nonzero. If one cannot find the Wigner solution of the quark gap equation in the case of nonzero current quark mass, this will mean that we cannot talk about the coexistence of these two phases. This is obviously an unsettled and important problem in the study of QCD phase transitions. The authors of Ref. [6] first discussed this problem and asked whether the quark gap equation has a Wigner solution in the case of nonzero current quark mass. Subsequently, the authors of Refs. [7–9] further investigated the problem of possible multisolutions of the quark gap equation. However, as far as we know, this problem has not been solved satisfactorily in the literature. In the present paper we try to propose a new approach to investigate this problem.

The main motivation of the present work is to study the Wigner solution of the quark gap equation at finite current quark mass and provide a new viewpoint on partial restoration of chiral symmetry at finite chemical potential. This paper is organized as follows: in Sec. II we analyze the reason why in the previous literature the Wigner solution of the quark gap equation does not exist in the case of the finite current quark mass and propose a new approach to discuss this question. In Sec. III, based on such an approach, we show in the framework of the Nambu-Jona-Lasinio (NJL) model that the quark DSE has a Wigner solution at finite current quark mass. Then, in Sec. IV we generalize this approach to the case of finite chemical potential to study partial restoration of chiral symmetry

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and compare our results with the corresponding ones in previous literature. The results are summarized in Sec. V.

II. QUARK GAP EQUATION AND ITS SOLUTIONS

In order to illustrate our new approach more clearly, let us now briefly recall the usual arguments which exclude the existence of the Wigner solution of the quark gap equation when $m \neq 0$. The quark DSE under rainbow approximation reads as follows (in the present paper we will always work in Euclidean space, and take the number of flavors, $N_f = 2$ and the number of colors, $N_c = 3$):

$$G^{-1}(p) = G_0^{-1}(p) + \frac{4}{3}\int \frac{d^4q}{(2\pi)^4} g^2 D_{\mu\nu}(p-q)\gamma_\mu G(q)\gamma_\nu, \tag{1}$$

where $G(p)$ is the dressed quark propagator, $G_0(p) = (i\gamma \cdot p + m)^{-1}$ is the free quark propagator, g is the strong coupling constant, and $D_{\mu\nu}(q)$ is the effective dressed gluon propagator. According to Lorentz structure analysis, one has

$$G^{-1}(p) = i\not{p}A(p^2) + B(p^2), \tag{2}$$

where $A(p^2)$ and $B(p^2)$ are scalar functions of p^2. Substituting Eq. (2) into Eq. (1), one has

$$[A(p^2) - 1]p^2 = \frac{4}{3}\int \frac{d^4q}{(2\pi)^4} \frac{g^2 D(p-q)A(q^2)}{q^2A^2(q^2) + B^2(q^2)} \times \left[p\cdot q + 2\frac{p\cdot(p-q)q\cdot(p-q)}{(p-q)^2}\right], \tag{3}$$

$$B(p^2) = m + \int \frac{d^4q}{(2\pi)^4} \frac{4g^2 D(p-q)B(q^2)}{q^2A^2(q^2) + B^2(q^2)}, \tag{4}$$

where Landau gauge has been employed. From Eqs. (3) and (4), one can find when $m = 0$ there are two distinct solutions for $B(p^2)$. One solution is $B(p^2) \neq 0$ which describes the Nambu phase, and the other one is $B(p^2) \equiv 0$ which describes the Wigner phase. However, when $m \neq 0$, it can be easily seen that $B(p^2) \equiv 0$ is not a solution of Eqs. (3) and (4). From this observation one often concludes that, when $m \neq 0$, the quark DSE has only one solution corresponding to the Nambu phase and the Wigner solution does not exist. Here, it should be noted that in obtaining this conclusion one has assumed that the dressed gluon propagators in these two phases are the same. However, since the features of these two phases are so different, it is reasonable to expect that the behavior of the dressed gluon propagator should be different in these two phases (for example, in the familiar liquid-solid phase transition of water, the effective interactions between molecules are different in the two phases). To see this more clearly, let us look at the graphical representation of the DSE for the dressed gluon propagator given in Fig. 1. From Fig. 1 it can be seen that the quark propagator can affect the gluon propagator through quark-loop insertions. Therefore, in principle, since the quark propagators in the Nambu phase and the Wigner phase are quite different, one naturally expects that the gluon propagators in these two phases should be different, too. Here we would like to stress that this observation is model independent. Besides, this observation has been verified in the study of quantum electrodynamics in $2+1$ dimensions (QED_3) by using the coupled DSE for the fermion and photon propagators with a range of fermion-photon vertices [10] (QED_3 has many features similar to QCD, such as spontaneous chiral symmetry breaking in the massless fermion limit and confinement. Because of these reasons it can serve as a toy model of QCD.) This indicates that one should choose different forms of gluon propagator as input to solve the quark propagators in the two different phases. Now, the key problem is how to choose appropriate model gluon propagators as input to calculate the dressed quark propagator in the Nambu phase and the Wigner phase, respectively.

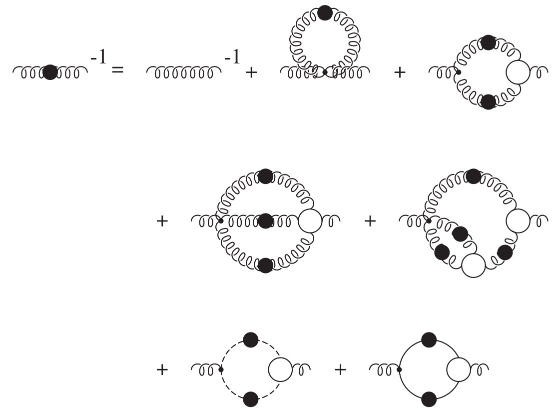

FIG. 1. The DSE for the dressed gluon propagator.

From Fig. 1 it can be seen that one can formally split the full gluon propagator into two parts as follows:

$$D_{\mu\nu}(q) = D^{\mathrm{YM}}_{\mu\nu}(q) + D^{Q}_{\mu\nu}(q), \tag{5}$$

where $D^{\mathrm{YM}}_{\mu\nu}$ is the pure Yang-Mills (YM) part which includes all diagrams without quark-loop insertions (which is usually called quenched gluon propagator in lattice QCD) and $D^{Q}_{\mu\nu}$ is the quark-affected part which includes all diagrams with quark-loop insertions. Obviously, the pure Yang-Mills part in the Wigner phase should be same as that in the Nambu phase, whereas in principle the quark-affected parts in these two phases should be different. At present it is impossible to calculate the two parts $D^{\mathrm{YM}}_{\mu\nu}(q)$ and $D^{Q}_{\mu\nu}(q)$ from the first principle of QCD. So one has to resort to various nonperturbative QCD models to express them phenomenologically.

Over the past few years, considerable progress has been made in the framework of the QCD sum rule [11], which provides a successful description of various nonperturbative aspects of strong interaction physics. We naturally expect that it might provide some useful clue to the study of the model gluon propagator. From the QCD sum rule approach the lowest-order contribution of quark condensate to the gluon propagator is [12]

$$\Delta_{\mu\nu}(p) = -g^2 \int d^4(y-z) \int \frac{d^4q}{(2\pi)^4} e^{i(p-q)\cdot(y-z)} \times \mathrm{tr}\left[\gamma_\mu \frac{-i\not{q}+m}{q^2+m^2}\gamma_\nu \langle\bar{\psi}(y)\psi(z)\rangle\right] \sim -\delta_{\mu\nu}\frac{mg^2\langle\bar{\psi}\psi\rangle}{3p^2} + \cdots, \tag{6}$$

where $\langle\bar{\psi}(y)\psi(z)\rangle$ is the nonlocal quark condensate and $\langle\bar{\psi}\psi\rangle$ is the ordinary two-quark condensate; the ellipsis represents terms of higher orders in $\frac{m^2}{p^2}$ which we neglect in the present work since we limit our discussion to two light flavors u and d. It is evident that the value of quark condensate $\langle\bar{\psi}\psi\rangle$ is different in the Nambu phase and the Wigner phase. This makes the gluon propagators in these two phases to be different. Therefore, in the following calculation we can phenomenologically identify $\Delta_{\mu\nu}(p)$ in Eq. (6) as a good approximation of the $D^Q_{\mu\nu}(q)$ part in Eq. (5).

III. NJL-LIKE MODEL AND TWO DISTINCT SOLUTIONS AT ZERO CHEMICAL POTENTIAL

Now, we should specify a model framework to calculate the quark propagators in the Nambu phase and the Wigner phase. The dressed quark propagators are the most elementary of the n-point Green functions of QCD. It is evident that the Dyson-Schwinger equations (DSEs) are the natural tool for investigating it in the continuum. In particular, it has been shown that DSEs are capable to describe the chiral phase transition and deconfinement phase transition at finite temperatures and chemical potential [2,13–17]. However, as is shown in Ref. [18], the Nambu-Jona-Lasinio (NJL) model can capture the main physical features of QCD at finite temperature and chemical potential. For example, partial restoration of chiral symmetry, the critical end point, and color superconductivity are all first studied in the framework of the NJL model. This is the reason why the NJL model is the most widely used QCD model in the study of QCD phase transition at finite temperature and chemical potential (although this model has two defects, namely, it can neither accommodate confinement nor is renormalizable). Therefore, as a first step, for simplicity in this paper we shall employ the NJL model to study the quark propagators in the Nambu phase and the Wigner phase.

In the normal NJL model the following model gluon propagator,

$$g^2 D_{\mu\nu}(p-q) = \delta_{\mu\nu}\frac{1}{M_G^2}\theta(\Lambda^2 - q^2), \tag{7}$$

is employed to calculate the quark propagator, where M_G is some effective gluon mass scale and Λ serves as a cutoff and is set to be 1.015 GeV in Ref. [18]. This model gluon propagator concentrates on the infrared region of the interaction which is believed to be vital for DCSB of QCD. With such a model gluon propagator, Eq. (1) becomes

$$i\not{p}A(p^2) + B(p^2) = i\not{p} + m + \frac{4}{3M_G^2}\int \frac{d^4q}{(2\pi)^4}\theta(\Lambda^2 - q^2) \times \frac{\gamma_\mu[-i\not{q}A(q^2) + B(q^2)]\gamma_\mu}{A^2(q^2)q^2 + B^2(q^2)}. \tag{8}$$

The solution of Eq. (8) is $A(p^2) \equiv 1$ and $B(p^2) \equiv M$ with M being a constant satisfying the following equation:

$$M = m + \frac{M}{3\pi^2 M_G^2} D_1(M^2, \Lambda^2), \tag{9}$$

where $D_1(M^2, \Lambda^2) = \Lambda^2 - M^2 \ln[1 + \Lambda^2/M^2]$. From Eq. (9) it is easy to find that, when $M_G^2 < 1/3\pi^2$, this equation has two different solutions in the chiral limit. However, when $m \neq 0$, one could only find one solution, the Nambu solution, which satisfies $M > 0$. This result is consistent with the one derived from the analysis of the quark DSE under rainbow approximation.

Obviously, the vacuum of the Wigner phase should be different from that of the Nambu phase and their difference can be characterized by the quark condensate which is regarded as the order parameter for chiral phase transition. Therefore, the gluon propagator should be different due to different values of quark condensate in these two phases. In order to reflect this fact, we introduce the quark condensate contribution [Eq. (6)] to the gluon self-energy and modify the effective gluon propagator in the normal NJL model as follows:

$$\begin{aligned} g^2 D_{\mu\nu}(q) &= \delta_{\mu\nu}\frac{1}{M_G^2}\theta(\Lambda^2 - q^2) \\ &\quad - \delta_{\mu\nu}\frac{1}{M_G^2}\frac{m\langle\bar{\psi}\psi\rangle}{\Lambda_q^2}\frac{1}{M_G^2}\theta(\Lambda^2 - q^2) \\ &= \delta_{\mu\nu}\frac{1}{M_{\mathrm{eff}}^2}\theta(\Lambda^2 - q^2), \end{aligned} \tag{10}$$

where the first term in the right-hand side of Eq. (10) is the usual model gluon propagator employed in the NJL model which has the same form in both the Nambu phase and the Wigner phase and can be regarded as the pure Yang-Mills part $D^{\mathrm{YM}}_{\mu\nu}(q)$ in the present work; the second term, which is inspired by the result of QCD sum rules [12], is the leading order nonperturbative contribution from quark condensate through quark-loop insertions. Here, it should be noted that, according to the usual approximation of NJL model, in obtaining the current quark mass dependent term of Eq. (10) we have taken all the momentum dependence of the effective interaction in momentum space as a constant. For this purpose, we have introduced a momentum scale

Λ_q which reflects the large distance behavior of QCD. For the external momentum squared much larger than Λ_q^2, the current quark mass dependent term of Eq. (10) can be neglected, whereas for external momentum squared approaching Λ_q^2 the contribution of quark condensate which has been neglected in the normal NJL model must be considered. Just as will be shown below, the current mass dependent term in Eq. (10) plays an important role in searching for the Wigner solution at finite current quark mass and the study of partial restoration of chiral symmetry at finite chemical potential.

The value of M_G which accounts for the pure Yang-Mills gauge field contribution could be fixed by requiring the amount of the intensity of the effective interaction to be $M_{\rm eff}/\Lambda = 0.17$ for the Nambu solution $M = 238$ MeV which is determined by fitting the observables such as pion decay constant and pion mass (in the present paper we set the current quark mass $m = 5$ MeV) [18]. With the modified gluon propagator given by Eq. (10), the quark gap equation, Eq. (9), becomes

$$M = m + \frac{M}{3\pi^2}\left[\frac{1}{M_G^2} + \frac{1}{M_G^2}\frac{3MmD_1(M^2,\Lambda^2)}{2\pi^2\Lambda_q^2}\frac{1}{M_G^2}\right]D_1(M^2,\Lambda^2). \tag{11}$$

Now let us turn to the calculation of Eq. (11). To illustrate how the solution of Eq. (11) varies with different Λ_q, let us define

$$F(M) = M - m - \frac{M}{3\pi^2}\left[\frac{1}{M_G^2} + \frac{1}{M_G^2}\frac{3MmD_1(M^2,\Lambda^2)}{2\pi^2\Lambda_q^2}\frac{1}{M_G^2}\right] \times C(M^2,\Lambda^2), \tag{12}$$

and the solution of $F(M) = 0$ is just the solution of the quark gap equation [Eq. (11)]. In Fig. 2 $F(M)$ is plotted as a function of M with different Λ_q. From Fig. 2 it can be seen that, when Λ_q is larger than about 100 MeV, the equation $F(M) = 0$ has only one solution $M = 238$ MeV, which is similar to the situation discussed in Ref. [7]. When $\Lambda_q < 100$ MeV, the equation $F(M) = 0$ has three solutions. Specifically, when 70 MeV $< \Lambda_q <$ 100 MeV, one solution is the required Nambu solution $M = 238$ MeV, and the other two solutions are all smaller than it; when $\Lambda_q <$ 70 MeV, among the two solutions other than the Nambu one, one is smaller than it and the other one is larger than it. Here we note that physical observables require the Nambu solution to be $M = 238$ MeV and the stability condition of the Nambu solution would exclude the existence of solutions larger than it. Therefore, the parameter Λ_q should be constrained within the range 70 MeV $\le \Lambda_q \le$ 100 MeV. For Λ_q in this range, the smallest solution of Eq. (11) is not very large compared with the current quark mass, and when the current quark mass m tends to zero, this solution will continuously tend to zero, which is just the Wigner solution in the chiral limit. Therefore, this solution might be identified as the Wigner solution in the case of $m \neq 0$ which describes the perturbative dressing effect.

The result in Fig. 2 shows that the scale at which the current quark mass dependent term of Eq. (10) would affect the effective interaction can change the pattern of the solutions of the quark gap equation. If the current quark mass dependent term plays an important role in the infrared region in the effective interaction ($\Lambda_q <$ 100 MeV), then the intensity of the pure Yang-Mills field would be weakened and the Wigner solution will appear. On the contrary, when $\Lambda_q >$ 100 MeV, the pure Yang-Mills part would be dominating and strong in the infrared region, and therefore the Wigner solution cannot exist due to strong interaction. From physical consideration we choose $\Lambda_q = 70$ MeV, because in this case the gap equation has just two solutions which can be identified as the Nambu solution and the Wigner solution. For $\Lambda_q = 70$ MeV, we plot the $F(M)$ versus M curve for different current quark mass m in Fig. 3. It can be seen that as m increases, the effective mass M of dressed quark in the Wigner phase will increase and at last coincide with the Nambu solution when $m \sim 60$ MeV.

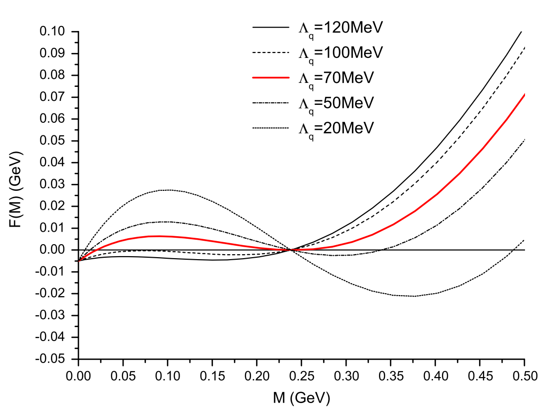

FIG. 2 (color online). Solutions of the gap equation with different Λ_q (m is fixed to be 5 MeV).

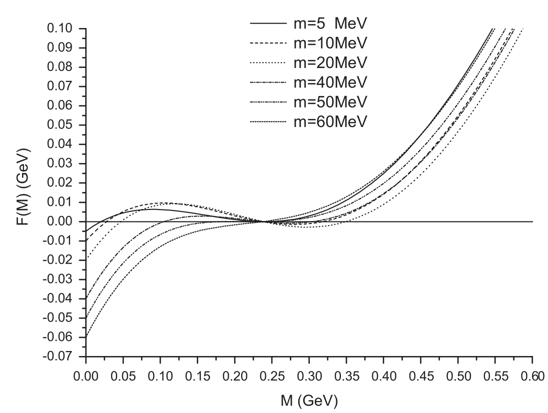

FIG. 3. Solutions of the gap equation with different m (Λ_q is fixed to be 70 MeV).

TABLE I. The Nambu and Wigner solution.

$\Lambda_q = 70$ (MeV)	M (GeV)	$-\langle\bar{\psi}\psi\rangle$ (GeV^3)	$\mathcal{P}$ (GeV^4)
Nambu phase	0.238	3.13×10^{-2}	1.797×10^{-3}
Wigner phase	0.02	3.16×10^{-3}	3.06×10^{-6}

As usual, the quark condensate is defined as

$$\langle\bar{\psi}\psi\rangle = -\int^{\Lambda} \frac{d^4p}{(2\pi)^4} \mathrm{Tr}[G(p)] = -\frac{3MD_1(M^2, \Lambda^2)}{2\pi^2} \tag{13}$$

and its value for the two solutions is listed in Table I. [In Table I we list the solution of Eq. (11) with $\Lambda_q = 70$ MeV and $m = 5$ MeV.] It can be seen that the value of the quark condensate in the Nambu phase is larger than that of the Wigner phase by 1 order of magnitude, which represents DCSB of the Nambu phase. Here it should be pointed out that the quark condensate of the Wigner solution is small but nonzero because it reflects the explicit chiral symmetry breaking due to nonzero current quark mass.

Of course, in order to determine which solution is the real one, one should compare the pressure (thermodynamical potential) of the different solutions. The vacuum pressure $\mathcal{P}$ of the two solutions is also listed in Table I which is calculated via "steepest descent" approximation as follows [4]:

$$\mathcal{P} = \int^{\Lambda} \frac{d^4p}{(2\pi)^4} \mathrm{Tr}\Big\{\ln[G^{-1}(p)G_0(p)] + \frac{1}{2}[G_0^{-1}(p)G(p) - 1]\Big\}. \tag{14}$$

From Table I it can be seen that the vacuum pressure of the Nambu phase is much larger than that of the Wigner phase (more than 2 orders of magnitude), which means the Nambu phase is more stable than the Wigner phase when temperature and density are zero. The vacuum pressure difference of the two phases can be regarded as the bag constant B_{bag} and the results in Table I correspond to $B_{\mathrm{bag}} \sim (206\ \mathrm{MeV})^4$ which is consistent with the value used in the literature [3]. One may expect that with increasing temperature and/or density this quantity may change and chiral phase transition would happen. We will discuss this question in the next section.

IV. PARTIAL RESTORATION OF CHIRAL SYMMETRY AT FINITE CHEMICAL POTENTIAL

Now we can generalize the previous treatment to the case of finite density. The quark propagator at finite quark chemical potential μ could be expressed as follows:

$$G^{-1}(p, \mu) = iA\not{p} + B - C\mu\gamma_4, \tag{15}$$

where $A(\vec{p}^2, p_4, \mu)$, $B(\vec{p}^2, p_4, \mu)$, and $C(\vec{p}^2, p_4, \mu)$ are scalar functions of $\vec{p}^2$, p_4, and μ. With the model gluon propagator in Eq. (10) the DSE of quark propagator at finite chemical potential is

$$iA\not{p} + B - C\mu\gamma_4 = i\not{p} + m - \mu\gamma_4 + \frac{4}{3M_{\mathrm{eff}}^2}\int \frac{d^4q}{(2\pi)^4} \times \frac{2iA\not{q} + 4B - 2C\mu\gamma_4}{A^2q^2 + B^2 - C^2\mu^2 + 2iAC\mu q_4}. \tag{16}$$

From the above equation one could easily find the solution should be $A = 1$ and B and C are constant. The constant $C\mu$ plays the role of effective chemical potential and therefore lets us set $\mu^* = C\mu$ and $B = M$ which satisfy the following combined equations:

$$\begin{aligned} M &= m + \frac{4}{3M_{\mathrm{eff}}^2}\int \frac{d^4q}{(2\pi)^4} \frac{4M}{q^2 + M^2 - \mu^{*2} + 2i\mu^* q_4} \\ &= m + \frac{4M}{3M_{\mathrm{eff}}^2\pi^3}\int_0^{\Lambda} d|\vec{q}|\frac{\vec{q}^2}{E_{qM}}\Big[\arctan\Big(\frac{\sqrt{\Lambda^2 - \vec{q}^2}}{E_{qM} + \mu^*}\Big) \\ &\quad + \arctan\Big(\frac{\sqrt{\Lambda^2 - \vec{q}^2}}{E_{qM} - \mu^*}\Big)\Big], \end{aligned} \tag{17}$$

$$\mu^* = \mu - \frac{2\rho(\mu^*)}{3N_cN_fM_{\mathrm{eff}}^2}, \tag{18}$$

with quark number density $\rho(\mu^*)$ defined as follows [19]:

$$\rho(\mu^*) = -N_cN_f\int \frac{d^4q}{(2\pi)^4} \mathrm{tr}[G(q, \mu)\gamma_4], \tag{19}$$

and $E_{qM} = \sqrt{\vec{q}^2 + M^2}$. Equations (17) and (18) are numerically solved and the results are shown in Fig. 4. From Fig. 4, one could find the effective mass of the dressed quark in the Wigner phase decreases with increasing μ, which means with increasing density the dressing effect of quarks becomes more and more weak. On the other hand, the corresponding one in the Nambu phase decreases with increasing μ until $\mu \sim 160$ MeV, and when $\mu > 160$ MeV the effective mass of the dressed quark in the Nambu phase increases with increasing μ.

In the previous literature (see, e.g., Refs. [16,20]) one usually employs the maximum of the susceptibility $\frac{\partial\langle\bar{\psi}\psi\rangle}{\partial m}$ to

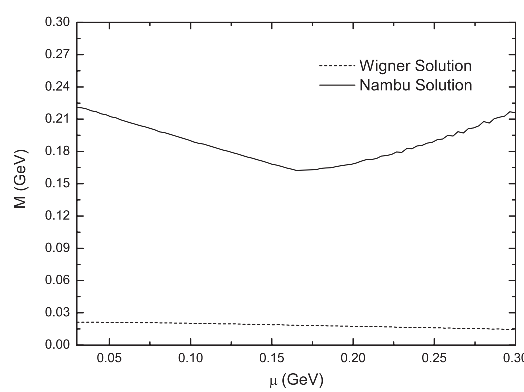

FIG. 4. Solutions of the gap equation at finite chemical potential.

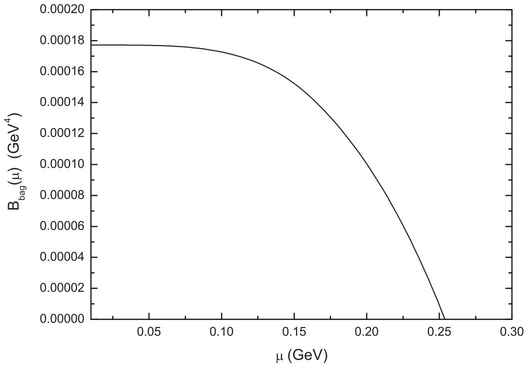

FIG. 5. The $B_{\mathrm{bag}}(\mu)$ at finite μ.

determine the transition temperature. In fact, a more reliable criterion for the chiral phase transition is the pressure difference of the Nambu phase and the Wigner phase, i.e., the bag constant $B_{\mathrm{bag}}(\mu)$. The pressure density of the two solutions at finite chemical potential could be calculated as follows [19]:

$$\mathcal{P}(\mu) = \mathcal{P}(\mu = 0) + \int_0^{\mu} d\mu' \rho(\mu'), \tag{20}$$

where the pressure density of the vacuum $\mathcal{P}(\mu = 0)$ can be calculated through Eq. (14). The $B_{\mathrm{bag}}(\mu)$ is plotted in Fig. 5 in which one can see when $\mu < \mu_c = 260$ MeV the Nambu solution is more stable and when $\mu > \mu_c = 260$ MeV the Wigner solution is more stable. At $\mu_c = 260$ MeV the pressure of the two phases is equal and the two phases could coexist at this point. Here it should be noted that no one has calculated the $B_{\mathrm{bag}}(\mu)$ in the case of nonzero current quark mass in the past. This is due to lack of knowledge about the Wigner solution of the quark gap equation at finite current quark mass in the previous literature.

Here it is interesting to compare our results with those of the normal NJL model. The first-order phase transition point μ_c in our modified NJL model is smaller than the one obtained in the normal NJL model which is about 354 MeV or 500 MeV corresponding to different parameters [21]. It should also be pointed out that, in the normal NJL model, the second solution appears when μ is big enough [18], but the magnitude of this solution at the phase transition point is much bigger (about 110 or 130 MeV, see Ref. [21]) than the Wigner solution obtained in the present paper (about 15 MeV). In addition, we want to stress that the Wigner solution at finite current quark mass in the normal NJL model is due to density effect. When the chemical potential tends to zero, this solution disappears. This shows $\mu = 0$ is a singularity of the Wigner solution at finite current quark mass. If this is real, it means that one cannot study the Wigner solution by means of small μ expansion, while the method of small μ expansion is usually employed in the study of lattice QCD at finite density.

V. SUMMARY

To summarize, based on the general analysis that the dressed gluon propagator in the Wigner phase should be different from that in the Nambu phase, we introduce the contribution of quark condensate to the gluon propagator and investigate the solution of quark DSE in the case of nonzero current quark mass. With such a modified model gluon propagator, in the framework of the NJL model we show that the quark DSE indeed has a Wigner solution in the case of nonzero current quark mass. We then generalize this approach to the case of finite chemical potential and discuss partial restoration of chiral symmetry at finite chemical potential. From the calculated result of the bag constant, we find that when $\mu < \mu_c = 260$ MeV the Nambu solution is more stable and when $\mu > \mu_c = 260$ MeV the Wigner solution is more stable. At $\mu_c = 260$ MeV, the pressure of the two phases is equal and the two phases could coexist at this point. We also compare our results with those of the normal NJL model. It is found that the first-order phase transition point μ_c in our modified NJL model is smaller than the one obtained in the normal NJL model which is about 354 or 500 MeV corresponding to different parameters. In addition, in the normal NJL model, the second solution appears when μ is big enough, but the magnitude of this solution at the phase transition point is much bigger (about 110 or 130 MeV) than the Wigner solution obtained in the present paper (about 15 MeV). Finally, we would like to point out that the results obtained in this paper are based on a simple NJL model. It is well known that the NJL model is far from QCD. In order to obtain a more solid result, we need to further discuss this problem in the framework of a model with better QCD foundation, such as DSE of QCD [2].

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