Addendum to "General maximum entropy principle for self-gravitating perfect fluid"

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A maximum entropy principle was proven in my recent paper [S. Gao, Phys. Rev. D **84**, 104023 (2011)]. I show that a crucial condition $p = p(\rho)$ used in the proof can be removed and the conclusion remains valid.

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In Ref. [1], I considered a self-gravitating system consisting of perfect fluid with spherical symmetry. By extremizing the total entropy of the fluid, I derived the well-known Tolman-Oppenheimer-Volkoff (TOV) equation. This work showed that the Einstein equation can be derived from ordinary thermodynamic laws. An important assumption used in the proof was that pressure p only depends on density ρ, i.e.,

$$p = p(\rho). \tag{1}$$

Because of this assumption, the results in Ref. [1] cannot be extended to a large class of fluid, e.g., fluid consisting of relativistic particles. The purpose of this addendum is to show that the assumption (1) is not necessary for the derivation in Ref. [1].

The fluid considered in Ref. [1] satisfies the general Gibbs-Duhem relation

$$p = Ts + \mu n - \rho \tag{2}$$

and the first law

$$ds = \frac{1}{T} d\rho - \frac{\mu}{T} dn. \tag{3}$$

By differentiating Eq. (2) and using Eq. (3), we find

$$dp = sdT + nd\mu. \tag{4}$$

This is eq. (47) of Ref. [1]. Now, we shall use a modified argument which does not require the use of Eq. (1). It follows immediately from Eq. (4) that

$$p'(r) = sT'(r) + n\mu'(r). \tag{5}$$

By using the method of Lagrange multipliers, we derived [see eq. (34) of Ref. [1]]

$$\frac{\mu}{T} = \lambda, \tag{6}$$

where λ is a constant. Combining Eqs. (5), (6), and (2), we have

$$T' = \frac{Tp'}{p + \rho}. \tag{7}$$

From Eq. (7), we can obtain the desired TOV equation [1]:

$$p' = -\frac{(p + \rho)(4\pi r^3 p + m)}{r(r - 2m)}. \tag{8}$$

Note that in Ref. [1], Eq. (1) of the current work was used to derive Eq. (7) [eq. (51) of Ref. [1]]. In the above derivation, Eq. (1) is no longer needed, which means that pressure may depend on ρ and another thermodynamic variable like μ, n, etc. By similar arguments, we also derived the generalized TOV equation for a charged perfect fluid in Ref. [1], where Eq. (1) was used to derive Eq. (5) [eq. (80) of [1]]. Now, we see that Eq. (5) can be obtained directly from Eq. (4). So Eq. (1) is not relevant, and the result remains valid. By removing condition (1), the conclusion of Ref. [1] has been significantly strengthened.

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[1] S. Gao, Phys. Rev. D **84**, 104023 (2011).

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