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$\mathcal{N} = 3$ supersymmetric effective action of D2-branes in massive IIA string theory

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We obtain a new type of $\mathcal{N} = 3$ Yang-Mills Chern-Simons theory from the Mukhi-Papageorgakis Higgs mechanism of the $\mathcal{N} = 3$ Gaiotto-Tomasiello theory. This theory has $\mathcal{N} = 1$ BPS fuzzy funnel solution, which is expressed in terms of the seven generators of SU(3), excluding T_8. We propose that this is an effective theory of multiple D2-branes with D6- and D8-branes background in massive IIA string theory.

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I. INTRODUCTION

Three-dimensional Yang-Mills Chern-Simons (YM CS) theories can be realized on brane configurations in type II string theory in two ways. On one hand, one can start with the Hanany-Witten-type brane configuration, which contains D3-branes stretched between two parallel NS5-branes in type IIB string theory [1]. The corresponding gauge theory is $(2 + 1)$-dimensional $\mathcal{N} = 4$ YM theory. When one of the NS5-branes is replaced by a $(1, k)$ 5-brane, a CS term with CS level k arises in the corresponding gauge theories and the supersymmetry is broken to $\mathcal{N} = 1, 2, 3$ depending on the orientation of the $(1, k)$ 5-brane with respect to the other NS5-brane [2,3]. For further progress on this issue, see [4–7]. This method of generating the CS term was also used in the type IIB brane configuration of the Aharony-Bergman-Jafferis-Maldacena (ABJM) theory [8], which describes the dynamics of M2-branes on $\mathbb{C}^4/\mathbb{Z}_k$ orbifold singularity. See also [9–11]. On the other hand, CS terms are also needed in describing D3(2)-branes dynamics in the background of D7(8)-branes [12,13]. In this case, the CS term is generated by the monodromy due to the presence of the D7-brane [14] in type IIB brane configurations. The corresponding CS term in massive IIA brane configurations is obtained by *massive* T duality [15]. This phenomenon is closely related with the brane configuration of the Gaiotto-Tomasiello (GT) theories [16,17]. In particular, by introducing D7-branes to the type IIB brane configuration of ABJM theory, Bergman and Lifschytz constructed the brane configuration of the $\mathcal{N} = 0$ GT theory [18]. For $\mathcal{N} = 3$, see [19].[1]

The dimensional reduction of the ABJM theory with $U(N) \times U(N)$ gauge symmetry [8] via the Mukhi-Papageorgakis (MP) Higgs mechanism [23] results in the $(2 + 1)$-dimensional $\mathcal{N} = 8$ supersymmetric YM theory with $U(N)$ gauge symmetry [24,25]. The $\mathcal{N} = 3$ GT theory [16] was obtained from the ABJM theory by shifting the CS levels of the two gauge groups, so that $k_1 + k_2 \neq 0$. Apparently, the $\mathcal{N} = 3$ GT theory is a minor deformation of the ABJM theory; however, there are unanswered questions about this theory. This is mainly because of the fact that there is no clear argument about the related brane system. In order to clarify this point, we apply the MP Higgs mechanism to the $\mathcal{N} = 3$ GT theory and obtain $(2 + 1)$-dimensional $\mathcal{N} = 3$ YM CS theory with $U(N)$ gauge symmetry and CS level $k_1 + k_2 = F_0$. This $\mathcal{N} = 3$ YM CS theory is different from the one studied in [26,27] because it contains four massless scalar fields and their fermionic superpartners in addition to the three massive scalar fields in the massive vector multiplet, which are also present in the latter theory. It is also true that our theory has $\mathcal{N} = 1$ BPS fuzzy funnel solution and $(R^4 \times S^1)^N/S_N$ vacuum moduli space while these are trivial in the theory in [26,27].

Even though the brane configuration for the original $\mathcal{N} = 3$ GT theory is unclear, the structure of the moduli space and the fuzzy funnel solution provide an insight into the branes configuration for our $\mathcal{N} = 3$ YM CS theory. In this paper, we argue that in massive IIA string theory, YM CS theories with $U(N)$ gauge symmetry describe the low-energy dynamics of N coincident D2-branes in the background of D8-brane.[2] We also show that the presence of three massive and four massless scalar fields, which are matter contents of our $\mathcal{N} = 3$ YM CS theory, implies the branes system should contain D6-branes. More precisely, the brane system includes N

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[1]Some aspects of $\mathcal{N} = 2, 3$ GT theories were also discussed in [20–22].

[2]See [28,29] for earlier considerations in the case of single D2-brane.

coincident D2-branes in the background of $|F_0|$ D8-branes, which have one common spacial direction with the D2-branes, and $|F_0|$ D6-branes, which have two common spacial directions with the D2-branes. The four massless scalar fields represent the position of the D2-branes inside the worldvolume of the D6-branes while the three massive scalar fields represent the position of the D2-branes along the directions transverse to the D6-branes in the presence of the background D8-branes.

The remaining part of the paper is organized as follows. In Sec. II, we apply the MP Higgs mechanism to the $\mathcal{N}=3$ GT theory and obtain $\mathcal{N}=3$ YM CS theory. In Sec. III, we find the vacuum moduli space and $\mathcal{N}=1$ BPS fuzzy funnel solution. In Sec. IV, we propose the brane configuration of the $\mathcal{N}=3$ GT theory and that of our $\mathcal{N}=3$ YM CS theory. In Sec. V, we discuss our results.

II. $\mathcal{N}=3$ YM CS THEORY

A. $\mathcal{N}=3$ GT theory

Based on the $\mathcal{N}=2$ superfield formulation of [16], the component field expansions of the GT theories were obtained in [22]. For clarity of presentation, we copy the Lagrangian of the $\mathcal{N}=3$ GT theory,

$$\mathcal{L}_{\mathcal{N}=3} = \mathcal{L}_0 + \mathcal{L}_{\rm CS} + \mathcal{L}_{\rm ferm} + \mathcal{L}_{\rm bos}, \tag{2.1}$$

where

$$\begin{aligned}
\mathcal{L}_0 &= {\rm tr}[-D_\mu Z_A^\dagger D^\mu Z^A - D_\mu W^{\dagger A} D^\mu W_A + i\xi_A^\dagger \gamma^\mu D_\mu \xi^A + i\omega^{\dagger A}\gamma^\mu D_\mu \omega_A],\\
\mathcal{L}_{\rm CS} &= \frac{k_1}{4\pi}\epsilon^{\mu\nu\rho}{\rm tr}\left(A_\mu\partial_\nu A_\rho + \frac{2i}{3}A_\mu A_\nu A_\rho\right) + \frac{k_2}{4\pi}\epsilon^{\mu\nu\rho}{\rm tr}\left(\hat{A}_\mu\partial_\nu\hat{A}_\rho + \frac{2i}{3}\hat{A}_\mu\hat{A}_\nu\hat{A}_\rho\right),\\
\mathcal{L}_{\rm ferm} &= -\frac{2\pi i}{k_1}{\rm tr}[(\xi^A\xi_A^\dagger - \omega^{\dagger A}\omega_A)(Z^B Z_B^\dagger - W^{\dagger B}W_B) + 2(Z^A\xi_A^\dagger - \omega^{\dagger A}W_A)(\xi^B Z_B^\dagger - W^{\dagger B}\omega_B)]\\
&\quad - \frac{2\pi i}{k_2}{\rm tr}[(\xi_A^\dagger\xi^A - \omega_A\omega^{\dagger A})(Z_B^\dagger Z^B - W_B W^{\dagger B}) + 2(Z_A^\dagger\xi^A - \omega_A W^{\dagger A})(\xi_B^\dagger Z^B - W_B\omega^{\dagger B})]\\
&\quad - \frac{2\pi}{k_1}{\rm tr}(Z^A\omega_A Z^B\omega_B + \xi^A W_A\xi^B W_B + 2Z^A W_A\xi^B\omega_B + 2Z^A\omega_A\xi^B W_B\\
&\quad - \omega^{\dagger A}Z_A^\dagger\omega^{\dagger B}Z_B^\dagger - W^{\dagger A}\xi_A^\dagger W^{\dagger B}\xi_B^\dagger - 2\omega^{\dagger A}\xi_A^\dagger W^{\dagger B}Z_B^\dagger - 2W^{\dagger A}\xi_A^\dagger\omega^{\dagger B}Z_B^\dagger)\\
&\quad - \frac{2\pi}{k_2}{\rm tr}(\omega_A Z^A\omega_B Z^B + W_A\xi^A W_B\xi^B + 2\omega_A Z^A W_B\xi^B + 2W_A Z^A\omega_B\xi^B\\
&\quad - Z_A^\dagger\omega^{\dagger A}Z_B^\dagger\omega^{\dagger B} - \xi_A^\dagger W^{\dagger A}\xi_B^\dagger W^{\dagger B} - 2\xi_A^\dagger W^{\dagger A}Z_B^\dagger\omega^{\dagger B} - 2\xi_A^\dagger\omega^{\dagger A}Z_B^\dagger W^{\dagger B}),\\
\mathcal{L}_{\rm bos} &= -\frac{4\pi^2}{k_1^2}{\rm tr}[(Z^A Z_A^\dagger + W^{\dagger A}W_A)(Z^B Z_B^\dagger - W^{\dagger B}W_B)(Z^C Z_C^\dagger - W^{\dagger C}W_C)]\\
&\quad - \frac{8\pi^2}{k_1k_2}{\rm tr}[(Z^A Z_A^\dagger - W^{\dagger A}W_A)Z^B(Z_C^\dagger Z^C - W_C W^{\dagger C})Z_B^\dagger\\
&\quad + (Z^A Z_A^\dagger - W^{\dagger A}W_A)W^{\dagger B}(Z_C^\dagger Z^C - W_C W^{\dagger C})W_B]\\
&\quad - \frac{4\pi^2}{k_2^2}{\rm tr}[(Z_A^\dagger Z^A + W_A W^{\dagger A})(Z_B^\dagger Z^B - W_B W^{\dagger B})(Z_C^\dagger Z^C - W_C W^{\dagger C})]\\
&\quad - 4{\rm tr}\Bigg[\left(\frac{2\pi}{k_1}W_A Z^B W_B + \frac{2\pi}{k_2}W_B Z^B W_A\right)\left(\frac{2\pi}{k_1}W^{\dagger C}Z_C^\dagger W^{\dagger A} + \frac{2\pi}{k_2}W^{\dagger A}Z_C^\dagger W^{\dagger C}\right)\\
&\quad + \left(\frac{2\pi}{k_1}Z^B W_B Z^A + \frac{2\pi}{k_2}Z^A W_B Z^B\right)\left(\frac{2\pi}{k_1}Z_A^\dagger W^{\dagger C}Z_C^\dagger + \frac{2\pi}{k_2}Z_C^\dagger W^{\dagger C}Z_A^\dagger\right)\Bigg].
\end{aligned} \tag{2.2}$$

In $\mathcal{N}=2$ superfield formalism, Z^A and W_A ($A=1,2$) are the scalar components of chiral superfields $\mathcal{Z}^A$ and $\mathcal{W}_A$, respectively, whereas ξ^A and ω_A are their fermionic superpartners. A_μ and $\hat{A}_\mu$ are the vector components of the vector superfields $\mathcal{V}_1$ and $\mathcal{V}_2$, respectively. The $\mathcal{N}=3$ supersymmetry transformation rules for these component fields are as follows [22]:

$$\delta Z^A = i\bar{\epsilon}\xi^A - \eta\omega^{\dagger A}, \qquad \delta W_A = i\bar{\epsilon}\omega_A + \eta\xi_A^\dagger,$$
$$\delta\xi^A = -D_\mu Z^A\gamma^\mu\epsilon - \sigma_1 Z^A\epsilon + Z^A\sigma_2\epsilon - \frac{4\pi i}{k_1}\bar{\epsilon}W^{\dagger B}Z_B^\dagger W^{\dagger A} - \frac{4\pi i}{k_2}\bar{\epsilon}W^{\dagger A}Z_B^\dagger W^{\dagger B} + iD_\mu W^{\dagger A}\gamma^\mu\eta$$
$$+ i\eta\sigma_1 W^{\dagger A} - i\eta W^{\dagger A}\sigma_2 + \frac{4\pi i}{k_1}\eta W^{\dagger B}Z_B^\dagger Z^A + \frac{4\pi i}{k_2}\eta Z^A Z_B^\dagger W^{\dagger B},$$
$$\delta\omega_A = -D_\mu W_A\gamma^\mu\epsilon + W_A\sigma_1\epsilon - \sigma_2 W_A\epsilon - \frac{4\pi i}{k_1}\bar{\epsilon}Z_A^\dagger W^{\dagger B}Z_B^\dagger - \frac{4\pi i}{k_2}\bar{\epsilon}Z_B^\dagger W^{\dagger B}Z_A^\dagger - iD_\mu Z_A^\dagger\gamma^\mu\eta$$
$$+ i\eta Z_A^\dagger\sigma_1 - i\eta\sigma_2 Z_A^\dagger - \frac{4\pi i}{k_1}\eta W_A W^{\dagger B}Z_B^\dagger - \frac{4\pi i}{k_2}\eta Z_B^\dagger W^{\dagger B}W_A,$$
$$\delta A_\mu = \tfrac{1}{2}(\bar{\epsilon}\gamma_\mu\bar{\chi}_1 + \chi_1\gamma_\mu\epsilon) - \tfrac{1}{2}(\eta\gamma_\mu\zeta_1 - i\bar{\zeta}_1\gamma_\mu\eta), \qquad \delta\hat{A}_\mu = \tfrac{1}{2}(\bar{\epsilon}\gamma_\mu\bar{\chi}_2 + \chi_2\gamma_\mu\epsilon) + \tfrac{1}{2}(\eta\gamma_\mu\zeta_2 - i\bar{\zeta}_2\gamma_\mu\eta), \tag{2.3}$$

where ϵ and $\bar{\epsilon}$ are a complex two-component spinor and its complex conjugate, whereas η is a complex spinor satisfying $\bar{\eta} = -i\eta$. Here, we also defined

$$\sigma_1 \equiv \frac{2\pi}{k_1}(Z^B Z_B^\dagger - W^{\dagger B}W_B), \quad \sigma_2 \equiv -\frac{2\pi}{k_2}(Z_B^\dagger Z^B - W_B W^{\dagger B}),$$
$$\chi_1 \equiv -\frac{4\pi}{k_1}(Z^A\xi_A^\dagger - \omega^{\dagger A}W_A), \quad \chi_2 \equiv \frac{4\pi}{k_2}(\xi_A^\dagger Z^A - W_A\omega^{\dagger A}),$$
$$\zeta_1 \equiv \frac{4\pi}{k_1}(\xi^A W_A + Z^A\omega_A), \quad \zeta_2 \equiv \frac{4\pi}{k_2}(W_A\xi^A + \omega_A Z^A). \tag{2.4}$$

In the next subsection, we apply the MP Higgs mechanism to the Lagrangian (2.1) and the corresponding supersymmetry transformation rules (2.3) and obtain the $\mathcal{N} = 3$ YM CS theory.

B. MP Higgs mechanism of the $\mathcal{N} = 3$ GT theory

An important step in the MP Higgs mechanism is to turn on vacuum expectation value v for a scalar field along which the bosonic potential is flat. The only flat directions for the bosonic potential in the $\mathcal{N} = 3$ GT theory are the tilted directions, $Z^1 \pm W^{\dagger 2}$ and $Z^2 \pm W^{\dagger 1}$. In order to turn on the vacuum expectation value for a specific field, it is convenient to make field redefinitions that align the scalars along the flat directions of the potential. The appropriate field redefinitions for bifundamental fields are

$$Z^A = \frac{X^A - Y^{\dagger A}}{\sqrt{2}}, \qquad W^{\dagger A} = \frac{\sigma_B^A(X^B + Y^{\dagger B})}{\sqrt{2}},$$
$$\xi^A = \frac{\chi^A - i\lambda^{\dagger A}}{\sqrt{2}}, \qquad \omega^{\dagger A} = \frac{\sigma_B^A(\lambda^{\dagger B} - i\chi^B)}{\sqrt{2}}, \tag{2.5}$$

where σ_B^A is the Pauli matrix σ_1. The $\mathcal{N} = 3$ GT Lagrangian is rewritten in terms of the redefined fields in Appendix A.

The MP Higgs mechanism of the ABJM theory involves a double-scaling limit of large vacuum expectation value and CS level k, keeping the ratio v/k finite. This can be applicable to the GT theory by setting $k_1 = k$ and $k_2 = -k + F_0$ and taking the same scaling limit.[3] The appearance of the Chern-Simons levels in the fermionic and bosonic potentials suggests the following expansions in powers of $1/k$ for finite F_0:

$$\frac{1}{k_2} = -\frac{1}{k}\left(1 + \frac{F_0}{k} + \cdots\right),$$
$$\frac{1}{k_2^2} = \frac{1}{k^2}\left(1 + \frac{2F_0}{k} + \frac{3F_0^2}{k^2} + \cdots\right), \tag{2.6}$$
$$\frac{1}{k_1 k_2} = -\frac{1}{k^2}\left(1 + \frac{F_0}{k} + \frac{F_0^2}{k^2} + \cdots\right).$$

We proceed by turning on the vacuum expectation value for a scalar field, which breaks the gauge symmetry from $U(N) \times U(N)$ to $U(N)$ as follows:

$$X^A = \tilde{X}^A + i\tilde{X}^{A+4},$$
$$Y^{\dagger A} = \frac{v}{2}T^0\delta^{A2} + \tilde{X}^{A+2} + i\tilde{X}^{A+6},$$
$$\chi^A = \psi_A + i\psi_{A+4}, \qquad \lambda^{\dagger A} = \psi_{A+2} + i\psi_{A+6}, \tag{2.7}$$

where T^0 is the generator of U(1). Here, the fields $\tilde{X}^i (i = 1, \cdots, 8)$ and $\psi_r (r = 1, \cdots, 8)$ are Hermitian and transform in the adjoint representation of the unbroken $U(N)$ gauge group. In the double-scaling limit of $v, k \to \infty$ with finite v/k, the covariant derivatives for the bosonic and fermionic fields are written as

$$D_\mu Y^{\dagger 2} = \tilde{D}_\mu\tilde{X}^4 + iv\left(A_\mu^- + \frac{1}{v}\tilde{D}_\mu\tilde{X}^8\right) \to \tilde{D}_\mu\tilde{X}^4 + ivA_\mu^-,$$
$$D_\mu Y^{\dagger 1} = \tilde{D}_\mu\tilde{X}^3 + i\tilde{D}_\mu\tilde{X}^7,$$
$$D_\mu X^A = \tilde{D}_\mu\tilde{X}^A + i\tilde{D}_\mu\tilde{X}^{A+4}, \tag{2.8}$$
$$D_\mu\xi^A = \tilde{D}_\mu\psi^A + i\tilde{D}_\mu\psi^{A+4},$$
$$D_\mu\omega^{\dagger A} = \tilde{D}_\mu\psi^{A+2} + i\tilde{D}_\mu\psi^{A+6},$$

where $A_\mu^\pm = \frac{1}{2}(A_\mu \pm \hat{A}_\mu)$, $\tilde{D}_\mu\tilde{X} = \partial_\mu\tilde{X} + i[A_\mu^+, \tilde{X}]$, and we have made the gauge choice $A_\mu^- \to A_\mu^- - \frac{1}{v}\tilde{D}_\mu\tilde{X}^8$ in

[3] In massive type IIA gravity, which is the gravity dual of the GT theory, F_0 is identified as the Romans mass [30].

the first line. In writing (2.8), we have also used the fact that the auxiliary field A^-_μ is of the order $\frac{1}{v}$ and neglected terms of this order or higher.

Using (2.7) and (2.8), from the kinetic and the Chern-Simons terms in the $\mathcal{N} = 3$ GT Lagrangian we obtain

$$\begin{aligned}\mathcal{L}_0 + \mathcal{L}_{\rm CS} = {\rm tr}\Big[& -\tilde{D}_\mu \tilde{X}^i \tilde{D}^\mu \tilde{X}^i - v^2 A^-_\mu A^{-\mu} \\ & + \frac{k}{2\pi}\epsilon^{\mu\nu\rho} A^-_\mu \tilde{F}_{\nu\rho} + i\psi_r \gamma^\mu \tilde{D}_\mu \psi_r \\ & + \frac{F_0}{4\pi}\epsilon^{\mu\nu\rho}\Big(A^+_\mu \partial_\nu A^+_\rho + \frac{2i}{3}A^+_\mu A^+_\nu A^+_\rho\Big)\Big] \\ & + \mathcal{O}\Big(\frac{1}{v}\Big),\end{aligned} \tag{2.9}$$

where $\tilde{F}_{\mu\nu} = \partial_\mu A^+_\nu - \partial_\nu A^+_\mu + i[A^+_\mu, A^+_\nu]$ and $i = 1, \cdots, 7$ since $\tilde{X}^8$ is eliminated by the gauge choice. Solving the equation of motion for the auxiliary gauge field A^-_μ, we can expresses it in terms of the field strength of the dynamical gauge field A^+_μ as

$$A^-_\mu = \frac{k}{4\pi v^2}\epsilon_\mu{}^{\nu\rho}\tilde{F}_{\nu\rho} = \frac{1}{2gv}\epsilon_\mu{}^{\nu\rho}\tilde{F}_{\nu\rho}, \tag{2.10}$$

where $g = \frac{2\pi v}{k}$ is the Yang-Mills coupling. For dimensional reasons, it is also necessary to rescale all the matter fields as $\phi \rightarrow \frac{1}{g}\phi$. Then, we obtain

$$\begin{aligned}\mathcal{L}_0 + \mathcal{L}_{\rm CS} &= \tilde{\mathcal{L}}_{\rm YM} + \tilde{\mathcal{L}}_0 + \tilde{\mathcal{L}}_{\rm CS} \\ &= \frac{1}{g^2}{\rm tr}\Big[-\frac{1}{2}\tilde{F}_{\mu\nu}\tilde{F}^{\mu\nu} - \tilde{D}_\mu \tilde{X}^i \tilde{D}^\mu \tilde{X}^i \\ &\quad + i\psi_r\gamma^\mu \tilde{D}_\mu \psi_r + \frac{F_0 g^2}{4\pi}\epsilon^{\mu\nu\rho}\Big(A^+_\mu \partial_\nu A^+_\rho \\ &\quad + \frac{2i}{3}A^+_\mu A^+_\nu A^+_\rho\Big)\Big].\end{aligned} \tag{2.11}$$

Using (2.6) and (2.7), the Higgs mechanism of the potential terms is tedious but straightforward. In particular, from the fermionic potential we obtain Yukawa-type coupling and fermionic mass term, which are given by

$$\tilde{\mathcal{L}}_{\rm ferm} = {\rm tr}\Big(\frac{iF_0}{4\pi}\mu^{rs}\psi_r\psi_s - \frac{1}{g^2}\Gamma^{rs}_i \psi_r[\tilde{X}^i, \psi_s]\Big), \tag{2.12}$$

where Γ^{rs}_i's are seven-dimensional Euclidian gamma matrices in a particular representation that is determined by the Higgs mechanism (see Appendix B), and μ^{rs} is fermionic mass matrix given by

$$\mu = \frac{1}{2}\begin{pmatrix} -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{pmatrix}.$$

For convenience, we write the Lagrangian in terms of the fermionic fields that are eigenstates of this mass matrix. The mass matrix can be diagonalized by an orthogonal matrix as follows:

$$\tilde{\mu} = O^T \mu O = {\rm diag}(-1, 1, 1, 1, 0, 0, 0, 0), \tag{2.13}$$

where O is given by

$$O = \frac{1}{\sqrt{2}}\begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}.$$

Then, the fermionic mass eigenstates are

$$\begin{aligned}\tilde{\psi}_r &= \psi_s O^{sr} \\ &= \frac{1}{\sqrt{2}}(\psi_5 - \psi_1, \psi_8 + \psi_4, \psi_7 - \psi_3, \psi_6 + \psi_2, \psi_8 \\ &\quad - \psi_4, \psi_7 + \psi_3, \psi_6 - \psi_2, \psi_5 + \psi_1).\end{aligned} \tag{2.14}$$

This transformation also modifies the gamma matrices as

$$\tilde{\Gamma}_i = O^T \Gamma_i O. \tag{2.15}$$

Then, we can write

$$\tilde{\mathcal{L}}_{\rm ferm} = \frac{1}{g^2}{\rm tr}\Big(\frac{iF_0 g^2}{4\pi}\tilde{\mu}^{rs}\tilde{\psi}_r\tilde{\psi}_s - \tilde{\Gamma}^{rs}_i\tilde{\psi}_r[\tilde{X}^i, \tilde{\psi}_s]\Big). \tag{2.16}$$

The fermionic kinetic term in (2.11) is invariant under the transformation (2.14).

The Higgs mechanism of the bosonic potential is even more involved than that of the fermionic potential; however, the algebraic procedure is similar. As a result of such lengthy algebra, we obtain[4]

$$\tilde{\mathcal{L}}_{\rm bos} = \frac{1}{g^2}\,{\rm tr}\bigg(-\frac{F_0^2 g^4}{16\pi^2} M_{ij}\tilde{X}^i\tilde{X}^j - \frac{iF_0 g^2}{2\pi}\tilde{T}_{ijk}\tilde{X}^i[\tilde{X}^j,\tilde{X}^k] + \frac{1}{2}[\tilde{X}^i,\tilde{X}^j]^2\bigg), \quad (2.17)$$

where the nonvanishing components of the bosonic mass matrix M_{ij} and the antisymmetric constant tensor $\tilde{T}_{ijk}$ are

$$M_{33} = M_{44} = M_{55} = 1,$$
$$\tilde{T}_{567} = -\tilde{T}_{468} = \tilde{T}_{369} = \tilde{T}_{345} = \tilde{T}_{378} = \tilde{T}_{479} = \tilde{T}_{589} = \tfrac{1}{6}. \quad (2.18)$$

In summary, the total Lagrangian after the Higgs mechanism is written as

$$\tilde{\mathcal{L}} = \tilde{\mathcal{L}}_{\rm YM} + \tilde{\mathcal{L}}_{F_0}, \quad (2.19)$$

where

$$\tilde{\mathcal{L}}_{\rm YM} = \frac{1}{g^2}\,{\rm tr}\bigg(-\frac{1}{2}\tilde{F}_{\mu\nu}\tilde{F}^{\mu\nu} - \tilde{D}_\mu\tilde{X}^i\tilde{D}^\mu\tilde{X}^i + i\psi_r\gamma^\mu\tilde{D}_\mu\psi_r - \tilde{\Gamma}_i^{rs}\tilde{\psi}_r[\tilde{X}^i,\tilde{\psi}_s] + \frac{1}{2}[\tilde{X}^i,\tilde{X}^j]^2\bigg),$$
$$\tilde{\mathcal{L}}_{F_0} = \frac{F_0}{4\pi}\,{\rm tr}\bigg(\epsilon^{\mu\nu\rho}\Big(A_\mu^+\partial_\nu A_\rho^+ + \frac{2i}{3}A_\mu^+A_\nu^+A_\rho^+\Big) + i\tilde{\mu}^{rs}\tilde{\psi}_r\tilde{\psi}_s - 2i\tilde{T}_{ijk}\tilde{X}^i[\tilde{X}^j,\tilde{X}^k] - \frac{F_0 g^2}{4\pi}M_{ij}\tilde{X}^i\tilde{X}^j\bigg). \quad (2.20)$$

This is the $\mathcal{N} = 3$ YM CS theory anticipated at the end of the previous subsection. For vanishing F_0, this reduces to $\mathcal{N} = 8$ super YM theory as expected. In literature, (2 + 1)-dimensional $\mathcal{N} = 3$ YM CS theory was already studied [26,27]. In this case, the theory can be obtained from the $\mathcal{N} = 4$ YM theory by adding a CS term, which breaks one supersymmetry. The field contents of the later differ from the field contents of our $\mathcal{N} = 3$ YM CS theory by four massless scalars and their superpartners. The Lagrangian of [26,27] can also be obtained by turning off four scalar fields $\tilde{X}^{6,7,8,9}$ and four Majorana fermions $\tilde{\psi}_{5,6,7,8}$ in our YM CS Lagrangian.

The supersymmetry transformation rules of (2.19) are obtained as a result of the Higgs mechanism of the corresponding transformation rules in the original GT theory given in (2.3),

$$\delta A_\mu^+ = i\epsilon_r\gamma_\mu\tilde{\psi}_r, \qquad \delta\tilde{X}^i = i\tilde{\Gamma}_i^{rs}\epsilon_r\tilde{\psi}_s,$$
$$\delta\tilde{\psi}_r = i\tilde{F}_{\mu\nu}\sigma^{\mu\nu}\epsilon_r + \tilde{\Gamma}_i^{rs}\gamma^\mu\epsilon_s\tilde{D}_\mu\tilde{X}^i - \tilde{\Gamma}_{ij}^{rs}\epsilon_s[\tilde{X}^i,\tilde{X}^j] - \frac{F_0 g^2}{4\pi}\tilde{\mu}^{rs}\tilde{\Gamma}_i^{st}\epsilon_t\tilde{X}^i, \quad (2.21)$$

where the nonvanishing supersymmetry parameters are

$$\epsilon_2 = -\frac{1+i}{2\sqrt{2}}(\bar{\epsilon} - i\epsilon), \qquad \epsilon_3 = -\frac{1-i}{\sqrt{2}}\eta, \qquad \epsilon_4 = \frac{1-i}{2\sqrt{2}}(\bar{\epsilon} + i\epsilon), \quad (2.22)$$

and

$$\sigma^{\mu\nu} = -\frac{i}{4}(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu), \qquad \tilde{\Gamma}_{ij} = \frac{i}{4}(\tilde{\Gamma}_i\tilde{\Gamma}_j - \tilde{\Gamma}_j\tilde{\Gamma}_i). \quad (2.23)$$

Actually, the Higgs mechanism of (2.3) gives the supersymmetric transformation rules of the dynamical fields, which are the seven scalar fields $\tilde{X}^i$, the eight fermionic fields $\tilde{\psi}_r$, and the gauge field A_μ^+, as well as the transformation rules for the auxiliary gauge field A_μ^-, and the would-be Goldstone boson $\tilde{X}^8$. However, the fields A_μ^- and $\tilde{X}^8$ are integrated out from the action and their transformation rules, which are not listed in (2.21), are irrelevant.

III. VACUUM MODULI SPACE AND FUZZY FUNNEL SOLUTION

A. Vacuum moduli space

In order to understand the brane configuration for our $\mathcal{N} = 3$ YM CS theory in (2.19), we start by figuring out the vacuum moduli space of the theory. The bosonic potential in (2.17) can be written in a positive-definite form as follows:

$$V_{\rm bos} = \frac{1}{4g^2}\sum_{r=1}^{8}\Big|((1-i)\tilde{\Gamma}_{ij}^{r2} - (1+i)\tilde{\Gamma}_{ij}^{r4})[\tilde{X}^i,\tilde{X}^j] + \beta\tilde{\mu}^{rs}((1-i)\tilde{\Gamma}_i^{s2} - (1+i)\tilde{\Gamma}_i^{s4})\tilde{X}^i\Big|^2, \quad (3.24)$$

where $\beta \equiv \frac{F_0 g^2}{4\pi}$ and we have introduced the notation $|\mathcal{O}|^2 \equiv {\rm tr}\mathcal{O}^\dagger\mathcal{O}$. We obtain the vacuum equations from this positive-definite potential,

$$[\tilde{X}^a,\tilde{X}^b] = 0, \qquad [\tilde{X}^a,\tilde{X}^p] = 0,$$
$$\beta\tilde{X}^3 + i([\tilde{X}^6,\tilde{X}^9] + [\tilde{X}^7,\tilde{X}^8]) = 0,$$
$$\beta\tilde{X}^4 - i([\tilde{X}^6,\tilde{X}^8] - [\tilde{X}^7,\tilde{X}^9]) = 0, \quad (3.25)$$
$$\beta\tilde{X}^5 + i([\tilde{X}^6,\tilde{X}^7] + [\tilde{X}^8,\tilde{X}^9]) = 0,$$

where $a, b = 3, 4, 5$, $p = 6, 7, 8, 9$. The solution of (3.25) is

$$\tilde{X}^a = 0, \qquad \tilde{X}^p = \text{diagonal matrices}. \quad (3.26)$$

[4] For later convenience, we have made renaming of scalar fields as follows: $\tilde{X}^1 \to \tilde{X}^6$, $\tilde{X}^2 \to \tilde{X}^3$, $\tilde{X}^3 \to \tilde{X}^4$, $\tilde{X}^4 \to \tilde{X}^7$, $\tilde{X}^5 \to \tilde{X}^5$, $\tilde{X}^6 \to \tilde{X}^8$, $\tilde{X}^7 \to \tilde{X}^9$. The same renaming applies to the gamma matrices $\tilde{\Gamma}_i$.

The diagonal matrices $\tilde{X}^p$'s represent the full moduli space of the theory. The fact that the $N \times N$ scalar fields are diagonal on the vacuum moduli indicates that the U(N) gauge symmetry of the theory is broken to $\mathrm{U}(1)^N \times S_N$, where the S_N permutes the diagonal elements of the matrices. Thus, the moduli space including the effect of the dual photon in (2 + 1) dimensions is given by

$$\mathcal{M} = \frac{(R^4 \times S^1)^N}{S_N}. \tag{3.27}$$

B. Fuzzy funnel solution

In this subsection, we will obtain fuzzy funnel solution of BPS equations in our $\mathcal{N} = 3$ YM CS theory. The Killing spinor equation of the supersymmetry variation (2.21) is written as

$$\delta\tilde{\psi}_r = i\tilde{F}_{\mu\nu}\sigma^{\mu\nu}\epsilon_r + \tilde{\Gamma}_i^{rs}\gamma^\mu\epsilon_s\tilde{D}_\mu\tilde{X}^i - \tilde{\Gamma}_{ij}^{rs}\epsilon_s[\tilde{X}^i, \tilde{X}^j] - \beta\tilde{\mu}^{rs}\tilde{\Gamma}_i^{st}\epsilon_t\tilde{X}^i = 0. \tag{3.28}$$

In order to obtain a fuzzy funnel solution, we consider the following projection to the supersymmetry parameters, $\gamma^2\epsilon_{2,3} = \epsilon_{2,3}$, and also set $\epsilon_4 = 0$. The resulting fuzzy funnel solution reduces the number of supersymmetries by 1/3, i.e., it has $\mathcal{N} = 1$ supersymmetry. We also assume the vanishing gauge field and a static configuration. Under these conditions, the BPS equations are

$$\begin{aligned} &\tilde{\Gamma}_i^{rs}\partial_1\tilde{X}^i = 0, \\ &\tilde{\Gamma}_i^{rs}\partial_2\tilde{X}^i - \tilde{\Gamma}_{ij}^{rs}[\tilde{X}^i, \tilde{X}^j] - \beta\mu^{rt}\tilde{\Gamma}_i^{ts}\tilde{X}^i = 0. \end{aligned} \tag{3.29}$$

The first line of (3.29) can be satisfied by choosing a configuration which does not depend on x_1 direction. From the second line of (3.29), we have

$$\begin{aligned} &\partial_2\tilde{X}^3 - i[\tilde{X}^4, \tilde{X}^5] = 0, \qquad \partial_2\tilde{X}^5 - i[\tilde{X}^3, \tilde{X}^4] = 0, \\ &\partial_2\tilde{X}^4 - \beta\tilde{X}^4 + i([\tilde{X}^6, \tilde{X}^8] + [\tilde{X}^3, \tilde{X}^5] - [\tilde{X}^7, \tilde{X}^9]) = 0, \\ &\beta\tilde{X}^3 + i([\tilde{X}^6, \tilde{X}^9] + [\tilde{X}^7, \tilde{X}^8]) = 0, \\ &\beta\tilde{X}^5 + i([\tilde{X}^6, \tilde{X}^7] + [\tilde{X}^8, \tilde{X}^9]) = 0, \\ &\partial_2\tilde{X}^6 - i[\tilde{X}^4, \tilde{X}^8] = 0, \qquad \partial_2\tilde{X}^7 + i[\tilde{X}^4, \tilde{X}^9] = 0, \\ &\partial_2\tilde{X}^8 - i[\tilde{X}^6, \tilde{X}^4] = 0, \qquad \partial_2\tilde{X}^9 - i[\tilde{X}^4, \tilde{X}^7] = 0, \\ &[\tilde{X}^6, \tilde{X}^5] - [\tilde{X}^3, \tilde{X}^8] = 0, \qquad [\tilde{X}^6, \tilde{X}^3] + [\tilde{X}^5, \tilde{X}^8] = 0, \\ &[\tilde{X}^3, \tilde{X}^7] + [\tilde{X}^5, \tilde{X}^9] = 0, \qquad [\tilde{X}^3, \tilde{X}^9] + [\tilde{X}^7, \tilde{X}^5] = 0. \end{aligned} \tag{3.30}$$

Comparing the equation in the second line with the remaining equations, it appears natural to divide it into the following two equations:

$$\begin{aligned} &\beta\tilde{X}^4 - i([\tilde{X}^6, \tilde{X}^8] - [\tilde{X}^7, \tilde{X}^9]) = 0, \\ &\partial_2\tilde{X}^4 + i[\tilde{X}^3, \tilde{X}^5] = 0. \end{aligned} \tag{3.31}$$

Then, from the first and the second lines of (3.30) we obtain

$$\partial_2\tilde{X}^a = i\epsilon^{abc}[\tilde{X}^b, \tilde{X}^c], \qquad (a, b, c = 3, 4, 5). \tag{3.32}$$

These are the Nahm equations with the fuzzy two-sphere solution, in which the scalar fields $\tilde{X}^{3,4,5}$ are proportional to the generators of SU(2). However, the fuzzy two sphere configuration does not satisfy the remaining equations in (3.30). It is also important to notice that there is no nontrivial solution satisfying the Eqs. (3.30) in the case of U(2) gauge group. For $N \geq 3$, an interesting solution exists and it can be expressed in terms of seven generators of SU(3). Explicitly, we can write

$$\tilde{X}^3 = g(x_2)T_1, \qquad \tilde{X}^4 = g(x_2)T_2, \qquad \tilde{X}^5 = g(x_2)T_3, \tag{3.33}$$

where $T_{1,2,3}$'s are the SU(2) subgroup elements of N-dimensional representation of SU(3). Then, from (3.32) we easily obtain

$$g(x_2) = \frac{1}{x_2}.$$

The remaining equations of (3.30) can be solved by choosing $\tilde{X}^{6,7,8,9}$ in terms of the rest of generators of SU(3), excluding T_8,

$$\begin{aligned} &\tilde{X}^6 = h(x_2)T_4, \qquad \tilde{X}^7 = h(x_2)T_5, \\ &\tilde{X}^8 = -h(x_2)T_6, \qquad \tilde{X}^9 = h(x_2)T_7, \end{aligned} \tag{3.34}$$

where

$$h(x_2) = \pm\sqrt{\beta g(x_2)} = \pm\sqrt{\frac{\beta}{x_2}}.$$

Here, we would like to point out an important difference between our $\mathcal{N} = 3$ YM CS theory and a similar theory in [26,27]. As we pointed out before, the latter theory can be obtained from ours by turning off four massless scalar fields, $\tilde{X}^{6,7,8,9}$, which means in that case the fuzzy funnel solution in (3.33) and (3.34) is not allowed for nonvanishing β. As we will see in the next section, together with the vacuum moduli space, this $\mathcal{N} = 1$ BPS solution provides useful insights about the brane configuration of our theory.

IV. BRANE CONFIGURATION

A. Generation of CS terms

In order to pave a way for the understanding of the brane configuration, which can be described by our YM CS theory obtained in Sec. II, we briefly summarize some brane configurations in the literature. These brane configurations are described by gauge theories involving CS terms. We start with a type IIB brane system where two parallel NS5-branes are separated along one direction of the worldvolume of N D3-branes. The remaining two worldvolume coordinates of the D3-branes are parallel to the corresponding coordinates of NS5-branes. In the low- energy limit, this configuration is described by $(2+1)$-dimensional $\mathcal{N} = 4$ YM theory with gauge group $U(N)$ [1], where all fields transform in the adjoint representations. Since the two NS5-branes are parallel, the three scalar fields, representing the positions of the D3-branes inside the worldvolume of NS5-branes, are massless.

Now, we replace one of the NS5-branes with a $(1, k)$ 5-brane (a bound state of an NS5-brane and k D5-branes) in a tilted direction with respect to the other NS5-brane. Then, the D3-branes cannot move freely and this fact translates into mass terms for the three scalar fields on the field theory side. The $\mathcal{N} = 4$ supersymmetry of the original theory is broken to $\mathcal{N} = 1, 2, 3$ theories, depending on the choice of the direction of the $(1, k)$ 5-brane. The corresponding effective field theories for these cases are obtained by including the CS terms with CS level k in supersymmetric ways [2,3]. Such CS term is introduced in order to cancel the surface term originated from the boundary condition of the $(1, k)$ 5-brane in the equation of motion of the gauge field [2,3].

The brane configuration of the ABJM theory [8] is based on the brane system of the $\mathcal{N} = 3$ YM CS theory [26,27]. An important difference is the fact that the two parallel NS5-branes are separated along a compact direction of the worldvolume of N D3-branes. In this case, the D3-branes, which wind around the compact direction, can break on the NS5-branes resulting in a $(2+1)$-dimensional $\mathcal{N} = 3$ YM CS gauge theory with gauge group $U_k(N) \times U_{-k}(N)$ [8]. At the infrared fixed point, this becomes conformal and the supersymmetry is enhanced to $\mathcal{N} = 6$. One can also add l fractional D3-branes, suspended on one side of the interval between the NS5-brane and the $(1, k)$ 5-brane. Then, the corresponding effective field theory becomes $\mathcal{N} = 3$ YM CS theory with gauge group $U(N+l)_k \times U(N)_{-k}$ or $U(N)_k \times U(N+l)_{-k}$ depending on the side on which the fractional D3-branes are added [10].

Along a different line of thought, CS terms are also required in order to describe brane systems involving D7- or D8-branes [12,13,18]. The configuration with D8-branes can be understood by the massive T dualization of that of D7-branes [15]. In [18], a D7-brane was added to the brane configuration of the ABJM theory as follows:

	0	1	2	3	4	5	6	7	8	9
N D3	•	•	•				•			
1 D7	•	•	•	•	•			•	•	•

Here, we have omitted 5-branes for simplicity. This configuration breaks the entire supersymmetry. Since the D7-brane is a pointlike object in the (x_5, x_6) plane, it sources a $SL(2, \mathbb{Z})$ monodromy on the plane, $\tau \to \tau + 1$,[5] i.e., $C_0 \to C_0 + 2\pi$ for the axion. This monodromy is the result of a branch cut emanated from the D7-brane with the direction of the cut chosen to cross the D3-branes. Then, the Wess-Zumino-type coupling for the D3-branes generates a CS term:

$$\int_{R^{2+1}} \int_{x_6} C_0 \mathrm{tr}(F \wedge F) \sim S^{\mathrm{CS}}_{R^{2+1}}(A). \qquad (4.35)$$

To summarize, we have seen two ways to generate the CS term in the descriptions of brane configurations in $(2+1)$-dimensional gauge theories. The CS term in our $\mathcal{N} = 3$ YM CS theory is related to the configuration involving D7- or D8-branes. In the next subsection, we construct the brane configuration for our $\mathcal{N} = 3$ YM CS theory starting with type IIB brane system involving D7-branes.

B. Massive IIA brane configuration

The type IIA string theory on $\mathrm{AdS}_4 \times \mathbb{CP}^3$ with q D8-branes ($q = |F_0|$) wrapped on $\mathbb{CP}^3$ was proposed as a dual gravity of the $\mathcal{N} = 3$ GT theory [19]. Based on this and the type IIB brane configuration of the $\mathcal{N} = 6$ ABJM theory, we propose the type IIB brane configuration of the $\mathcal{N} = 3$ GT theory as in Table I: where $\hat{6}$ represents a compact direction and θ is the orientation of the $(1, k)$ 5-brane relative to NS5-brane in (x_3, x_7), (x_4, x_8), and (x_5, x_9) planes, and $\tan\theta = k$, assuming the string coupling $g_s = 1$ and RR axion is vanishing. In addition to the brane configuration of the ABJM theory, this configuration contains q D7-branes and additional q D5-branes in a supersymmetric way. The D7-branes are results of the T dualization of the D8-branes in the proposal of [19], while the additional D5-branes are included in our proposed brane configuration for the reason that we will explain below.

The MP Higgs mechanism in ABJM theory includes two important steps, which are identification of the two gauge fields with each other and moving the M2-branes far away from the orbifold singularity. In the corresponding brane configuration, these actions are interpreted as separating the D3-branes from the five-branes and moving them far away in the transverse directions. After the separation, the T duality along the compact direction will give the brane configuration with coincident D2-branes, and the

TABLE I. The NS5-brane, q D5-branes, and q D7-branes are located at the same point along the x_6 direction.

	0	1	2	3	4	5	$\hat{6}$	7	8	9
N D3	•	•	•				•			
NS5	•	•	•	•	•	•				
$(1, k)$ 5	•	•	•	$\cos\theta$	$\cos\theta$	$\cos\theta$		$\sin\theta$	$\sin\theta$	$\sin\theta$
q D5	•	•	•					•	•	•
q D7	•	•		•	•	•		•	•	•

[5] We define the complex combination of the axion field C_0 and the dilaton field ϕ as $\tau \equiv \frac{C_0}{2\pi} + ie^{-\phi}$.

TABLE II. Type IIB brane configuration for $\mathcal{N} = 3$ GT theory after the MP Higgs mechanism.

	0	1	2	3	4	5	$\hat{6}$	7	8	9
N D3	•	•	•				•			
q D5	•	•	•					•	•	•
q D7	•	•		•	•	•		•	•	•

corresponding effective field theory is the $\mathcal{N} = 8$ YM theory in $(2 + 1)$ dimensions. This procedure does not break supersymmetry.

Even though the M theory uplifting of our proposed brane configuration is unclear, we can apply the MP Higgs mechanism to this brane configuration as well. This corresponds to moving the NS5- and $(1, k)$ 5-brane far away from the D7-D3-D5-brane system in the transverse directions. This results in the type IIB brane configuration with N D3-branes intersecting q D7-branes along one common spatial direction and q D5-branes along two common spatial directions as in Table II: Unlike the brane configuration in [18], ours is supersymmetric. Based on the discussion in the previous subsection, q D7-branes generate a CS term with CS level $\pm q$ depending on the relative orientation of D3- and D7-branes.

Applying massive T duality along $x_{\hat{6}}$ direction from IIB configuration with D7-branes to massive IIA configuration with D8-branes [15], we obtain the brane configuration of Table III: where 6 denotes the new direction appeared after the T dualization along $x_{\hat{6}}$ direction. This brane configuration is expected to coincide with the brane configuration described by the $\mathcal{N} = 3$ CS YM theory discussed in Sec. II.

Next, we use the vacuum moduli space and the fuzzy funnel solution in Sec. III to discuss the importance of D6-branes in the brane configuration of massive IIA string theory in Table III. From the vacuum moduli space in (3.27), we can infer that there are three massive directions for which $\langle \tilde{X}^{3,4,5} \rangle = 0$ and four flat directions for which $\langle \tilde{X}^{6,7,8,9} \rangle =$ diagonal. The former indicates the fact that the D2-branes are not free to move in these directions, while they are free to move in the remaining four transverse directions. This moduli space and the supersymmetry structure in the massive IIA gravity [19] suggest the presence of D6-branes parallel to the D2-branes in addition to D8-branes. Moreover, the $\mathcal{N} = 1$ BPS fuzzy funnel solution, in which the seven transverse scalar fields are proportional to the seven generators (excluding T_8) of SU(3) with x_2-dependent coefficients, also seems to support our brane configuration. The solution is given by $\tilde{X}^{3,4,5} \sim (1/x_2) T_{1,2,3}$ and $\tilde{X}^{6,7,8,9} \sim (1/\sqrt{x_2}) T_{4,5,6,7}$. The $(1/x_2)$ dependence of $\tilde{X}^{3,4,5}$ indicates the localization of the D8-branes along those directions without any interference from the D6-branes. On the other hand, the $(1/\sqrt{x_2})$ dependence of $\tilde{X}^{6,7,8,9}$ indicates mild localization of the D8-branes along those directions due to an interference from the D6-branes that span the x_2 direction. Further evidence for this brane configuration should come from the BPS solutions in the massive IIA gravity. We leave this possibility for future investigation.

TABLE III. Massive IIA brane configuration for $\mathcal{N} = 3$ YM CS theory.

	0	1	2	3	4	5	6	7	8	9
N D2	•	•	•							
q D6	•	•	•				•	•	•	•
q D8	•	•		•	•	•	•	•	•	•

V. CONCLUSION

In this paper, we carried out the MP Higgs mechanism of the $\mathcal{N} = 3$ GT theory and obtained $\mathcal{N} = 3$ YM CS theory in $(2 + 1)$ dimensions with U(N) gauge symmetry. We also verified that the MP Higgs mechanism of the supersymmetry transformation rules of the GT theory results in the corresponding rules in the YM CS theory. Compared to the MP Higgs mechanism of the ABJM theory, the present case is more subtle because of two reasons. First, none of the four complex scalars in the GT theory represent the flat direction of the bosonic potential and they cannot acquire infinitely large vacuum expectation values. We overcame this problem by introducing field redefinitions that rotate the scalars to the flat directions of the bosonic potential. Second, in the GT theory we have two CS levels k_1 and k_2 and it is not clear how to take the large CS level limit. We took k_1, $k_2 \rightarrow \pm\infty$ limit under the assumption that $k_1 + k_2 = F_0$ and F_0 is a finite dimensionless parameter. It turns out that the F_0 is the CS level in the resulting YM CS theory.

Earlier, $\mathcal{N} = 3$ YM CS theory was studied from different perspectives [26,27]. This theory is a deformation of the $\mathcal{N} = 4$ YM theory in $(2 + 1)$ dimensions by a CS term. On the other hand, our $\mathcal{N} = 3$ YM CS theory is a similar deformation of the $\mathcal{N} = 8$ YM theory in $(2 + 1)$ dimensions. By comparing these two theories, one can realize that the former is obtained from the latter by turning off four massless scalars and their fermionic superpartners. An interesting difference between these two theories is the fact that in our theory we could find a nontrivial fuzzy funnel solution to the BPS equations while in their theory such BPS solution does not exist. In addition, the vacuum moduli space in our theory is $(R^4 \times S^1)^N / S_N$, while it is trivial in their theory.

Since the $\mathcal{N}=3$ YM CS theory we obtained in this paper is new, we found it interesting to figure out the brane configuration that can be described by this theory. We proposed that the theory describes the dynamics of N coincident D2-branes in the background of q D6-branes and the same number of D8-branes, q being the absolute value of the CS level F_0. More precisely, the branes system contains N D2-branes extending along the directions $x_{0,1,2}$, q D6-branes along the directions $x_{0,1,2,6,7,8,9}$, and q D8-branes along the directions $x_{0,1,3,4,5,6,7,8,9}$. As a confirmation of our brane configuration, we obtained $\mathcal{N}=1$ BPS fuzzy funnel solution, which indicates the localization of the D8-branes along the x_2 direction and supports the presence of D6-branes.

The massive IIA supergravity [30] is the low-energy limit of the massive IIA string theory. This supergravity theory has many (non)supersymmetric solutions of the form $AdS_4 \times \mathcal{M}_6$ [20,30–37], where $\mathcal{M}_6$ represents a six-dimensional manifold. The supersymmetries of these solutions are less than $\mathcal{N}=3$. Since our massive IIA brane configuration in Sec. IV B has $\mathcal{N}=3$ supersymmetry, finding the corresponding solution in gravity side will be interesting.

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APPENDIX A: $\mathcal{N}=3$ GT LAGRANGIAN AFTER FIELD REDEFINITION

After the field redefinition (2.5), the $\mathcal{N}=3$ GT Lagrangian in (2.1) is rewritten as

$$
\begin{aligned}
\mathcal{L}_0 &= \mathrm{tr}\Big[-D_\mu X_A^\dagger D^\mu X^A - D_\mu Y^{\dagger A} D^\mu Y_A + i\chi_A^\dagger \gamma^\mu D_\mu \chi^A + i\lambda^{\dagger A}\gamma^\mu D_\mu \lambda_A\Big],\\
\mathcal{L}_{\mathrm{CS}} &= \frac{k_1}{4\pi}\epsilon^{\mu\nu\rho}\mathrm{tr}\Big(A_\mu \partial_\nu A_\rho + \frac{2i}{3}A_\mu A_\nu A_\rho\Big) + \frac{k_2}{4\pi}\epsilon^{\mu\nu\rho}\mathrm{tr}\Big(\hat{A}_\mu \partial_\nu \hat{A}_\rho + \frac{2i}{3}\hat{A}_\mu \hat{A}_\nu \hat{A}_\rho\Big),\\
\mathcal{L}_{\mathrm{ferm}} &= \frac{2\pi}{k_1}\,\mathrm{tr}[(\lambda^{\dagger A}\chi_A^\dagger - \chi^A\lambda_A)(X^B Y_B + Y^{\dagger B}X_B^\dagger) + \frac{1}{2}(\lambda^{\dagger A}X_A^\dagger - i\chi^A Y_A - iX^A\lambda_A + Y^{\dagger A}\chi_A^\dagger)\\
&\quad \times(\lambda^{\dagger B}X_B^\dagger - i\chi^B Y_B - iX^B\lambda_B + Y^{\dagger B}\chi_B^\dagger) + \frac{1}{2}(\chi^A X_A^\dagger + i\lambda^{\dagger A}Y_A - Y^{\dagger A}\lambda_A - iX^A\chi_A^\dagger)\\
&\quad \times(\chi^B X_B^\dagger + i\lambda^{\dagger B}Y_B - Y^{\dagger B}\lambda_B - iX^B\chi_B^\dagger) + \sigma^A{}_C\sigma^B{}_D\Big\{(\chi^C\lambda_A + \lambda^{\dagger C}\chi_A^\dagger)(Y^{\dagger D}X_B^\dagger - X^D Y_B)\\
&\quad + i(\lambda^{\dagger C}\lambda_A - \chi^C\chi_A^\dagger)(X^D X_B^\dagger - Y^{\dagger D}Y_B) - \frac{1}{2}(\chi^C X_A^\dagger - i\lambda^{\dagger C}Y_A - Y^{\dagger C}\lambda_A + iX^C\chi_A^\dagger)\\
&\quad \times(\chi^D X_B^\dagger - i\lambda^{\dagger D}Y_B - Y^{\dagger D}\lambda_B + iX^D\chi_B^\dagger) + \frac{1}{2}(\lambda^{\dagger C}X_A^\dagger + i\chi^C Y_A + iX^C\lambda_A + Y^{\dagger C}\chi_A^\dagger)\\
&\quad \times(\lambda^{\dagger D}X_B^\dagger + i\chi^D Y_B + iX^D\lambda_B + Y^{\dagger D}\chi_B^\dagger)\Big\}\Big] + \frac{2\pi}{k_2}\,\mathrm{tr}\Big[(\chi_A^\dagger\lambda^{\dagger A} - \lambda_A\chi^A)(Y_B X^B + X_B^\dagger Y^{\dagger B})\\
&\quad + \frac{1}{2}(X_A^\dagger\lambda^{\dagger A} - iY_A\chi^A - i\lambda_A X^A + \chi_A^\dagger Y^{\dagger A})(X_B^\dagger\lambda^{\dagger B} - iY_B\chi^B - i\lambda_B X^B + \chi_B^\dagger Y^{\dagger B})\\
&\quad + \frac{1}{2}(X_A^\dagger\chi^A + iY_A\lambda^{\dagger A} - \lambda_A Y^{\dagger A} - i\chi_A^\dagger X^A)(X_B^\dagger\chi^B + iY_B\lambda^{\dagger B} - \lambda_B Y^{\dagger B} - i\chi_B^\dagger X^B)\\
&\quad + \sigma_A^C\sigma_B^D\Big\{(\lambda_C\chi^A + \chi_C^\dagger\lambda^{\dagger A})(X_D^\dagger Y^{\dagger B} - Y_D X^B) + i(\lambda_C\lambda^{\dagger A} - \chi_C^\dagger\chi^A)(X_D^\dagger X^B - Y_D Y^{\dagger B})\\
&\quad - \frac{1}{2}(X_C^\dagger\chi^A - iY_C\lambda^{\dagger A} - \lambda_C Y^{\dagger A} + i\chi_C^\dagger X^A)(X_D^\dagger\chi^B - iY_D\lambda^{\dagger B} - \lambda_D Y^{\dagger B} + i\chi_D^\dagger X^B)\\
&\quad + \frac{1}{2}(X_C^\dagger\lambda^{\dagger A} + iY_C\chi^A + i\lambda_C X^A + \chi_C^\dagger Y^{\dagger A})(X_D^\dagger\lambda^{\dagger B} + iY_D\chi^B + i\lambda_D X^B + \chi_D^\dagger Y^{\dagger B})\Big\}\Big],
\end{aligned}
$$

and

$$
\begin{aligned}
\mathcal{L}_{\rm bos} = &-\frac{4\pi^2}{k_1^2}\,\mathrm{tr}\Big[(X^AX_A^\dagger + Y^{\dagger A}Y_A)(X^BY_B + Y^{\dagger B}X_B^\dagger)(X^CY_C + Y^{\dagger C}X_C^\dagger)\\
&+\frac{1}{2}\sigma_B{}^D\sigma^C{}_E(X^BX_D^\dagger + X^BY_D - Y^{\dagger B}X_D^\dagger - Y^{\dagger B}Y_D)\\
&\times(X^AX_A^\dagger - X^AY_A - Y^{\dagger A}X_A^\dagger + Y^{\dagger A}Y_A)\\
&\times(X^EX_C^\dagger - X^EY_C + Y^{\dagger E}X_C^\dagger - Y^{\dagger E}Y_C)\\
&+\frac{1}{2}\sigma_B{}^D\sigma^C{}_E(X_A^\dagger X^B + Y_AX^B - X_A^\dagger Y^{\dagger B} - Y_AY^{\dagger B})\\
&\times(X_D^\dagger X^E + Y_DX^E + X_D^\dagger Y^{\dagger E} + Y_DY^{\dagger E})\\
&\times(X_C^\dagger X^A - Y_CX^A + X_C^\dagger Y^{\dagger A} - Y_CY^{\dagger A})\Big]\\
&-\frac{4\pi^2}{k_2^2}\,\mathrm{tr}\Big[(X_A^\dagger X^A + Y_AY^{\dagger A})(X_B^\dagger Y^{\dagger B} + Y_BX^B)(X_C^\dagger Y^{\dagger C} + Y_CX^C)\\
&+\frac{1}{2}\sigma_B{}^D\sigma^C{}_E(X_D^\dagger X^B + Y_DX^B - X_D^\dagger Y^{\dagger B} - Y_DY^{\dagger B})\\
&\times(X_A^\dagger X^A + Y_AX^A + X_A^\dagger Y^{\dagger A} + Y_AY^{\dagger A})\\
&\times(X_C^\dagger X^E - Y_CX^E + X_C^\dagger Y^{\dagger E} - Y_CY^{\dagger E})\\
&+\frac{1}{2}\sigma_B{}^D\sigma^C{}_E(X^AX_D^\dagger + X^AY_D - Y^{\dagger A}X_D^\dagger - Y^{\dagger A}Y_D)\\
&\times(X^BX_C^\dagger - X^BY_C - Y^{\dagger B}X_C^\dagger + Y^{\dagger B}Y_C)\\
&\times(X^EX_A^\dagger - X^EY_A + Y^{\dagger E}X_A^\dagger - Y^{\dagger E}Y_A)\Big]\\
&-\frac{8\pi^2}{k_1k_2}\,\mathrm{tr}\Big[(X^AY_A + Y^{\dagger A}X_A^\dagger)\{X^B(X_C^\dagger Y^{\dagger C} + Y_CX^C)X_B^\dagger + Y^{\dagger B}(X_C^\dagger Y^{\dagger C} + Y_CX^C)Y_B\}\\
&\times+\frac{1}{4}\sigma_B{}^D\sigma^C{}_E(X^BX_D^\dagger + X^BY_D - Y^{\dagger B}X_D^\dagger - Y^{\dagger B}Y_D)\\
&\times(X^AX_C^\dagger - X^AY_C - Y^{\dagger A}X_C^\dagger + Y^{\dagger A}Y_C)\\
&\times(X^EX_A^\dagger - X^EY_A + Y^{\dagger E}X_A^\dagger - Y^{\dagger E}Y_A)\\
&+\frac{1}{4}\sigma_B{}^D\sigma^C{}_E(X_A^\dagger X^B + Y_AX^B - X_A^\dagger Y^{\dagger B} - Y_AY^{\dagger B})\\
&\times(X_D^\dagger X^A + Y_DX^A + X_D^\dagger Y^{\dagger A} + Y_DY^{\dagger A})(X_C^\dagger X^E - Y_CX^E + X_C^\dagger Y^{\dagger E} - Y_CY^{\dagger E})\\
&+\frac{1}{4}\sigma_B{}^D\sigma^C{}_E(X_D^\dagger X^B + Y_DX^B - X_D^\dagger Y^{\dagger B} - Y_DY^{\dagger B})\\
&\times(X_A^\dagger X^E + Y_AX^E + X_A^\dagger Y^{\dagger E} + Y_AY^{\dagger E})\\
&\times(X_C^\dagger X^A - Y_CX^A + X_C^\dagger Y^{\dagger A} - Y_CY^{\dagger A})\\
&+\frac{1}{4}\sigma_B{}^D\sigma^C{}_E(X^AX_D^\dagger + X^AY_D - Y^{\dagger A}X_D^\dagger - Y^{\dagger A}Y_D)\\
&\times(X^BX_A^\dagger - X^BY_A - Y^{\dagger B}X_A^\dagger + Y^{\dagger B}Y_A)(X^EX_C^\dagger - X^EY_C + Y^{\dagger E}X_C^\dagger - Y^{\dagger E}Y_C)\Big].
\end{aligned}
\tag{A1}
$$

APPENDIX B: SEVEN-DIMENSIONAL EUCLIDEAN GAMMA MATRICES

In Sec. II B, we have seen that the MP Higgs mechanism of the fermionic potential gives the fermionic mass term and Yukawa-type coupling that is expressed in terms of seven-dimensional Euclidean gamma matrices. In this Appendix, we list the gamma matrices that were determined by the Higgs mechanism,

$$\Gamma_1 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \end{pmatrix}, \quad \Gamma_2 = \begin{pmatrix} 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

$$\Gamma_3 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}, \quad \Gamma_4 = \begin{pmatrix} 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \end{pmatrix},$$

$$\Gamma_5 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \quad \Gamma_6 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \end{pmatrix},$$

$$\Gamma_7 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

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