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Supersonic velocities in noncommutative acoustic black holes

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In this paper we derive Schwarzschild and Kerr-like noncommutative acoustic black hole metrics in the (3 + 1)-dimensional noncommutative Abelian Higgs model. We have found that the changing $\Delta T_{\rm H}$ in the Hawking temperature $T_{\rm H}$ due to spacetime noncommutativity accounts for supersonic velocities v_g, whose deviation with respect to the sound speed c_s is given in the form $(v_g - c_s)/c_s = \Delta T_{\rm H}/8T_{\rm H}$.

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I. INTRODUCTION

Acoustic black holes possess many of the fundamental properties of the black holes of general relativity and have been extensively studied in the literature [1–3]. The connection between black hole physics and the theory of supersonic acoustic flow was established in 1981 by Unruh [3] and has been developed to investigate the Hawking radiation and other phenomena for understanding quantum gravity. Hawking radiation is an important quantum effect of black hole physics. In 1974, Hawking, combining Einstein's general relativity and quantum mechanics, announced that classically a black hole does not radiate, but when we consider quantum effects emits thermal radiation at a temperature proportional to the horizon surface gravity.

Since the Hawking radiation showed by Unruh [3] is a purely kinematic effect of quantum field theory, we can study the Hawking radiation process in completely different physical systems. For example, acoustic horizons are regions where a moving fluid exceeds the local sound speed through a spherical surface and possesses many of the properties associated with the event horizons of general relativity. In particular, the acoustic Hawking radiation when quantized appears as a flux of thermal phonons emitted from the horizon at a temperature proportional to the acoustic black hole "surface gravity"—the normal derivative of the local sound speed combined with the normal derivative of the normal component of the fluid velocity at the horizon [1]. Many fluid systems have been investigated on a variety of analog models of acoustic black holes, including gravity wave [4], water [5], slow light [6], optical fiber [7], and electromagnetic waveguide [8]. The models of superfluid helium II [9], atomic Bose-Einstein condensates [10,11], and one-dimensional Fermi degenerate noninteracting gas [12] have been proposed to create an acoustic black hole geometry in the laboratory.

The purpose of this paper is to investigate the relativistic version of acoustic black holes from the noncommutative Abelian Higgs model. Various gravitational black hole solutions of noncommutative spacetime have been found in [13], thermodynamic properties of the noncommutative black hole were studied in [14], the evaporation of the noncommutative black hole was studied in [15], quantum tunneling of noncommutative Kerr black hole was studied in [16], and quantized entropy was studied in [17].

A relativistic version of acoustic black holes has been presented in [18,19] (see also [20–22]). Differently from most cases studied, we consider the acoustic black hole metrics obtained from a relativistic fluid in a noncommutative spacetime, where we are assuming that the fluid approximation is valid as long as the energy scale $E \sim k \ll 1/\sqrt{\theta}$, where $1/\sqrt{\theta} \sim M_{\rm Planck}$ is the noncommutativity parameter. The effects of this setup are such that the fluctuations of the fluids are also affected. The sound waves inherit spacetime noncommutativity of the fluid and may lose the Lorentz invariance. As a consequence the Hawking temperature is directly affected by the spacetime noncommutativity. Analogously to Lorentz-violating gravitational black holes [23,24], the effective Hawking temperature of the noncommutativity acoustic black holes now is not universal for all species of particles. It depends on the maximal attainable velocity of this species. Furthermore, the acoustic black hole metric can be identified with an acoustic Kerr-like black hole. The spacetime noncommutativity affects the rate of loss of mass of the black hole. Thus for suitable values of the spacetime noncommutativity parameter a wider or narrower spectrum of particle wave function can be scattered with increased amplitude by the acoustic black hole. This increases or decreases the superresonance phenomenon previously studied in [25,26].

The paper is organized as follows. In Sec. II we obtain a general acoustic black hole metric in the noncommutative Abelian Higgs model that is revealed to be similar to Lorentz-violating acoustic black holes [19,27] of the Lorentz-violating Abelian Higgs model [28]. In Sec. III we address the issue of group velocity, which shows a deviation on the maximal attainable particle velocity on the fluid. The magnitude of the deviation is consistent with that found in Lorentz-violating models [29,30]. In Sec. IV we find explicitly Schwarzschild and Kerr-like noncommutative acoustic black holes and address the issue of

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supersonic velocities in terms of the variation of the Hawking temperature as a response to the spacetime noncommutativity. Such a response is given as

$$\frac{\Delta T_{\rm H}}{T_{\rm H}} = 4\theta_3 B_3. \tag{1}$$

As we shall see this allows us to write the deviation on the maximal attainable particle velocity as simply given by $(v_g - c_s)/c_s = \Delta T_{\rm H}/8T_{\rm H}$.

II. THE ACOUSTIC METRIC IN NONCOMMUTATIVE ABELIAN HIGGS MODEL

In this section we make an extension of the Abelian Higgs model by modifying its scalar and gauge sector with the Moyal product [31–34]. Thus, the Lagrangian of the noncommutative Abelian Higgs model in flat space is

$$\hat{\mathcal{L}} = -\tfrac{1}{4}\hat{F}_{\mu\nu} * \hat{F}^{\mu\nu} + (D_\mu\hat{\phi})^\dagger * D^\mu\hat{\phi} + m^2\hat{\phi}^\dagger * \hat{\phi} - b\hat{\phi}^\dagger * \hat{\phi} * \hat{\phi}^\dagger * \hat{\phi}. \tag{2}$$

Now we use the Seiberg-Witten map [31], up to the lowest order in the spacetime noncommutative parameter $\theta^{\mu\nu}$, given by

$$\begin{aligned}\hat{A}_\mu &= A_\mu + \theta^{\nu\rho}A_\rho(\partial_\nu A_\mu - \tfrac{1}{2}\partial_\mu A_\nu),\\ \hat{F}_{\mu\nu} &= F_{\mu\nu} + \theta^{\rho\beta}(F_{\mu\rho}F_{\nu\beta} + A_\rho\partial_\beta F_{\mu\nu}),\\ \hat{\phi} &= \phi - \tfrac{1}{2}\theta^{\mu\nu}A_\mu\partial_\nu\phi.\end{aligned} \tag{3}$$

This very useful map leads to the corresponding theory in a commutative spacetime [32]

$$\begin{aligned}\hat{\mathcal{L}} = &-\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}(1 + \tfrac{1}{2}\theta^{\alpha\beta}F_{\alpha\beta}) + (1 - \tfrac{1}{4}\theta^{\alpha\beta}F_{\alpha\beta})\\ &\times (|D_\mu\phi|^2 + m^2|\phi|^2 - b|\phi|^4)\\ &+ \tfrac{1}{2}\theta^{\alpha\beta}F_{\alpha\mu}[(D_\beta\phi)^\dagger D^\mu\phi + (D^\mu\phi)^\dagger D_\beta\phi],\end{aligned} \tag{4}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and $D_\mu\phi = \partial_\mu\phi - ieA_\mu\phi$. We are using Planck units, $\hbar = c = k_B = 1$. As one knows the parameter $\theta^{\alpha\beta}$ is a constant, real-valued antisymmetric $D\times D$ matrix in D-dimensional spacetime with dimensions of length squared. For a review see [34].

Now, in order to find the noncommutative acoustic black hole metric, let us use the decomposition $\phi = \sqrt{\rho(x,t)}\exp(iS(x,t))$ in the original Lagrangian to obtain

$$\begin{aligned}\mathcal{L} = &-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}(1 - 2\vec{\theta}\cdot\vec{B}) + \tilde{\theta}[\partial_\mu S\partial^\mu S\\ &- 2eA_\mu\partial^\mu S + e^2A_\mu A^\mu + m^2]\rho - \tilde{\theta}b\rho^2\\ &+ \Theta^{\mu\nu}[\partial_\mu S\partial_\nu S - eA_\mu\partial_\nu S - eA_\nu\partial_\mu S + e^2A_\mu A_\nu]\rho\\ &+ \frac{\rho}{\sqrt{\rho}}[\tilde{\theta}\partial_\mu\partial^\mu + \Theta^{\mu\nu}\partial_\mu\partial_\nu]\sqrt{\rho},\end{aligned} \tag{5}$$

where $\tilde{\theta} = (1 + \vec{\theta}\cdot\vec{B})$, $\vec{B} = \nabla\times\vec{A}$, and $\Theta^{\mu\nu} = \theta^{\alpha\mu}F^\nu_{\ \alpha}$. In our calculations we shall consider the case where there is no noncommutativity between spatial and temporal coordinates, that is, $\theta^{0i} = 0$, but $\theta^{ij} = \varepsilon^{ijk}\theta^k$, $F^{i0} = E^i$, and $F^{ij} = \varepsilon^{ijk}B^k$.

By using the Lagrangian (5) we can find the following equations of motion,

$$-\partial_\mu\Big[\tilde{\theta}\rho(\partial^\mu S - eA^\mu) + \frac{\rho}{2}(\Theta^{\mu\nu} + \Theta^{\nu\mu})(\partial_\nu S - eA_\nu)\Big] = 0, \tag{6}$$

that is,

$$\begin{aligned}&-\partial_t\Big[\tilde{\theta}\rho(\dot{S} - eA_t) + \frac{\rho\Theta^{jt}}{2}(\partial_j S - eA_j)\Big]\\ &+ \partial_i\Big[-\rho\tilde{\theta}(\partial^i S - eA^i) - \frac{\rho\Theta^{it}}{2}(\dot{S} - eA_t)\\ &- \frac{\rho}{2}(\Theta^{il} + \Theta^{li})(\partial_l S - eA_l)\Big] = 0,\end{aligned} \tag{7}$$

and

$$\begin{aligned}&\frac{(\tilde{\theta}\partial_\mu\partial^\mu + \Theta^{\mu\nu}\partial_\mu\partial_\nu)\sqrt{\rho}}{\sqrt{\rho}} + \tilde{\theta}(\partial_\mu S - eA_\mu)^2\\ &+ \Theta^{\mu\nu}(\partial_\mu S - eA_\mu)(\partial_\nu S - eA_\nu) + \tilde{\theta}m^2 - 2\tilde{\theta}b\rho = 0,\end{aligned} \tag{8}$$

which can also be given as

$$\begin{aligned}&\frac{(\tilde{\theta}\partial_\mu\partial^\mu + \Theta^{\mu\nu}\partial_\mu\partial_\nu)\sqrt{\rho}}{\sqrt{\rho}} + \tilde{\theta}(\dot{S} - eA_t)^2\\ &+ \tilde{\theta}(\partial_i S - eA_i)(\partial^i S - eA^i) + \Theta^{jt}(\partial_j S - eA_j)(\dot{S} - eA_t)\\ &+ \Theta^{jl}(\partial_j S - eA_j)(\partial_l S - eA_l) + \tilde{\theta}m^2 - 2\tilde{\theta}b\rho = 0.\end{aligned} \tag{9}$$

For the gauge field A_μ, we obtain the modified Maxwell's equations

$$\begin{aligned}&\partial_\mu F^{\mu\nu} + \frac{1}{4}\partial_\mu(\theta^{\mu\nu}F_{\alpha\beta}F^{\alpha\beta} + 2\theta^{\alpha\beta}F_{\alpha\beta}F^{\mu\nu})\\ &- \frac{1}{2}\theta^{\mu\nu}\partial_\mu(u_\alpha u^\alpha\rho + m^2\rho - b\rho^2)\\ &- \partial_\mu[u_\beta(\theta^{\mu\beta}u^\nu - \theta^{\nu\beta}u^\mu)\rho]\\ &+ \partial_\mu\Big[\frac{\rho}{\sqrt{\rho}}\Big(\frac{1}{2}\theta^{\mu\nu}\partial_\alpha\partial^\alpha - \theta^{\mu\beta}\partial_\beta\partial^\nu + \theta^{\nu\beta}\partial_\beta\partial^\mu\Big)\sqrt{\rho}\Big]\\ &= 2e\rho(1 + \vec{\theta}\cdot\vec{B})u^\nu + e\rho u^\mu(\theta^{\alpha\nu}F_{\alpha\mu} + \theta_{\alpha\mu}F^{\alpha\nu}),\end{aligned} \tag{10}$$

where we have defined $u^\mu = \partial^\mu S - eA^\mu = (-w, -u^i)$.

That is, there exist changes in the Gauss and Ampère laws

$$\begin{aligned}&\nabla\cdot[(1 - 2\vec{\theta}\cdot\vec{B})\vec{E}] - \theta^{ij}\partial_i(u_j w\rho) + \partial_i\Big[\frac{\rho}{\sqrt{\rho}}(\theta^{ij}\partial_j\partial^0)\sqrt{\rho}\Big]\\ &= 2e\rho(1 + \vec{\theta}\cdot\vec{B})w + e\rho(\vec{\theta}\times\vec{E})\cdot\vec{u},\end{aligned} \tag{11}$$

and

$$
\begin{aligned}
&(\nabla\times\vec{B})^j-\partial_t E^j-\frac{1}{4}\theta^{ij}\partial_i(F_{\mu\nu}F^{\mu\nu})+2\partial_t[(\vec{\theta}\cdot\vec{B})E^j]\\
&\quad+2\partial_i[(\vec{\theta}\cdot\vec{B})F^{ij}]+\frac{1}{2}\theta^{ij}\partial_i(u_\alpha u^\alpha\rho+m^2\rho-b\rho^2)\\
&\quad+\theta^{il}\partial_i(u_l u^j\rho)-\theta^{jl}\partial_\mu(u_l u^\mu\rho)\\
&\quad-\partial_\mu\left[\frac{\rho}{\sqrt{\rho}}\left(\frac{1}{2}\theta^{\mu j}\partial_\alpha\partial^\alpha-\theta^{\mu\beta}\partial_\beta\partial^j+\theta^{j\beta}\partial_\beta\partial^\mu\right)\sqrt{\rho}\right]\\
&=2e\rho(1+\vec{\theta}\cdot\vec{B})u^j+e\rho w(\vec{\theta}\times\vec{E})^j\\
&\quad+2e\rho(\vec{\theta}\cdot\vec{B})u^j-e\rho(\vec{\theta}\cdot\vec{u})B^j-e\rho(\vec{u}\cdot\vec{B})\theta^j. \qquad (12)
\end{aligned}
$$

We shall consider plane wave solutions or background fields satisfying the gauge field equations [33]. This allows us to write our acoustic black hole metric in terms of a constant parameter $\vec{\theta}\cdot\vec{B}$ with some freedom to choose it arbitrarily small (or large) depending on the regime where the spacetime noncommutativity takes place—such a regime is well assumed to happen in the UV regime where a very small distance around $\sqrt{\theta}$ can be probed.

Equation (7) is the continuity equation and Eq. (9) is an equation describing a hydrodynamical fluid with a term $[(\tilde{\theta}\partial_\mu\partial^\mu+\Theta^{\mu\nu}\partial_\mu\partial_\nu)\sqrt{\rho}]/\sqrt{\rho}$ called "quantum potential" (quantum correction term) [1], which can be negligible in the hydrodynamic region. Now we consider perturbations around the background (ρ_0, S_0), which are solutions of the previous equations of motion, with $\rho=\rho_0+\rho_1$ and $S=S_0+S_1$, so we can rewrite (7) and (9) as

$$
\begin{aligned}
&-\partial_t\left[\rho_0\left(\tilde{\theta}\dot{S}_1+\frac{\Theta^{jt}}{2}\partial_j S_1\right)-\rho_1\left(\tilde{\theta}w_0-\frac{\Theta^{jt}}{2}v_0^j\right)\right]\\
&\quad-\partial_i\left[\rho_0\left(\tilde{\theta}\partial^i S_1+\frac{\Theta^{it}}{2}\dot{S}_1+\frac{1}{2}(\Theta^{il}+\Theta^{li})\partial_l S_1\right)\right.\\
&\quad\left.+\rho_1\left(-\tilde{\theta}v_0^i-\frac{\Theta^{it}}{2}w_t+\frac{1}{2}(\Theta^{il}+\Theta^{il})v_0^l\right)\right]=0, \qquad (13)
\end{aligned}
$$

and

$$
\begin{aligned}
&-2\tilde{\theta}w_0\dot{S}_1-2\tilde{\theta}v_0^i\partial_i S_1+\Theta^{lt}(v_0^l\dot{S}_1-w_0\partial_l S_1)\\
&\quad+\Theta^{lj}(v_0^l\partial_j S_1+v_0^j\partial_l S_1)-\tilde{\theta}b\rho_1=0, \qquad (14)
\end{aligned}
$$

where we have defined $w_0=-\dot{S}_0+eA_t$ and $\vec{v}_0=\nabla S_0+e\vec{A}$ (the local velocity field). Thus, the wave equation for the perturbations S_1 around the background S_0 becomes

$$\partial_t[a^{tt}\dot{S}_1+a^{tj}\partial_j S_1]+\partial_i[a^{it}\dot{S}_1+a^{ij}\partial_j S_1]=0, \qquad (15)$$

where

$$a^{tt}=-\tilde{\theta}\rho_0-\frac{2}{b}(\tilde{\theta}w_0^2-\Theta^{jt}v_0^j w_0), \qquad (16)$$

$$
\begin{aligned}
a^{tj}=&-\frac{1}{2}\rho_0\Theta^{jt}-\frac{2}{b}\left[v_0^j\left(\tilde{\theta}w_0-\frac{\Theta^{lt}}{2}v_0^l\right)+\frac{\Theta^{jt}}{2}w_0^2\right.\\
&\left.-\frac{1}{2}(\Theta^{lj}+\Theta^{jl})w_0v_0^l\right], \qquad (17)
\end{aligned}
$$

$$
\begin{aligned}
a^{it}=&-\frac{1}{2}\rho_0\Theta^{it}-\frac{2}{b}\left[v_0^i\left(\tilde{\theta}w_0-\frac{\Theta^{lt}}{2}v_0^l\right)+\frac{\Theta^{it}}{2}w_0^2\right.\\
&\left.-\frac{1}{2}(\Theta^{li}+\Theta^{il})w_0v_0^l\right], \qquad (18)
\end{aligned}
$$

$$
\begin{aligned}
a^{ij}=&\tilde{\theta}\rho_0\delta^{ij}-\frac{\rho_0}{2}(\Theta^{ij}+\Theta^{ji})\\
&-\frac{2}{b}\left(\tilde{\theta}v_0^iv_0^j+\frac{\Theta^{it}}{2}w_0v_0^j+\frac{\Theta^{jt}}{2}v_0^iw_0\right)\\
&+\frac{2}{b}\left[\frac{1}{2}(\Theta^{lj}+\Theta^{jl})v_0^iv_0^l+\frac{1}{2}(\Theta^{li}+\Theta^{il})v_0^jv_0^l\right]. \qquad (19)
\end{aligned}
$$

Notice that one can readily identify the local sound speed in the fluid and velocity of the flow as $c_s^2=b\rho_0/2w_0^2$ and $v^i=v_0^i/w_0$ in the linearized equation of motion describing the fluctuations (the sound) in the fluid—see below. Equation (15) can be seen as the Klein-Gordon equation in a curved spacetime and can be written as [3]

$$\frac{1}{\sqrt{-g}}\partial_\mu\sqrt{-g}g^{\mu\nu}\partial_\nu S_1=0, \qquad (20)$$

being the metric components given in the form

$$\sqrt{-g}g^{\mu\nu}\equiv\frac{b\rho_0}{2c_s^2}\begin{bmatrix} g^{tt} & \vdots & g^{tj}\\ \cdots\cdots & \cdot & \cdots\cdots\\ g^{it} & \vdots & g^{ij}\end{bmatrix}, \qquad (21)$$

where

$$
\begin{aligned}
g^{tt}=&-\tilde{\theta}c_s^2-(\tilde{\theta}-\Theta^{jt}v^j),\\
g^{tj}=&-\frac{\Theta^{jt}}{2}c_s^2-\left[\tilde{\theta}v^j-\frac{\Theta^{lt}}{2}v^lv^j+\frac{\Theta^{jt}}{2}-\frac{\Theta^{lj}}{2}v^l-\frac{\Theta^{jl}}{2}v^l\right],\\
g^{it}=&-\frac{\Theta^{it}}{2}c_s^2-\left[\tilde{\theta}v^i-\frac{\Theta^{lt}}{2}v^lv^i+\frac{\Theta^{it}}{2}-\frac{\Theta^{li}}{2}v^l-\frac{\Theta^{il}}{2}v^l\right],\\
g^{ij}=&\left[\tilde{\theta}\delta^{ij}-\frac{1}{2}(\Theta^{ij}+\Theta^{ji})\right]c_s^2-\left[\tilde{\theta}v^iv^j+\frac{\Theta^{it}}{2}v^j+\frac{\Theta^{jt}}{2}v^i\right]\\
&+\left[\frac{1}{2}(\Theta^{lj}+\Theta^{jl})v^iv^l+\frac{1}{2}(\Theta^{li}+\Theta^{il})v^jv^l\right]. \qquad (22)
\end{aligned}
$$

In terms of the inverse of $g^{\mu\nu}$ we have the metric of a noncommutative acoustic black hole

$$g_{\mu\nu}\equiv\frac{\frac{b\rho_0}{2c_s}}{\sqrt{f}}\begin{bmatrix} g_{tt} & \vdots & g_{ti}\\ \cdots\cdots & \cdot & \cdots\cdots\\ g_{jt} & \vdots & g_{ij}\end{bmatrix}, \qquad (23)$$

where

$$g_{tt} = -[(\tilde{\theta} - \Theta^{jj})c_s^2 - \tilde{\theta}v^2 + 2\Theta^{jl}v^jv^l - \Theta^{jt}v^j],$$
$$g_{tj} = -\frac{\Theta^{jt}}{2}c_s^2 - \left[\tilde{\theta}v^j - \frac{\Theta^{lt}}{2}v^lv^j + \frac{\Theta^{jt}}{2} - \frac{\Theta^{lj}}{2}v^l - \frac{\Theta^{jl}}{2}v^l\right],$$
$$g_{it} = -\frac{\Theta^{it}}{2}c_s^2 - \left[\tilde{\theta}v^i - \frac{\Theta^{lt}}{2}v^lv^i + \frac{\Theta^{it}}{2} - \frac{\Theta^{li}}{2}v^l - \frac{\Theta^{il}}{2}v^l\right],$$
$$g_{ij} = [\tilde{\theta}(1 + c_s^2) - \tilde{\theta}v^2 - \Theta^{lt}v^l]\delta^{ij} + \tilde{\theta}v^iv^j. \tag{24}$$

Here $\Theta^{jt} = \theta^{ij}F_i^t = -\theta^{ij}F^{it} = \theta^{ij}E^i$ and $\Theta^{jl} = \theta^{ij}F_i^l = -\theta^{ij}F^{il}$. Thus, we find the following components:

$$g_{tt} = -[(1 - 3\vec{\theta}\cdot\vec{B})c_s^2 - (1 + 3\vec{\theta}\cdot\vec{B})v^2 + 2(\vec{\theta}\cdot\vec{v})(\vec{B}\cdot\vec{v}) - (\vec{\theta}\times\vec{E})\cdot\vec{v}],$$
$$g_{tj} = -\frac{1}{2}(\vec{\theta}\times\vec{E})^j(c_s^2 + 1) - [2(1 + 2\vec{\theta}\cdot\vec{B}) - (\vec{\theta}\times\vec{E})\cdot\vec{v}]\frac{v^j}{2} + \frac{B^j}{2}(\vec{\theta}\cdot\vec{v}) + \frac{\theta^j}{2}(\vec{B}\cdot\vec{v}),$$
$$g_{it} = -\frac{1}{2}(\vec{\theta}\times\vec{E})^i(c_s^2 + 1) - [2(1 + 2\vec{\theta}\cdot\vec{B}) - (\vec{\theta}\times\vec{E})\cdot\vec{v}]\frac{v^i}{2} + \frac{B^i}{2}(\vec{\theta}\cdot\vec{v}) + \frac{\theta^i}{2}(\vec{B}\cdot\vec{v}),$$
$$g_{ij} = [(1 + \vec{\theta}\cdot\vec{B})(1 + c_s^2) - (1 + \vec{\theta}\cdot\vec{B})v^2 - (\vec{\theta}\times\vec{E})\cdot\vec{v}]\delta^{ij} + (1 + \vec{\theta}\cdot\vec{B})v^iv^j.f$$
$$= [(1 - 2\vec{\theta}\cdot\vec{B})(1 + c_s^2) - (1 + 4\vec{\theta}\cdot\vec{B})v^2] - 3(\vec{\theta}\times\vec{E})\cdot\vec{v} + 2(\vec{B}\cdot\vec{v})(\vec{\theta}\cdot\vec{v}). \tag{25}$$

This acoustic metric describes a relativistic noncommutative acoustic black hole, and depends simply on the density ρ_0, the local sound speed in the fluid c_s, the velocity of flow $\vec{v}$, the noncommutativity parameter $\vec{\theta}$, and gauge field components $\vec{E}$, $\vec{B}$. Notice that the sound speed c_s is a function of the electromagnetic field A_t. The acoustic line element can be written as

$$ds^2 = \frac{b\rho_0}{2c_s\sqrt{f}}[g_{tt}dt^2 + g_{it}dx^idt + g_{jt}dtdx^j + g_{ij}dx^idx^j],$$
$$= \frac{b\rho_0}{2c_s\sqrt{f}}[-\mathcal{F}(v)dt^2 - \vec{\xi}(v)\cdot d\vec{x}dt + \Lambda(v)dx^2 + (1 + \vec{\theta}\cdot\vec{B})(\vec{v}\cdot d\vec{x})^2], \tag{26}$$

where

$$\mathcal{F}(v) = (1 - 3\vec{\theta}\cdot\vec{B})c_s^2 - (1 + 3\vec{\theta}\cdot\vec{B})v^2 - (\vec{\theta}\times\vec{E})\cdot\vec{v} + 2(\vec{\theta}\cdot\vec{v})(\vec{B}\cdot\vec{v}), \tag{27}$$

$$\Lambda(v) = (1 + \vec{\theta}\cdot\vec{B})(1 + c_s^2 - v^2) - (\vec{\theta}\times\vec{E})\cdot\vec{v}, \tag{28}$$

$$\vec{\xi}(v) = [2(1 + 2\vec{\theta}\cdot\vec{B}) - (\vec{\theta}\times\vec{E})\cdot\vec{v}]\vec{v} + (1 + c_s^2)(\vec{\theta}\times\vec{E}) - (\vec{B}\cdot\vec{v})\vec{\theta} - (\vec{\theta}\cdot\vec{v})\vec{B}. \tag{29}$$

Now changing the time coordinate as

$$d\tau = dt + \frac{\vec{\xi}(v)\cdot d\vec{x}}{2\mathcal{F}(v)}, \tag{30}$$

we find the acoustic metric in the stationary form

$$ds^2 = \frac{b\rho_0}{2c_s\sqrt{f}}\left[-\mathcal{F}(v)d\tau^2 + \Lambda\left(\frac{v^iv^j\Gamma + \Sigma^{ij}}{\Lambda\mathcal{F}(v)} + \delta^{ij}\right)dx^idx^j\right], \tag{31}$$

where

$$\Gamma(v) = 1 + 4\vec{\theta}\cdot\vec{B} + (1 - 2\vec{\theta}\cdot\vec{B})c_s^2 - (1 + 4\vec{\theta}\cdot\vec{B})v^2 - 2(\vec{\theta}\times\vec{E})\cdot\vec{v} + 2(\vec{\theta}\cdot\vec{v})(\vec{B}\cdot\vec{v}), \tag{32}$$

$$\Sigma^{ij}(v) = [(1 + c_s^2)(\vec{\theta}\times\vec{E})^i - (\vec{B}\cdot\vec{v})\theta^i - (\vec{\theta}\cdot\vec{v})B^i]v^j. \tag{33}$$

III. THE DISPERSION RELATION

The sound waves are usually governed by an effective Lorentzian spacetime geometry. In order to study the effect of the spacetime noncommutativity in such a structure we should investigate the dispersion relation. So let us now discuss the dispersion relation.

We now derive the dispersion relation from Eq. (15). Since the field S_1 is real we use the notation

$$S_1 \sim \mathrm{Re}[e^{(i\omega t - i\vec{k}\cdot\vec{x})}], \qquad \omega = \frac{\partial S_1}{\partial t}, \qquad \vec{k} = \vec{\nabla}S_1. \tag{34}$$

In this case, the Klein-Gordon equation (15) in terms of momenta and frequency becomes

$$a\omega^2 + \sigma\omega + d = 0, \tag{35}$$

where

$$a = (1 + \vec{\theta}\cdot\vec{B})(1 + c_s^2) - (\vec{\theta}\times\vec{E})\cdot\vec{v}, \tag{36}$$

$$\sigma = [2(1 + 2\vec{\theta}\cdot\vec{B}) - (\vec{\theta}\times\vec{E})\cdot\vec{v}](\vec{v}\cdot\vec{k}) + (c_s^2 + 1)(\vec{\theta}\times\vec{E})\cdot\vec{k} - (\vec{\theta}\cdot\vec{v})(\vec{B}\cdot\vec{k}) - (\vec{B}\cdot\vec{v})(\vec{\theta}\cdot\vec{k}), \tag{37}$$

$$d = -\{[(1+2\vec{\theta}\cdot\vec{B})k^2 - (\vec{\theta}\cdot\vec{k})(\vec{B}\cdot\vec{k})]c_s^2 - (1+3\vec{\theta}\cdot\vec{B})v^2k^2 - [(\vec{\theta}\times\vec{E})\cdot\vec{k}](\vec{v}\cdot\vec{k}) + (\vec{\theta}\cdot\vec{v})(\vec{B}\cdot\vec{k})(\vec{v}\cdot\vec{k}) + (\vec{B}\cdot\vec{v})(\vec{\theta}\cdot\vec{k})(\vec{v}\cdot\vec{k})\}. \quad (38)$$

Here we choose $k^i = k\delta^{i1}$ ($i = 1, 2, 3$). Thus, the dispersion relation can be easily found and reads

$$\omega = \frac{-\sigma \pm \sqrt{\sigma^2 - 4ad}}{2a}, \quad (39)$$

where

$$\Delta = \sigma^2 - 4ad = 4k^2c_s^2\{[(1+3\vec{\theta}\cdot\vec{B})(1+c_s^2) - (1+4\vec{\theta}\cdot\vec{B})v_1^2 - (\vec{\theta}\times\vec{E})\cdot\vec{v}] - \theta_1B_1(1+c_s^2) + (\vec{B}\cdot\vec{v})\theta_1v_1 + (\vec{\theta}\cdot\vec{v})B_1v_1\}. \quad (40)$$

We can simplify our result using the following projections, $\vec{\theta}\cdot\vec{B} = \theta_3B_3$ and $\vec{\theta}\times\vec{E} = 0$, or even for a pure magnetic background, i.e., $E = 0$, so that

$$\omega = \frac{-(1+2\theta_3B_3)(v_1k) \pm c_sk\sqrt{(1+3\theta_3B_3)(1+c_s^2) - (1+4\theta_3B_3)v_1^2}}{(1+\theta_3B_3)(1+c_s^2)}, \quad (41)$$

and for $\theta = 0$, we recover the result obtained in [18]. In the nonrelativistic limit $c_s^2 \ll 1$, $v_1^2 \ll 1$ and for small θ_3B_3, the dispersion relation is simply given by

$$\omega \approx \pm\frac{c_s\sqrt{(1+3\theta_3B_3)}}{(1+\theta_3B_3)}k = \pm c_s\left(1+\frac{1}{2}\theta_3B_3\right)k. \quad (42)$$

This means the group velocity that measures the maximal attainable velocity of a particle in the medium is given by

$$v_g = \left|\frac{d\omega}{dk}\right| = c_s\left(1+\frac{1}{2}\theta_3B_3\right), \quad (43)$$

or in terms of deviations in relation to the sound speed c_s, the maximal attainable velocity in the medium, then we have

$$\frac{v_g - c_s}{c_s} = \frac{1}{2}\theta_3B_3. \quad (44)$$

This deviation also appears in recent scenarios with Lorentz-violating parameters [19,27,29,30] with magnitude around $|\beta| \sim |\theta_3B_3| \sim 10^{-6}$, a bound found in Bose-Einstein condensation physics [29,30]—see also [35]. Notice this implies a supersonic behavior of a particle with maximal attainable velocity v_g.

IV. CANONICAL ACOUSTIC BLACK HOLES

In this section, we shall address the issue of Hawking temperature. For this we consider an incompressible fluid with spherical symmetry. In this case the density ρ is a position-independent quantity and the continuity equation implies that $v \sim \frac{1}{r^2}$. The sound speed is also a constant.

The relativistic and noncommutative acoustic metric can be written as a Schwarzschild metric type, up to an irrelevant position-independent factor, as follows,

$$ds^2 = -\tilde{\mathcal{F}}(v_r)d\tau^2 + \frac{[v_r^2\Gamma + \Sigma + \tilde{\mathcal{F}}(v_r)\Lambda]}{\tilde{\mathcal{F}}(v_r)}dr^2 + \frac{(1+c_s^2)r^2(d\vartheta^2 + \sin^2\vartheta d\phi^2)}{\sqrt{f}}, \quad (45)$$

where $\tilde{\mathcal{F}}(v_r) = \frac{\mathcal{F}(v_r)}{\sqrt{f}}$. In the nonrelativistic limit $c_s^2 \ll 1$ and $v^2 \ll 1$, we have

$$\tilde{\mathcal{F}}(v_r) = \frac{[(1-3\vec{\theta}\cdot\vec{B})c_s^2 - (1+3\vec{\theta}\cdot\vec{B})v_r^2 - (\vec{\theta}\times\vec{E})_rv_r + 2(\theta_rB_rv_r^2)]}{\sqrt{(1-2\vec{\theta}\cdot\vec{B}) - 3(\vec{\theta}\times\vec{E})_rv_r}}. \quad (46)$$

Now using the relation $v_r = c_s\frac{r_h^2}{r^2}$ in the equation above, where r_h is the event horizon, the radius at which the flow speed exceeds the sound speed in the fluid, we have

$$\tilde{\mathcal{F}}(r) = \frac{c_s^2[(1-3\vec{\theta}\cdot\vec{B}) - (1+3\vec{\theta}\cdot\vec{B} - 2\theta_rB_r)\frac{r_h^4}{r^4} - (\vec{\theta}\times\vec{E})_r\frac{r_h^2}{c_sr^2}]}{\sqrt{(1-2\vec{\theta}\cdot\vec{B}) - 3[(\vec{\theta}\times\vec{E})_r]\frac{r_h^2}{c_sr^2}}}. \quad (47)$$

In this case the Hawking temperature is given by

$$\tilde{T}_H = \frac{\tilde{\mathcal{F}}'(r_h)}{4\pi} = \frac{[1+3\vec{\theta}\cdot\vec{B} - 2\theta_rB_r + (\vec{\theta}\times\vec{E})_r/2c_s]}{\sqrt{1-2\vec{\theta}\cdot\vec{B} - 3(\vec{\theta}\times\vec{E})_r/c_s}}T_H, \quad (48)$$

where $T_{\rm H} = c_s^2/(\pi r_{\rm h})$ is the Hawking temperature in the commutative case, and that for $\theta_r = 0$, $\vec{\theta}\cdot\vec{B} = \theta_3 B_3$, $\vec{\theta}\times\vec{E} = 0$ (or $E = 0$) with small $\theta_3 B_3$ is

$$\tilde{T}_{\rm H} = \frac{(1+3\theta_3 B_3)}{\sqrt{1-2\theta_3 B_3}} T_{\rm H} = (1+4\theta_3 B_3) T_{\rm H}. \tag{49}$$

In the limit $\theta \to 0$ the usual (commutative) result is obtained; otherwise one can see from (43) that the Hawking temperature in terms of group velocity goes like $\tilde{T}_{\rm H} \simeq v_g^8 T_{\rm H}$, for $\theta_3 B_3$ small enough.

Let us now analyze this result more carefully. The formula above can be rewritten in terms of variations of the Hawking temperature with respect to the changing of medium due to spacetime noncommutativity and "strong" magnetic field component as follows:

$$\frac{\Delta T_{\rm H}}{T_{\rm H}} = 4\theta_3 B_3. \tag{50}$$

This allows us to write the interesting formula for the group velocity deviation with respect to the sound speed in terms of the variation of the Hawking temperature:

$$\frac{v_g - c_s}{c_s} = \frac{1}{2}\theta_3 B_3 = \frac{1}{8}\frac{\Delta T_{\rm H}}{T_{\rm H}}. \tag{51}$$

The noncommutative acoustic metric can also be written in a Kerr-like form. We now address the issue of rotating black holes by using the projections above such that we can rewrite Eq. (26) as

$$\begin{aligned} ds^2 = {} & \frac{b\rho_0}{2c_s\sqrt{f}}[-[(1-3\theta_z B_z)c_s^2 - (1+3\theta_z B_z)(v_r^2+v_\phi^2)]dt^2 \\ & - 2(1+2\theta_z B_z)v_r dr dt - 2(1+2\theta_z B_z)v_\phi r d\phi dt \\ & + (1+\theta_z B_z)[v_r^2 dr^2 + r^2 v_\phi^2 d\phi^2 + 2v_r v_\phi r dr d\phi] \\ & + [(1+\theta_z B_z)(1+c_s^2 - v_r^2 - v_\phi^2)] \\ & \times (dr^2 + r^2 d\phi^2 + dz^2)]. \end{aligned} \tag{52}$$

However, exploring the original solutions as spherically symmetric solutions with $v_z = 0$, $v_r \neq 0$, and $v_\phi \neq 0$, one can show that they can be written in a Kerr-like form:

$$\begin{aligned} ds^2 = \frac{b\rho_0}{2c_s}\bigg[& -N^2 d\tau^2 + M^2 dr^2 + Q^2 r^2 d\varphi^2 + Z^2 dz^2 \\ & + \frac{(1+\theta_z B_z)[(1+\theta_z B_z)v_\phi d\tau - r d\varphi]^2}{\sqrt{f}}\bigg], \end{aligned} \tag{53}$$

where we have the Kerr-like components as

$$N^2 = \frac{(1-3\theta_z B_z)c_s^2 - (1+3\theta_z B_z)v_r^2}{\sqrt{f}}, \qquad M^2 = \frac{\mathcal{F} c_s^2}{[(1-3\theta_z B_z)c_s^2 - (1+3\theta_z B_z)v_r^2]\sqrt{f}}, \tag{54}$$

$$Q^2 = \frac{(1+\theta_z B_z)(c_s^2 - v_r^2)}{\sqrt{f}}, \qquad Z^2 = \frac{(1+\theta_z B_z)(1+c_s^2 - v^2)}{\sqrt{f}}, \tag{55}$$

$$\mathcal{F} = (1-2\theta_z B_z)(1+c_s^2) - (1+4\theta_z B_z)v^2 + \frac{6\theta_z B_z v_r^2 v_\phi^2}{c_s^2 - v_r^2}, \tag{56}$$

$$f = (1-2\theta_z B_z)(1+c_s^2) - (1+4\theta_z B_z)v^2 \tag{57}$$

and the coordinate transformations we have used are

$$d\tau = dt + \frac{(1+2\theta_z B_z)v_r dr}{[(1-3\theta_z B_z)c_s^2 - (1+3\theta_z B_z)v_r^2]}, \qquad d\varphi = d\phi + \frac{v_r v_\phi dr}{r[c_s^2 - v_r^2]}. \tag{58}$$

Now we find the important components

$$g_{\tau\tau} = \frac{-(1-3\theta_z B_z)c_s^2 + (1+3\theta_z B_z)(v_r^2+v_\phi^2)}{\sqrt{f}}, \qquad g_{rr} = \frac{\mathcal{F} c_s^2}{[(1-3\theta_z B_z)c_s^2 - (1+3\theta_z B_z)v_r^2]\sqrt{f}}, \tag{59}$$

where we have made the approximation $(1-\theta_z B_z)^3 \simeq (1-3\theta_z B_z)$ which is valid for $\theta_z B_z$ sufficiently small. For a planar solution (assuming $z = 0$) the velocities assume the form $v_r = \frac{A}{r}$ and $v_\phi = \frac{B}{r}$. After substituting this into the equations above we are able to find the ergosphere radius and the horizon via coordinate singularity through the following equations, $g_{\tau\tau}(r_e) = 0$ and $g_{rr}(r_{\rm h}) = 0$, respectively. The corresponding radii read

$$r_e = (1+3\theta_3 B_3)\frac{(A^2+B^2)^{1/2}}{c_s}, \qquad r_{\rm h} = (1+3\theta_3 B_3)\frac{|A|}{c_s}. \tag{60}$$

Notice that $3\theta_z B_z$ stands for the Lorentz-violating parameter β in our previous results [19,27] for Lorentz-violating acoustic black holes. Starting from this point all analysis made in [19,27] applies here. Many interesting studies can be followed from this point. One of them is the superresonance [25,27,36] which is an analog of the superradiance phenomenon in gravitational black holes, but a detailed study on this subject is outside the scope of this paper. We shall consider this study in a forthcoming publication.

V. CONCLUSIONS

One of the main results of the present paper is that supersonic particles can be understood in terms of $\Delta T_{\rm H}/T_{\rm H}$, where $T_{\rm H}$ is the Hawking temperature. We have considered this study in noncommutative acoustic black holes in a noncommutative Abelian Higgs model. The Abelian Higgs model is good for describing high energy physics and the noncommutative Abelian Higgs model can also describe Lorentz symmetry violation in particle physics in high energy. Thus our results suggest that in addition to the expected gravitational mini black holes formed in high energy experiments one can also expect the formation of acoustic black holes together.

The model also develops several similarities with respect to Lorentz-violating acoustic black holes studied in [19,27]. One of the consequences is that the acoustic Hawking temperature is changed such that it depends on the group speed, which means that, analogously to the gravitational case [23,24], the Hawking temperature is not universal for all species of particles. It depends on the maximal attainable velocity of the species. In the context of gravitational black holes this has been previously studied and appointed as a sign of possible violation of the second law of thermodynamics. Furthermore, the acoustic black hole metric in our model can be identified with an acoustic Kerr-like black hole. As we explicitly have shown in [27], using a similar Lorentz-violating setup, the spacetime noncommutativity should also affect the rate of loss of mass (energy) of the black hole. Thus for suitable values of the noncommutative parameter a wider or lower spectrum of particle wave function can be scattered with increased amplitude by the acoustic black hole. The superresonance and its increasing/decreasing phenomenon have been previously studied in [25–27,36].

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