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Minimal extension of tribimaximal mixing and generalized $Z_2 \times Z_2$ symmetries

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We discuss consequences of combining the effective $Z_2 \times Z_2$ symmetry of the tribimaximal neutrino mass matrix with the CP symmetry. Imposition of such generalized $Z_2 \times Z_2$ symmetries leads to predictive neutrino mass matrices determined in terms of only four parameters and leads to a nonzero θ_{13} and maximal atmospheric mixing angle and CP violating phase. It is shown that an effective generalized $Z_2 \times Z_2$ symmetry of the mass matrix can arise from the A_4 symmetry with specific vacuum alignment. The neutrino mass matrix in the considered model has only three real parameters and leads to determination of the absolute neutrino mass scale as a function of the reactor angle θ_{13}.

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I. INTRODUCTION

Recent ν_e–ν_μ oscillation observations by T2K [1] and MINOS [2] and double CHOOZ [3] have led to a search of alternatives to the tribimaximal (TBM) leptonic mixing [4] pattern among neutrinos.

$$U_{\rm TBM} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}. \quad (1)$$

The above pattern corresponds to $\sin^2\theta_{12} = 1/3$, $\sin^2\theta_{23} = 1/2$, and $\sin^2\theta_{13} = 0$ for the three mixing angles involved in neutrino oscillations. It is theoretically well founded and can be obtained using flavor symmetries in the leptonic sector; see [5] for a review and original references. While the predicted values of the θ_{12} and θ_{23} in TBM agree nearly within 1σ of the latest global analysis [6,7] of the neutrino oscillation data, prediction $\theta_{13} = 0$ is at variance with T2K [1] (MINOS [2]) results by $2.5\sigma(1.6\sigma)$ and with the global analysis [6,7] by about 3σ. This suggests that one should look either for perturbations to TBM affecting mainly θ_{13} or for alternative flavor symmetries which imply a nonzero θ_{13}. Recently, several attempts [8–10] have been made in these directions. Some of these works [8,9] discuss the possible schemes of perturbations to TBM while some [10] provide the models also. The minimal scheme would be the one in which θ_{13} gets generated but θ_{23} and θ_{12} remain close to their predictions in the TBM scheme. We show that this can be achieved by generalizing the $Z_2 \times Z_2$ symmetry of the TBM mass matrix and identify appropriate flavor symmetry which can lead to the modified pattern.

The paper is organized as follows. In the next section, we discuss the generalized $Z_2 \times Z_2$ symmetry of the neutrino mass matrix which minimally modifies the TBM mixing pattern and leads to a nonzero θ_{13}. In Sec. III, we present possible modifications in the well-known A_4 model which lead to the neutrino mass matrix invariant under the proposed symmetry and discuss its phenomenology. Finally, we summarize in Sec. IV.

II. GENERALIZED $Z_2 \times Z_2$ SYMMETRY AND LEPTON MIXING ANGLES

A well-known property of the TBM pattern is the presence of a specific $Z_2 \times Z_2$ symmetry [11] enjoyed by the corresponding neutrino mass matrix $\mathcal{M}_{\nu f}$ in the flavor basis. This symmetry is defined in general by the operators S_i, $i = 1, 2, 3$:

$$(S_i)_{jk} = \delta_{jk} - 2U_{ji}U^*_{ki}, \quad (2)$$

where U is the matrix diagonalizing $\mathcal{M}_{\nu f}$. Each S_i defines a Z_2 group and satisfies

$$S_iS_j = -S_k, \qquad i \neq j \neq k. \quad (3)$$

The S_i also leave the neutrino mass matrix invariant

$$S_i^T \mathcal{M}_{\nu f} S_i = \mathcal{M}_{\nu f}, \quad (4)$$

as can be verified from the Eq. (2) and the property $\mathcal{M}_{\nu f} = U^* D_\nu U^\dagger$, D_ν being the diagonal neutrino mass matrix.

The explicit forms for S_2 and S_3 in the TBM case are given by

$$S_2 = \frac{1}{3}\begin{pmatrix} 1 & -2 & -2 \\ -2 & 1 & -2 \\ -2 & -2 & 1 \end{pmatrix} \text{ and } S_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}. \quad (5)$$

S_2 and S_3, respectively, are determined by the second and the third column of the TBM mixing matrix (1) using Eq. (2). The element S_1 can be obtained using relation (3). In particular, S_3 corresponds to the well-known μ-τ symmetry which is responsible for two of the three predictions, namely $\theta_{13} = 0$, $\theta_{23} = \frac{\pi}{4}$ of the TBM pattern.

A desirable replacement of the $\mu - \tau$ symmetry would be the one which retains maximality of θ_{23} but allows a

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nonzero θ_{13}. Such a symmetry is already known [12] and is obtained by combining the μ-τ symmetry with the CP transformation. The neutrino mass matrix gets transformed to its complex conjugate under the action of the generalized μ-τ:

$$S_3^T \mathcal{M}_{\nu f} S_3 = \mathcal{M}_{\nu f}^*. \tag{6}$$

A neutrino mass matrix with this property leads to two predictions [12]:

$$\sin^2\theta_{23} = \tfrac{1}{2}, \tag{7}$$

$$\sin\theta_{13}\cos\delta = 0. \tag{8}$$

One needs a nonzero θ_{13} in which case the above equation leads to a prediction $\delta = \frac{\pi}{2}$ for the CP violating Dirac phase. Equation (6) does not put any restrictions on the solar angle. In order to do this, we would like to combine the generalized μ-τ symmetry with the "magic symmetry" corresponding to invariance under S_2 and define a generalized $Z_2 \times Z_2$ symmetry. This can be done in two independent ways.

Case I: Let us first assume that the neutrino mass matrix in flavor basis simultaneously satisfies

$$S_{1,3}^T \mathcal{M}_{\nu f} S_{1,3} = \mathcal{M}_{\nu f}^*. \tag{9}$$

Both these conditions together imply that

$$S_2^T \mathcal{M}_{\nu f} S_2 = \mathcal{M}_{\nu f}. \tag{10}$$

The above condition fixes the second column of the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix U to be $1/\sqrt{3}(1, 1, 1)^T$. This form of U has been studied before and known as a trimaximal mixing pattern [13,14]. When compared with the standard form [15], it leads to the relation

$$|\sin\theta_{12}\cos\theta_{13}| = \frac{1}{\sqrt{3}} \Rightarrow \sin^2\theta_{12} = \frac{1}{3}(1 + \tan^2\theta_{13}), \tag{11}$$

which provides the lower limit $\sin^2\theta_{12} \geq 1/3$. The neutrino mass matrix in the flavor basis $\mathcal{M}_{\nu f}$ that satisfies (9) can be written as

$$\mathcal{M}_{\nu f} = \begin{pmatrix} y+z-x & x+ix' & x-ix' \\ x+ix' & y-ix' & z \\ x-ix' & z & y+ix' \end{pmatrix}, \tag{12}$$

where all the parameters are real. Note that $\mathrm{Re}(\mathcal{M}_{\nu f})$ is invariant under (4) and so it is in the TBM form while $\mathrm{Im}(\mathcal{M}_{\nu f})$ follows the condition

$$S_{1,3}^T \mathrm{Im}(\mathcal{M}_{\nu f}) S_{1,3} = -\mathrm{Im}(\mathcal{M}_{\nu f}).$$

The neutrino mass matrix in Eq. (12) can be diagonalized by the matrix

$$U^I = U_{\mathrm{TBM}} P R_{13}(\theta), \tag{13}$$

where $P = \mathrm{diag.}(1, 1, i)$ and $R_{13}(\theta)$ denotes a rotation by an angle θ in the 1–3 plane.

Case II: The second possibility is

$$S_{2,3}^T \mathcal{M}_{\nu f} S_{2,3} = \mathcal{M}_{\nu f}^*, \tag{14}$$

which leads to

$$S_1^T \mathcal{M}_{\nu f} S_1 = \mathcal{M}_{\nu f}. \tag{15}$$

This fixes the first column of U to be $1/\sqrt{6}(2, -1, -1)^T$ which implies

$$|\cos\theta_{12}\cos\theta_{13}| = \sqrt{\frac{2}{3}} \Rightarrow \sin^2\theta_{12} = \frac{1}{3}(1\text{–}2\tan^2\theta_{13}). \tag{16}$$

In contrast to the previous case, this provides an upper bound on the solar angle $\sin^2\theta_{12} \leq 1/3$. The neutrino mass matrix in the flavor basis $\mathcal{M}_{\nu f}$ in this case can be written as

$$\mathcal{M}_{\nu f} = \begin{pmatrix} y+z-x & x+ix' & x-ix' \\ x+ix' & y+2ix' & z \\ x-ix' & z & y-2ix' \end{pmatrix}. \tag{17}$$

The above $\mathcal{M}_{\nu f}$ can be diagonalized by the matrix

$$U^{II} = U_{\mathrm{TBM}} P R_{23}(\theta), \tag{18}$$

where $R_{23}(\theta)$ denotes a rotation by an angle θ in the 2–3 plane. The third possibility is to consider $S_{1,2}^T \mathcal{M}_{\nu f} S_{1,2} = \mathcal{M}_{\nu f}^*$ and this results in the μ-τ symmetric $\mathcal{M}_{\nu f}$ which leads to $\theta_{13} = 0$, so it is not the case of our interest. Both the above scenarios predict small deviations in $\sin^2\theta_{12}$ from its tribimaximal value, but in opposite directions. While both of them are consistent with the present 3σ ranges of θ_{12} and θ_{13} obtained from the global fits to the recent neutrino oscillation data [7], prediction (16) is more favored if only 1σ is considered. Note that both these scenarios lead to the trivial Majorana phases (0 or π) and do not restrict the masses of neutrinos.

The mass matrices in Eqs. (12) and (17) based on the generalized $Z_2 \times Z_2$ symmetry are different and more predictive compared to most other proposed modifications of the TBM structure [8–10]. Let us emphasize the main differences:

(i) Equations (12) and (17) contain four real parameters and hence lead to five predictions among nine observables. These are two trivial Majorana phases, a Dirac phase $\delta = \pm\pi/2$, an atmospheric mixing angle $\theta_{23} = \pi/4$, and the solar mixing angle predicted by Eq. (11) or (16).

(ii) Grimus and Lavoura [13] and He and Zee [9] proposed a mixing matrix similar to Eq. (13). The differences being the absence of P, the presence of the Majorana phase matrix, and the replacement of R_{13} by a unitary transformation in the 1–3 plane with an undetermined Dirac CP phase δ. In the process, δ and Majorana phases become unpredictable and θ_{23} deviates from the TBM value by a term of $\mathcal{O}(\theta_{13})$.

(iii) Likewise, Ma in his classic paper [16] considered a modification to TBM analogous to Eq. (18). Here also R_{23} gets replaced by a unitary transformation in the 2–3 plane with an undetermined phase resulting in less predictivity than the present case.

(iv) A special case of Eq. (12) was considered by Grimus and Lavoura [14]. This corresponds to choosing

$$x' = -\frac{1}{\sqrt{3}}(z - x).$$

As a result, $\mathcal{M}_{\nu f}$ contains only three parameters and allows determination of the absolute neutrino mass scale in addition to the five predictions mentioned above. It is also shown in [14] that such a mass matrix can arise in a model based on the Δ_{27} group. So far we have not appealed to any flavor symmetry at the Lagrangian level and considered only the effective $Z_2 \times Z_2$ symmetry of the neutrino mass matrix. We now propose to realize this effective symmetry from an underlying flavor symmetry. In the process, we find that the use of flavor symmetry also leads us to a three parameter neutrino mass matrix as in the case proposed by Grimus and Lavoura [14].

III. MODEL AND PHENOMENOLOGY

We use the flavor symmetry A_4. Several versions of this symmetry are proposed [5] to obtain a neutrino mass matrix which exhibits the TBM mixing. Here we show that a simple modification of the existing A_4 schemes can lead to the more predictive mass matrix given in Eq. (17). For definiteness, we concentrate on a specific A_4 model of He, Keum, and Volkas [17]. We propose two possible schemes, one based on the type-I seesaw and the other using a combination of both the type-I and type-II seesaw mechanisms.

Let us first outline the basic features of the A_4 model proposed in [17]. Though it was proposed to explain both quark and lepton mixing patterns, we here discuss only the lepton sector of it. The matter and Higgs field content of the model with their assignments under the standard model (SM) gauge group $G_{\rm SM} \equiv SU(3)_c \times SU(2)_L \times U(1)_Y$ and A_4 group are given in Table I.

The renormalizable $G_{\rm SM} \times A_4$ Yukawa interactions of the model can be written as

$$-\mathcal{L}_Y = y_e(\bar{l}_L\tilde{\Phi})_{\mathbf{1}}e_R + y_\mu(\bar{l}_L\tilde{\Phi})_{\mathbf{1}''}\mu_R + y_\tau(\bar{l}_L\tilde{\Phi})_{\mathbf{1}'}\tau_R + y_D(\bar{l}_L\nu_R)_{\mathbf{1}}\phi + \tfrac{1}{2}M\bar{\nu}_R\nu_R^c + \tfrac{1}{2}B'(\bar{\nu}_R\nu_R^c)_{\mathbf{3}}\chi + \text{H.c.}, \tag{19}$$

where $\tilde{\Phi} \equiv i\tau_2\Phi^*$ and $(\ldots)_R$ denotes the R-dimensional representation of A_4. Note that in [17], an additional $U(1)_X$ symmetry is also imposed so that an unwanted $G_{\rm SM} \times A_4$ invariant term $\bar{l}_L\nu_R\Phi$ can be forbidden when it is assumed that l_L, e_R, μ_R, τ_R, and ϕ carry $X = 1$ and other fields are chargeless under $U(1)_X$. A specific choice of the A_4 vacuum $\langle\Phi\rangle = v(1,1,1)^T$ leads to the charged lepton mass matrix:

$$M_l = \sqrt{3}vU(\omega)\text{Diag}(y_e, y_\mu, y_\tau), \tag{20}$$

where

$$U(\omega) = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{pmatrix} \tag{21}$$

and $\omega = e^{2i\pi/3}$ is a cube root of unity. The Dirac neutrino mass matrix is proportional to the identity matrix

$$M_D = y_D v_\phi \mathbf{I}, \tag{22}$$

where $v_\phi = \langle\phi\rangle$. Further, assuming that the field χ develops a vacuum expectation value (vev) in the direction $\langle\chi\rangle = v_\chi(1,0,0)^T$, the heavy neutrino mass matrix can be written as

$$M_R = \begin{pmatrix} A & 0 & 0 \\ 0 & A & B \\ 0 & B & A \end{pmatrix}, \tag{23}$$

where $B = B'v_\chi$. After the seesaw, Eqs. (22) and (23) lead to the light neutrino mass matrix

$$\mathcal{M}_\nu = -M_D M_R^{-1} M_D^T = \begin{pmatrix} \frac{(a^2-b^2)}{a} & 0 & 0 \\ 0 & a & b \\ 0 & b & a \end{pmatrix}, \tag{24}$$

where $a = -[y_D^2 v_\phi^2/(A^2 - B^2)]A$ and $b = [y_D^2 v_\phi^2/(A^2 - B^2)]B$.

As is well-known, Eqs. (21) and (24) lead to an $\mathcal{M}_{\nu f} = U(\omega)^T\mathcal{M}_\nu U(\omega)$ in the form exhibiting the TBM mixing:

$$\mathcal{M}_{\nu f} = \frac{1}{3a}\begin{pmatrix} (a+b)(3a-b) & -b(a+b) & -b(a+b) \\ -b(a+b) & b(2a-b) & 3a^2 - b(a+b) \\ -b(a+b) & 3a^2 - b(a+b) & b(2a-b) \end{pmatrix}. \tag{25}$$

We need to change the existing model in two ways to obtain a more predictive form of Eq. (17). First, we require that all the Yukawa couplings in Eq. (19) as well as the

TABLE I. Various fields and their representations under $G_{\rm SM} \times A_4$.

	l_L	e_R	μ_R	τ_R	ν_R	Φ	ϕ	χ
$G_{\rm SM}$	(1, 2, −1)	(1, 1, −2)	(1, 1, −2)	(1, 1, −2)	(1, 1, 0)	(1, 2, −1)	(1, 2, −1)	(1, 1, 0)
A_4	**3**	**1**	**1′**	**1″**	**3**	**3**	**1**	**3**

vacuum expectation values are real. Equation (25) then coincides with the real part of (17) with

$$z = a - \frac{b}{3a}(a+b); \quad y = \frac{b}{3a}(2a-b); \quad x = -\frac{b}{3a}(a+b). \tag{26}$$

We need to enlarge the model to introduce the imaginary part. This can be done by adding additional $SU(2)_L$ singlet or triplet fields transforming as an A_4 triplet. Conventionally, one uses CP symmetry to obtain the real Yukawa couplings. The reality of Yukawa couplings follows if the definition of CP is generalized in a manner analogous to [12]. This generalized CP combines the CP and μ-τ symmetry as follows:

$$\begin{aligned} &(l_l, \nu_R, \Phi, \chi) \rightarrow S_3(l_L^c, \nu_R^c, \Phi^\dagger, \chi^\dagger), \\ &(e_R, \mu_R, \tau_R) \rightarrow (e_R^c, \mu_R^c, \tau_R^c), \end{aligned} \tag{27}$$

where superfix c on a field denotes its CP conjugate and S_3 is defined in Eq. (5). Note that the above symmetry behaves like ordinary CP on the A_4 singlet right-handed charged leptons and is thus slightly different from the generalized μ-τ symmetry introduced in [13].

The required imaginary part in $\mathcal{M}_{\nu f}$ can be generated in two ways:

A. Type-II extension

Add three copies of $SU(2)_L$ triplet fields Δ which form a triplet of A_4 with the $U(1)_X$ charge $X = 2$. This modifies the Yukawa interaction by an additional triplet seesaw term

$$-\mathcal{L}_Y^\Delta = y_L(\bar{l}_L l_L^c)_{\mathbf{3}}\Delta + \text{H.c.} \tag{28}$$

y_L becomes real if $\Delta \rightarrow S_3\Delta^\dagger$ under the generalized CP. Let us now assume that Δ takes the vev along the following direction:

$$\langle\Delta\rangle = v_\Delta(0, -1, 1)^T. \tag{29}$$

Such vacuum alignment can be achieved through some terms that break A_4 softly and an explicit example is discussed in [16]. Equation (28) gives rise to a type-II contribution in the neutrino mass matrix. Combining this with the type-I contribution, Eq. (24), we get the following:

$$\mathcal{M}_\nu = \begin{pmatrix} \frac{(a^2-b^2)}{a} & c & -c \\ c & a & b \\ -c & b & a \end{pmatrix}. \tag{30}$$

Parameters a, b, c are real but when transformed to the flavor basis one obtains a complex $\mathcal{M}_{\nu f}$ coinciding exactly with Eq. (17) with x, y, z defined in Eq. (26) and

$$x' = -\frac{c}{\sqrt{3}}.$$

The generalized $Z_2 \times Z_2$ symmetry emerges here as an effective symmetry. The type-II contribution (characterized by the parameter c) in the above neutrino mass matrix generates a nonzero reactor angle and modifies the solar mixing angle as in Eq. (16).

B. Type-I extension

Another viable extension of the model is obtained by adding the A_4 triplet, $SU(2)_L$ singlet field χ' in addition to the χ already present. χ' introduces the following term in Eq. (19):

$$-\mathcal{L}_Y^{\chi'} = \tfrac{1}{2}y_R(\bar{\nu}_R\nu_R^c)_{\mathbf{3}}\chi' + \text{H.c.} \tag{31}$$

y_R can be made real using the similar generalized CP symmetry defined in Eq. (27). Assuming that χ' takes a vev along the same direction as Δ in the previous case, i.e., $\langle\chi'\rangle = v_{\chi'}(0, -1, 1)^T$, we get

$$M_R = \begin{pmatrix} A & C & -C \\ C & A & B \\ -C & B & A \end{pmatrix}, \tag{32}$$

where $C = y_R v_{\chi'}$. After the seesaw the light neutrino mass matrix can be suitably written as

$$\mathcal{M}_\nu = \begin{pmatrix} \frac{(a^2-b^2+c^2)}{a} & c & -c \\ c & a & b \\ -c & b & a \end{pmatrix}. \tag{33}$$

This matrix also exhibits the generalized $Z_2 \times Z_2$ symmetry and is determined by three real parameters as before. The only difference from the previous case is a small contribution of the $\sim\mathcal{O}(a\theta_{13}^2)$ in the 11 entry in $\mathcal{M}_\nu$. As a result the phenomenology of neutrino masses in both cases is very similar and we now turn to this discussion.

C. Phenomenology

We now derive the phenomenological consequences of the generalized $Z_2 \times Z_2$ structures Eqs. (30) and (33) obtained using the A_4 symmetry and specific vacuum alignment. While the most general, $Z_2 \times Z_2$ invariant structure, Eq. (17), has four parameters, the specific realization obtained here has only three parameters. This follows from Eq. (26) which shows that x, y, z are not independent but are related by

$$z = -y + \frac{x(x+5y)}{2x+y}. \tag{34}$$

The situation here is similar to the original A_4 models in which specific realizations of the TBM schemes lead to a more constrained mass pattern than the most general one and lead to sum rules among neutrino masses [18]. Specifically, Eq. (26) leads to a mass sum rule [18,19]

$$\frac{2}{m_2} + \frac{1}{m_3} = \frac{1}{m_1}, \tag{35}$$

where m_i are the neutrino masses. Note that m_i are real in our case since all the parameters in the neutrino mass matrix (24) are real. The phenomenological implications

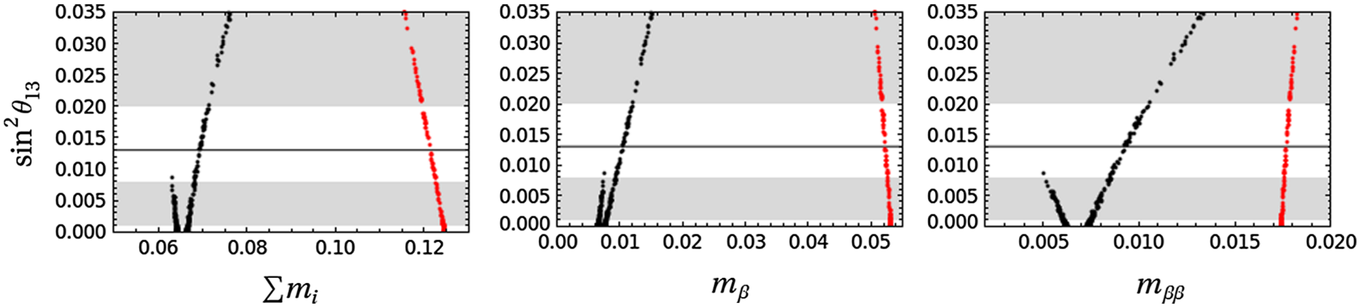

FIG. 1 (color online). Correlations between the reactor angle and different neutrino mass dependent observables implied by the neutrino mass matrix in Eq. (30). The black [lighter (red)] points correspond to the normal (inverted) hierarchy in neutrino masses. The black horizontal line shows the mean value of $\sin^2\theta_{13}$ obtained from the global fits. The unshaded and the shaded regions correspond to 1σ and 3σ ranges of $\sin^2\theta_{13}$, respectively.

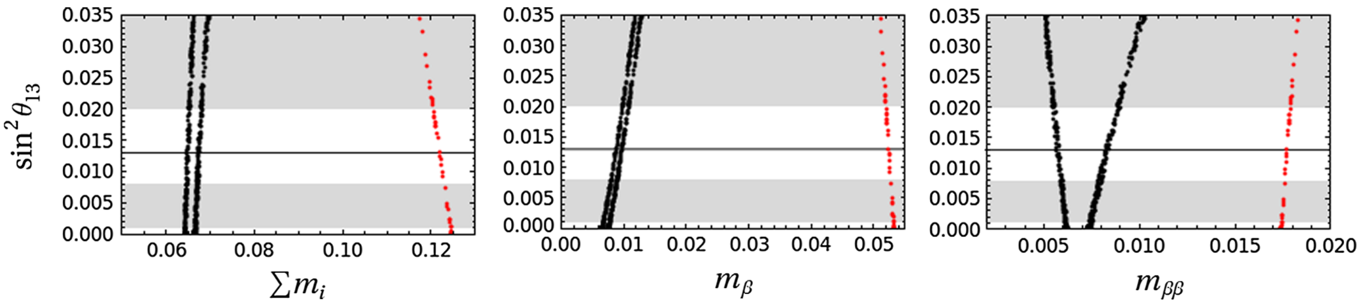

FIG. 2 (color online). Correlations between the reactor angle and different neutrino mass dependent observables arise in the neutrino mass matrix given by Eq. (33). The black [lighter (red)] points correspond to the normal (inverted) hierarchy in neutrino masses. The black horizontal line shows the mean value of $\sin^2\theta_{13}$ obtained from the global fits. The unshaded and the shaded regions correspond to 1σ and 3σ ranges of $\sin^2\theta_{13}$, respectively.

of this neutrino mass sum rule are already considered in [18,19]. The generalization introduced through Eq. (30) modifies this sum rule to

$$\frac{2}{m_2+3(m_3-m_2)s_{13}^2}+\frac{1}{m_3+3(m_2-m_3)s_{13}^2}=\frac{1}{m_1}. \tag{36}$$

The above sum rule allows determination of the absolute neutrino mass scale as a function of s_{13}^2. This determination depends on the type of hierarchy and approximate analytic form for the lightest neutrino mass given in the limit $s_{13}=0$ by [18]

$$\text{For normal hierarchy } |m_1|\approx\sqrt{\frac{\Delta m_{\rm sol}^2}{3}}\left(1\pm\frac{4\sqrt{3}}{9}\sqrt{\frac{\Delta m_{\rm sol}^2}{\Delta m_{\rm atm}^2}}\right), \tag{37}$$

$$\text{For inverted hierarchy} |m_3|\approx\sqrt{\frac{\Delta m_{\rm atm}^2}{8}}\left(1+\frac{1}{3}\frac{\Delta m_{\rm sol}^2}{\Delta m_{\rm atm}^2}\right). \tag{38}$$

Using the values of $\Delta m_{\rm sol}^2$ and $\Delta m_{\rm atm}^2$ obtained from recent global fits [7] to the neutrino oscillation data, the above equations imply

$$\begin{aligned}\text{For normal hierarchy} |m_1| &\approx 5.7\times10^{-3}\ \text{eV}\\ \text{or}\quad &\approx 4.4\times10^{-3}\ \text{eV},\end{aligned} \tag{39}$$

$$\text{For inverted hierarchy } |m_3|\approx 0.0179\ \text{eV}. \tag{40}$$

Further, the three mass dependent neutrino observables, namely, (1) the sum of absolute neutrino masses Σm_i, (2) the kinematic electron neutrino mass in beta decay m_β, and (3) the effective mass for the neutrinoless double beta decay $m_{\beta\beta}$, can also be obtained by their approximated expressions given in [18,19]. The presence of a nonzero θ_{13} modifies the predicted values of the observable compared to the models in [18,19]. We determine the effect of a nonzero θ_{13} numerically using Eq. (30). The results of such an analysis are given in Fig. 1.

As can be seen from Fig. 1, all the mass dependent observables vary slightly with the reactor angle. These variations are smaller for inverted hierarchy compared to the normal hierarchy. Results of a similar numerical analysis for purely type-I extension, Eq. (33), are given in Fig. 2.

IV. SUMMARY

The evidence of a possible nonzero θ_{13} requires the modification of the TBM patterns motivated by A_4 and other discrete symmetries. We have proposed a minimal modification which retains the prediction of the maximality of θ_{23}, allows a nonzero θ_{13}, and introduces small

$\mathcal{O}(\theta_{13}^2)$ deviation from the θ_{12} predicted in the TBM. The basis of our proposal is the $Z_2 \times Z_2$ symmetry of the TBM mass matrix. It is shown that combination of this symmetry with the *CP* gives rise to a very predictive structure determined in terms of only four real parameters. The generalized $Z_2 \times Z_2$ can emerge from a simple extension of the standard A_4 schemes if Yukawa couplings and vev are real. The resulting neutrino mass matrix is quite predictive and is determined in terms of only three parameters making it one of the simplest modifications of the TBM scheme consistent with the present information on neutrino masses and mixing angles.

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