Analysis of $B_{d,s}$ mixing angles in the presence of new physics and an update of $B_s \rightarrow \bar{K}^{0*} K^{0*}$

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We discuss a simple approach to measure the weak mixing angles ϕ_s and ϕ_d of the B_s and B_d systems in the presence of New Physics. We present a new expression that allows one to measure directly $\phi_{d,s}^{\rm NP}$ if New Physics contributes significantly to the mixing only. We apply the method to specific penguin-mediated $B \rightarrow PP$, $B \rightarrow PV$ and $B \rightarrow VV$ modes. We provide a very stringent and simple bound on the direct CP asymmetries of all these modes, the violation of which is a signal of New Physics in decay. Within the same theoretical framework, an updated prediction for the branching ratio of $B_s \rightarrow K^{0*} \bar{K}^{0*}$ is presented, which can be compared with a recent LHCb analysis.

In our quest for New Physics (NP) in flavor processes, the weak angles ϕ_M involved in the mixing of $B_d(\phi_d)$ and $B_s(\phi_s)$ mesons have been studied with a great attention recently, since they have shown interesting discrepancies with respect to the standard model (SM)[1,2]. On the one hand, there is a tension in the SM fit between the measurement of ϕ_d and the branching ratio of $B \rightarrow \tau\nu$ (see also Refs. [3,4] where the discrepancies with respect to other inputs are discussed, e.g., inclusive or exclusive semileptonic decays, or mixing-dependent observables). On the other hand, the most recent angular analyses of $B_s \rightarrow J/\Psi\phi$ by CDF, D0 and LHCb [5–7] seem to reduce substantially the space for New Physics in ϕ_s. Finally, there still remains an important discrepancy with the standard model through the dimuon asymmetry measured at D0 [8,9].

In this context, it is useful to devise alternative methods to extract the mixing angle of both $B_d(\phi_d)$ and $B_s(\phi_s)$ systems from experiment. In this paper we will take the point of view of assuming that New Physics contributes significantly only to the mixing phases whereas the formalism of the CKM matrix can still be used to analyze decay amplitudes. Under this assumption we present a clean method to extract the NP contribution to the weak mixing phase $\phi_{d,s}$ from selected $B_{d,s} \rightarrow PP, VP$, and VV decays.

Some time ago [10], we proposed a test on the value of $\sin\phi_s$ in the SM by measuring certain longitudinal branching ratios and CP asymmetries of $B_{d,s}$ mesons decaying into vectors, for modes mediated by penguin diagrams—we focused on the potential of the $B_s \rightarrow \bar{K}^{0*} K^{0*}$ decay at that time. The method has advantages both from the theoretical and experimental points of view. On the theoretical side, it reduces the required theoretical input mainly to a single hadronic quantity Δ, defined as the difference between "tree" and "penguin" contributions. More precisely, for a $\bar{B}_Q$ meson decaying through a $b \rightarrow q$ penguin-mediated process, the decomposition:

$$\bar{A} \equiv A(\bar{B}_Q \rightarrow M_1 M_2) = \lambda_u^{(q)} T + \lambda_c^{(q)} P, \quad (1)$$

with the CKM factors $\lambda_U^{(q)} = V_{Ub} V_{Uq}^*$, can be used to define Δ as the difference of the hadronic matrix elements that are multiplied by $\lambda_u^{(q)}$ and $\lambda_c^{(q)}$ respectively, i.e.,

$$\Delta = T - P. \quad (2)$$

In the case of penguin-mediated decays, the evaluation of this quantity using QCD factorization (QCDF) [11] is expected to be particularly robust, as it was built specifically to cancel the infrared divergences coming from spectator-quark and annihilation contributions up to next-to-leading order [12,13]. On the experimental side, this test of weak mixing angles within the standard model can be performed simply by measuring a CP-averaged branching ratio and an untagged rate. This allows us to avoid tagging, although a time-dependent analysis is still required.

In the present article, we extend this approach beyond the simple SM test on the weak mixing angles ϕ_s and ϕ_d presented in Ref. [10]. Indeed, we show that we can also use penguin-mediated decays to measure the size of NP contributions to neutral-meson mixing, assuming that its contribution to $\Delta B = 1$ decays is negligible. We also update our predictions for the branching ratio of $B_s \rightarrow \bar{K}^{*0} K^{*0}$ which has been recently measured. The paper is organized in the following way. In Sec. I, we present a formula to pin down the NP contribution to ϕ_s and ϕ_d, obtained from $b \rightarrow d, s$ transitions, under the assumption that NP provides significant contributions only to the meson-mixing phases. In Sec. II, we discuss the main theoretical input in this method. In Sec. III, we consider three different examples of the method, corresponding to $B \rightarrow PP$, PV and VV decays. We update our prediction in Ref. [10] for $BR^{\rm long}(B_s \rightarrow \bar{K}^{*0} K^{*0})$ in Sec. IV, and compare it with its recent measurement by the LHCb collaboration [14]. In Appendix A we provide a dictionary between the experimental measurements [15,16] and the theoretical quantities [10] defined for longitudinal observables in $B \rightarrow VV$ decays. In Appendix B, we discuss differences in the determination of the branching ratio for neutral

mesons for tagged or untagged analyses, highlighting the role played by the width difference $\Delta\Gamma$ (a problem particularly relevant for B_s mesons).

I. FORMULAS FOR NP MIXING ANGLES

Using the unitarity relation $\lambda_u^{(q)} + \lambda_c^{(q)} + \lambda_t^{(q)} = 0$, we can write Eq. (1) in terms of $\lambda_c^{(q)}$ and $\lambda_t^{(q)}$:

$$\bar{A} \equiv A(\bar{B}_Q \rightarrow M_1 M_2) = -\lambda_t^{(q)} T - \lambda_c^{(q)} \Delta. \quad (3)$$

The weak phase in $\lambda_t^{(q)}$ is the angle β_q, defined as

$$\beta_q \equiv \arg\left(-\frac{V_{tb}V_{tq}^*}{V_{cb}V_{cq}^*}\right) = \arg\left(-\frac{\lambda_t^{(q)}}{\lambda_c^{(q)}}\right), \quad (4)$$

whereas $\lambda_c^{(q)}$ is real to a very good approximation for both $q = d$, s. Following the definitions in Appendix A, we introduce the observables: a branching ratio $BR = g_{ps}(|A|^2 + |\bar{A}|^2)/2$ (where g_{ps} is the phase space factor) and three CP asymmetries A_{dir}, A_{mix} and $A_{\Delta\Gamma}$:

$$A_{dir} \equiv \frac{|A|^2 - |\bar{A}|^2}{|A|^2 + |\bar{A}|^2}, \quad A_{mix} \equiv -2\eta_f \frac{\text{Im}(e^{-i\phi_Q}A^*\bar{A})}{|A|^2 + |\bar{A}|^2},$$

$$A_{\Delta\Gamma} \equiv -2\eta_f \frac{\text{Re}(e^{-i\phi_Q}A^*\bar{A})}{|A|^2 + |\bar{A}|^2}, \quad (5)$$

defined in terms of $\bar{A}$ and its CP-conjugate A, as well as the B_Q meson-mixing phase ϕ_Q (η_f is the CP parity of the final state).

These four observables can be written, using Eq. (3), in terms of $\lambda_{c,t}^{(q)}$, T, Δ, and ϕ_Q. One can then eliminate the hadronic parameter T to obtain a relationship between these observables:

$$2g_{ps}|\Delta|^2|\lambda_c^{(q)}|^2\sin^2\beta_q = BR(1 - \eta_f \sin\Phi_{Qq}A_{mix}$$
$$+ \eta_f \cos\Phi_{Qq}A_{\Delta\Gamma}), \quad (6)$$

with Φ_{Qq} defined as:

$$\Phi_{Qq} = 2\beta_Q - 2\beta_q + \phi_Q^{NP}, \quad (7)$$

being ϕ_Q^{NP} the New Physics contribution to the mixing angle of the B_Q system: $\phi_Q = 2\beta_Q + \phi_Q^{NP}$. In deriving Eq. (6) we have assumed that New Physics could alter only the mixing phase of the neutral B_Q meson, but not the CKM matrix elements involved in the decay process (the validity of this hypothesis could be tested through a consistency check between different channels, as proposed in Ref. [17]). Equation (6) is a generalization of similar formulas developed in Refs. [10,12] in the context of the SM. This relation is the starting point of our analysis.

Collecting all terms on the left-hand side in Eq. (6), and defining:

$$C = \frac{2g_{ps}|\lambda_c|^2\sin^2\beta_q|\Delta|^2}{BR}, \quad (8)$$

we can solve Eq. (6) for Φ_{Qq}:

$$\sin\Phi_{Qq} = z\eta_f\hat{A}_{mix} \pm \sqrt{1 - z^2}\eta_f\hat{A}_{\Delta\Gamma},$$
$$\cos\Phi_{Qq} = -z\eta_f\hat{A}_{\Delta\Gamma} \pm \sqrt{1 - z^2}\eta_f\hat{A}_{mix}. \quad (9)$$

Here, $z = (1 - C)/\sqrt{1 - A_{dir}^2}$, $\hat{A}_{mix} = A_{mix}/\sqrt{1 - A_{dir}^2}$, and $\hat{A}_{\Delta\Gamma} = A_{\Delta\Gamma}/\sqrt{1 - A_{dir}^2}$, with the relation:

$$\hat{A}_{mix}^2 + \hat{A}_{\Delta\Gamma}^2 = 1. \quad (10)$$

There is a twofold ambiguity in Eq. (9). In practice, we will see that $z \simeq 1$ (or equivalently $C \ll 1$), so that the two solutions are very close and they can be considered as one single solution within current theoretical and experimental uncertainties. Notice also that $z^2 \leq 1$, which provides a very strong and interesting constraint on the size of A_{dir}:

$$A_{dir}^2 < C(2 - C). \quad (11)$$

In general, a pair of values for BR and Δ imply an upper bound on $|A_{dir}|$ [10,18] which is much tighter than the experimental value for A_{dir}. A violation of this bound would be an indication of NP in decay. In the following we will take $A_{dir} = 0 \pm \sqrt{2C - C^2}$ in the numerical analyses. For example, in the case of $B \rightarrow \phi K_S$ we have $C = (8.9 \pm 7.7) \times 10^{-5}$, which implies $|A_{dir}| < 0.019$.

If there is no New Physics and if the process considered is such that $Q = q$, one recovers $\sin\Phi_{Qq} \rightarrow 0$ and $\cos\Phi_{Qq} \rightarrow 1$, as can be checked from Eq. (7). In this case, Eq. (6) reduces to the simple relation:

$$z^{SM} = -\eta_f\hat{A}_{\Delta\Gamma}^{SM}. \quad (12)$$

This compact equation can be easily rewritten as a SM test of the angles of the unitarity triangle and corresponds to Eqs. (33)–(36) of Ref. [10].

II. THEORETICAL INPUT

The theoretical input in this formalism is limited to the quantity Δ defined in Eq. (2). As discussed in Refs. [10,12], this quantity can be computed safely within QCD factorization [11] for penguin-mediated decays because of the cancellation of long-distance contributions. As a consequence of this cancellation, only penguin contractions contribute to Δ, as can be seen by inspection of the formulas in Ref. [19]. The general form of Δ for a $B_Q \rightarrow M_1 M_2$ decay is [10,12]:

$$\Delta_{M_1M_2} = A_{M_1M_2}^Q \frac{C_F\alpha_s}{4\pi N}C_1[\bar{G}_{M_2}(m_c^2/m_b^2) - \bar{G}_{M_2}(0)], \quad (13)$$

where M_1 picks up the spectator quark of the B_Q meson. The normalization involves the M_2 decay constant and a $B \rightarrow M_1$ form factor:

$$A_{M_1M_2}^Q = \frac{G_F}{\sqrt{2}}m_{B_Q}^2 f_{M_2}A^{B_Q \rightarrow M_1}(0). \quad (14)$$

TABLE I. Values of Δ for the various decays of interest, and the corresponding values for $C \times BR$, defined in Eq. (8). In the case of two vector mesons these numbers should be understood as referring to longitudinal polarizations (the third column means $C \times BR^{\text{long}}$ in this case).

Channel	$|\Delta|(10^{-7}\ \text{GeV})$	$C \times BR$
$B_d \to K\bar{K}$	(3.23 ± 1.16)	$(29.8 \pm 21.9) \times 10^{-9}$
$B_s \to \bar{K}K$	(3.05 ± 1.11)	$(1.21 \pm 0.89) \times 10^{-9}$
$B_d \to K\phi$	(2.32 ± 1.00)	$(0.74 \pm 0.64) \times 10^{-9}$
$B_d \to K\bar{K}^*$	(2.29 ± 0.93)	$(14.7 \pm 12.1) \times 10^{-9}$
$B_d \to K^*\bar{K}$	(0.41 ± 0.60)	$(0.47 \pm 1.38) \times 10^{-9}$
$B_s \to \bar{K}K^*$	(2.16 ± 0.89)	$(0.60 \pm 0.50) \times 10^{-9}$
$B_s \to \bar{K}^*K$	(0.36 ± 0.53)	$(0.02 \pm 0.05) \times 10^{-9}$
$B_d \to K^*\bar{K}^*$	(1.85 ± 0.93)	$(9.37 \pm 9.53) \times 10^{-9}$
$B_s \to \bar{K}^*K^*$	(1.62 ± 0.81)	$(0.33 \pm 0.33) \times 10^{-9}$
$B_d \to K^*\phi$	(1.92 ± 1.03)	$(0.49 \pm 0.53) \times 10^{-9}$
$B_s \to \phi K^*$	(1.87 ± 0.94)	$(8.80 \pm 8.96) \times 10^{-9}$
$B_s \to \phi\phi$	(3.86 ± 2.09)	$(0.92 \pm 1.00) \times 10^{-9}$

C_1 is the relevant Wilson coefficient of the effective Hamiltonian at a scale of order m_b. $\bar{G}_{M_2} = G_{M_2} \pm r_\chi^{M_2} \hat{G}_{M_2}$ is the penguin function with a u or a c quark running in the loop and the sign depends on whether M_1 is a pseudoscalar or a vector meson (see [19,20] for the precise definition of the quantities entering Δ). For identical particles in the final state (for instance $B_s \to \phi\phi$) an extra factor of 2 must be included in Δ.

In the case in which both M_1 and M_2 are vector mesons, there is a different Δ for each of the three transversity amplitudes. As discussed in Ref. [20], only longitudinal amplitudes can be handled safely within QCD factorization, so we shall focus on longitudinal polarizations for vector-vector modes, in the spirit of Ref. [10] (see also Ref. [21] for a discussion of transverse amplitudes in QCDF). This requires the definition of suitable longitudinal observables, together with a procedure to extract them from the observables that are actually *measured*, as discussed in more detail in the appendices of the present article.

In Table I we present the values of Δ for $B_Q \to XY$ penguin-mediated decay channels, with X, $Y = P$, V_L (where V_L stands for longitudinally polarised vectors), using the same hadronic inputs as in Ref. [10].

III. NEUTRAL-MESON MIXING ANGLES FROM $B_Q \to PP$, PV, AND $V_L V_L$

In this section we illustrate the determination of the NP contributions to both weak mixing angles ϕ_d and ϕ_s using Eqs. (6) and (9). We consider channels of three different types: $B_s \to K^0\bar{K}^0$ ($B \to PP$), $B_d \to \phi K_S$ ($B \to PV$), and $B_d \to \phi K^*$ ($B \to V_L V_L$). We will see that in the case of $B_d \to \phi K_S$ and $B_d \to \phi K^*$, all the required observables are already measured, which allows us to perform a complete analysis to extract ϕ_d^{NP}. There is no experimental data

for $B_s \to K^0\bar{K}^0$, so we will only be able to present a case study, which can be exploited immediately as soon as data is available.

Besides the theory input Δ and the measured branching ratio, we need additional inputs for the SM CKM parameters. Since we are assuming that meson mixing could be affected by NP, we must use values of the CKM elements extracted using modes insensitive to mixing. The fit in Table 11 of Ref. [1] satisfies this requirement, as the mixing-related observables included in this fit are used essentially to determine the size of NP in meson mixing and have a very small impact on the CKM parameter themselves, which are thus determined from tree-dominated quantities. Indeed we checked that a fit limited to the tree-dominated inputs of Ref. [1] yields almost identical results for the CKM parameters. From Ref. [1] we obtain, upon symmetrizing errors:

$$|\lambda_u^{(d)}| = (4.1 \pm 0.3) \times 10^{-3}, \qquad |\lambda_u^{(s)}| = (9.5 \pm 0.6) \times 10^{-4},$$

$$|\lambda_c^{(d)}| = (9.2 \pm 0.2) \times 10^{-3}, \qquad |\lambda_c^{(s)}| = 0.041 \pm 0.001,$$

$$\beta_d = (26.2 \pm 2.1)^\circ, \qquad \beta_s = -(1.26 \pm 0.07)^\circ,$$

$$\gamma = (69.9 \pm 4.4)^\circ, \tag{15}$$

which will be used in the following studies.

A. First example: $B_d \to \phi K_S$

We first consider $B_d \to \phi K_S$. Using the formulas in Eq. (9), together with Δ and C in Table I, and the experimental numbers $BR = (8.3 \pm 1.0) \times 10^{-6}$ and $A_{\text{mix}} = -0.56 \pm 0.16$ [22] (the product of intrinsic parities is $\eta_{\phi K_S} = -1$), we find the two solutions for the NP contribution to the mixing angle:

$$\phi_d^{\text{NP}} = (-0.38 \pm 0.21) \vee (-0.35 \pm 0.21) \text{ rad}, \tag{16}$$

from which we can give an averaged result:

$$\phi_{d,\text{aver}}^{\text{NP}} = -0.36 \pm 0.22 \text{ rad}. \tag{17}$$

This result is (marginally) compatible with the SM value $\phi_d^{\text{NP}} = 0$. However, the error on ϕ_d^{NP} is almost completely dominated by the experimental uncertainty in A_{mix}: any improvement on the latter would impact our knowledge of ϕ_d^{NP} and its agreement with SM expectations.

In Fig. 1 we show the regions in the A_{mix}-$A_{\Delta\Gamma}$ plane corresponding to different values of ϕ_d^{NP}, the NP contribution to the mixing angle. The easiest way to understand this plot is through the two constraints on A_{mix} and $A_{\Delta\Gamma}$: Eq. (6) is a linear equation at fixed ϕ_Q^{NP}, whereas $A_{\text{mix}}^2 + A_{\Delta\Gamma}^2 \leq 1$ yields a radial constraint. For a fixed ϕ_Q^{NP}, the solution for A_{mix} and $A_{\Delta\Gamma}$ is thus given by the intersection of a line with the unit circle. If this line is tangent, the solution is limited to *one* point, and our determination is perfectly precise. The opposite situation occurs if the line goes through the origin. It turns out that the distance of this line from the
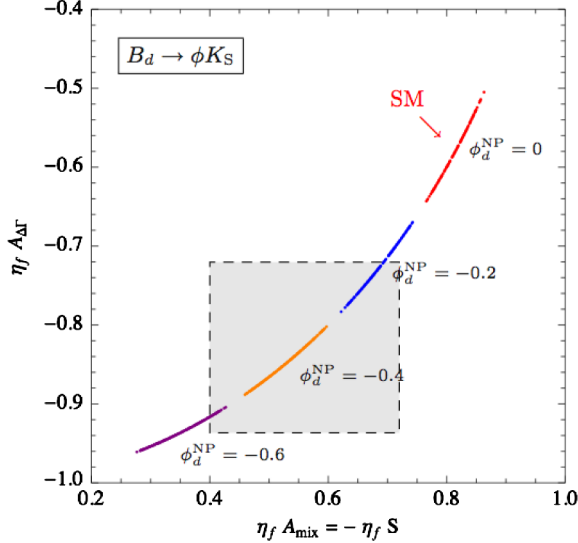
FIG. 1 (color online). A_{mix} vs $A_{\Delta\Gamma}$, for several values of the NP mixing angle ϕ_d^{NP} for $B_d \to \phi K_S$, where $BR = (8.3 \pm 1.1) \times 10^{-6}$, $\Delta = (2.32 \pm 1.00) \times 10^{-7}$ GeV and $g_{\mathrm{ps}} = 8.4 \times 10^9$ GeV^{-2}. The box indicates the experimental value for the asymmetry $A_{\mathrm{mix}} = -0.56 \pm 0.16$ and the range obtained for $A_{\Delta\Gamma}$ using $A_{\mathrm{dir}} = 0 \pm 0.019$ ($\eta_f = -1$).

origin is given exactly by $1 - C$, so C measures the precision with which we can determine ϕ_Q^{NP}:

The smaller the value of C, the more precisely we can pin down the value of ϕ_Q^{NP}.

The SM solution is shown explicitly in Fig. 1, and the gray box indicates the current experimental situation. One can see there is marginal agreement, in line with the result in Eq. (17). We stress that these results include all known hadronic uncertainties, and the errors are relatively small because the chosen theoretical input is robust. A more precise value for the angle β_d would achieve a substantial reduction of the regions of fixed ϕ_d^{NP} in Fig. 1.

It should be mentioned that, since we have no way of determining the sign of $A_{\Delta\Gamma}$, there is a second box in Fig. 1 that has not been drawn, corresponding to $\eta_f A_{\Delta\Gamma} \sim +0.8$. We have discarded this possibility because it leads to a very large New Physics mixing angle.

B. Second example: $B_d \to \phi K^*$

An angular analysis of the decay $B_d \to \phi K^*$ is available from both *BABAR* and Belle collaborations [15,16], with an additional time-dependent analysis for the former experiment. The averaged results read [15,22]

$$BR = (9.8 \pm 0.7) \times 10^{-6}, \quad \mathcal{A}_{CP} = 0.01 \pm 0.05,$$
$$f_L = 0.48 \pm 0.03, \quad \mathcal{A}_{CP}^0 = 0.04 \pm 0.06, \quad (18)$$
$$\Delta\phi_0 = 0.28 \pm 0.42, \quad \Delta\delta_0 = 0.27 \pm 0.16.$$

Using Eqs. (A9)–(A12) in Appendix A (and noting that $\eta_{\phi K^*} = +1$), we obtain the longitudinal observables:

$$BR^{\mathrm{long}} = (4.7 \pm 0.4) \times 10^{-6},$$
$$A_{\mathrm{dir}}^{\mathrm{long}} = -0.05 \pm 0.08,$$
$$A_{\mathrm{mix}}^{\mathrm{long}} = 0.96 \pm 0.25, \quad (19)$$
$$A_{\Delta\Gamma}^{\mathrm{long}} = \pm(0.27 \pm 0.86).$$

Compared to the case of $B_d \to \phi K_S$, the uncertainty on $A_{\Delta\Gamma}^{\mathrm{long}}$ is so large that one cannot distinguish, among the two solutions, which one is disfavored by too large a value of the NP mixing angle. For illustration we focus on the negative solution of $A_{\Delta\Gamma}^{\mathrm{long}}$.

For this decay, Eq. (11) yields the following range for the direct asymmetry:

$$A_{\mathrm{dir}}^{\mathrm{long}} = 0 \pm 0.015, \quad (20)$$

which is more precise than the experimental value in Eq. (19), and which is used in the following. Equation (9) then yields the two solutions:

$$\phi_d^{\mathrm{NP}} = (0.31 \pm 0.90) \vee (0.34 \pm 0.90) \text{ rad}, \quad (21)$$

which can be averaged as:

$$\phi_{d,\mathrm{aver}}^{\mathrm{NP}} = 0.33 \pm 0.90 \text{ rad}. \quad (22)$$

This result is compatible with the SM ($\phi_d^{\mathrm{NP}} = 0$), and also with the result obtained from $B_d \to \phi K_S$ (Eq. (17)), within large uncertainties.

In Fig. 2, we show the regions in the A_{mix}-$A_{\Delta\Gamma}$ plane corresponding to different values of the NP contribution to the mixing angle. The SM solution is shown explicitly, and the gray box indicates the current experimental situation.
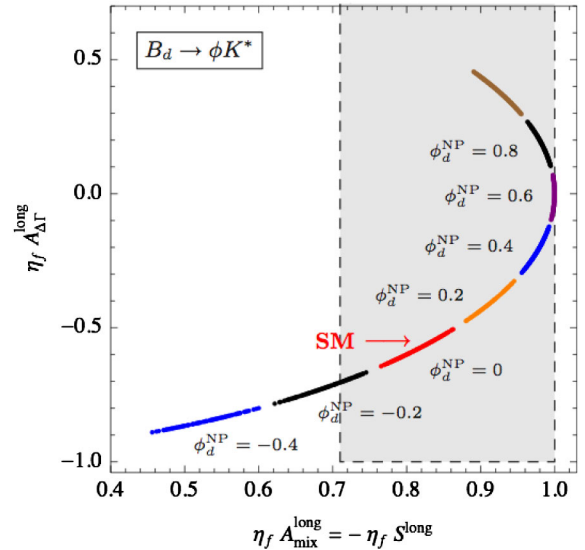

FIG. 2 (color online). A_{mix} vs $A_{\Delta\Gamma}$, for several values of the NP mixing angle ϕ_d^{NP} for $B_d \to \phi K^*$, where $BR^{\mathrm{long}} = (4.7 \pm 0.4) \times 10^{-6}$, $\Delta = (1.92 \pm 1.03) \times 10^{-7}$ GeV and $g_{\mathrm{ps}} = 8.2 \times 10^9$ GeV^{-2}. The box indicates the experimental values for the asymmetries given in Eq. (19) ($\eta_f = +1$).

From this plot we see that a more accurate knowledge of $A_{\text{mix}}^{\text{long}}$ or $A_{\Delta\Gamma}^{\text{long}}$ would be very useful to test the SM hypothesis.

C. Third example: $B_s \rightarrow K^0 \bar{K}^0$

There is currently no information on this mode. However, the U-spin related decay $B_d \rightarrow K^0\bar{K}^0$ has the measured branching ratio [22]:

$$BR(B_d \rightarrow K^0\bar{K}^0) = (0.96 \pm 0.20) \times 10^{-6}. \quad (23)$$

As explained in Ref. [12], if there is no NP affecting the decay $B_d \rightarrow K^0\bar{K}^0$, one can predict the SM branching ratio for the decay $B_s \rightarrow K^0\bar{K}^0$ quite precisely. This prediction is not affected by a possible NP contribution to ϕ_d (see Section 3 of Ref. [10]), and since we are assuming that there is no NP affecting $\Delta B = 1$ processes, it is a good option to take this prediction as an input for our analysis [12]:

$$BR(B_s \rightarrow K^0\bar{K}^0) = (18.2 \pm 7.3) \times 10^{-6}. \quad (24)$$

The present analysis of this decay should be considered as a case study, waiting for data to be available.

Without further experimental information on CP asymmetries, we cannot use Eq. (9) to extract ϕ_s^{NP}, the NP contribution to the mixing angle. But it is still possible to determine the regions in the A_{mix}-$A_{\Delta\Gamma}$ plane that correspond to each value of ϕ_s^{NP}, similarly to Figs. 1 and 2. Notice that in the present case we have simply $\Phi_{Qq} = \phi_s^{\text{NP}}$, since $Q = q = s$ here. The case $B_s \rightarrow K^0\bar{K}^0$ is shown in Fig. 3. Whenever the branching ratio for this mode is measured, this plot can be remade, although we do not expect it to change in an appreciable way. A precise
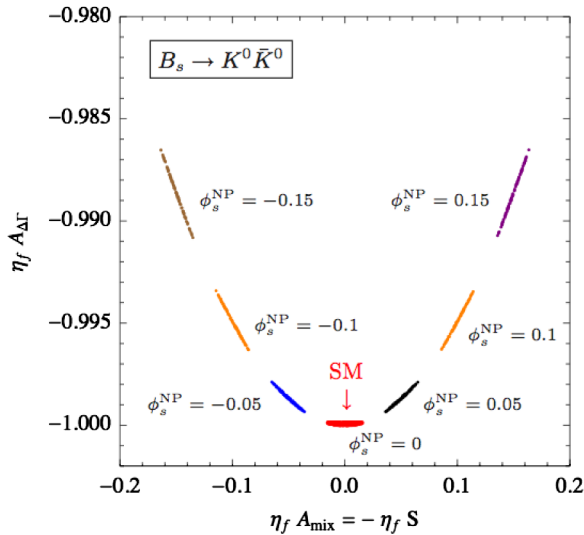

FIG. 3 (color online). A_{mix} vs $A_{\Delta\Gamma}$, for several values of the NP mixing angle ϕ_s^{NP} for $B_s \rightarrow K^0\bar{K}^0$, where we take the estimate $BR = (18.2 \pm 7.3) \times 10^{-6}$ (see the text). In addition, we used $\Delta = (3.05 \pm 1.11) \times 10^{-7}$ GeV and $g_{\text{ps}} = 8.03 \times 10^9$ GeV^{-2}.

measurement of the time-dependent CP asymmetry will then provide an accurate determination of ϕ_s^{NP}.

One may wonder why we did not consider the decay $B_d \rightarrow K^0\bar{K}^0$ at a first stage, since its branching ratio is known. However, the prospects for this particular decay are not very alluring. The mixing angle involved in the expression for C [Eq. (8)] is β_d, and the branching ratio is not particularly large, so that the value of C is small but not tiny ($C \sim 0.03$). As mentioned above, the value of C determines the potential accuracy of our procedure, which turns out to be rather poor in the present case. This is true in general for all $b \rightarrow d$ transitions in our list (see Table I); however, the situation may change if the branching ratio of some of the modes is found to be large enough, as it might be for $B_d \rightarrow K^{*0}\bar{K}^{*0}$.

The three plots in Figs. 1–3 can be reversed and reinterpreted as predictions for the A_{mix} and $A_{\Delta\Gamma}$ asymmetries if the range for $\phi_{d,s}^{\text{NP}}$ is extracted from other processes, always under the assumption that there is no significant New Physics contribution to $\Delta B = 1$ processes.

IV. THE BRANCHING RATIOS $BR(B_{d,s} \rightarrow \bar{K}^{*0}K^{*0})$ WITHIN THE SM

In this section, we present a prediction for $BR^{\text{long}}(B_s \rightarrow \bar{K}^{*0}K^{*0})$ in the SM using our approach. This prediction can be easily turned, as shown later on, into a prediction for the total branching ratio, once f_L, $\mathcal{A}_{CP}$ and $\mathcal{A}_{CP}^0$ are measured (see Appendix A for details). This section is an update of results presented in Ref. [10].

The two decays $B_q \rightarrow \bar{K}^{*0}K^{*0}$, with $q = d, s$, are related by U-spin symmetry. The symmetry-breaking effects can be separated into a factorizable and a nonfactorizable part. This translates into the following relations involving the hadronic parameters T_q and P_q:

$$P_s = fP_d(1 + \delta_P), \qquad T_s = fT_d(1 + \delta_T), \quad (25)$$

with the factorizable factor f:

$$f = \frac{m_{B_s}^2 A_0^{B_s \rightarrow K^*}}{m_B^2 A_0^{B \rightarrow K^*}} = \frac{\Delta_{K^*K^*}^s}{\Delta_{K^*K^*}^d}, \quad (26)$$

and $\delta_{T,P}$ account for the nonfactorizable symmetry-breaking effects. There is no theoretically clean way to compute these quantities. Here we use QCD-factorization to *estimate* an upper bound on this corrections. We get $|\delta_P| < 0.09$, and keep its phase as a free parameter (even though QCD factorization would predict it to be small). It is very easy to show that the quantity δ_T is related to δ_P through:

$$T_d\delta_T = P_d\delta_P. \quad (27)$$

(This relation comes from Eq. (25) in the following way: $\Delta^s = P_s - T_s = f(P_d - T_d + P_d\delta_P - T_d\delta_T) = f(\Delta^d + P_d\delta_P - T_d\delta_T)$, but $\Delta^s/f = \Delta^d$ [see Eq. (26)], so that $\Delta^d = \Delta^d + P_d\delta_P - T_d\delta_T$).

TABLE II. Results for the ratio R_{sd}^{DMV} for three different values of $BR^{\mathrm{long}}(B_d \to K^{*0}\bar{K}^{*0})$. It can be seen that the dependence on this branching ratio is very mild.

BR_d	5×10^{-7}	5×10^{-6}	5×10^{-5}
R_{sd}^{DMV}	16.05 ± 4.87	16.38 ± 4.92	16.46 ± 4.93

Observables related to the decay $B_s \to \bar{K}^{*0}K^{*0}$ can be obtained if the ones for $B_d \to \bar{K}^{*0}K^{*0}$ are known. An important result discussed in Ref. [10] is that the ratio:

$$R_{\mathrm{sd}} \equiv \frac{BR^{\mathrm{long}}(B_s \to K^{*0}\bar{K}^{*0})}{BR^{\mathrm{long}}(B_d \to K^{*0}\bar{K}^{*0})}, \qquad (28)$$

is almost a constant if $BR^{\mathrm{long}}(B_d \to K^{*0}\bar{K}^{*0}) \gtrsim 10^{-7}$, even in the presence of NP in mixing. Indeed, when the two branching ratios in Eq. (28) are reexpressed in terms of $P_{s,d}$ and $\Delta_{s,d}$ (as computed in Table I), it becomes clear that the ratio R_{sd} is completely dominated by the penguin contributions P_s and P_d for large enough $BR^{\mathrm{long}}(B_d \to K^{*0}\bar{K}^{*0})$, and thus determined by f and δ_P essentially. This behavior is confirmed in Table II for several values of the branching ratio.

Combining all the error sources (including the variation of BR_d in the range given in Table II), we obtain:

$$R_{sd}^{\mathrm{DMV}} = 16.4 \pm 5.2, \qquad (29)$$

which updates Table IV in Ref. [10]. The improvement on the uncertainty comes, in particular, from Eq. (27) which was not used in this reference. We have added the superscript "DMV" to distinguish our determination from other ones that we will describe now.

Indeed, this number can be compared with the value obtained within the QCDF framework taking the usual model for $1/m_b$-suppressed corrections described in Ref. [20]. The corresponding predictions are for the B_s decay mode: $BR(B_s \to \bar{K}^{0*}K^{0*}) = (9.1^{+11.3}_{-6.8}) \times 10^{-6}$ and $f_L(B_s) = 0.63^{+0.42}_{-0.29}$, and for the corresponding B_d decay mode: $BR(B_d \to \bar{K}^{0*}K^{0*}) = (0.6^{+0.5}_{-0.3}) \times 10^{-6}$ and $f_L(B_d) = 0.69^{+0.34}_{-0.27}$. We write R_{sd} in terms of total branching ratios and polarization fractions, as described in Appendix A:

$$R_{sd} = \frac{BR(B_s)f_L(B_s)}{BR(B_d)f_L(B_d)} f_c, \qquad (30)$$

where we introduce the correcting factor

$$f_c = \frac{1 + \mathcal{A}_{CP}^0(B_s)\mathcal{A}_{CP}(B_s)}{1 + \mathcal{A}_{CP}^0(B_d)\mathcal{A}_{CP}(B_d)}. \qquad (31)$$

In Ref. [20], the predictions within QCDF are $\mathcal{A}_{CP}(B_s) \simeq 1\%$ and $\mathcal{A}_{CP}(B_d) \simeq -13\%$. Since the QCDF predictions are dominated by longitudinal polarization, we assume $\mathcal{A}_{CP}^0(B_s)$ and $\mathcal{A}_{CP}^0(B_d)$ to be of the same size, leading to a factor f_c ranging from 0.98 to 1, and to the QCDF prediction:

$$R_{sd}^{\mathrm{QCDF\text{-}I}} = 13.8 \pm 19.2. \qquad (32)$$

Alternatively, if data on $B \to K^*\phi$ is used to control annihilation rather than the usual model for these $1/m_b$-suppressed corrections, the predictions become [20]: $BR(B_s \to \bar{K}^{0*}K^{0*}) = (7.9^{+4.3}_{-3.9}) \times 10^{-6}$, $f_L(B_s) = 0.72^{+0.16}_{-0.21}$, $BR(B_d \to \bar{K}^{0*}K^{0*}) = (0.6^{+0.3}_{-0.2}) \times 10^{-6}$, and $f_L(B_d) = 0.69^{+0.16}_{-0.20}$. In this case the prediction is practically the same but with smaller errors:

$$R_{sd}^{\mathrm{QCDF\text{-}II}} = 13.7 \pm 10.5, \qquad (33)$$

which has a central value a bit lower than our results, but is compatible within its (larger) error bars.

The most recent experimental values [22] for $BR(B_d \to \bar{K}^{0*}K^{0*}) = (1.28^{+0.35}_{-0.30} \pm 0.11) \times 10^{-6}$ and $f_L(B_d) = 0.80^{+0.10}_{-0.12} \pm 0.06$ are consistent with the QCDF prediction, although both are on the high range. The LHCb collaboration has measured the B_s mode very recently, reporting the following numbers [14]: $BR(B_s \to \bar{K}^{0*}K^{0*}) = (2.81 \pm 0.46 \pm 0.45 \pm 0.34) \times 10^{-5}$ and $f_L(B_s) = 0.31 \pm 0.12 \pm 0.04$.

Following the discussion in Appendix B, we can express the ratio R_{sd} in terms of experimentally measured quantities:

$$R_{sd} = \frac{BR_{\mathrm{LHCb}}(B_s)f_{L,\mathrm{LHCb}}(B_s)}{BR_{B\text{-fact}}(B_d)f_{L,B\text{-fact}}(B_d)} \frac{1 - y^2}{1 + y\cos\phi_s}, \qquad (34)$$

where we define $y = \Delta\Gamma_s/(2\Gamma_s)$. In both cases, it was assumed in the experimental analysis that there is no CP-violation in decay, and therefore we do not include the corrections associated with the direct asymmetries. We take $y = 0.046 \pm 0.027$ [22], and the SM value of ϕ_s. These results imply a quite low value for R_{sd}:

$$R_{sd}^{\mathrm{exp\text{-}I}} = (8.1 \pm 3.3) \times \left(\frac{f_L(B_s)}{0.31}\right) = 8.1 \pm 4.7. \qquad (35)$$

The measured value of $f_L(B_s)$ is unexpectedly low with respect to the polarization fraction for the U-spin related channel: $f_L(B_d) \simeq 0.8$. If $f_L(B_s) \simeq f_L(B_d)$, we would obtain a ratio $R_{sd} \simeq 21$ in better agreement with Eq. (29), although $f_L(B_s) \simeq 0.6$ would fit better.

An alternative determination of this branching ratio using data from $B_s^0 \to D_s^{\pm}\pi^{\mp}$ yields the result [23] $BR(B_s \to \bar{K}^{0*}K^{0*}) = (2.64 \pm 0.61 \pm 0.42) \times 10^{-5}$ in fair agreement with the previous result:

$$R_{sd}^{\mathrm{exp\text{-}II}} = (7.6 \pm 3.2) \times \left(\frac{f_L(B_s)}{0.31}\right) = 7.6 \pm 4.5. \qquad (36)$$

A similar exercise with $f_L(B_s) \simeq 0.8$ yields $R_{sd} \simeq 19$, once again in better agreement with our expectations.

Finally, one can invert these relations and get a prediction for the total $BR(B_s \to \bar{K}^{0*}K^{0*})$, as measured at a hadronic machine (thereby the subscript "LHCb"):

$$BR(B_s \to \bar{K}^{0*}K^{0*})^{\text{DMV}}_{\text{LHCb}}$$

$$= R^{\text{DMV}}_{sd} BR_{B\text{-fact}}(B_d) \frac{f_{L,B\text{-fact}}(B_d)}{f_{L,\text{LHCb}}(B_s)} \frac{1 + y\cos\phi_s}{1 - y^2}$$

$$= (5.7 \pm 2.5) \times 10^{-5} \times \left(\frac{0.31}{f_L(B_s)}\right)$$

$$= (5.7 \pm 3.4) \times 10^{-5}, \tag{37}$$

which can be trivially updated if the longitudinal polarization of the B_s mode changes.

We remind the reader that, as opposed to the rest of the paper, all the predictions in this section have been obtained within the SM. A detailed analysis of the impact of New Physics on these observables is certainly worthwhile, but lies beyond the scope of the present article.

V. CONCLUSIONS

We have shown how to exploit $B \to PP$, PV, and VV penguin-mediated decays in order to obtain the New Physics contribution to the weak mixing phase of both B_d and B_s systems, under the assumption of no significant New Physics contribution to the decay amplitudes. The main theoretical input consists of the infrared-safe quantity Δ, which can be evaluated within QCD factorization with a good theoretical control. It is interesting that the experimental knowledge of the branching ratio by itself yields a stringent bound on the direct CP asymmetry, Eq. (11). As a consequence, the knowledge of BR and A_{mix} is sufficient to determine the New Physics contribution to the weak mixing angles. Different examples are discussed and the corresponding values for ϕ^{NP}_Q presented; however, more precise data is required before a clear conclusion can be reached.

These results should be compared with the one obtained from charmonium modes, where one is also sensitive to the NP mixing angle. Time-dependent analyses of $B \to J/\Psi K_S$ give the average [1,22]: $\phi^{c\bar{c}}_d = (21.2 \pm 0.9)°$. Using the value quoted in Eq. (15) for the SM contribution β_d leads to the estimate:

$$\phi^{\text{NP}}_d(c\bar{c}) = -0.09 \pm 0.04 \text{ rad.} \tag{38}$$

In Ref. [1], the analysis of NP contributions to both B_d and B_s mixings (so-called scenario I) including the experimental information available at that time led to the following value for the NP mixing phase in the B_d sector:

$$\phi^{\text{NP}}_d([1]) = -0.22^{+0.07}_{-0.05} \text{ rad.} \tag{39}$$

The main ingredient for this value is the discrepancy between $\phi^{c\bar{c}}_d$ and $BR(B \to \tau\nu)$ in the SM, and thus it will only be marginally affected by the CDF, D0, and LHCb updates concerning the B_s sector. The comparison of these numbers with the ones obtained in Sec. III is shown in Fig. 4.
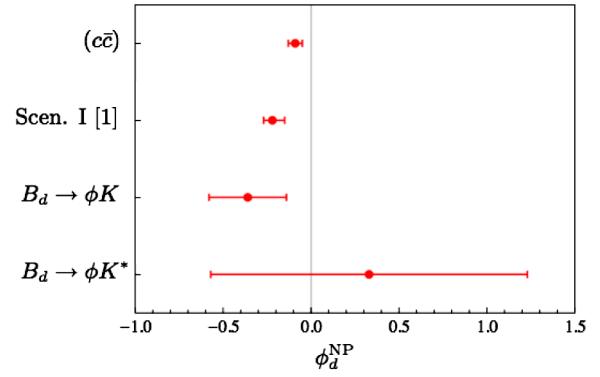

FIG. 4 (color online). Comparison between the NP mixing angles obtained from $c\bar{c}$, from $B_d \to \phi K_S$, from $B_d \to \phi K^*$, and from the fit in Scenario I of Ref. [1]. The line $\phi^{\text{NP}}_d = 0$ corresponds to the standard model.

Finally, an updated result for the longitudinal observables of the decay mode $B_s \to \bar{K}^{0*}K^{0*}$ has been presented. For the ratio of B_s to B_d longitudinal branching ratios, defined in Eq. (28), we find:

$$R^{\text{DMV}}_{sd} = 16.4 \pm 5.2. \tag{40}$$

This result can be compared with recent experimental analyses, and with similar theoretical predictions obtained with alternative approaches:

(1) Our result is compatible with the QCDF prediction [see Eq. (32)], although our error is about 4 times smaller. If the QCDF is supplemented with $B_d \to K^*\phi$ data to fix $1/m_b$-suppressed corrections of phenomenological relevance, the error decreases, but still is twice as large as ours.

(2) Our result is compatible with the recent LHCb measurement, Eq. (35), although the experimental number seems rather low. We tentatively identify this anomaly with the surprisingly low value for the longitudinal polarization fraction $f_L(B_s) \simeq 0.3$: a value of $f_L \simeq 0.8$, similar to the one measured for the U-spin related B_d mode, would lead to a $R_{sd} \simeq 20$, more in agreement with our prediction. If the LHCb measurement of $f_L(B_s)$ is confirmed, its interpretation would constitute a theoretical challenge, as it would require a type of NP that violates the flavour symmetry relating B_d and B_s decays very significantly.

We hope that these results will trigger more precise experimental analyses of penguin-mediated decays both in B_d and B_s sectors, considering the potential of these channels to identify New Physics in meson mixing.

APPENDIX A: DICTIONARY FOR LONGITUDINAL OBSERVABLES IN $B \to VV$ DECAYS

In the case of two-body B decays into vector mesons V_1 and V_2, there are three different configurations of helicity available $h = +1, 0, -1$. This is described by three transversity amplitudes $A_{0,\perp,\|}$ corresponding to linearly polarized states, with definite CP properties ($A_{0,\|}$ correspond to a CP-parity $\eta_{0,\|} = \eta_{V_1}\eta_{V_2}$, whereas $A_\perp$ has $\eta_\perp = -\eta_{V_1}\eta_{V_2}$). As discussed extensively in Refs. [20,21], the transversal amplitudes $A_{\perp,\|}$ are suppressed by powers of $1/m_B$ compared to the longitudinal amplitude A_0, leading to an expected longitudinal polarization f_L close to 1. In turn, this implies that predictions based on the heavy-quark limit, such as those from QCD factorization or SCET, are much more reliable for longitunal observables than for parallel and perpendicular ones, where only rough estimates can be derived.

This theoretical situation explains why we considered only longitudinal observables for VV modes in Ref. [10] as well as in the present article. We defined longitudinal observables as the CP observables constructed by considering as final CP eigenstate the two-meson state with longitudinal polarizations, i.e., the CP-averaged branching ratio and the CP-asymmetries:

$$BR^{\mathrm{long}} = g_{\mathrm{ps}}\frac{|A_0|^2 + |\bar{A}_0|^2}{2}, \tag{A1}$$

$$A^{\mathrm{long}}_{\mathrm{dir}} = \frac{|A_0|^2 - |\bar{A}_0|^2}{|A_0|^2 + |\bar{A}_0|^2}, \tag{A2}$$

$$A^{\mathrm{long}}_{\Delta\Gamma} + iA^{\mathrm{long}}_{\mathrm{mix}} = -2\eta_0\frac{e^{-i\phi_Q}A_0^*\bar{A}_0}{|A_0|^2 + |\bar{A}_0|^2}, \tag{A3}$$

with the phase space factor relating an amplitude to the corresponding branching ratio:

$$g_{\mathrm{ps}} = \frac{\tau_B\sqrt{[m_B^2 - (m_{V_1} + m_{V_2})^2][m_B^2 - (m_{V_1} - m_{V_2})^2]}}{16\pi m_B^3}. \tag{A4}$$

A_0 represents the longitudinal decay amplitude for B_Q, $\bar{A}_0$ its CP conjugate, and ϕ_Q the meson-mixing angle.

In principle, these quantities can be obtained from the angular analysis of the differential decay width (with respect to the usual angular variables θ_1, θ_2, ψ [20]):

$$\frac{d^3\Gamma_{B_q \to V_1 V_2}}{d\cos\theta_1 d\cos\theta_2 d\psi} = \frac{9}{8\pi}\Big[|a_0|^2\cos^2\theta_1\cos^2\theta_2 + |a_\||^2\frac{1}{2}\sin^2\theta_1\sin^2\theta_2\cos^2\psi + |a_\perp|^2\frac{1}{2}\sin^2\theta_1\sin^2\theta_2\sin^2\psi$$

$$+ \mathrm{Re}[a_0^*a_\|]\frac{1}{2\sqrt{2}}\sin2\theta_1\sin2\theta_2\cos\psi - \mathrm{Im}[a_0^*a_\perp]\frac{1}{2\sqrt{2}}\sin2\theta_1\sin2\theta_2\sin\psi$$

$$- \mathrm{Im}[a_\|^*a_\perp]\frac{1}{2}\sin^2\theta_1\sin^2\theta_2\sin2\psi\Big]. \tag{A5}$$

If the initial meson is charged, or if we neglect the difference of lifetimes between neutral mesons ($\Delta\Gamma = 0$), the coefficients a_i and the transversity amplitudes A_i can be identified up to a global normalization factor, and the quantities (A1)–(A3) are easy to obtain from an angular analysis of the decay, as explained in the following.

For the moment, the only available detailed time-dependent analyses for VV modes of interest to us concerns $B_d \to \phi K^*$ [15,16], providing CP-averaged parameters and CP-asymmetries for the 3 transversity amplitudes (these analyses for the B_d meson set $\Delta\Gamma_{B_d} = 0$). Using a $-$ superscript for B_d observables and a $+$ superscript for $\bar{B}_d$ observables, one has values for the branching ratios:

$$BR^+ = \frac{\bar{\Gamma}}{\Gamma_{\mathrm{total}}} = g_{\mathrm{ps}}\sum_\lambda|\bar{A}_\lambda|^2,$$
$$BR^- = \frac{\Gamma}{\Gamma_{\mathrm{total}}} = g_{\mathrm{ps}}\sum_\lambda|A_\lambda|^2, \tag{A6}$$

and for the polarization fractions:

$$f_L^+ = \frac{|\bar{A}_0|^2}{\sum_\lambda|\bar{A}_\lambda|^2}, \qquad f_L^- = \frac{|A_0|^2}{\sum_\lambda|A_\lambda|^2}. \tag{A7}$$

We have, therefore, the relationships:

$$BR = \frac{1}{2}\frac{1}{\Gamma_{\mathrm{total}}}(\bar{\Gamma} + \Gamma), \qquad \mathcal{A}_{CP} = \frac{\bar{\Gamma} - \Gamma}{\bar{\Gamma} + \Gamma},$$
$$f_L = \frac{1}{2}(f_L^+ + f_L^-), \qquad \mathcal{A}_{CP}^0 = \frac{f_L^+ - f_L^-}{f_L^+ + f_L^-}, \tag{A8}$$

and we can easily define the longitudinal observables in terms of these observables:

$$BR^{\mathrm{long}} = BR \cdot f_L \cdot [1 + \mathcal{A}_{CP}^0 \cdot \mathcal{A}_{CP}], \tag{A9}$$

$$A^{\mathrm{long}}_{\mathrm{dir}} = -\frac{\mathcal{A}_{CP}^0 + \mathcal{A}_{CP}}{1 + \mathcal{A}_{CP}^0 \cdot \mathcal{A}_{CP}}, \tag{A10}$$

$$A^{\mathrm{long}}_{\mathrm{mix}} = \eta\sqrt{1 - (A^{\mathrm{long}}_{\mathrm{dir}})^2}\sin(2\beta + \arg(A_0/\bar{A}_0)), \tag{A11}$$

$$A_{\Delta\Gamma}^{\text{long}} = -\eta\sqrt{1 - (A_{\text{dir}}^{\text{long}})^2}\cos(2\beta + \arg(A_0/\bar{A}_0)), \quad (A12)$$

where the relative phase between A_0 and $\bar{A}_0$ is defined following the notation of Ref. [15]

$$\arg(A_0/\bar{A}_0) = 2\Delta\delta_0 + 2\Delta\phi_0. \quad (A13)$$

In this analysis, the mixing angle ϕ_d was assumed to have its standard model value $2\beta = 0.75 \pm 0.03$ rad, which we used to translate the measured quantities into the longitudinal observables $A_{\text{mix}}^{\text{long}}$ and $A_{\Delta\Gamma}^{\text{long}}$. In the experimental analysis of Ref. [15], there is only sensitivity to $\sin(2\beta + \arg(A_0/\bar{A}_0))$, which means that there is a sign ambiguity in $A_{\Delta\Gamma}^{\text{long}}$, as seen in Eq. (19).

APPENDIX B: NEUTRAL-MESON OBSERVABLES WITH A FINITE WIDTH DIFFERENCE

Since we consider the B_s meson in the present article, we have to discuss the modifications induced by a finite lifetime difference, before distinguishing its impact on flavour-tagged and flavor-untagged analyses for the longitudinal observables of interest. We will see that we must include different $O(\Delta\Gamma_s/\Gamma_s)$ corrections in each case to connect the measured quantities and the observables (A1)–(A3). As will become clear in the following, most of this discussion applies not only to two-body vector modes but more generally to any decay of a neutral meson into a CP eigenstate.

1. Branching ratios in presence of meson mixing

The B-$\bar{B}$ systems can be described in terms of CP-conjugate flavour states $|B_Q\rangle$ and $|\bar{B}_Q\rangle$ ($Q = d, s$). The time evolution of an isolated neutral B_Q meson of a given flavour at $t = 0$ and decaying at a later time t into a CP eigenstate f is given as [24,25]:

$$\Gamma(B_Q(t) \to f) = N_f \frac{|A_f|^2 + |\bar{A}_f|^2}{2} e^{-\Gamma t}$$
$$\times \left[\cosh\frac{\Delta\Gamma t}{2} + A_{\text{dir}}\cos(\Delta M t)\right.$$
$$\left. - A_{\Delta\Gamma}\sinh\frac{\Delta\Gamma t}{2} + A_{\text{mix}}\sin\Delta M t\right], \quad (B1)$$

$$\Gamma(\bar{B}_Q(t) \to f) = N_f \frac{|A_f|^2 + |\bar{A}_f|^2}{2} e^{-\Gamma t}$$
$$\times \left[\cosh\frac{\Delta\Gamma t}{2} - A_{\text{dir}}\cos(\Delta M t)\right.$$
$$\left. - A_{\Delta\Gamma}\sinh\frac{\Delta\Gamma t}{2} - A_{\text{mix}}\sin\Delta M t\right], \quad (B2)$$

where the Hamitonian eigenvalues $\mu_{L,H} = M_{L,H} - i\Gamma_{L,H}/2$ define $\Delta\mu = \mu_H - \mu_L = \Delta M - i\Delta\Gamma/2$. Also, $\Gamma \equiv (\Gamma_H + \Gamma_L)/2$ and $A_f = \langle f|B_Q\rangle$. N_f is a time-independent, but state-dependent normalization factor,

corresponding to the integration over phase space. The mixing ratio $q/p = \exp(-i\phi_Q)$ is assumed to be a pure phase in the present article (as suggested by the very small values of the flavor specific asymmetries both for B_d and B_s [22]).

The normalization factor N_f comes from:

$$\Gamma(B_Q(t) \to f) = N_f|\langle f|B_Q(t)\rangle|^2. \quad (B3)$$

Let us notice that this "definition" is rather ambiguous, as one generally considers initial and final states that are asymptotic (mass eigen)states, which B_Q is not. However, one can still determine this factor by going back to the derivation of Fermi's golden rule. This is generally done for an initial mass eigenstate of the unperturbed Hamiltonian but can be easily adapted to the superposition of two mass eigenstates $|B_L\rangle, |B_H\rangle$, provided that the difference of energy between the two transitions $\omega_L - \omega_H = \langle f|H_1|B_L\rangle - \langle f|H_1|B_H\rangle$ is small. In that case, the normalization N_f corresponds to the phase space available, computed for the mass eigenstates of the unperturbed Hamiltonian. We obtain, therefore, the same normalization as in the case of charged B-decays:

$$N_f = g_{\text{ps}}\Gamma, \quad (B4)$$

where g_{ps} is given by the phase space with an incoming meson of mass $M = (M_H + M_L)/2$.

Since there is no unambiguous definition of the CP-averaged branching ratio for neutral mesons as the states involved (B_Q and $\bar{B}_Q$) are not mass eigenstates, we should define what we call the CP-averaged branching ratio for B_q decays. We opt for the simple definition, inspired by the charged meson case, and corresponding to the value that we would obtain through a measurement at $t = 0$ (i.e., before neutral B-meson mixing could take place):

$$BR_f = g_{\text{ps}}(|A_f|^2 + |\bar{A}_f|^2)/2. \quad (B5)$$

We will show later that this definition coincides exactly with time-integrated CP-averaged decay widths in the limit $\Delta\Gamma \to 0$, but that the relationship is corrected by terms of order $O(\Delta\Gamma/\Gamma)$. This correction depends on the experimental setting, because each one has a different sensitivity to the time evolution of the neutral B-meson. This correction factor can thus be seen as a correction for the temporal acceptance of the considered experiment.

Let us now come to $B_Q \to V_1 V_2$ decays, and let us assume that all the direct asymmetries vanish, whereas $A_{0,\text{mix}} = A_{\parallel,\text{mix}} = -A_{\perp,\text{mix}} = \eta\sin\phi_Q$ and $A_{0,\Delta\Gamma} = A_{\parallel,\Delta\Gamma} = -A_{\perp,\Delta\Gamma} = -\eta\cos\phi_Q$, where $\eta = \eta_{V_1}\eta_{V_2}$. As discussed in Ref. [24], we can obtain the time-dependent decay width from Eq. (A5), upon the identification:

$$|a_f|^2 \to N|A_f|^2 e^{-\Gamma t} \times \left[\cosh\frac{\Delta\Gamma t}{2} + \eta_f\cos\phi_Q\sinh\frac{\Delta\Gamma t}{2}\right.$$
$$\left. + \eta_f\sin\phi_Q\sin\Delta mt\right], \quad (B6)$$

$$\text{Re}[a_0^* a_\parallel] \to N|A_0||A_\parallel| \cos(\delta_\parallel - \delta_0)e^{-\Gamma t}$$

$$\times \left[\cosh\frac{\Delta\Gamma t}{2} + \eta \cos\phi_Q \sinh\frac{\Delta\Gamma t}{2} \right.$$

$$\left. + \eta \sin\phi_Q \sin\Delta m t \right], \qquad (B7)$$

$$\text{Im}[a_f^* a_\perp] \to N|A_f||A_\perp|e^{-\Gamma t} \times \left[\sin\delta_f \cos(\Delta m t) \right.$$

$$- \eta \cos\delta_f \cos\phi_Q \sin(\Delta m t)$$

$$\left. + \eta \cos\delta_f \sin\phi_Q \sinh\frac{\Delta\Gamma t}{2} \right], \qquad (B8)$$

with $f = 0, \perp, \parallel$, and δ_0 and $\delta_\parallel$ the relative strong phases of A_0 and $A_\parallel$ with respect to $A_\perp$. There is a common normalization factor $N = g_{\text{ps}}\Gamma$ for all amplitudes. The CP-conjugate expression can be obtained by the replacement $\phi_Q \to -\phi_Q$ and by multiplying by (-1) the two imaginary parts involving $a_\perp$ in Eq. (B8).

2. Tagged analysis

Flavour-tagged analyses are particularly easy to perform in B-factories as they produce intricated $B\bar{B}$ pairs [15], and they allow one to study separately the time-dependence of B and $\bar{B}$ samples as well as to extract the modulus and relative phases of the three transversity amplitudes through an angular analysis. The probability of the process depends on the decay times t_{tag} and t_{CP} of both mesons (the one decaying into a tagging state f_{tag} and the one actually studied for CP-violation and decaying into f_{CP}). After integrating over the sum $t_{\text{tag}} + t_{CP}$, one obtains for $\Gamma_{\text{tag}}(B_Q(t) \to f)$ a structure similar to that for untagged decays provided that the exponential $\exp(-\Gamma t)$ is replaced by $\exp(-\Gamma|t|)$ and that $t = t_{CP} - t_{\text{tag}}$ is allowed to run from $-\infty$ to $+\infty$. Indeed, from Ch. 1 in Ref. [26], we have the joint decay amplitude $A(t_{\text{tag}}, t_{CP})$ which can be integrated over time to yield:

$$\int dt_{\text{tag}} \int dt_{CP} \int [dp] A(t_{\text{tag}}, t_{CP})$$

$$= \frac{\pi}{2} \frac{N_{\text{tag}}}{\Gamma} |\bar{A}_{\text{tag}}|^2 \int_{-\infty}^{\infty} dt N_{CP} \frac{|A_{CP}|^2 + |\bar{A}_{CP}|^2}{2} e^{-\Gamma|t|}$$

$$\times \left[\cosh\frac{\Delta\Gamma t}{2} + A_{\text{dir}} \cos(\Delta m t) - A_{\Delta\Gamma} \sinh\frac{\Delta\Gamma t}{2} \right.$$

$$\left. + A_{\text{mix}} \sin\Delta m t \right]$$

$$= \frac{\pi}{2} BR(\bar{B}_Q \to f_{\text{tag}}) \times \int_{-\infty}^{\infty} dt \Gamma_{\text{tagged}}(B_Q(t) \to f_{CP})$$

$$(B9)$$

where the first factor comes from the angular integral describing the $e^+ e^- \to B_Q \bar{B}_Q$ transition, and we have defined $\Gamma_{\text{tagged}}(B_Q(t) \to f_{CP})$ as the integrand of Eq. (B9).

In the case of a nonvanishing width difference, and assuming the normalization Eq. (B5), we can determine the CP-averaged branching ratio by considering:

$$BR_{f,\text{tagged}} = \int_{-\infty}^{+\infty} [\Gamma_{\text{tagged}}(B_Q(t) \to f)$$

$$+ \Gamma_{\text{tagged}}(\bar{B}_Q(t) \to f)] = BR_f \times \frac{\Gamma^2}{\Gamma_H \Gamma_L}.$$

$$(B10)$$

We have, therefore, a correction of the time-integrated branching ratio with respect to the branching ratio in absence of mixing. This correction is due to the difference of the widths between the two neutral states. This is typically a small correction: if we define $y \equiv \Delta\Gamma/(2\Gamma)$, we can write [22]:

$$\frac{\Gamma^2}{\Gamma_H \Gamma_L} = \frac{1}{1 - y^2} = \begin{cases} 1 + (0 \pm 2) \times 10^{-4} & \text{for } B_d, \\ 1 + (2 \pm 2) \times 10^{-3} & \text{for } B_s. \end{cases} \quad (B11)$$

3. Untagged analysis

In the case of hadronic machines, such as CDF, D0 and LHCb, we encounter a rather different situation with no flavour tags available and an integral over time being performed. Since there is no information on the second B-meson being produced, one must consider Eqs. (B1) and (B2) for $t \geq 0$. The CP-averaged branching ratio can then be determined through:

$$BR_{f,\text{untag}} = \int_0^{+\infty} \frac{1}{2} [\Gamma_{\text{untag}}(B_Q(t) \to f)$$

$$+ \Gamma_{\text{untag}}(\bar{B}_Q(t) \to f)]$$

$$= BR_f \times \frac{\Gamma^2}{\Gamma_L \Gamma_H} \left[1 - A_{\Delta\Gamma} \frac{\Delta\Gamma}{2\Gamma} \right]. \quad (B12)$$

Compared to the tagged case, we see that there is a further term coming from $A_{f,\Delta\Gamma}$. The resulting correction for the branching ratio is larger than the one for B-factories, since it is linear in the small quantity $\Delta\Gamma/(2\Gamma)$, and not quadratic.

For vector-vector channels, assuming that the production rate for B_Q and $\bar{B}_Q$ is the same and that there is no CP-violation in decay, $BR_{V_1 V_2,\text{untag}}$ yields an angular structure identical to Eq. (A5), with the replacement:

$$|a_f|^2 \to g_{\text{ps}}|A_f|^2 \frac{\Gamma^2}{\Gamma_L \Gamma_H} \left[1 + \eta_f \frac{\Delta\Gamma}{2\Gamma} \cos\phi_Q \right], \quad (B13)$$

$$\text{Re}[a_0^* a_\parallel] \to g_{\text{ps}}|A_0||A_\parallel| \cos(\delta_\parallel - \delta_0)$$

$$\times \frac{\Gamma^2}{\Gamma_L \Gamma_H} \left[1 + \eta \frac{\Delta\Gamma}{2\Gamma} \cos\phi_Q \right], \quad (B14)$$

$$\text{Im}[a_f^* a_\perp] \rightarrow g_{\text{ps}} |A_f||A_\perp|$$

$$\times \frac{\Gamma^2}{\Gamma_L \Gamma_H} \left[\eta_f \frac{\Delta\Gamma}{2\Gamma} \cos\delta_f \sin\phi_Q \right], \quad \text{(B15)}$$

where the $\cos\phi$ terms are the remnants of $A_{\Delta\Gamma}$. After integration over the angles, we obtain the CP-averaged width:

$$\Gamma_{B_Q \rightarrow V_1 V_2} = g_{\text{ps}} \frac{\Gamma^3}{\Gamma_L \Gamma_H} \times \left[|A_0|^2 + |A_\parallel|^2 + |A_\perp|^2 \right.$$

$$\left. + \eta \frac{\Delta\Gamma}{2\Gamma} \cos\phi_Q (|A_0|^2 + |A_\parallel|^2 - |A_\perp|^2) \right],$$

$$\text{(B16)}$$

which is expressed in terms of the longitudinal and transverse CP-averaged branching ratios.

In the case of the LHCb analysis for $B_s \rightarrow K^{*0} \bar{K}^{*0}$ ($\eta = 1$), an angular analysis was performed to extract the total branching ratio as well as the longitudinal polarization. The expression in Ref. [7] for the latter quantity was given in the SM case with a vanishing mixing angle ϕ_s. However, the angular analysis performed does not rely on this assumption, so that we can just identify the coefficients with the same angular dependence in the differential decay width, leading to: $f_{L,\text{LHCb}} = |a_0^2|/\Gamma_{B \rightarrow V_1 V_2}$. We can thus derive the relation, valid in the absence of CP-violation in decay:

$$f_{L,\text{LHCb}} \times BR_{\text{LHCb}} = BR^{\text{long}} \frac{\Gamma_s^2}{\Gamma_L \Gamma_H} \left[1 + \frac{\Delta\Gamma_s}{2\Gamma_s} \cos\phi_s \right].$$

$$\text{(B17)}$$

We see that even in the SM case where ϕ_s is tiny, there is a small contribution from $\Delta\Gamma_s/\Gamma_s$ to the relationship between the polarization measured at LHCb and BR^{long}.

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