Transport coefficients of the D1-D5-P system and the membrane paradigm

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I discuss a correspondence between string theory and the black hole membrane paradigm in the context of the D1-D5-P system. By using the Kubo formula, I calculate transport coefficients of the effective string model induced by two kinds of minimal scalars. Then, I show that these transport coefficients *exactly* agree with the corresponding membrane transport coefficients of a five-dimensional near-extremal black hole with three charges.

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I. INTRODUCTION

In recent decades, much progress has been made on a correspondence between a black hole and string theory. In [1,2], the Bekenstein-Hawking entropy of a black hole has been derived from a highly excited fundamental string up to a numerical factor. These works have been generalized and it has been conjectured that a highly excited fundamental string becomes a black hole with the same mass and charges when the string coupling is increased, and becomes a critical value which is called the correspondence point [3].

Although the correct numerical factor of the black hole entropy could not be reproduced from the fundamental string, the Bekenstein-Hawking entropy of a five-dimensional extremal black hole with three charges has been found to be exactly equal to the degeneracy of BPS states in a system which is composed of n_1 D1-branes wrapped on S^1 and n_5 D5-branes wrapped on $S^1 \times M_4$, where $M_4 = K3$ or T^4 [4,5]. This system is called the D1-D5-P system. In the case of $M_4 = T^4$, the microscopic states are effectively described by a single D1-brane with winding number $n_1 n_5$ which vibrates only inside T^4. It has been shown that the correct Bekenstein-Hawking entropy of the near extremal five dimensional black hole is reproduced by counting the number of states in the effective string model [5–7]. In addition, the Hawking radiation of minimal scalars has been correctly explained by the effective string model [8–14]. Although the effective string model does not correctly produce fixed scalar emissions of the black hole [15], it is still useful to discuss a coupling of the black hole with some minimal scalars on the string theory side. More appropriate treatment of the D1-D5-P system is given by a $\mathcal{N} = (4, 4)$ superconformal field theory living on a circle [16].

In this paper, I discuss a correspondence between string theory and the black hole membrane paradigm in the context of the D1-D5-P system. The membrane paradigm states that a distant observer sees a fictitious membrane or fluid with some transport coefficients such as viscosities and conductivities on a stretched horizon of a black hole [17,18]. Recently, we have found that the membrane shear viscosity of a neutral black hole agrees with the shear viscosity of highly excited fundamental string states at the correspondence point if the central charge c is 6 [19]. This work has been generalized and I have shown that except for the bulk viscosity, the membrane transport coefficients of an electric NS-NS 2-charged black hole correspond to the transport coefficients of the fundamental string states with a Kaluza-Klein momentum and a winding number at the correspondence point if $c = 6$ [20]. From these results, we can guess that in the D1-D5-P system, the membrane paradigm can be correctly explained by the effective string model because the central charge of the effective string model is 6. In fact, I show that the membrane transport coefficients of the D1-D5-P black hole induced by two kinds of minimal scalars are exactly the same as the corresponding transport coefficients of the effective string model.

This paper is organized as follows. In Sec. II, I review the D1-D5-P black hole and calculate the membrane transport coefficients induced by the minimal scalars. In Sec. III, I introduce the effective string model of the D1-D5-P system and calculate the transport coefficients induced by the minimal scalars by using the Kubo formula. Then, I find that both the transport coefficients are exactly equal. The final section is devoted to the summary and comments.

II. MEMBRANE TRANSPORT COEFFICIENTS

A. D1-D5-P black hole

Let us consider type IIB string theory compactified on $T^4 \times S^1$ and wrap n_5 D5-branes on $T^4 \times S^1$ and n_1 D1-branes on S^1. We also put $\frac{n_p}{R}$ left-moving momentum along the D1-branes, where R is the radius of S^1. This system becomes a five-dimensional extremal black hole with three charges at strong string coupling g_s [5].

The Einstein metric of the five-dimensional extremal black hole is given by [5,16]

$$ds^2 = -f(r)^{-2/3} dt^2 + f(r)^{1/3}(dr^2 + r^2 d\Omega_3^2), \quad (1)$$

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where

$$f(r) = f_1(r) f_5(r) f_p(r), \quad (2)$$

$$f_x(r) = 1 + \frac{r_x^2}{r^2}, \qquad (x = 1, 5, p), \quad (3)$$

$$r_x^2 = c_x n_x, \quad (4)$$

$$c_1 = \frac{g_s \alpha'}{\tilde{v}}, \qquad c_5 = g_s \alpha', \qquad c_p = \frac{g_s^2 \alpha'^2}{\tilde{v} R^2}. \quad (5)$$

Here, $V = (2\pi)^4 \alpha'^2 \tilde{v}$ is the volume of T^4. The event horizon is located at $r = 0$.

To discuss the membrane paradigm, we need a finite radius of the event horizon. The generalization to the nonextremal case is given by the following Einstein metric [6,16]:

$$ds^2 = -h(r) f(r)^{-2/3} dt^2 + f(r)^{1/3} (h(r)^{-1} dr^2 + r^2 d\Omega_3^2), \quad (6)$$

where

$$h(r) = 1 - \frac{r_0^2}{r^2}, \quad (7)$$

$$f(r) = f_1(r) f_5(r) f_p(r), \quad (8)$$

$$f_x(r) = 1 + \frac{r_x^2}{r^2}, \qquad (x = 1, 5, p), \quad (9)$$

$$r_x^2 = r_0^2 \sinh^2 \alpha_x, \quad (10)$$

and r_0 is the horizon radius. The mass and three charges are

$$M = \frac{R \tilde{v} r_0^2}{2 \alpha'^2 g_s^2} (\cosh 2\alpha_1 + \cosh 2\alpha_5 + \cosh 2\alpha_p), \quad (11)$$

$$Q_x = \frac{r_0^2 \sinh 2\alpha_x}{2 c_x}. \quad (12)$$

The extremal limit corresponds to the limit $r_0 \to 0$ with at least one of $\alpha_x \to \infty$, keeping R, $\tilde{v}$ and the associated charges fixed.

This nonextremal black hole can be formally viewed as a system which is composed of noninteracting branes, anti-branes and left-right moving momentum [6]. The numbers of D1-branes, D5-branes, left-moving momentum and their anticounterparts ($\overline{\text{D1}}$-branes, $\overline{\text{D5}}$-branes and right-moving momentum) are

$$n_x = \frac{r_0^2 e^{2\alpha_x}}{4 c_x}, \qquad \bar{n}_x = \frac{r_0^2 e^{-2\alpha_x}}{4 c_x}. \quad (13)$$

In terms of these numbers, the mass and charges are expressed by

$$M = \frac{R}{g_s \alpha'} (n_1 + \bar{n}_1) + \frac{\tilde{v} R}{g_s \alpha'} (n_5 + \bar{n}_5) + \frac{1}{R} (n_p + \bar{n}_p), \quad (14)$$

$$Q_x = n_x - \bar{n}_x. \quad (15)$$

The area of the horizon is

$$\begin{aligned} A_H &= 2\pi^2 r_0^3 \cosh\alpha_1 \cosh\alpha_5 \cosh\alpha_p \\ &= 8\pi G_5 \left(\sqrt{n_1} + \sqrt{\bar{n}_1}\right)\left(\sqrt{n_5} + \sqrt{\bar{n}_5}\right)\left(\sqrt{n_p} + \sqrt{\bar{n}_p}\right), \end{aligned} \quad (16)$$

where

$$G_5 = \frac{\pi g_s^2 \alpha'^2}{4 \tilde{v} R} \quad (17)$$

is the five-dimensional Newton constant. Thus, the Bekenstein-Hawking entropy is [6]

$$\begin{aligned} S_{\text{BH}} &= \frac{A_H}{4 G_5} \\ &= 2\pi \left(\sqrt{n_1} + \sqrt{\bar{n}_1}\right)\left(\sqrt{n_5} + \sqrt{\bar{n}_5}\right)\left(\sqrt{n_p} + \sqrt{\bar{n}_p}\right). \end{aligned} \quad (18)$$

In this paper, we assume the dilute gas regime [12],

$$r_1, \qquad r_5 \gg r_0, \qquad r_p, \quad (19)$$

and the near extremality,

$$n_p \gg \bar{n}_p. \quad (20)$$

The near extremality will be necessary for perturbative string calculations to be valid at the strong coupling regime.

Then, the area of the horizon and the Bekenstein-Hawking entropy become

$$A_H = 8\pi G_5 \left(\sqrt{n_1 n_5 n_p} + \sqrt{n_1 n_5 \bar{n}_p}\right), \quad (21)$$

$$S_{\text{BH}} = 2\pi \left(\sqrt{n_1 n_5 n_p} + \sqrt{n_1 n_5 \bar{n}_p}\right), \quad (22)$$

because $\bar{n}_1, \bar{n}_5 = 0$.

B. Membrane transport coefficients induced by minimal scalars

We consider the fluctuations of the off-diagonal metric components of T^4 and the six-dimensional dilaton around the near extremal black hole solution, which are denoted by $h_{ij} \equiv f_1^{-1/2} f_5^{1/2} \delta G_{ij}$ $(i, j = 6, 7, 8, 9)$ and ϕ, respectively. They are called minimal scalars. The action for these scalars is given by [13,16]

$$S = \frac{1}{16\pi G_5} \int d^5 x \sqrt{-g} \left[-\frac{1}{4} \sum_{\substack{i,j=6 \\ i\neq j}}^{9} \partial_\mu h_{ij} \partial^\mu h_{ij} - \partial_\mu \phi \partial^\mu \phi \right], \tag{23}$$

where $\mu, \nu = 0, 1, 2, 3, 4$.

Let us calculate the membrane transport coefficient of the near extremal black hole induced by h_{ij} [18,20,21]. It is enough to show it in the case of h_{67}. Let us set $h_{67} \equiv h$. By varying the action with respect to h, one finds the following boundary term on the horizon surface Σ:

$$\delta S = \frac{1}{16\pi G_5} \int_\Sigma d^4 x \sqrt{-\gamma} n^\mu \delta h \nabla_\mu h, \tag{24}$$

where $\gamma_{\mu\nu}$ is the induced metric on Σ. This boundary term is unnecessary for the bulk equation of motion to hold on Σ. To cancel this boundary term, we add the following surface term to the action:

$$S_{\rm surf} = \int_\Sigma d^4 x \sqrt{-\gamma} J_h h. \tag{25}$$

Then, we find

$$J_h = -\frac{1}{16\pi G_5} n^\mu \nabla_\mu h. \tag{26}$$

J_h is interpreted as a charge density on the stretched horizon induced by the bulk field h. Since the Einstein metric of the black hole solution (6) takes the following form,

$$ds^2 = -g_{tt}(r) dt^2 + g_{rr}(r) dr^2 + f^{1/3}(r) r^2 d\Omega_3^2, \tag{27}$$

the membrane charge density becomes

$$J_h = -\frac{1}{16\pi G_5} \frac{1}{\sqrt{g_{rr}}} \partial_r h|_\Sigma. \tag{28}$$

In general, fields measured by a free-falling observer must be regular at an event horizon [17,18]. This is equivalent to the fact that the fields at the event horizon depend only on the ingoing null coordinate v defined by [21]

$$dv = dt + \sqrt{\frac{g_{rr}}{g_{tt}}} dr. \tag{29}$$

Thus, near the horizon, we find

$$\partial_r h \simeq \sqrt{\frac{g_{rr}}{g_{tt}}} \partial_t h. \tag{30}$$

Therefore, the membrane charge density becomes

$$J_h \simeq -\frac{1}{16\pi G_5} \frac{1}{\sqrt{g_{tt}}} \partial_t h|_\Sigma = -\frac{1}{16\pi G_5} U^\mu \partial_\mu h, \tag{31}$$

where U^μ is the velocity vector of an observer at the stretched horizon.

If we assume that h is isotropic, the total membrane charge induced by h per unit time is found to be

$$\begin{aligned} J_h^{\rm tot} &= -\frac{A_H}{16\pi G_5} U^\mu \partial_\mu h \\ &= -\frac{1}{2}\left(\sqrt{n_1 n_5 n_p} + \sqrt{n_1 n_5 \bar{n}_p}\right) U^\mu \partial_\mu h, \end{aligned} \tag{32}$$

where we have used (21). Therefore, the membrane transport coefficient induced by h is[1]

$$\chi_h^{mb} = \frac{1}{2}(\sqrt{n_1 n_5 n_p} + \sqrt{n_1 n_5 \bar{n}_p}). \tag{33}$$

Divided by the Bekenstein-Hawking entropy (22), we obtain

$$\frac{\chi_h^{mb}}{S_{\rm BH}} = \frac{1}{4\pi}. \tag{34}$$

In the same way, the membrane transport coefficient induced by ϕ is

$$\chi_\phi^{mb} = \sqrt{n_1 n_5 n_p} + \sqrt{n_1 n_5 \bar{n}_p}, \tag{35}$$

$$\frac{\chi_\phi^{mb}}{S_{\rm BH}} = \frac{1}{2\pi}. \tag{36}$$

III. TRANSPORT COEFFICIENTS OF D1-D5-P SYSTEM

A. Effective string model

The effective string model of the D1-D5-P system is described by a single D1-brane wrapped $n_1 n_5$ times on S^1. The D1-brane has $\frac{n_p}{R}$ left-moving momentum and $\frac{\bar{n}_p}{R}$ right-moving momentum which are carried by the open strings attached on the D1-brane. These open strings are assumed to oscillate only inside T^4. This model is valid when $\tilde{v} \sim \mathcal{O}(1)$, $R \gg \sqrt{\alpha'}$ and the energy scale is much lower than the string scale [16].

The low energy effective dynamics in our interest is given by the following DBI action [9,13],

$$S = -T_{\rm eff} \int d^2\sigma e^{-\phi_{10}} \sqrt{-\det \gamma_{\alpha\beta}}, \tag{37}$$

where $T_{\rm eff}$ is the effective tension of the D1-brane, ϕ_{10} is the 10-dimensional dilaton and $\gamma_{\alpha\beta}(\alpha, \beta = 0, 1)$ is the induced metric on the D1-brane.

Let us choose the static gauge $\sigma^0 \equiv \tau = X^0$, $\sigma^1 \equiv \sigma = X^5$. Expand the action around the flat backgrounds

[1] Since the conventional definition of the membrane transport coefficient is given by the membrane charge density (31), the conventional membrane transport coefficient is $\frac{1}{16\pi G_5}$. However, we use (32) to compare the membrane paradigm with the transport coefficient of the effective string model.

and carrying out the Kaluza-Klein reduction of the external fields, we find [13]

$$S = S_0 + S_1 + \cdots, \tag{38}$$

$$S_0 = \frac{T_{\rm eff}}{2}\int d^2\sigma(\dot{X}^i\dot{X}_i - X'^i X'_i), \tag{39}$$

$$S_1 = \frac{T_{\rm eff}}{2}\int d^2\sigma[h_{ij}(X^\mu)P^{ij} - \phi(X^\mu)P^i{}_i], \tag{40}$$

where

$$\dot{X}^i = \frac{\partial X^i}{\partial\tau}, \qquad X'^i = \frac{\partial X^i}{\partial\sigma}, \tag{41}$$

$$P^{ij} = \dot{X}^i\dot{X}^j - X'^i X'^j, \tag{42}$$

and S_1 is the leading source terms of $h_{ij}(i \neq j)$ and ϕ. Assuming that the external fields h_{ij} and ϕ depend only on time t [20], S_1 becomes

$$\begin{aligned} S_1 &= \frac{T_{\rm eff}}{2}\int dt \int_0^{2\pi R n_1 n_5} d\sigma[h_{ij}(t)P^{ij} - \phi(t)P^i_i]|_{\tau=t}, \\ &= \int dt\Big[\frac{1}{2}h_{ij}(t)\mathcal{J}^{ij}_h(t) + \phi(t)\mathcal{J}_\phi(t)\Big], \end{aligned} \tag{43}$$

where

$$\mathcal{J}^{ij}_h(t) = T_{\rm eff}\int_0^{2\pi R n_1 n_5} d\sigma P^{ij}|_{\tau=t}, \tag{44}$$

$$\mathcal{J}_\phi(t) = -\frac{T_{\rm eff}}{2}\int_0^{2\pi R n_1 n_5} d\sigma P^i_i|_{\tau=t}. \tag{45}$$

We note that the mass dimension of $\mathcal{J}^{ij}_h$ and $\mathcal{J}_\phi$ is 1, which is the same as (32).

From the kinetic term (39), we can quantize X^i in the same way as the bosonic string theory. Since σ is identified with $\sigma + 2\pi R n_1 n_5$, the mode expansion of X^i becomes

$$\begin{aligned} X^i(\tau,\sigma) = i(4\pi T_{\rm eff})^{-1/2}\sum_{m\neq 0}\Big[&\frac{\alpha^i_m}{m}e^{-i(m/Rn_1n_5)(\tau-\sigma)} \\ &+ \frac{\tilde{\alpha}^i_m}{m}e^{-i(m/Rn_1n_5)(\tau+\sigma)}\Big], \end{aligned} \tag{46}$$

where

$$[\alpha^i_m, \alpha^j_n] = [\tilde{\alpha}^i_m, \tilde{\alpha}^j_n] = m\delta_{m+n,0}\delta^{ij}, \qquad [\alpha^i_m, \tilde{\alpha}^j_n] = 0. \tag{47}$$

Inserting the mode expansion into (44) and (45), we find

$$\mathcal{J}^{ij}_h(t) = \frac{1}{Rn_1n_5}\sum_{m\neq 0}(\alpha^i_m\tilde{\alpha}^j_m + \tilde{\alpha}^i_m\alpha^j_m)e^{-i(2m/Rn_1n_5)t}, \tag{48}$$

$$\mathcal{J}_\phi(t) = -\frac{1}{2Rn_1n_5}\sum_{m\neq 0}(\alpha^i_m\tilde{\alpha}_{mi} + \tilde{\alpha}^i_m\alpha_{mi})e^{-i(2m/Rn_1n_5)t}. \tag{49}$$

The mode expansion shows that each quantum which is labeled by m and i carries the momentum $\frac{m}{Rn_1n_5}$. Therefore, the total left-moving momentum and right-moving momentum are

$$\frac{n_p}{R} = \frac{N_L}{Rn_1n_5}, \qquad \frac{\bar{n}_p}{R} = \frac{N_R}{Rn_1n_5}, \tag{50}$$

where N_L and N_R are the excitation levels of the left movers and right movers, respectively. Thus, we obtain

$$N_L = n_1n_5n_p, \qquad N_R = n_1n_5\bar{n}_p. \tag{51}$$

Because of the near extremality (20), we find $N_L \gg N_R$.

The Hamiltonian of this system is

$$H = \frac{1}{Rn_1n_5}(N_L + N_R) = \frac{n_p}{R} + \frac{\bar{n}_p}{R}. \tag{52}$$

B. Transport coefficients of effective string model

To describe the mixed states of the effective string model, we introduce the following density matrix [22]:

$$\rho = Z^{-1}\exp(-\beta_L N_L - \beta_R N_R), \tag{53}$$

where $Z = {\rm tr}[\exp(-\beta_L N_L - \beta_R N_R)]$ and $\beta_{L,R}$ are the conjugate parameters of $N_{L,R}$, respectively. The mean values of the oscillation levels and the entropy are

$$\bar{N}_L \equiv \langle N_L\rangle = \frac{c\pi^2}{6\beta_L^2}, \qquad \bar{N}_R \equiv \langle N_R\rangle = \frac{\tilde{c}\pi^2}{6\beta_R^2}, \tag{54}$$

$$S = -\langle \ln\rho\rangle = 2\pi\Big(\sqrt{\frac{c\bar{N}_L}{6}} + \sqrt{\frac{\tilde{c}\bar{N}_R}{6}}\Big), \tag{55}$$

where $\langle\mathcal{O}\rangle \equiv {\rm tr}(\rho\mathcal{O})$. Since there are four bosonic oscillations and four fermionic oscillations, the central charges are $c = \tilde{c} = 6$. Therefore, the entropy becomes

$$S = 2\pi(\sqrt{n_1n_5n_p} + \sqrt{n_1n_5\bar{n}_p}), \tag{56}$$

which exactly agrees with the Bekenstein-Hawking entropy (22) [5,6]. The statistical description is valid if $\beta_{L,R} \ll 1$ [20,22]. Thus, together with the near extremality, we need $\beta_L \ll \beta_R \ll 1$ or $1 \ll \bar{n}_p \ll n_p$. This gives the microscopic reason of why the membrane paradigm does not exist in the extremal black hole.

Let us define the following function:

$$f_{\mathcal{A}\mathcal{B}}(t-t') = \frac{1}{2}\langle :[\mathcal{A}(t), \mathcal{B}(t')]: \rangle, \tag{57}$$

where $\mathcal{A}(t), \mathcal{B}(t)$ are some operators and :: denotes the normal ordering.[2] A transport coefficient is obtained by

$$\mathcal{X}_{\mathcal{A}\mathcal{B}} = \lim_{\omega\to 0}\frac{f_{\mathcal{A}\mathcal{B}}(\omega)}{\omega}, \tag{58}$$

where $f_{\mathcal{A}\mathcal{B}}(\omega)$ is the Fourier transformation of $f_{\mathcal{A}\mathcal{B}}(t)$. This is known as the Kubo formula [20,23].

Let us calculate the transport coefficient of the effective string model induced by $h \equiv h_{67}$. Using the following formulas,

$$\langle :\alpha^i_m \alpha^j_n: \rangle = \frac{|n|}{e^{\beta_L |n|} - 1}\delta^{ij}\delta_{m+n,0}, \tag{59}$$

$$\langle :\tilde{\alpha}^i_m \tilde{\alpha}^j_n: \rangle = \frac{|n|}{e^{\beta_R |n|} - 1}\delta^{ij}\delta_{m+n,0}, \tag{60}$$

we find

$$\begin{aligned} f_{\mathcal{J}^{ij}_h \mathcal{J}^{i'j'}_h}(t-t') &= \frac{1}{2}\langle :[\mathcal{J}^{ij}_h(t), \mathcal{J}^{i'j'}_h(t'):] \rangle = \frac{1}{(Rn_1n_5)^2}\delta^{ij,i'j'}\sum_{m\neq 0} e^{-i(2m/Rn_1n_5)(t-t')} m\left(\frac{|m|}{e^{\beta_L|m|}-1} + \frac{|m|}{e^{\beta_R|m|}-1}\right) \\ &= \frac{-2i}{(Rn_1n_5)^2}\delta^{ij,i'j'}\sum_{m=1}^{\infty} m^2\left(\frac{1}{e^{\beta_L m}-1} + \frac{1}{e^{\beta_R m}-1}\right)\sin\left(\frac{2m}{Rn_1n_5}(t-t')\right), \end{aligned} \tag{61}$$

where $\delta^{ij,i'j'} \equiv \delta^{ii'}\delta^{jj'} + \delta^{ij'}\delta^{ji'}$. The Fourier transformation of $f_{\mathcal{J}^{ij}_h \mathcal{J}^{i'j'}_h}(t)$ is

$$f_{\mathcal{J}^{ij}_h \mathcal{J}^{i'j'}_h}(\omega) = \int_{-\infty}^{\infty} dt f_{\mathcal{J}^{ij}_h \mathcal{J}^{i'j'}_h}(t) e^{i\omega t} = \frac{\pi R n_1 n_5}{4}\delta^{ij,i'j'}\omega^2\left(\frac{1}{e^{\beta_L Rn_1n_5\omega/2}-1} + \frac{1}{e^{\beta_R Rn_1n_5\omega/2}-1}\right). \tag{62}$$

Therefore, using (54) and (51), the transport coefficient induced by h is

$$\mathcal{X}^{\rm str}_h = \lim_{\omega\to 0}\frac{f_{\mathcal{J}^{67}_h \mathcal{J}^{67}_h}(\omega)}{\omega} = \frac{1}{2}\left(\sqrt{n_1 n_5 n_p} + \sqrt{n_1 n_5 \bar{n}_p}\right), \tag{63}$$

which exactly agrees with the membrane transport coefficient (33).

In the same way, we obtain the transport coefficient induced by ϕ,

$$\mathcal{X}^{\rm str}_\phi = \sqrt{n_1 n_5 n_p} + \sqrt{n_1 n_5 \bar{n}_p}, \tag{64}$$

which exactly agree with (35).

IV. SUMMARY AND COMMENTS

I have calculated the transport coefficients of the D1-D5-P system induced by two kinds of minimal scalars h_{ij} and ϕ by using the effective string model. Then, I have found that these transport coefficients exactly agree with the corresponding membrane transport coefficients of the D1-D5-P black hole.

Two comments are in order. First, there are the other kinds of minimal scalars whose coupling with the D1-D5-P system can not be found in the effective string model [16]. Also, generically we can not use the effective string model to study the coupling with fixed scalars [15,16]. Since it is known that the correct couplings of the D1-D5-P system with these scalars are given by a $\mathcal{N} = (4, 4)$ superconformal field theory [16], we should use the superconformal field theory to calculate the remaining transport coefficients induced by the scalar fields.

Finally, the effective string model does not possess the viscosities and conductivities because there is no fluctuation of the effective string in the noncompact space and therefore the effective string can not couple to the bulk metric and gauge fields. This seems to conflict with the membrane paradigm because there exists the membrane viscosities and conductivities in the D1-D5-P black hole. This discrepancy comes from the fact that the energy scale at which the effective string model is valid is much smaller than the string energy scale. It is known that the Hawking radiation of spin-1 and spin-2 particles are suppressed at low energy compared to the case of the scalar particles. On the string theory side, this situation corresponds to the fact that the effective string does not couple to the bulk metric and gauge fields [24]. Thus, to discuss the viscosities and conductivities of the D1-D5-P system, we will need to study the string scale physics of the D1-D5-P system.

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[2]The normal ordering must be evaluated after the calculation of the commutator.

[1] L. Susskind, C. Teitelboim, *The Black Hole* (World Scientific, Singapore, 1998), p. 118.
[2] A. Sen, Mod. Phys. Lett. A **10**, 2081 (1995).
[3] G. T. Horowitz and J. Polchinski, Phys. Rev. D **55**, 6189 (1997).
[4] A. Strominger and C. Vafa, Phys. Lett. B **379**, 99 (1996).
[5] C. G. Callan and J. M. Maldacena, Nucl. Phys. **B472**, 591 (1996).
[6] G. T. Horowitz, J. M. Maldacena, and A. Strominger, Phys. Lett. B **383**, 151 (1996).
[7] J. M. Maldacena and L. Susskind, Nucl. Phys. **B475**, 679 (1996).
[8] A. Dhar, G. Mandal, and S. R. Wadia, Phys. Lett. B **388**, 51 (1996).
[9] S. R. Das and S. D. Mathur, Nucl. Phys. **B478**, 561 (1996).
[10] S. R. Das and S. D. Mathur, Nucl. Phys. **B482**, 153 (1996).
[11] S. S. Gubser and I. R. Klebanov, Nucl. Phys. **B482**, 173 (1996).
[12] J. M. Maldacena and A. Strominger, Phys. Rev. D **55**, 861 (1997).
[13] C. G. Callan, Jr., S. S. Gubser, I. R. Klebanov, and A. A. Tseytlin, Nucl. Phys. **B489**, 65 (1997).
[14] I. R. Klebanov and S. D. Mathur, Nucl. Phys. **B500**, 115 (1997).
[15] M. Krasnitz and I. R. Klebanov, Phys. Rev. D **56**, 2173 (1997).
[16] J. R. David, G. Mandal, and S. R. Wadia, Phys. Rep. **369**, 549 (2002).
[17] K. S. Thorne, R. H. Price, and D. A. Macdonald, *Black Holes: The Membrane Paradigm* (Yale Univ. Pr., New Haven, USA, 1986), p. 367.
[18] M. Parikh and F. Wilczek, Phys. Rev. D **58**, 064011 (1998).
[19] Y. Sasai and A. Zahabi, Phys. Rev. D **83**, 026002 (2011).
[20] Y. Sasai, Phys. Rev. D **84**, 105006 (2011).
[21] N. Iqbal and H. Liu, Phys. Rev. D **79**, 025023 (2009).
[22] E. Halyo, B. Kol, A. Rajaraman, and L. Susskind, Phys. Lett. B **401**, 15 (1997).
[23] P. M. Chaikin and T. C. Lubensky, *Principles of Condensed Matter Physics* (Cambridge Univ. Pr., Cambridge, UK, 2000), p. 699.
[24] S. D. Mathur, Nucl. Phys. **B529**, 295 (1998).