PHYSICAL REVIEW D **85,** 023530 (2012)

Radio bursts from superconducting strings

Yi-Fu Cai,* Eray Sabancilar,† and Tanmay Vachaspati‡

Physics Department, Arizona State University, Tempe, Arizona 85287, USA

(Received 17 October 2011; revised manuscript received 4 January 2012; published 25 January 2012)

We show that radio bursts from cusps on superconducting strings are linearly polarized, thus providing a signature that can be used to distinguish them from astrophysical sources. We write the event rate of string-generated radio transients in terms of observational variables, namely, the event duration and flux. Assuming a canonical set of observational parameters, we find that the burst event rate can be quite reasonable, e.g., order ten a year for grand unified strings with 100 TeV currents, and a lack of observed radio bursts can potentially place strong constraints on particle physics models.

DOI: 10.1103/PhysRevD.85.023530 PACS numbers: 98.80.Cq, 11.27.+d, 95.85.Bh, 95.85.Fm

Cosmic strings are possible relics from the early Universe. Their discovery would substantiate our hot big bang cosmological model and also provide tremendous insight into the nature of fundamental interactions.

There are a number of different ways to look for cosmic strings, mostly based on their gravitational interactions, and negative searches so far impose constraints on particle physics models and cosmology. If the strings are superconducting [1], their electromagnetic emission provides yet another signature that can be used to search for them. The electromagnetic emission from a cosmic string loop is not steady and can have sharp bursts that can be seen as transient events. In Ref. [2], it was pointed out that it might be fruitful to look for superconducting strings by searching for bursts at radio wavelengths. There is a simple reason for choosing to look in the radio band. Cosmic strings are large objects and their fundamental frequency of emission is very low. The power emitted at higher frequencies generally falls off with increasing harmonic. Thus, there is more power emitted in the radio than in other bands such as the optical. Also, as we shall see in Sec. I, the emission in the burst is beamed, with the beam being widest at lower frequencies. Thus, the event rate in radio bursts can be expected to be larger than those at higher frequencies. On the other hand, propagation effects in the radio band are stronger, and these have to be included when evaluating the signature.

Besides superconducting cosmic strings, there are other strong motivations for looking at transient radio phenomenon from pulsars, supernovae, black hole evaporation, gamma-ray bursts, active galactic nuclei, and extraterrestrial life. A radio burst from a superconducting cosmic string will have to be distinguished among bursts from other potential astrophysical sources. With this in mind, we recalculate the characteristics of the string burst and show that it is linearly polarized in a direction that is independent of the frequency.

*ycai21@asu.edu
†Eray.Sabancilar@asu.edu
‡tvachasp@asu.edu

The feasibility of observations depends on the event rates for radio bursts. Here, we focus on evaluating the event rate in variables that are most useful to observers. The burst at source occurs with a certain duration and flux. However, the observed duration and flux depend on the redshift of the source. We transform the event rate from source variables to observer variables. These results will be useful in the ongoing search for radio transients at the Parkes [3], ETA [4], and LWA [5] telescopes, and the new generation large radio telescopes such as LOFAR [6] and SKA [7].

This paper is organized as follows. In Sec. I, we calculate the characteristics of a burst from a superconducting string, including the polarization. In Sec. II, we find the event rate in observer variables, followed by a numerical evaluation in Sec. IV. We conclude in Sec. V.

I. BURST CHARACTERISTICS

The electromagnetic field due to a superconducting cosmic string is given by Maxwell's equations

$$\partial_\nu \partial^\nu A^\mu = 4\pi J^\mu, \tag{1}$$

where

$$J^\mu(t, \vec{x}) = I \int d\sigma f^\mu_{,\sigma} \delta^{(3)}(\vec{x} - \vec{f}(t, \sigma)), \tag{2}$$

and we have assumed that the string denoted by $f^\mu(t, \sigma)$ carries a uniform and constant current I.

A string loop oscillates and the general solution in its center-of-mass frame can be written as

$$f^0 = t, \qquad \vec{f}(t, \sigma) = \tfrac{1}{2}[\vec{f}_+(\sigma_+) + \vec{f}_-(\sigma_-)],$$
$$\sigma_\pm = \sigma \pm t, \qquad |\vec{f}'_\pm| = 1, \qquad \int d\sigma_\pm \vec{f}_\pm = 0, \tag{3}$$

where a prime denotes derivate with respect to the argument.

The power emitted in electromagnetic radiation from superconducting strings has been analyzed in [8] but the polarization has not been studied. So, we repeat earlier analyses to show that the radiation is linearly polarized. This also leads into the analysis of the event rate in Sec. II.

The string dynamics is periodic and so is the current. Hence, we work with discrete Fourier transforms

$$A^\mu(t,\vec{x}) = \sum_\omega e^{-i\omega t} A^\mu_\omega(\vec{x}), \tag{4}$$

$$J^\mu(t,\vec{x}) = \sum_\omega \int d^3k e^{-i(\omega t - \vec{k}\cdot\vec{x})} J^\mu_\omega(\vec{k}), \tag{5}$$

where $\omega = 4\pi n/L$ and n is an integer. Then,

$$A^\mu_\omega(\vec{x}) = \int d^3x' \int d^3k \frac{e^{i\vec{k}\cdot\vec{x}'}}{|\vec{x}-\vec{x}'|} J^\mu_\omega(\vec{k}) e^{i\omega|\vec{x}-\vec{x}'|}, \tag{6}$$

and $J^\mu_\omega(\vec{k})$ follows from Eq. (2)

$$J^\mu_\omega(\vec{k}) = \frac{2I}{(2\pi)^3 L}\int_0^{L/2} dt \int_0^L d\sigma e^{i(\omega t - \vec{k}\cdot\vec{f})} f'^\mu(t,\sigma), \tag{7}$$

where the delta function that appeared in Eq. (2) has been integrated out. In terms of the left and right movers of Eq. (3), we get

$$J^\mu_\omega(\vec{k}) = \frac{2I}{(2\pi)^3 L}(J_+{}^\mu J_-{}^0 + J_+{}^0 J_-{}^\mu), \tag{8}$$

where

$$J^\mu_\pm(\vec{k}) = \int_0^L d\sigma_\pm e^{ik\cdot f_\pm/2} f'^\mu_\pm. \tag{9}$$

The loops that will give us the strongest observational signatures will have lengths that are much larger than the radio wavelengths at which observations are made. Hence, $\omega L = 4\pi n \gg 1$, and only high harmonics are of interest. The integrals $J_\pm$ can be evaluated using the saddle point approximation. If the saddle point has a nonvanishing imaginary piece in the complex $\sigma_\pm$ plane, the integrals fall off exponentially fast with n, and the electromagnetic radiation in those directions, $\vec{k}$, is negligible. The interesting situation is when the saddle point is real in the evaluation of both $J^\mu_\pm$. This can happen if

$$k\cdot f'_\pm = 0, \tag{10}$$

and corresponds to a "cusp" on the string loop as discussed in earlier work, and more recently in some generality in Ref. [9]. The integrals can be evaluated by expansion around the real saddle point and lead to

$$J^\mu_\omega(\vec{k}) \simeq i\frac{2I}{(2\pi)^3\omega} e^\mu_\omega, \tag{11}$$

where

$$e^\mu_\omega = -\frac{i}{L\alpha_+\alpha_-}\left[\frac{f''^\mu_+}{\alpha_+}\int du_+ u_+ e^{iu_+^3}\int du_- e^{-iu_-^3} + \frac{f''^\mu_-}{\alpha_-}\int du_+ e^{iu_+^3}\int du_- u_- e^{-iu_-^3}\right], \tag{12}$$

and

$$u_+ = \omega^{1/3}\alpha_+\sigma_+, \qquad \alpha_+ = \left(\frac{l_\mu f'''^\mu_+}{12}\right)^{1/3},$$
$$u_- = \omega^{1/3}\alpha_-\sigma_-, \qquad \alpha_- = \left(\frac{l_\mu f'''^\mu_-}{12}\right)^{1/3}, \tag{13}$$

where $k \equiv \omega l^\mu$.

Notice that e^μ_ω depends on the frequency since the range of integration is proportional to $\omega^{1/3}$. However, recall that we are interested in high harmonics and so $\omega L \gg 1$. In this limit, the term e^μ_ω approaches a frequency-independent form

$$e^\mu_\omega \to e^\mu \equiv \frac{\Gamma(\frac{1}{3})\Gamma(\frac{2}{3})}{3L\alpha_+\alpha_-}\left(\frac{f''^\mu_+}{\alpha_+} - \frac{f''^\mu_-}{\alpha_-}\right). \tag{14}$$

Then, Eq. (6) gives

$$A^\mu_\omega(\vec{x}) \propto e^\mu. \tag{15}$$

The corresponding electric and magnetic fields are

$$\vec{E}_\omega(\vec{x}) \propto \vec{e}, \qquad \vec{B}_\omega(\vec{x}) \propto \vec{e}\times\hat{k}, \tag{16}$$

where $\hat{k}$ is the unit vector in the direction of the beam emitted from the cusp. We have used $\vec{e}\cdot\hat{k} = 0$ because $\vec{f}'_+ = -\vec{f}'_- = \hat{k}$ at a cusp and $\vec{f}''_\pm \cdot \vec{f}'_\pm = 0$ because $|\vec{f}'_\pm| = 1$ [see Eq. (3)].

The form of the electric field shows that the radiation from cusps is linearly polarized in the direction $\vec{e}$. Furthermore, the direction of linear polarization is independent of the frequency of observation.

The above analysis applies for radiation exactly along the direction of the beam. Slightly off the direction of the beam, the saddle point in the integrals of Eq. (9) will acquire small imaginary components, and this causes the beam to die off exponentially fast outside an angle [8,10]

$$\theta_\omega \simeq (\omega L)^{-1/3}. \tag{17}$$

Therefore, the width of the beam is given by θ_ω. Similarly, the beam at frequency ω is emitted for a duration given by

$$\delta t_\omega \simeq \frac{L^{2/3}}{\omega^{1/3}}. \tag{18}$$

This is not the *observed* duration of the beam which we will discuss in the next section.

Within the beam, the energy radiated in a burst per unit frequency per unit solid angle is

$$\frac{d^2E_{\rm rad}}{d\omega d\Omega} \sim 2I^2L^2|\vec{e}|^2, \qquad \theta < \theta_\omega, \tag{19}$$

where θ denotes the angle measured from the direction of the beam. The energy arriving at a distance r is given by

$$\frac{1}{r^2}\frac{d^2E_{\rm rad}}{d\omega d\Omega} \sim 2\frac{I^2L^2}{r^2}|\vec{e}|^2, \qquad \theta < \theta_\omega. \tag{20}$$

Cosmic string loops are large objects and the fundamental frequency of radiation, given by $\sim L^{-1}$, is very small. Hence, radiation that can be observed is due to emission at very high harmonics. Although the energy per solid angle does not depend on the frequency, the width of the beam θ_ω does become smaller with increasing frequency. This suggests that the event rate will be largest at lower frequencies where the beam is wider. Hence, it seems favorable to seek bursts from strings in the radio band, though the dependence of the event rate on frequency can be more complicated because the more numerous small loops produce higher frequencies.

We now examine the event rate in more detail.

II. BURST EVENT RATE

Arguments of scale invariance and simulations of a cosmic string network indicate that the loop distribution function in the radiation-dominated epoch is

$$dn_{L_0} \sim \kappa \frac{dL_0}{L_0^{5/2} t^{3/2}}, \tag{21}$$

where n_L is the number density of loops of size L at cosmic time $t < t_{\rm eq}$, where $t_{\rm eq}$ is the time of radiation-matter equality, and $\kappa \sim 1$. In the matter-dominated epoch, $t > t_{\rm eq}$, there will be two components to the loop distribution. The first is the loops that were produced in the radiation-dominated era but survived into the matter era. The second is the loops that were produced during the matter-dominated era and these are expected to have a $1/L^2$ distribution. The total loop distribution is a sum of these two components,

$$dn_{L_0} \sim \left(\kappa_M + \kappa_R \sqrt{\frac{t_{\rm eq}}{L_0}}\right) \frac{dL_0}{L_0^2 t^2}, \tag{22}$$

where κ_R and κ_M are order 1 coefficients relevant for the radiation and matter era distributions. We will take $\kappa_M \sim \kappa_R \equiv \kappa$.

Radiative losses from loops imply that the loops shrink with time and so

$$L(t) = L_0 - \Gamma(t - t_i), \tag{23}$$

where Γ is a parameter and we will use $t \gg t_i$, i.e., we consider a time much later than the time when the loop was produced. For shrinkage due to gravitational radiation, $\Gamma \sim 100 G\mu$, where μ is the string tension, e.g., for strings produced at the scale of 10^{14} GeV, $G\mu \approx 10^{-10}$. Therefore, the loop distribution function, taking energy losses into account, is

$$dn(L, t) = \frac{\kappa C_L dL}{t^2 (L + \Gamma t)^2}, \tag{24}$$

where

$$C_L \equiv 1 + \sqrt{\frac{t_{\rm eq}}{L + \Gamma t}}. \tag{25}$$

For $L \ll \Gamma t_0$, the radiation era loops are more important because we will be interested in $\Gamma < 10^{-6}$ whereas $t_{\rm eq}/t_0 \approx 10^{-5}$. For larger L, the matter era loops dominate.

We now write this formula in terms of the redshift, z, in the matter-dominated era

$$dn(L, z) \simeq \frac{\kappa C_L (1+z)^6 dL}{t_0^2 [(1+z)^{3/2} L + \Gamma t_0]^2}, \tag{26}$$

where

$$1 + z = \left(\frac{t_0}{t}\right)^{2/3}, \tag{27}$$

and

$$C_L = 1 + (1+z)^{3/4} \sqrt{\frac{t_{\rm eq}}{(1+z)^{3/2} L + \Gamma t_0}}. \tag{28}$$

The current age of the Universe is $t_0 \simeq 4 \times 10^{17}$ s.

If a loop has a cusp, there will be a burst in every period of oscillation. So, the rate of cusps on a loop of length L is c/L where $c \sim 1$ is the probability that a loop will contain a cusp [11]. If the loop is at cosmological redshift, the observed rate of cusps on a given loop will be $c/(L(1+z))$ due to time dilation.

The radiation from a cusp can be emitted in any direction. Only the bursts pointing in the direction of the observer are relevant. Since the beam width at frequency ω is $\theta_\omega \sim (\omega L)^{-1/3}$ [Eq. (17)], the event rate will be suppressed by a factor θ_ω^2.

Combining all these factors gives an event "production" rate in a spatial volume dV,

$$d\dot{N} \simeq c \frac{\theta_\omega^2}{L(1+z)} dn(L, z) dV. \tag{29}$$

Note that the beam of radiation emitted from a cusp is wider at lower frequencies. Thus, if a burst is observed at a particular frequency ω_e, it will also be observed at all lower frequencies.

The volume element is converted to a redshift element using the distance-redshift relation assuming a matter-dominated, flat cosmology

$$H_0 dr = \frac{dz}{(1+z)^{3/2}}, \tag{30}$$

where $H_0 = 2/(3t_0) = 72$ km/sec/Mpc. Then,

$$r = \frac{2}{H_0}\left[1 - \frac{1}{\sqrt{1+z}}\right], \tag{31}$$

and the physical volume is

$$dV = \frac{16\pi}{H_0^3}\left[1 - \frac{1}{\sqrt{1+z}}\right]^2 \frac{dz}{(1+z)^{9/2}}, \tag{32}$$

where we have integrated over the angular coordinates.

As a consequence, the burst production rate is

$$d\dot{N} \simeq \frac{At_0\nu_e C_L}{(\nu_e L)^{5/3}} \frac{(1+z)^{-1/2}[\sqrt{1+z}-1]^2}{[(1+z)^{3/2}L + \Gamma t_0]^2} dLdz, \tag{33}$$

where all the numerical factors have been consolidated in $A \sim 50$, and the subscript "e" on ν_e denotes that it is the frequency at emission.

From an observer's point of view, the burst production rate is not relevant; instead, we must find the event rate that observers can expect to see within the parameters of their instruments. Thus, the event rate must be expressed in terms of quantities such as the energy flux per frequency interval, S, to which the instrument is sensitive, and the burst duration, Δ, that can be detected. So, we must transform the variables (L, z) occuring in Eq. (33) to (S, Δ). That will give the event rate in terms of variables that are relevant to observation.

The observed frequency is related to the emitted frequency by a redshift factor

$$\nu_o = \frac{\nu_e}{1+z}. \tag{34}$$

The energy flux per frequency interval can be found from the radiated energy in Eq. (20), which gives the total energy radiated from the cusp. To get the energy radiated per unit time, we need to divide that expression by the observed duration of the burst, Δ. So, the observed energy per unit time per unit area per unit frequency interval is

$$S \simeq \frac{I^2L^2}{r^2\Delta}, \tag{35}$$

where r is given in terms of z in Eq. (31). We have chosen to normalize the cosmological scale factor to be one today, $a(t_0) = 1$, and hence the coordinate distance r is also the physical distance at the present epoch. As a simplifying assumption, we will only consider the case when the current, I, is a constant. In general, the current will depend on the cosmological epoch because it can build up due to string interactions with a cosmological magnetic field and dissipate due to scattering of the charge carriers on the string.

The duration of the burst is determined by a combination of the duration at emission ("intrinsic" duration), the cosmological redshift, and the time delays due to scattering with the cosmological medium. This last factor is important for bursts at long wavelengths such as in the radio. The intrinsic burst duration at the emission point is given in Eq. (18). To obtain the burst duration at the observation point, we have to include a factor of $1/\gamma^2$ where $\gamma \sim (\omega L)^{1/3}$ is the Lorentz factor at the cusp. This factor was first derived in Ref. [12] and is also seen in synchrotron radiation [13]; it was, however, missed in Ref. [2]. It arises because the cusp is moving toward the observer and so photons emitted over a time interval δt arrive at the observer in the interval $\delta t(1-v)$ where v is the speed of the string in the emitting region near the cusp. In addition, we need to include a cosmological redshift factor to account for time dilation. This gives the intrinsic beam duration at the observation point

$$\Delta t_{\rm in} \simeq \frac{(1+z)L^{2/3}}{\nu_e^{1/3}} \frac{1}{(\nu_e L)^{2/3}} \simeq \frac{1}{\nu_o}. \tag{36}$$

The burst duration due to scattering with the turbulent intergalactic medium at given frequency, ν_o, and redshift, z, is modeled as a power law [14,15] (for a review, see [16])

$$\Delta t_s \simeq \delta t_1 \left(\frac{1+z}{1+z_1}\right)^{1-\beta} \left(\frac{\nu_o}{\nu_1}\right)^{-\beta}, \tag{37}$$

where, empirically,

$$\delta t_1 = 5\,{\rm ms}, \quad z_1 = 0.3, \quad \nu_1 = 1.374\,{\rm GHz}, \quad \beta = +4.8. \tag{38}$$

Note that with our conventions in Eq. (37), $\beta > 0$. (In Ref. [2], the sign conventions were such that β was negative.)

The total burst duration, Δ, is a sum in quadratures of the intrinsic time width and the width due to scattering

$$\Delta = \sqrt{\Delta t_{\rm in}^2 + \Delta t_s^2}. \tag{39}$$

Inserting the expressions in Eqs. (36) and (37) leads to

$$1 + z = \frac{(\Delta^2\nu_o^2 - 1)^{1/2(1-\beta)}}{\delta_1^{1/(1-\beta)}\nu_o}, \tag{40}$$

where

$$\delta_1 \equiv \nu_1^\beta \delta t_1 (1+z_1)^{\beta-1}. \tag{41}$$

Inserting numerical values from Eq. (38) gives

$$1 + z \simeq \frac{82}{(\Delta^2\nu_o^2 - 1)^{1/2(\beta-1)}} \left(\frac{\nu_1}{\nu_o}\right). \tag{42}$$

Our calculations assume that $z < z_{\rm rec} \simeq 1100$, the redshift at recombination. Then, the constraints $0 < z < z_{\rm rec}$ give

$$\Delta_{\rm min} < \Delta < \Delta_{\rm max}, \tag{43}$$

where

$$\Delta_{\rm min} = \frac{1}{\nu_o}\left\{1 + \left[0.075\left(\frac{\nu_1}{\nu_o}\right)\right]^{2(\beta-1)}\right\}^{1/2}, \tag{44}$$

$$\Delta_{\rm max} = \frac{1}{\nu_o}\left\{1 + \left[82\left(\frac{\nu_1}{\nu_o}\right)\right]^{2(\beta-1)}\right\}^{1/2}. \tag{45}$$

For example, with $\nu_o = \nu_1$, $\Delta_{\rm min} \sim 7 \times 10^{-10}$ s and $\Delta_{\rm max} \sim 1.3 \times 10^{-2}$ s.

To transform from intrinsic variables (L, z) to observer variables (S, Δ), we need to calculate the Jacobian of the transformation. We have already obtained $z(\Delta, S)$ in Eq. (40). From Eq. (35), we also obtain

$$L = \frac{r}{I}\sqrt{S\Delta}, \tag{46}$$

and r is a function of z [Eq. (31)] which is a function of (S, Δ) as in (40). Some algebra then leads to the Jacobian factor

$$\left|\frac{\partial(L, z)}{\partial(S, \Delta)}\right| = \frac{\nu_o L\Delta}{2(\beta - 1)\delta_1^{1/(1-\beta)} S(\Delta^2\nu_o^2 - 1)^{1-1/2(1-\beta)}}, \tag{47}$$

where $L = L(S, \Delta)$ via Eq. (46).

Now, we can get the event rate in observer variables from the production rate of Eq. (33),

$$d\dot{N} \simeq \frac{At_0}{2(\beta - 1)} \frac{C_L \nu_o^2 \Delta}{S(\nu_o L)^{2/3}} \frac{1}{\Delta^2\nu_o^2 - 1} \times \frac{[\sqrt{1+z} - 1]^2}{(1+z)^{1/6}[(1+z)^{3/2}L + \Gamma t_0]^2} dSd\Delta, \tag{48}$$

where we have used Eq. (34), L is given by Eq. (46), and $r(z)$ by Eq. (31). Note that $\Delta_{\min}^2\nu_o^2 > 1$ and so the event rate does not have a singularity for $\Delta \in [\Delta_{\min}, \Delta_{\max}]$.

Equation (48) is our final expression for the differential event rate. We will now analyze the expression to extract certain closed form results.

III. EVENT RATE ANALYSIS

Even though the emitted burst duration in Eq. (18) depends on the length of the loop, the observed burst duration in Eq. (39) is independent of the length of the loop. Hence, for a given burst duration at a certain frequency, a loop (of any length) has to be at the redshift given by Eq. (40). This fixes the distance to the loop. The energy flux from a loop, however, does depend on its length and Eq. (46) gives $L \propto \sqrt{S}$. The only implicit dependence of the event rate in Eq. (48) on S occurs through L which also appears in C_L. If we consider the limit $S \to \infty$, we have $(1+z)^{3/2}L \gg \Gamma t_0$ and $C_L \to 1$, and simple power counting gives

$$d\dot{N} \propto \frac{dS}{S^{7/3}}, \qquad S \to \infty. \tag{49}$$

For smaller S, such that $(1+z)^{3/2}L \ll \Gamma t_0$, the power counting gives

$$d\dot{N} \propto \frac{dS}{S^{4/3}}. \tag{50}$$

The dependence of the event rate on the burst duration is less apparent. Note that long duration bursts, $\Delta \gg 1/\nu_o$, are only possible if the loop is very close, and then too it is not possible to have bursts of arbitrarily long duration at some fixed observation frequency. The maximum possible duration occurs at $z = 0$ and is given in Eq. (45). So, to find the event rate for duration bursts close to $\Delta_{\max}$, we expand the event rate around $z = 0$. First, we obtain

$$z \simeq \frac{\nu_o^{2\beta}\Delta_{\max}}{(\beta - 1)\delta_1^2}(\Delta_{\max} - \Delta), \tag{51}$$

which leads to

$$d\dot{N} \propto \left(\frac{\Delta_{\max} - \Delta}{S}\right)^{4/3} dSd\Delta, \tag{52}$$

where we have assumed $L \ll \Gamma t_0$ which implies that S cannot be too large.

Having obtained these limiting forms for the event rate, we now turn to a numerical evaluation.

IV. NUMERICAL ESTIMATES

We now find the event rate as a function of the flux and the burst duration by numerically evaluating and integrating Eq. (48).

For our numerical estimates, we take the cosmological parameters

$$t_0 = 4 \times 10^{17}\ \text{s}, \qquad t_{\text{eq}} = 2.4 \times 10^{12}\ \text{s}. \tag{53}$$

We also assume the string parameters

$$I = 10^5\ \text{GeV}, \qquad \Gamma = 10^{-8}. \tag{54}$$

Typically, for string loop decay due to gravitational radiation, $\Gamma \sim 100G\mu$ where G is the gravitational Newton's constant and μ is the string tension. Therefore, our choice of Γ corresponds to $G\mu \sim 10^{-10}$ or a symmetry breaking energy scale of 10^{14} GeV, which is a scale at which grand unification may occur.

The scattering contribution to the burst duration in Eq. (37) contains a number of parameters that are determined empirically, and are shown in Eq. (38). In exploring parameter space, we shall assume a range of parameters motivated by the Parkes survey [3],

$$\nu_o \in (1.230, 1.518)\ \text{GHz}, \qquad \Delta \in (10^{-3}, 1)\ \text{s}, \qquad S \in (10^{-5}, 10^{+5})\ \text{Jy}. \tag{55}$$

Note the conversion

$$1\ \text{Jy} = 10^{-23}\frac{\text{ergs}}{\text{cm}^2\text{-s-Hz}}. \tag{56}$$

In the following figures, we show the differential event rate of Eq. (48) as functions of the flux and burst duration. First, in Fig. 1 we plot the differential event rate as a function of S for several different choices of ν_o and Δ. The plot is made on a log-log scale to accommodate the wide range of scales, and shows two different power law behaviors, consistent with the analytical results in Eqs. (49) and (50).

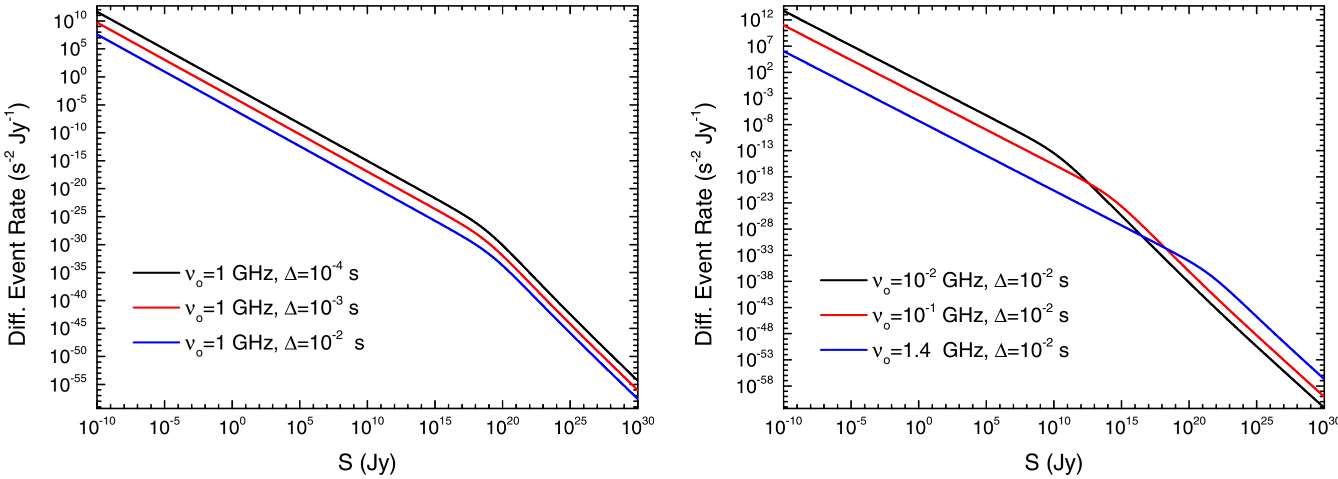

FIG. 1 (color online). The differential event rate of radio bursts emitted from superconducting cosmic strings as a function of flux S. In the left panel, the observed frequency is fixed, $\nu_o = 1$ GHz, and the duration is chosen to be $\Delta = 10^{-4}, 10^{-3}, 10^{-2}$ s (top to bottom curves). In the right panel, the duration is fixed, $\Delta = 0.01$ s, and the observed frequency is chosen to be $\nu_o = 0.01, 0.1, 1.4$ GHz (top to bottom curves for small S).

In Fig. 2, we show the dependence of the differential event rate on the burst duration for a variety of values of S and ν_o.

The integrated event rate as a function of the flux S and burst duration Δ is shown in the left and right panels of Fig. 3. The asymptotic fits to these plots are

$$\frac{d\dot{N}}{dS} \simeq 10^{-7}\left(\frac{S}{1\ \mathrm{Jy}}\right)^{-4/3}\ \mathrm{s}^{-1}\,\mathrm{Jy}^{-1}, \tag{57}$$

$$\frac{d\dot{N}}{d\Delta} \simeq 10^{-2}\left(\frac{\Delta}{1\ \mathrm{ms}}\right)^{-9/4}\ \mathrm{s}^{-2}. \tag{58}$$

Hence, an experiment that integrates events over the ranges of Δ in Eq. (55), and is sensitive to milli Jansky fluxes, will observe on the order of 1 radio bursts per month, if there are superconducting cosmic strings with the chosen parameters. Turning this figure around, a search for cosmological radio transients can place stringent constraints on superconducting cosmic strings. If we consider radio bursts emitted by superconducting strings with observable frequency 1.23 GHz and flux greater than 300 mJy, the event rate is $\sim 10^{-3}$ per hour, and is a factor of 10 smaller than the upper bound given by the Parkes survey [3], 0.025 per hour. Since the predicted event rate depends on the string parameters, this result implies that current radio

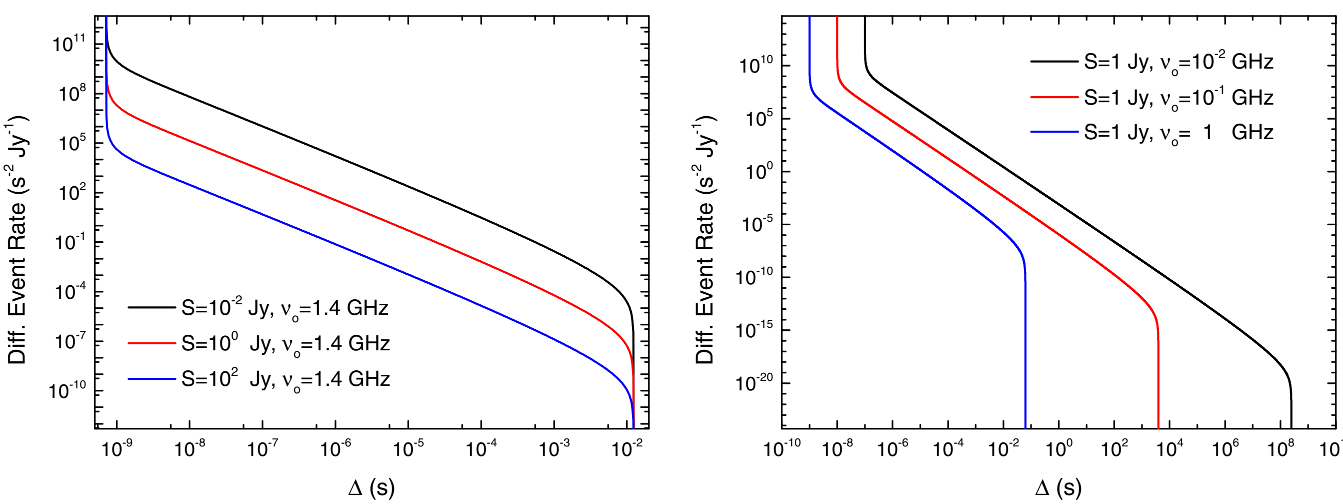

FIG. 2 (color online). The differential event rate of radio bursts from superconducting cosmic strings as a function of duration Δ. In the left panel, the observed frequency is fixed, $\nu_o = 1.4$ GHz, and the flux is chosen to be $S = 10^{-2}, 1, 10^2$ Jy (top to bottom curves). In the right panel, the flux is fixed as $S = 1$ Jy, and the observed frequency is chosen to be $\nu_o = 0.01, 0.1, 1$ GHz (top to bottom curves). The range of Δ lies in $[\Delta_{\min}, \Delta_{\max}]$ as defined in Eqs. (44) and (45).

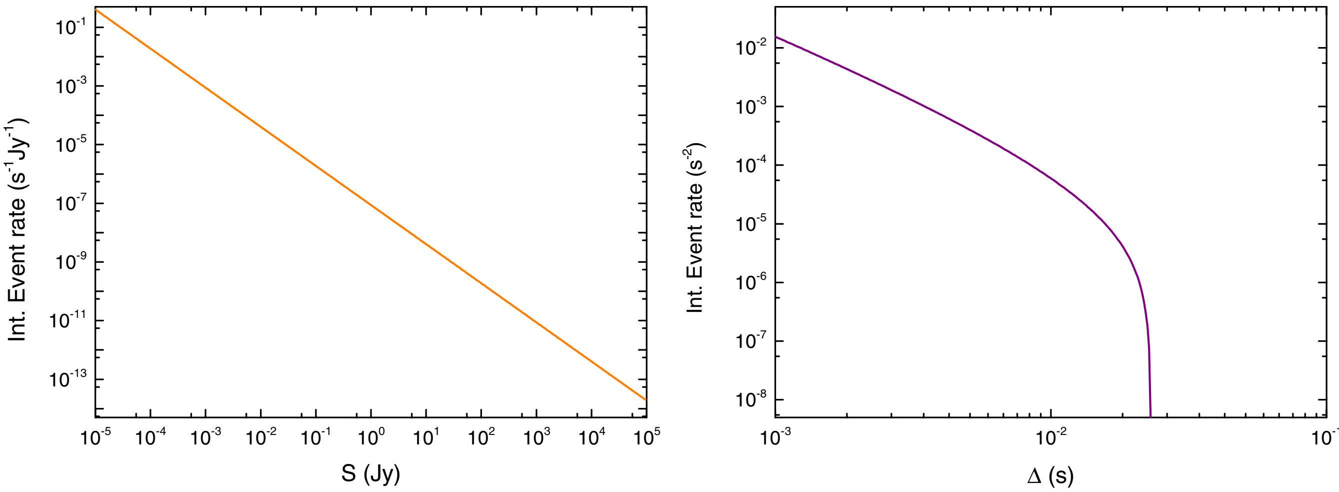

FIG. 3 (color online). The integrated event rate of radio bursts with fixed observable frequency, $\nu_o = 1.23$ GHz, from superconducting cosmic strings as functions of the flux S (left panel) and the duration Δ (right panel). The intervals of integration and other parameters are given in Sec. IV.

experiments already rule out an interesting part of parameter space (current and string tension).

V. CONCLUSIONS

We have addressed two observational aspects of radio transients produced by cusps on superconducting strings. First, we have shown that the radiation emitted along the beam direction is linearly polarized, and the direction of polarization is independent of the frequency. The polarization can be used as a discriminating signature for radio bursts from superconducting strings, though a more detailed study should also consider the dependence of the polarization as a function of angle from the direction of the beam and the variation in the polarization over the duration of the event. Second, we have calculated the event rate of radio bursts from cusps on superconducting strings in terms of observational variables, namely, the burst duration and the flux. Our calculation includes the Jacobian that results from the transformation from string variables to observational variables.

Unlike burst events in higher energy parts of the electromagnetic spectrum, a novelty of the calculation for radio bursts is that the burst duration depends on the redshift of the burst event due to two contributions: the cosmological redshift and the scattering due to intervening matter. As is well understood, the former grows with redshift as $1 + z$ and when the redshifting of the frequency is also taken into account, gives a duration of $1/\nu_o$. The contribution of scattering is given by Eq. (37) [14,15] and is somewhat counterintuitive because it diminishes with increasing redshift. To understand this (partially), we note that the burst duration increases due to scattering because scattering allows photons to bend into the direction of the observer. If the relevant scattering can be thought to occur at roughly half the distance to the source, for a source that is farther away, the halfway scattering point is also more distant. Therefore, for fixed observational frequency, the frequency at the scattering point is also higher, and hence scattering is less efficient. Thus, more distant bursts get a smaller contribution to their duration from the scattering. The two contributions to the burst duration are added in quadrature, yielding Eq. (39).

We have also found the integrated event rate as a function of the flux and burst duration. For the canonical set of parameters listed in Sec. IV, the integrated event rates are reasonable, at the level of one event per month. Such event rates indicate that the search for radio bursts can serve as excellent probes of the superconducting string model.

Our analysis has been performed under some simplifying assumptions that may need to be reexamined in the future. Our formula for the burst duration due to scattering of radio waves, Eq. (37), should be reexamined in the cosmological context, since the relevant cosmological epochs are concurrent with reionization, formation of large scale structure, and other astrophysical activity. Note that we have also neglected the cosmological acceleration which will dilute the number density of cosmic strings and thus reduce the event rate of radio bursts at low redshifts. We have also sharply cut off all radio bursts prior to the epoch of recombination. In principle, there will be a gradual cutoff, though this may not make much difference to the final results. From the string side, we have assumed a constant current on all strings, whereas we expect the current to grow as a string cuts through ambient magnetic fields. If a primordial magnetic field exists, our assumption may be justified. In the absence of a primordial magnetic field, currents on strings will build up only after structures have generated magnetic fields. We have also assumed that the dominant energy loss from strings is due to gravitational radiation and not due to electromagnetic losses, i.e.,

$\Gamma\mu = 100G\mu^2 \gg 10I\sqrt{\mu}$. For $I \sim 10^5$ GeV, this is valid if the string energy scale is larger than 10^{14} GeV, i.e., $G\mu > 10^{-10}$. For yet lighter strings, Γ will be set by electromagnetic losses, and for very light strings, $\mu \sim (1\ \mathrm{TeV})^2$, the strings are dragged by the cosmological plasma, at least on large length scales, and the string dynamics will be very different. In the regime where gravitational losses dominate and radio bursts due to short loops dominate the event rate, our numerical results give

$$\dot{N} \simeq 2\times 10^{-5}\mu_{-8}^{-5/2} I_5^{2/3}\ \mathrm{s}^{-1}, \quad 100G\mu^2 > 10I\sqrt{\mu}, \qquad (59)$$

where $\mu_{-8} \equiv G\mu/(10^{-8})$ and $I_5 \equiv I/(10^5\ \mathrm{GeV})$. If the string parameters are such that the power lost to electromagnetic radiation is larger than that to gravitational radiation, we should replace the expression for gravitational power emission, $100G\mu^2$, by the electromagnetic power $10I\sqrt{\mu}$. This occurs when $I > 1.2\times 10^8 \mu_{-8}^{3/2}$ GeV. Then,

$$\dot{N} \simeq 2\times 10^{-3}\mu_{-8}^{5/4} I_8^{-11/6}\ \mathrm{s}^{-1}, \quad 100G\mu^2 < 10I\sqrt{\mu}. \qquad (60)$$

There are several radio telescopes currently in operation searching for radio transients, e.g., Parkes [3], ETA [4], LWA [5], LOFAR [6], and others under construction, e.g., SKA [7]. It would be useful to tailor the analysis in our paper to the specific range of observational parameters that will be employed in these searches.

Cosmic string cusps also produce gravitational wave bursts [17], which can be detectable by sensitive interferometers such as LIGO, VIRGO, and LISA, ultrahigh energy neutrino bursts [18], which can be detectable by the space-based cosmic ray detector JEM-EUSO and by radio telescopes LOFAR and SKA via Askaryan effect [19]. There has already been some initiative to look for electromagnetic counterparts of gravitational wave bursts at LIGO and VIRGO [20]. Linearly polarized radio signal and simultaneous detection of accompanying bursts from the same cusp can help distinguish cosmic strings from astrophysical sources, and hence help to discover cosmic strings or to put constraints on superconducting string parameters.

ACKNOWLEDGMENTS

We are especially grateful to Ken Olum for pointing out an error in an earlier version, and to Jose Blanco-Pillado, Robert Brandenberger, Mike Kavic, Ben Shlaer, John Simonetti, Daniele Steer, Alex Vilenkin, and Zheng Zheng for discussions. T. V. is grateful to the Institute for Advanced Study for hospitality. This work was supported by a grant from the Department of Energy at ASU.

[1] E. Witten, Nucl. Phys. **B249**, 557 (1985).
[2] T. Vachaspati, Phys. Rev. Lett. **101**, 141301 (2008).
[3] D. R. Lorimer, M. Bailes, M. A. McLaughlin, D. J. Narkevic, and F. Crawford, Science **318**, 777 (2007).
[4] C. D. Patterson *et al.*, ACM Transactions on Reconfigurable Technology and Systems **1**, 1 (2009).
[5] P. Henning *et al.*, Proc. Sci., ISKAF2010 (2010) 024 [arXiv:1009.0666].
[6] R. Fender *et al.* (The LOFAR Collaboration), Proc. Sci., MQW6 (2006) 104 [arXiv:astro-ph/0611298]; DYNAMIC (2007) 030 [arXiv:0805.4349].
[7] J. M. Cordes, Bull. Am. Astron. Soc. **39**, 999 (2007) [http://adsabs.harvard.edu/abs/2007AAS...21114604C]; P. E. Dewney *et al.*, Proc. IEEE **97**, 1482 (2009).
[8] A. Vilenkin and T. Vachaspati, Phys. Rev. Lett. **58**, 1041 (1987).
[9] D. A. Steer and T. Vachaspati, Phys. Rev. D **83**, 043528 (2011).
[10] J. J. Blanco-Pillado and K. D. Olum, Nucl. Phys. **B599**, 435 (2001).
[11] C. J. Copi and T. Vachaspati, Phys. Rev. D **83**, 023529 (2011).
[12] A. Babul, B. Pacynzki, and D. Spergel, Astrophys. J. **316**, L49 (1987).
[13] G. B. Rybicki and A. P. Lightman, *Radiative Processes in Astrophysics* (Wiley, New York, 1979).
[14] L. C. Lee and J. R. Jokipii, Astrophys. J. **206**, 735 (1976).
[15] S. R. Kulkarni, E. O. Ofek, J. D. Neill, M. Juric, and Z. Zheng, "Giant Sparks at Cosmological Distances" (unpublished).
[16] B. J. Rickett, Annu. Rev. Astron. Astrophys. **15**, 479 (1977).
[17] T. Damour and A. Vilenkin, Phys. Rev. Lett. **85**, 3761 (2000); Phys. Rev. D **64**, 064008 (2001); **71**, 063510 (2005).
[18] V. Berezinsky, K. D. Olum, E. Sabancilar, and A. Vilenkin, Phys. Rev. D **80**, 023014 (2009).
[19] G. Askarian, Sov. Phys. JETP **14**, 441 (1962).
[20] J. Abadie *et al.* (The LIGO Scientific Collaboration and the Virgo Collaboration), arXiv:1109.3498.