Observing signals of the bulk matter Randall-Sundrum model through rare decays of supersymmetric particles

Toshifumi Yamada

Department of Particles and Nuclear Physics, The Graduate University for Advanced Studies (SOKENDAI), and Institute of Particle and Nuclear Studies, High Energy Accelerator Research Organization (KEK), 1-1 Oho, Tsukuba, Ibaraki 305-0801, Japan

(Received 16 September 2011; published 19 January 2012)

The bulk matter Randall-Sundrum (RS) model is a setup where standard model (SM) matter and gauge fields reside in the bulk of 5D warped spacetime while the Higgs field is confined on the IR brane. The wave functions of the 1st and 2nd generation matter particles are localized toward the UV brane and those of the 3rd generation toward the IR brane, so that the hierarchical structure of the Yukawa couplings arises geometrically without hierarchy in fundamental parameters. This paper discusses observing signals of this model in the case where the Kaluza-Klein scale is far above the collider scale, but the model is combined with the 5D minimal supersymmetric standard model (MSSM) and supersymmetric (SUSY) particles are in the reach of collider experiments. A general SUSY breaking mass spectrum consistent with the bulk matter RS model is considered: a SUSY breaking sector locates on the IR brane and its effects are mediated to 5D MSSM through a hybrid of gravity mediation, gaugino mediation, and gauge mediation. This paper argues that it is possible to observe the signals of the bulk matter RS model through rare decays of "almost SU(2) singlet mass eigenstates" that are induced by flavor-violating gravity mediation contributions to matter soft SUSY breaking terms.

DOI: 10.1103/PhysRevD.85.016007 PACS numbers: 12.90.+b, 12.60.Jv, 14.80.Ly

I. INTRODUCTION

The origin of the fermion mass hierarchy is a long-standing mystery in particle physics. In the standard model (SM), it is explained by the hierarchical Yukawa coupling constants, but the hierarchy itself is still introduced by hand. Many authors have proposed models beyond the SM where the Yukawa coupling hierarchy arises from nonhierarchical couplings of a fundamental theory. The bulk matter Randall-Sundrum (RS) model [1,2] is one of the successful models. In this model, SM fermions are identified with the zero modes of 5D Dirac fermions that live in 5D warped spacetime (bulk), whereas the Higgs field is confined on 4D spacetime of the infrared (IR) brane. With nonhierarchical values of 5D Dirac masses, the zero modes of the 5D fermions can be localized toward either the ultraviolet (UV) brane or the IR brane. In this way, the geometrical overlap between each zero mode in the bulk and the Higgs field on the IR brane gains exponential hierarchy, which gives rise to the hierarchical structure of the Yukawa coupling constants. As far as the model gives a natural explanation to the fermion mass hierarchy, neglecting the gauge hierarchy problem, it is sufficient that the warp factor be around $\sim m_e/m_t$, or equivalently the Kaluza-Klein (KK) scale be around $\sim M_* \cdot (m_e/m_t)$ (M_* indicates the 4D reduced Planck mass) if the 5D Planck scale is the same as that of 4D. Then the KK modes need not exist at the TeV scale. Futhermore, the physics of flavor has already imposed severe constraints on the mass of KK modes; for example, the 1st KK gluon in the bulk matter RS model induces flavor changing neutral current interactions. The data on the $K^0 - \bar{K}^0$ mixing require that its mass be larger than 21 TeV [3].

If the KK modes appear only far above the TeV scale, it is impossible to produce them at colliders and confirm the model. In this paper, I argue that it is possible to observe indirect signatures of the bulk matter RS model at near-future colliders. I consider the case where the KK scale is at an intermediate scale between Planck and TeV, but the 4D effective theory contains $\mathcal{N} = 1$ supersymmetry (SUSY) which is broken at the TeV scale and it can be described by the minimal supersymmetric standard model (MSSM). Hence the SUSY particles are accessible at colliders while the KK modes are beyond their reach. This is a natural situation because SUSY breaking at the TeV scale is necessary to solve the gauge hierarchy problem when the KK scale is at an intermediate scale. I consider a general SUSY breaking mediation mechanism that is consistent with the bulk matter RS setup, in contrast to the paper [4] where a simultaneous explanation to the SUSY breaking mediation mechanism and the Yukawa coupling hierarchy based on the RS spacetime was pursued. In the setup of this paper, the SUSY breaking sector locates on the IR brane and its effects are mediated to the MSSM in the bulk through contact terms on the IR brane (gravity mediation [5]), renormalization group evolutions below the KK scale (gaugino mediation [6]), and gauge interactions with messenger fields on the IR brane (gauge mediation [7]). (Anomaly mediation contributions [8] are suppressed at least by the warp factor compared to gaugino mediation ones and hence are negligible.)

Gravity mediation contributions are the key to observe signals of the bulk matter RS model. This is because they arise from contact terms on the IR brane in a similar way to the Yukawa coupling constants. The basic strategy for testing the model is as follows. Since the zero modes of matter hypermultiplets (SM fermions are their fermionic components) reside in the bulk, they have contact terms with the SUSY breaking sector on the IR brane which induce soft SUSY breaking terms through gravity mediation. The amount of the gravity mediation contribution to each term is proportional to the geometrical overlap between the zero-mode fields and the IR brane. In the bulk matter RS model, the same overlap also gives rise to the hierarchy of the Yukawa coupling constants. Hence the flavor structure of gravity mediation contributions and the Yukawa coupling hierarchy are related and one can predict the former from the latter. (A similar setup [9] was proposed on a different context as a 5D realization of ''flavorful supersymmtery'' [10].) Therefore the model can be tested through the measurement of gravity-mediation-origined soft SUSY breaking terms. Our task is then to extract gravity mediation contributions from experimentally observable quantities related to soft SUSY breaking terms. To be a realistic SUSY breaking model, the model must contain dominant flavor-conserving soft SUSY breaking mass terms that arise through gaugino mediation and gauge mediation. On the other hand, gravity mediation intrinsically violates flavor. Therefore one can indirectly measure the gravity-mediation-origined terms through flavor-violating interactions of SUSY matter particles. One obstacle is that the Yukawa coupling constants themselves induce flavor-violating soft SUSY breaking terms through renormalization group (RG) evolutions, as in models with minimal flavor violation (MFV). However, one can distinguish gravity mediation contributions from the RG effects of the Yukawa couplings by focusing on SU(2) singlet SUSY matter particles, for which flavor violation of gravity mediation contributions can be significantly larger than that of the RG effects. In this paper, I introduce three promising channels for observing signatures of the bulk matter RS model at colliders. One is that SU(2) singlet smuon mixes with stau through gravity-mediation-origined soft terms and one measures the branching ratio of the ''almost singlet smuon mass eigenstate'' decaying into the SM tau and another SUSY particle. Another channel is that the SU(2) singlet smuon mixes with a selectron and one measures the branching ratio of the almost singlet smuon mass eigenstate decaying into the SM electron and another SUSY particle. The third is that a SU(2) singlet scharm mixes with stop and one measures the branching ratio of the ''almost singlet scharm mass eigenstate'' decaying into the SM top and another SUSY particle. The bulk matter RS model predicts these branching ratios and can be tested through their measurements. In this setup, one cannot determine the exact values of the contact term couplings of the 5D theory. I here assume that the contact term couplings for gravity mediation are all $O(1)$, and estimate the orders of magnitudes of flavor-violating soft SUSY breaking terms. Still the bulk matter RS model gives predictions on the magnitudes of the branching ratios of sparticle rare decays.

This paper is organized as follows. In Sec. II, I review the bulk matter RS model and combine it with the 5D MSSM. 5D disposition of matter wave functions is determined so as to reproduce the hierarchical structure of the Yukawa coupling constants, the Cabbibo-Kobayashi-Maskawa (CKM) matrix, and the neutrino mass matrix with $O(1)$ couplings of the 5D theory. In Sec. III, I derive the SUSY particle mass spectrum with emphasis on its flavor-violating sector. In Sec, IV, I make predictions on the flavor mixings of SUSY particles and compare the bulk matter RS model with other models. In Sec. V, I discuss experimental methods to test these predictions.

II. THE BULK MATTER RS MODEL WITH THE 5D MSSM

A. Setup

Consider 5D warped spacetime with the metric [1]:

$$ds^2 = e^{-2k|y|}\eta_{\mu\nu}dx^\mu dx^\nu - dy^2, \tag{1}$$

where y is the 5th dimension compactified on the orbifold S^1/Z_2: $-\pi R \leq y \leq \pi R$, and k is the anti–de Sitter (AdS) curvature that is of the same order as the 5D Planck scale M_5. Assuming that the warp factor, $e^{-kR\pi}$, is much smaller than 1, we have the following relation for k and M_5:

$$M_*^2 = \frac{M_5^3}{k}(1 - e^{-2kR\pi}) \simeq \frac{M_5^3}{k}, \tag{2}$$

where M_* is the 4D reduced Planck mass, which implies $k \sim M_5 \sim M_*$. The UV brane is put at $y = 0$ and the IR brane at $y = \pi R$. The Planck scale on the UV brane is M_5, while that on the IR brane is $M_5 e^{-kR\pi}$. The IR scale, $ke^{-kR\pi} \sim M_5 e^{-kR\pi}$, is assumed to be at an intermediate scale between M_* and TeV. In particular, we assume that it is far above 21 TeV. Since the most severe constraint on the IR scale of the bulk matter RS model comes from the data on the $K^0 - \bar{K}^0$ mixing, which require it to be larger than 21 TeV [3], my model is free from any constraint on the bulk matter RS model itself. At the same time, it is hopeless to observe the effects of the Kaluza-Klein excitations by near-future experiments.

Consider the 5D MSSM [11] where the 4D $\mathcal{N} = 1$ Higgs superfields are confined on the IR brane, and the 5D $\mathcal{N} = 1$ gauge superfields and matter hypermultiplets live in the bulk. In the following, we use the 4D superfield formalism extended with the 5th dimension y. We introduce a chiral superfield, X, on the IR brane whose F component, F_X, develops a vacuum expectation value (VEV) to break 4D $\mathcal{N} = 1$ SUSY there. We consider both cases where there are one to several messenger pairs on the IR brane and there is no messenger pair at all. (It is easy to extend the model to cases where the messengers live in the bulk.) The gauge symmetries of the messenger pairs are not specified. The SU(2) doublet squark, the singlet up-type squark, the singlet down-type squark, the doublet slepton, and the singlet charged slepton hypermultiplets are denoted by Q_i, U_i, D_i, L_i, and E_i, respectively, with i being the flavor index. The up-type Higgs and the down-type Higgs superfields are denoted by H_u, H_d, respectively.

An off-shell 5D $\mathcal{N}=1$ gauge superfield consists of a 5D gauge field $A_M (M = 0, 1, 2, 3, 5)$, two 4D Weyl spinors λ_1, λ_2, a real scalar Σ, a real auxiliary field D, and a complex auxiliary field F, all of which transform as the adjoint representation of some gauge group. They combine to form one 4D $\mathcal{N}=1$ gauge superfield V and one 4D $\mathcal{N}=1$ chiral superfield χ that are

$$V = -\theta\sigma^\mu\bar{\theta}A_\mu - i\bar{\theta}^2\theta\lambda_1 + i\theta^2\bar{\theta}\bar{\lambda}_1 + \frac{1}{2}\bar{\theta}^2\theta^2 D,$$

$$\chi = \frac{1}{\sqrt{2}}(\Sigma + iA_5) + \sqrt{2}\theta\lambda_2 + \theta^2 F.$$

By Z_2: $y \to -y$ symmetry, they transform as

$$V \to V, \qquad \chi \to -\chi.$$

Assuming the invariance of the theory under the Z_2 symmetry, we obtain the following action for 5D $\mathcal{N}=1$ gauge superfields:

$$\begin{aligned} S_{\text{5D gauge}} = \int dy \int d^4x e^{-4k|y|}\bigg[&\frac{1}{4(g_5^a)^2}\int d^2\theta \\ &\times e^{k|y|}\,\mathrm{tr}\{(e^{(3/2)k|y|}W^{a\alpha})(e^{(3/2)k|y|}W^a_\alpha) + \text{H.c.}\} \\ &+ \frac{1}{(g_5^a)^2}\int d^4\theta e^{2k|y|}\,\mathrm{tr}\{(\sqrt{2}\partial_y + \chi^{a\dagger}) \\ &\times e^{-V}(-\sqrt{2}\partial_y + \chi^a)e^V - (\partial_y e^{-V})(\partial_y e^V)\}\bigg], \end{aligned} \tag{3}$$

where a labels gauge groups and $W^{a\alpha}$ denotes the field strength of V^a in 4D flat spacetime. When the unitary gauge, $A_5^a = 0$, is chosen, only V^a has a massless mode in the 4D picture. This mode has no dependence on y and will be written as $V_0(x, \theta, \bar{\theta})$.

A 5D $\mathcal{N}=1$ hypermultiplet is expressed in terms of two 4D $\mathcal{N}=1$ chiral superfields Φ, Φ^c that are in conjugate representaions of some gauge group. We assume that the former is Z_2 even and the latter Z_2 odd. Taking the basis of diagonal bulk masses, we have the following action for 5D $\mathcal{N}=1$ hypermultiplets:

$$\begin{aligned} S_{\text{5D chiral}} = \int dy \int d^4x e^{-4k|y|}\bigg[&\int d^4\theta \\ &\times e^{2k|y|}(\Phi_i^\dagger e^{-V}\Phi_i + \Phi_i^c e^V \Phi_i^{c\dagger}) \\ &+ \int d^2\theta e^{k|y|}\Phi_i^c\{\partial_y - \chi/\sqrt{2} - (3/2 - c_i)k\}\Phi_i \\ &+ \text{H.c.}\bigg], \end{aligned} \tag{4}$$

where i is a flavor index and c_i denotes the 5D bulk mass in units of AdS curvature k. Only Φ_i has a massless mode in the 4D picture, which will be written as $\phi_i(x, \theta)e^{(3/2-c_i)k|y|}$.

We write down the 4D effective action for the fields in the bulk in terms of the massless modes:

$$\begin{aligned} S_{\text{4D eff}} = \int d^4x\bigg[&\frac{2\pi R}{4g_5^{a2}}\int d^2\theta W^{a\alpha}W^a_\alpha + \text{H.c.} \\ &+ \int d^4\theta 2\frac{e^{(1-2c_i)kR\pi} - 1}{(1-2c_i)k}\phi_i^\dagger e^{-V}\phi_i\bigg], \end{aligned} \tag{5}$$

where the dimensionful 5D gauge coupling, g_5^a, is connected to the 4D gauge coupling g_4^a by the relation $g_5^a = \sqrt{2\pi R}g_4^a$. ϕ_i represents the zero mode of each of Q_i, U_i, D_i, L_i, and E_i.

We also introduce an IR-brane-localized action. Below are the parts of the action relevant to the topic of this paper.

MSSM term:

$$\begin{aligned} S_{\text{IR}} \supset \int d^4x\bigg[&\int d^4\theta e^{-2kR\pi}\{H_u^\dagger e^{-V}H_u + H_d^\dagger e^{-V}H_d\} \\ &+ \int d^2\theta e^{-3kR\pi}\bigg\{e^{(3-c_i-c_j)kR\pi}\frac{(y_u)_{ij}}{M_5}H_u U_i Q_j \\ &+ e^{(3-c_k-c_l)kR\pi}\frac{(y_d)_{kl}}{M_5}H_d D_k Q_l\bigg\} + \text{H.c.} \\ &+ \int d^2\theta e^{-3kR\pi}e^{(3-c_m-c_n)kR\pi}\frac{(y_e)_{mn}}{M_5}H_d E_m L_n + \text{H.c.}\bigg]. \end{aligned} \tag{6}$$

Gaugino mass term:

$$S_{\text{IR}} \supset \int d^4x\bigg[\int d^2\theta d_a \frac{X}{M_5}W^{a\alpha}W^a_\alpha + \text{H.c.}\bigg]. \tag{7}$$

Matter soft SUSY breaking mass term:

$$\begin{aligned} S_{\text{IR}} \supset \int d^4x\bigg[&\int d^4\theta e^{-2kR\pi}e^{(3-c_i-c_j)kR\pi} \\ &\times\bigg\{d_{Q1ij}\frac{X + X^\dagger}{M_5^2}Q_i^\dagger Q_j + d_{Q2ij}\frac{X^\dagger X}{M_5^3}Q_i^\dagger Q_j\bigg\}\bigg] \\ &+ (Q \to U, D, L, E). \end{aligned} \tag{8}$$

A-term-generating term:

$$S_{\rm IR} \supset \int d^4x \left[\int d^2\theta e^{-3kR\pi} \left\{ e^{(3-c_i-c_j)kR\pi} \frac{(a_u)_{ij}}{M_5^2} X H_u U_i Q_j + e^{(3-c_k-c_l)kR\pi} \frac{(a_d)_{kl}}{M_5^2} X H_d D_k Q_l + e^{(3-c_m-c_n)kR\pi} \frac{(a_e)_{mn}}{M_5^2} X H_d E_m L_n \right\} + \text{H.c.} \right]. \tag{9}$$

Messenger term:

$$S_{\rm IR} \supset \sum_I \int d^4x \left[\int d^4\theta e^{-2kR\pi} \{ \Xi_I^\dagger e^{-V} \Xi_I + \bar{\Xi}_I^\dagger e^{V} \bar{\Xi}_I \} + \int d^2\theta e^{-3kR\pi} \{ M_{{\rm mess}I} \Xi_I \bar{\Xi}_I + \lambda_{{\rm mess}I} X \Xi_I \bar{\Xi}_I \} + \text{H.c.} \right], \tag{10}$$

where $M_{{\rm mess}I}$ indicates the SUSY conserving mass for the messenger pair Ξ_I, $\bar{\Xi}_I$. Note that we did not necessarily assume the existence of messengers. In that case, only gaugino mediation gives rise to flavor-conserving soft masses, as is realized in the model [4].

In addition, the terms for the Higgs superfields exist on the IR brane. We simply assume that the μ term and the $B\mu$ term are somehow derived at the TeV scale.

We normalize X, H_u, H_d, Q_i, U_i, D_i, L_i, E_i, Ξ_I, and $\bar{\Xi}_I$ to make their kinetic terms in the 4D effective theory canonical. This is done by the following rescaling:

$$\begin{aligned} &X \to \tilde{X} = e^{-kR\pi} X, \quad H_u \to \tilde{H}_u = e^{-kR\pi} H_u, \\ &H_d \to \tilde{H}_d = e^{-kR\pi} H_d, \\ &\phi_i \to \tilde{\phi}_i = \sqrt{2\frac{e^{(1-2c_i)kR\pi} - 1}{(1-2c_i)k}} \phi_i, \\ &\Xi_I \to \tilde{\Xi}_I = e^{-kR\pi} \Xi_I, \quad \bar{\Xi}_I \to \tilde{\bar{\Xi}}_I = e^{-kR\pi} \bar{\Xi}_I. \end{aligned} \tag{11}$$

Then the MSSM term, the gaugino mass term, the matter soft SUSY breaking mass term, the A-term-generating term, and the messenger term become as follows:

$$S_{\rm IR} \supset \int d^4x \left[\int d^4\theta \{ \tilde{H}_u^\dagger e^{-V} \tilde{H}_u + \tilde{H}_d^\dagger e^{-V} \tilde{H}_d \} + \int d^2\theta \left\{ \sqrt{\frac{1-2c_i}{2\{1-e^{-(1-2c_i)kR\pi}\}}} \sqrt{\frac{1-2c_j}{2\{1-e^{-(1-2c_j)kR\pi}\}}} \frac{k}{M_5} (y_u)_{ij} \tilde{H}_u \tilde{U}_i \tilde{Q}_j + \sqrt{\frac{1-2c_k}{2\{1-e^{-(1-2c_k)kR\pi}\}}} \sqrt{\frac{1-2c_l}{2\{1-e^{-(1-2c_l)kR\pi}\}}} \frac{k}{M_5} (y_d)_{kl} \tilde{H}_d \tilde{D}_k \tilde{Q}_l + \sqrt{\frac{1-2c_m}{2\{1-e^{-(1-2c_m)kR\pi}\}}} \sqrt{\frac{1-2c_n}{2\{1-e^{-(1-2c_n)kR\pi}\}}} \frac{k}{M_5} \times (y_e)_{mn} \tilde{H}_d \tilde{E}_m \tilde{L}_n \right\} + \text{H.c.} \right]. \tag{12}$$

The gaugino mass term will be

$$S_{\rm IR} \supset \int d^4x \left[\int d^2\theta d_a \frac{\tilde{X}}{M_5 e^{-kR\pi}} W^{a\alpha} W^a_\alpha + \text{H.c.} \right]. \tag{13}$$

The matter soft mass term will be

$$S_{\rm IR} \supset \int d^4x \left[\int d^4\theta \sqrt{\frac{1-2c_i}{2\{1-e^{-(1-2c_i)kR\pi}\}}} \sqrt{\frac{1-2c_j}{2\{1-e^{-(1-2c_j)kR\pi}\}}} \frac{k}{M_5} \left\{ d_{Q1ij} \frac{\tilde{X} + \tilde{X}^\dagger}{M_5 e^{-kR\pi}} \tilde{Q}_i^\dagger \tilde{Q}_j + d_{Q2ij} \frac{\tilde{X}^\dagger \tilde{X}}{M_5^2 e^{-2kR\pi}} \tilde{Q}_i^\dagger \tilde{Q}_j \right\} \right] + (\tilde{Q} \to \tilde{U}, \tilde{D}, \tilde{L}, \tilde{E}). \tag{14}$$

The A-term-generating term will be

$$S_{\rm IR} \supset \int d^4x \left[\int d^2\theta \left\{ \sqrt{\frac{1-2c_i}{2\{1-e^{-(1-2c_i)kR\pi}\}}} \sqrt{\frac{1-2c_j}{2\{1-e^{-(1-2c_j)kR\pi}\}}} \frac{k}{M_5} \frac{(a_u)_{ij}}{M_5 e^{-kR\pi}} \tilde{X} \tilde{H}_u \tilde{U}_i \tilde{Q}_j + \sqrt{\frac{1-2c_k}{2\{1-e^{-(1-2c_k)kR\pi}\}}} \sqrt{\frac{1-2c_l}{2\{1-e^{-(1-2c_l)kR\pi}\}}} \frac{k}{M_5} \frac{(a_d)_{kl}}{M_5 e^{-kR\pi}} \tilde{X} \tilde{H}_d \tilde{D}_k \tilde{Q}_l + \sqrt{\frac{1-2c_m}{2\{1-e^{-(1-2c_m)kR\pi}\}}} \sqrt{\frac{1-2c_n}{2\{1-e^{-(1-2c_n)kR\pi}\}}} \frac{k}{M_5} \frac{(a_e)_{mn}}{M_5 e^{-kR\pi}} \tilde{X} \tilde{H}_d \tilde{E}_m \tilde{L}_n \right\} + \text{H.c.} \right]. \tag{15}$$

The messenger term will be

$$S_{\rm IR} \supset \sum_I \int d^4x \Big[\int d^4\theta \{ \tilde{\Xi}_I^\dagger e^{-V} \tilde{\Xi}_I + \tilde{\bar{\Xi}}_I^\dagger e^{V} \tilde{\bar{\Xi}}_I \} + \int d^2\theta \{ M_{{\rm mess}I} e^{-kR\pi} \tilde{\Xi}_I \tilde{\bar{\Xi}}_I + \lambda_{{\rm mess}I} \tilde{X} \tilde{\Xi}_I \tilde{\bar{\Xi}}_I \} + {\rm H.c.} \Big]. \qquad (16)$$

We introduce light neutrino masses by writing IR-scale-suppressed higher-dimensional operators or by adopting the seesaw mechanism [12]. In either case, we have the following term for light neutrino masses:

$$S_{\rm IR} \supset \int d^4x \int d^2\theta e^{-3kR\pi} e^{(3-c_p-c_q)kR\pi} (Y_\nu)_{pq} \frac{L_p H_u L_q H_u}{M_{\rm seesaw}} + {\rm H.c.}$$
$$= \int d^4x \int d^2\theta \sqrt{\frac{1-2c_p}{2\{1-e^{-(1-2c_p)kR\pi}\}}} \sqrt{\frac{1-2c_q}{2\{1-e^{-(1-2c_q)kR\pi}\}}} (Y_\nu)_{pq} \frac{\tilde{L}_p \tilde{H}_u \tilde{L}_q \tilde{H}_u}{M_{\rm seesaw} e^{-kR\pi}} + {\rm H.c.}, \qquad (17)$$

where $M_{\rm seesaw}$ indicates the mass scale relevant to the light neutrino mass. To maintain the generality of the model, we hereafter consider cases with SU(2) singlet neutrinos. Their Majorana masses in the *4D effective* theory are assumed to be around a common scale, denoted by $M_{\rm Maj}$, which is lower than $M_{{\rm mess}I} e^{-kR\pi}$ or $M_5 e^{-kR\pi}$. The results of this paper can be extended to cases without singlet neutrinos by dropping terms containing $M_{\rm Maj}$.

Now the MSSM Yukawa coupling constants are expressed as

$$(Y_u)_{ij} = \sqrt{\frac{1-2c_i}{2\{1-e^{-(1-2c_i)kR\pi}\}}} \sqrt{\frac{1-2c_j}{2\{1-e^{-(1-2c_j)kR\pi}\}}} \frac{k}{M_5} (y_u)_{ij},$$
$$(Y_d)_{kl} = \sqrt{\frac{1-2c_k}{2\{1-e^{-(1-2c_k)kR\pi}\}}} \sqrt{\frac{1-2c_l}{2\{1-e^{-(1-2c_l)kR\pi}\}}} \frac{k}{M_5} (y_d)_{kl}, \qquad (18)$$
$$(Y_e)_{mn} = \sqrt{\frac{1-2c_m}{2\{1-e^{-(1-2c_m)kR\pi}\}}} \sqrt{\frac{1-2c_n}{2\{1-e^{-(1-2c_n)kR\pi}\}}} \frac{k}{M_5} (y_e)_{mn},$$

and the neutrino mass matrix m_ν is given by

$$(m_\nu)_{pq} = \sqrt{\frac{1-2c_p}{2\{1-e^{-(1-2c_p)kR\pi}\}}} \sqrt{\frac{1-2c_q}{2\{1-e^{-(1-2c_q)kR\pi}\}}} (Y_\nu)_{pq} \frac{v_u^2}{M_{\rm seesaw} e^{-kR\pi}}. \qquad (19)$$

The geometrical factor $\sqrt{(1-2c)/(2\{1-e^{-(1-2c)kR\pi}\})}$ generates hierarchical couplings without fundamental hierarchy [2]; for $c < 1/2$, it is approximated by $\sqrt{1/2-c}$ and is $O(1)$, whereas for $c > 1/2$, it is approximated by $\sqrt{c-1/2}e^{-(c-1/2)kR\pi}$ and is exponentially suppressed. Note that this factor cannot be larger than $O(1)$. We assume that the components of 5D coupling matrices y_u, y_d, and y_e are all $O(1)$ and that the hierarchy of the Yukawa coupling constants stems solely from the following terms:

$$\sqrt{\frac{1-2c_i}{2\{1-e^{-(1-2c_i)kR\pi}\}}} \sqrt{\frac{1-2c_j}{2\{1-e^{-(1-2c_j)kR\pi}\}}}.$$

This is how the bulk matter RS model explains the Yukawa coupling hierarchy.

We further assume that the components of Y_ν are $O(1)$. Note that Y_ν arises by integrating out singlet Majorana neutrinos. If the components of the 5D neutrino Dirac coupling are $O(1)$, it is possible to take the value of $M_{\rm seesaw}$ such that $(Y_\nu)_{ij} \sim O(1)$ holds, regardless of the 5D disposition of singlet neutrino fields and the flavor structure of the Majorana mass term. Hence this is a natural assumption in the bulk matter RS model, in which all 5D couplings are considered $O(1)$. With this assumption, the hierarchy of the light neutrino mass matrix (19) arises only from the geometrical factors of SU(2) doublet lepton fields.

We hereafter use the following notations:

$$\alpha_i \equiv \sqrt{\frac{1-2c_{qi}}{2\{1-e^{-(1-2c_{qi})kR\pi}\}}} \quad \text{with} \quad i = 1, 2, 3 \qquad (20)$$

for SU(2) doublet quark superfields with flavor index i, and β_i, γ_i, δ_i, and ϵ_i for the SU(2) singlet up-type quark, the singlet down-type quark, the doublet lepton, and the singlet charged lepton, respectively. Then the hierarchical structures of the up-type quark Yukawa matrix Y_u, the down-type Yukawa matrix Y_d, and the charged lepton Yukawa matrix Y_e (in the basis of diagonal 5D bulk masses) are expressed as

$$(Y_u)_{ij} \sim \beta_i \alpha_j, \quad (Y_d)_{ij} \sim \gamma_i \alpha_j, \quad (Y_e)_{ij} \sim \epsilon_i \delta_j, \qquad (21)$$

and that of the neutrino mass matrix m_ν is expressed as

$$(m_\nu)_{ij} \sim \delta_i \delta_j \frac{v_u^2}{M_{\rm seesaw} e^{-kR\pi}}. \tag{22}$$

B. Determination of the geometrical factors

The magnitudes of the geometrical factors, α_i, β_i, γ_i, δ_i, and ϵ_i, can be almost determined by the data on the SM fermion masses, the CKM matrix, and the neutrino oscillations. The sole exception is the absolute scale of δ_i's, of which we know only the relative scales between different flavors. In this section, we will estimate these factors. The values that correspond to the model must be given at the KK scale, $M_5 e^{-kR\pi}$, where the 5D theory is connected to the 4D effective theory. However, as seen from [13], RG evolutions change the Yukawa coupling constants by at most 2 and the CKM matrix components by at most 1.2 through evolving from $\sim 10^{15}$ GeV to the electroweak scale. Also the neutrino mass matrix is affected only by $O(1)$ through RG evolutions [14]. Therefore we may estimate the magnitudes of α_i, β_i, γ_i, δ_i, and ϵ_i from the data at low energies.

We first derive the model's predictions on the eigenvalues of the Yukawa coupling matrices and the components of the CKM matrix. Let us diagonalize the Yukawa matrices:

$$V_u Y_u U_u^\dagger = \text{diag}, \quad V_d Y_d U_d^\dagger = \text{diag}, \quad V_e Y_e U_e^\dagger = \text{diag}.$$

For successful diagonalization of the hierarchical Yukawa matrices, the unitary matrices, U_u, U_d, V_u, V_d, U_e, and V_e, need to have the following structure:

$$U_u \sim U_d \sim \begin{pmatrix} 1 & 0 & 0 \\ \alpha_1/\alpha_2 & 1 & 0 \\ \alpha_1/\alpha_3 & \alpha_2/\alpha_3 & 1 \end{pmatrix}, \quad V_u \sim (\alpha \to \beta),$$
$$V_d \sim (\alpha \to \gamma), \quad U_e \sim (\alpha \to \delta), \quad V_e \sim (\alpha \to \epsilon), \tag{23}$$

which leads to

$$\begin{aligned} V_u Y_u U_u^\dagger &\sim \text{diag}(\beta_1\alpha_1, \beta_2\alpha_2, \beta_3\alpha_3), \\ V_d Y_d U_d^\dagger &\sim \text{diag}(\gamma_1\alpha_1, \gamma_2\alpha_2, \gamma_3\alpha_3), \\ V_e Y_e U_e^\dagger &\sim \text{diag}(\epsilon_1\delta_1, \epsilon_2\delta_2, \epsilon_3\delta_3). \end{aligned} \tag{24}$$

The hierarchical structure of the CKM matrix $U_{\rm CKM}$ is given by

$$U_{\rm CKM} = U_u U_d^\dagger \sim \begin{pmatrix} 1 & \alpha_1/\alpha_2 & \alpha_1/\alpha_3 \\ \alpha_1/\alpha_2 & 1 & \alpha_2/\alpha_3 \\ \alpha_1/\alpha_3 & \alpha_2/\alpha_3 & 1 \end{pmatrix}. \tag{25}$$

We next list the experimental data on the CKM matrix and the neutrino mass matrix. The absolute values of the CKM matrix components, $|U_{\rm CKM}|$, at the electroweak scale have been measured to be [15]

$$|U_{\rm CKM}[M_W]| = \begin{pmatrix} 0.974\,19 \pm 0.000\,22 & 0.2257 \pm 0.0010 & 0.003\,59 \pm 0.000\,16 \\ 0.2256 \pm 0.0010 & 0.973\,34 \pm 0.000\,23 & 0.0415 + 0.0010 - 0.0011 \\ 0.008\,74 + 0.000\,26 - 0.000\,37 & 0.0407 \pm 0.0010 & 0.999\,133 + 0.000\,044 - 0.000\,043 \end{pmatrix}.$$

We approximate this matrix by the following formula:

$$|U_{\rm CKM}| \simeq \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix} \quad \text{with} \quad \lambda = 0.22. \tag{26}$$

To discuss the neutrino mass matrix, we adopt the tri-bi-maximal mixing matrix:

$$U_{MNS} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ \sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \\ \sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \end{pmatrix},$$

and the following data on neutrino mass squared differences [15]:

$$\Delta m_{21}^2 = 7.59 \pm 0.20 \times 10^{-5}\ \text{eV}^2,$$
$$|\Delta m_{32}^2| = 2.43 \pm 0.13 \times 10^{-3}\ \text{eV}^2.$$

We assume that the mass of the lightest neutrino is negligible. Then the neutrino mass matrix, $U_{MNS}\text{diag}(m_{\nu 1}, m_{\nu 2}, m_{\nu 3}) \times U_{MNS}^\dagger$, is given by

$$U_{MNS}\text{diag}(m_{\nu 1}, m_{\nu 2}, m_{\nu 3})U_{MNS}^\dagger = \begin{pmatrix} 0.29 & 0.29 & 0.29 \\ 0.29 & 2.8 & -2.2 \\ 0.29 & -2.2 & 2.8 \end{pmatrix} \times 10^{-11}\ \text{GeV}$$

for normal hierarchy, (27)

$$U_{MNS}\text{diag}(m_{\nu 1}, m_{\nu 2}, m_{\nu 3})U_{MNS}^\dagger = \begin{pmatrix} 4.9 & 0.026 & 0.026 \\ 0.026 & 2.5 & 2.5 \\ 0.026 & 2.5 & 2.5 \end{pmatrix} \times 10^{-11}\ \text{GeV}$$

for inverted hierarchy. (28)

We now compare the predictions of the model with the data and estimate the magnitudes of α_i, β_i, γ_i, δ_i, and ϵ_i. For Yukawa eigenvalues, we simply have

$$\beta_1\alpha_1 \sim m_u/v\sin\beta,\quad \beta_2\alpha_2 \sim m_c/v\sin\beta,\quad \beta_3\alpha_3 \sim m_t/v\sin\beta, \tag{29}$$

$$\gamma_1\alpha_1 \sim m_d/v\cos\beta,\quad \gamma_2\alpha_2 \sim m_s/v\cos\beta,\quad \gamma_3\alpha_3 \sim m_b/v\cos\beta, \tag{30}$$

$$\epsilon_1\delta_1 \sim m_e/v\cos\beta,\quad \epsilon_2\delta_2 \sim m_\mu/v\cos\beta,\quad \epsilon_3\delta_3 \sim m_\tau/v\cos\beta, \tag{31}$$

where the mass values can be approximated by their pole values. Since the top Yukawa coupling is ~ 1, we have $\alpha_3\beta_3 \sim 1$, which leads to

$$\alpha_3 \sim 1, \qquad \beta_3 \sim 1. \tag{32}$$

Comparing (25) with (26), we find that putting

$$\alpha_1 \sim \lambda^3, \qquad \alpha_2 \sim \lambda^2 \tag{33}$$

works. We then have

$$\beta_1 \sim \lambda^{-3} m_u/v\sin\beta, \qquad \beta_2 \sim \lambda^{-2} m_c/v\sin\beta, \tag{34}$$

$$\gamma_1 \sim \lambda^{-3} m_d/v\cos\beta,\quad \gamma_2 \sim \lambda^{-2} m_s/v\cos\beta,\quad \gamma_3 \sim m_b/v\cos\beta. \tag{35}$$

Next compare the matrix (22) with the neutrino mass matrix. For the normal hierarchy case, it is possible to reproduce the hierachical structure of the neutrino mass matrix by assuming the relation:

$$3\delta_1 \sim \delta_2 \sim \delta_3, \tag{36}$$

and the ratio up to 3 among the components of 5D coupling Y_ν. In contrast, for the inverted hierarchy case, $\sim$200 ratio is required among the 5D coupling components no matter how we choose δ_i's, which makes it difficult to naturally explain the hierarchy of the neutrino mass matrix. The situation gets worse if we consider a non-negiligible mass of the lightest neutrino. In conclusion, the bulk matter RS model favors the normal hierarchy of neutrino masses and implies the relation (34) for δ_i's. We estimate ϵ_i's assuming the relation (36); we obtain

$$\epsilon_1 \sim 3\delta_3^{-1} m_e/v\cos\beta,\quad \epsilon_2 \sim \delta_3^{-1} m_\mu/v\cos\beta,\quad \epsilon_3 \sim \delta_3^{-1} m_\tau/v\cos\beta. \tag{37}$$

The magnitude of δ_3 is a free parameter because we do not specify the scale of $M_{\rm seesaw}$.

III. FLAVOR-VIOLATING SOFT SUSY BREAKING TERMS

In this model, flavor-conserving soft SUSY breaking terms arise from RG contributions of gaugino masses below the KK scale (gaugino mediation) and gauge interactions with messenger superfields (gauge mediation). On the other hand, flavor-violating terms arise from contact interactions with the SUSY breaking sector on the IR brane (gravity mediation) and RG contributions of the Yukawa couplings. Of particular importance are the gravity mediation contributions, which have a flavor structure unique to the bulk matter RS model. In this section, we separately estimate the gravity mediation contributions and the Yukawa coupling contributions to flavor-violaing soft SUSY breaking terms.

Remember that there are two scales of soft SUSY breaking terms, namely, the gravity mediation scale and the gauge mediation scale. Assuming that the messenger masses and couplings are around the same orders, we define the following two mass parameters:

$$M_{\rm grav} \equiv \frac{|\langle F_{\tilde{X}}\rangle|}{M_5 e^{-kR\pi}}, \tag{38}$$

$$M_{\rm gauge} \equiv \frac{1}{16\pi^2}\frac{\lambda_{\rm mess}|\langle F_{\tilde{X}}\rangle|}{M_{\rm mess}e^{-kR\pi}}, \tag{39}$$

where $M_{\rm mess}$ represents the typical scale of the SUSY conserving messenger masses $M_{{\rm mess}I}$, and $\lambda_{\rm mess}$ the typical scale of the messenger couplings to SUSY breaking sector $\lambda_{{\rm mess}I}$. Note that Yukawa RG contributions to flavor-violating terms depend on both $M_{\rm grav}$ and $M_{\rm gauge}$, whereas gravity mediation contributions depend only on $M_{\rm grav}$.

A. Flavor-violating gravity mediation contributions

Let us estimate the magnitudes of gravity mediation contributions in the bulk matter RS model.

From (14), we obtain the following formulas for gravity-mediation-origined soft SUSY breaking matter mass terms at the scale $M_5 e^{-kR\pi}$:

For SU(2) doublet squarks, we have

$$(m_Q^2)_{ij} = (-d_{Q2ij} + d_{Q1ij}^2)\frac{k}{M_5}\alpha_i\alpha_j M_{\rm grav}^2. \tag{40}$$

By substituting (U, β), (D, γ), (L, δ), and (E, ϵ) into (Q, α) in the above formula, we obtain similar expressions for SU(2) singlet up-type squarks, down-type squarks, SU(2) doublet sleptons, and singlet charged sleptons.

Assuming that the 5D couplings d_{*2ij}, d_{*1ij} are $O(1)$, we obtain the following estimates on the magnitudes at the scale $M_5 e^{-kR\pi}$:

$$(m_Q^2)_{ij} \sim \alpha_i\alpha_j M_{\rm grav}^2 \tag{41}$$

and similar formulas with (U, β), (D, γ), (L, δ), and (E, ϵ) replacing (Q, α) in the above formula.

Next we estimate the magnitudes of the A terms that are induced by gravity mediation. The terms (15) directly contribute to the A terms. Furthermore, since the Higgs superfields can couple to the SUSY breaking sector in the following way:

$$\int d^4\theta\left[d_{uA}\frac{\tilde{X}}{M_5 e^{-kR\pi}}\tilde{H}_u^\dagger\tilde{H}_u + d_{dA}\frac{\tilde{X}}{M_5 e^{-kR\pi}}\tilde{H}_d^\dagger\tilde{H}_d + \text{H.c.}\right],$$

the A terms also arise from the Higgs F terms via (12). Hence gravity-mediation-origined A terms at the scale $M_5 e^{-kR\pi}$ are given by

$$A_{uij} = -d_{uA}(y_u)_{ij}\beta_i\alpha_j\frac{k}{M_5}M_{\text{grav}} + (a_u)_{ij}\beta_i\alpha_j\frac{k}{M_5}M_{\text{grav}}, \\ = -d_{uA}(Y_u)_{ij}M_{\text{grav}} + (a_u)_{ij}\beta_i\alpha_j\frac{k}{M_5}M_{\text{grav}}, \tag{42}$$

$$A_{dij} = -d_{dA}(Y_d)_{ij}M_{\text{grav}} + (a_d)_{ij}\gamma_i\alpha_j\frac{k}{M_5}M_{\text{grav}}, \tag{43}$$

$$A_{eij} = -d_{dA}(Y_e)_{ij}M_{\text{grav}} + (a_e)_{ij}\epsilon_i\delta_j\frac{k}{M_5}M_{\text{grav}}. \tag{44}$$

Assuming that the components of 5D couplings d_{*A}, $(a_*)_{ij}$ are $O(1)$, we obtain the following estimates on the magnitudes at the scale $M_5 e^{-kR\pi}$:

$$A_{uij} \sim (Y_u)_{ij}M_{\text{grav}} + \beta_i\alpha_j M_{\text{grav}}, \tag{45}$$

$$A_{dij} \sim (Y_d)_{ij}M_{\text{grav}} + \gamma_i\alpha_j M_{\text{grav}}, \tag{46}$$

$$A_{eij} \sim (Y_e)_{ij}M_{\text{grav}} + \epsilon_i\delta_j M_{\text{grav}}. \tag{47}$$

B. RG contributions

Let us estimate the magnitudes of the flavor-violating soft SUSY breaking terms that arise from the RG equations involving the Yukawa couplings. In doing so, we take the specific flavor basis where Y_u or Y_d and Y_e are diagonal.

We first study how Y_u, Y_d, and Y_e-diagonal bases change through RG evolutions. We define the following unitary matrices U_*:

$$U_U Y_u U_{Qu} = (\text{diag}), \qquad U_D Y_d U_{Qd} = (\text{diag}), \\ U_E Y_e U_L = (\text{diag}).$$

Note that U_*'s depend on the renormalization scale because the Yukawa matrices receive RG corrections. We will calculate how U_*'s vary through RG evolutions. We have the RG equation

$$\mu\frac{d}{d\mu}(U_U Y_u U_{Qu}) = \left(\mu\frac{d}{d\mu}U_U\right)U_U^\dagger(U_U Y_u U_{Qu}) + U_U\left(\mu\frac{d}{d\mu}Y_u\right)U_{Qu} + (U_U Y_u U_{Qu})U_{Qu}^\dagger\left(\mu\frac{d}{d\mu}U_{Qu}\right) \\ = \left(\mu\frac{d}{d\mu}U_U\right)U_U^\dagger(U_U Y_u U_{Qu}) + \frac{1}{16\pi^2}U_U\Big\{Y_u Y_d^\dagger Y_d + 3Y_u Y_u^\dagger Y_u + 3\text{tr}[Y_u^\dagger Y_u]Y_u + \text{tr}[Y_D^\dagger Y_D]Y_u \\ - \left(\frac{13}{15}g_1^2 + 3g_2^2 + \frac{16}{3}g_3^2\right)Y_u\Big\}U_{Qu} + (U_U Y_u U_{Qu})U_{Qu}^\dagger\left(\mu\frac{d}{d\mu}U_{Qu}\right), \tag{48}$$

where Y_D is the neutrino Dirac coupling which appears if there are singlet neutrinos lighter than the KK scale, from which the RG equations are calculated. We hereafter adopt the grand unified theory normalization for g_1. From (48), we see that, to keep $U_U Y_u U_{Qu}$ diagonal through RG evolutions, it is sufficient to have

$$\mu\frac{d}{d\mu}U_U = 0, \tag{49}$$

$$\mu\frac{d}{d\mu}U_{Qu} \\ = -\frac{1}{16\pi^2}(\text{off-diagonal components of } Y_d^\dagger Y_d)U_{Qu}. \tag{50}$$

In a similar manner, we obtain the following sufficient conditions for other U_*'s:

$$\mu\frac{d}{d\mu}U_D = 0, \tag{51}$$

$$\mu\frac{d}{d\mu}U_{Qd} \\ = -\frac{1}{16\pi^2}(\text{off-diagonal components of } Y_u^\dagger Y_u)U_{Qd}, \tag{52}$$

$$\mu\frac{d}{d\mu}U_E = 0, \tag{53}$$

$$\mu\frac{d}{d\mu}U_L \\ = -\frac{1}{16\pi^2}(\text{off-diagonal components of } Y_D^\dagger Y_D)U_L. \tag{54}$$

Now that we know how Y_u, Y_d, and Y_e-diagonal bases change through RG evolutions, we estimate the RG contributions to the A terms in these bases. Below is the list of the MSSM RG equations for the A terms:

$$
\begin{aligned}
16\pi^2\mu\frac{d}{d\mu}(U_U A_u U_{Qu}) = {}& 3U_U A_u Y_u^\dagger Y_u U_{Qu} + 3U_U Y_u Y_u^\dagger A_u U_{Qu} + (U_U A_u U_{Qu})(\text{diagonal part of } U_{Qu}^\dagger Y_d^\dagger Y_d U_{Qu}) \\
& + 2U_U Y_u Y_d^\dagger A_d U_{Qu} + 2\left(3\,\mathrm{tr}[Y_u^\dagger A_u] - \frac{13}{15}g_1^2 M_{1/2}^{a=1} - 3g_2^2 M_{1/2}^{a=2} - \frac{16}{3}g_3^2 M_{1/2}^{a=3}\right)(U_U Y_u U_{Qu}) \\
& + \left(3\,\mathrm{tr}[Y_u^\dagger Y_u] - \frac{13}{15}g_1^2 - 3g_2^2 - \frac{16}{3}g_3^2\right)(U_U A_u U_{Qu}) + \mathrm{tr}[Y_D^\dagger Y_D](U_U A_u U_{Qu}) + \mathrm{tr}[Y_D^\dagger A_D](U_U Y_u U_{Qu}),
\end{aligned}
\tag{55}
$$

$$
\begin{aligned}
16\pi^2\mu\frac{d}{d\mu}(U_D A_d U_{Qd}) = {}& 3U_D A_d Y_d^\dagger Y_d U_{Qd} + 3U_D Y_d Y_d^\dagger A_d U_{Qd} + (U_D A_d U_{Qd})(\text{diagonal part of } U_{Qd}^\dagger Y_u^\dagger Y_u U_{Qd}) \\
& + 2U_D Y_d Y_u^\dagger A_u U_{Qd} + 2\left(3\mathrm{tr}[Y_d^\dagger A_d] + \mathrm{tr}[Y_e^\dagger A_e] - \frac{7}{15}g_1^2 M_{1/2}^{a=1} - 3g_2^2 M_{1/2}^{a=2} - \frac{16}{3}g_3^2 M_{1/2}^{a=3}\right) \\
& \times (U_D Y_d U_{Qd}) + \left(3\,\mathrm{tr}[Y_d^\dagger Y_d] + \mathrm{tr}[Y_e^\dagger Y_e] - \frac{7}{15}g_1^2 - 3g_2^2 - \frac{16}{3}g_3^2\right)(U_D A_d U_{Qd}),
\end{aligned}
\tag{56}
$$

$$
\begin{aligned}
16\pi^2\mu\frac{d}{d\mu}(U_E A_e U_L) = {}& 3U_E A_e Y_e^\dagger Y_e U_L + 3U_E Y_e Y_e^\dagger A_e U_L + 2\left(3\,\mathrm{tr}[Y_d^\dagger A_d] + \mathrm{tr}[Y_e^\dagger A_e] - \frac{9}{5}g_1^2 M_{1/2}^{a=1} - 3g_2^2 M_{1/2}^{a=2}\right) \\
& \times (U_E Y_e U_L) + \left(3\,\mathrm{tr}[Y_d^\dagger Y_d] + \mathrm{tr}[Y_e^\dagger Y_e] - \frac{9}{5}g_1^2 - 3g_2^2\right)(U_E A_e U_L) + (U_E A_e U_L) \\
& \times (\text{diagonal part of } U_L^\dagger Y_D^\dagger Y_D U_L) + 2U_E Y_e Y_D^\dagger A_D U_L,
\end{aligned}
\tag{57}
$$

where Y_D and A_D, respectively, indicate neutrino Dirac coupling and its corresponding A term.

Note that the magnitudes of the components of the Yukawa couplings in each basis are given by (δ_{ij} is the ordinary Kronecker's delta)

$$
\begin{aligned}
&(U_U Y_u U_{Qu})_{ij} \sim \beta_i \alpha_i \delta_{ij}, \qquad (U_U Y_d U_{Qu})_{ij} \sim \gamma_i \alpha_j, \qquad (U_D Y_u U_{Qd})_{ij} \sim \beta_i \alpha_j, \qquad (U_D Y_d U_{Qd})_{ij} \sim \gamma_i \alpha_i \delta_{ij}, \\
&(U_E Y_e U_L)_{ij} \sim \epsilon_i \delta_i \delta_{ij}, \qquad (U_E Y_D U_L)_{ij} \sim \zeta_i \delta_j,
\end{aligned}
$$

where ζ_i's indicate the geometrical factors for singlet neutrinos and satisfy $\zeta_i \leq 1$. Note also that the A terms receive RG corrections which are proportional to the corresponding Yukawa couplings and to the gaugino masses. We write these terms by M_u, M_d, M_e, and M_D, respectively, for A_u, A_d, A_e, and A_D. They depend on both $M_{\rm grav}$ and $M_{\rm gauge}$.

With these ingredients, we estimate the RG contributions to those parts of A terms which are not proportional to the corresponding Yukawa couplings, or equivalently the off-diagonal components of $(U_U A_u U_{Qu})$, $(U_D A_d U_{Qd})$, and $(U_E A_e U_L)$. On the right-hand sides of Eqs. (55)–(57), the second lines determine the magnitudes of the RG contributions. We thus obtain the following estimates ($i \neq j$):

$$
\Delta(U_U A_u U_{Qu})_{ij} \sim 2\beta_i(\alpha_i)^2(\gamma_3)^2\alpha_j \times \frac{1}{16\pi^2}\int d(\ln\mu) M_d,
\tag{58}
$$

$$
\Delta(U_D A_d U_{Qd})_{ij} \sim 2\gamma_i(\alpha_i)^2(\beta_3)^2\alpha_j \times \frac{1}{16\pi^2}\int d(\ln\mu) M_u,
\tag{59}
$$

$$
\Delta(U_E A_e U_L)_{ij} \sim 2\epsilon_i(\delta_i)^2(\zeta_3)^2\delta_j \times \frac{1}{16\pi^2}\int d(\ln\mu) M_D.
\tag{60}
$$

Since M_u, M_d, and M_D depend on $M_{\rm grav}$ and $M_{\rm gauge}$, so do the magnitudes of the RG contributions above.

Let us estimate the RG contributions to soft SUSY breaking matter mass terms. Below are those parts of the MSSM RG equations that give rise to flavor-violating soft SUSY breaking mass terms:

$$16\pi^2\mu\frac{d}{d\mu}(U_{Qu}^\dagger m_Q^2 U_{Qu}) \supset U_{Qu}^\dagger Y_u^\dagger Y_u m_Q^2 U_{Qu} + U_{Qu}^\dagger m_Q^2 Y_u^\dagger Y_u U_{Qu} + 2U_{Qu}^\dagger Y_u^\dagger m_U^2 Y_u U_{Qu} + 2(U_{Qu}^\dagger Y_u^\dagger Y_u U_{Qu})m_{H_u}^2 + (\text{diagonal parts of} U_{Qu}^\dagger Y_d^\dagger Y_d U_{Qu})(U_{Qu}^\dagger m_Q^2 U_{Qu}) + (U_{Qu}^\dagger m_Q^2 U_{Qu}) \times(\text{diagonal parts of } U_{Qu}^\dagger Y_d^\dagger Y_d U_{Qu}) + 2U_{Qu}^\dagger Y_d^\dagger m_D^2 Y_d U_{Qu} + 2(U_{Qu}^\dagger Y_d^\dagger Y_d U_{Qu})m_{H_d}^2 + 2U_{Qu}^\dagger A_u^\dagger A_u U_{Qu} + 2U_{Qu}^\dagger A_d^\dagger A_d U_{Qu}, \tag{61}$$

$$16\pi^2\mu\frac{d}{d\mu}(U_U m_U^2 U_U^\dagger) \supset 2U_U Y_u Y_u^\dagger m_U^2 U_U^\dagger + 2U_U m_U^2 Y_u Y_u^\dagger U_U^\dagger + 4U_U Y_u m_Q^2 Y_u^\dagger U_U^\dagger + 4(U_U Y_u Y_u^\dagger U_U^\dagger)m_{H_u}^2 + 4U_U A_u A_u^\dagger U_U^\dagger, \tag{62}$$

$$16\pi^2\mu\frac{d}{d\mu}(U_D m_D^2 U_D^\dagger) \supset 2U_D Y_d Y_d^\dagger m_D^2 U_D^\dagger + 2U_D m_D^2 Y_d Y_d^\dagger U_D^\dagger + 4U_D Y_d m_Q^2 Y_d^\dagger U_D^\dagger + 4(U_D Y_d Y_d^\dagger U_D^\dagger)m_{H_d}^2 + 4U_D A_d A_d^\dagger U_D^\dagger, \tag{63}$$

$$16\pi^2\mu\frac{d}{d\mu}(U_L^\dagger m_L^2 U_L) \supset U_L^\dagger Y_e^\dagger Y_e m_L^2 U_L + U_L^\dagger m_L^2 Y_e^\dagger Y_e U_L + 2U_L^\dagger Y_e^\dagger m_E^2 Y_e U_L + 2(U_L^\dagger Y_e^\dagger Y_e U_L)m_{H_d}^2 + 2U_L^\dagger A_e^\dagger A_e U_L + (\text{diagonal parts of } U_L^\dagger Y_D^\dagger Y_D U_L)(U_L^\dagger m_L^2 U_L) + (U_L^\dagger m_L^2 U_L)(\text{diagonal parts of } U_L^\dagger Y_D^\dagger Y_D U_L) + 2U_L^\dagger Y_D^\dagger m_N^2 Y_D U_L + 2(U_L^\dagger Y_D^\dagger Y_D U_L)m_{H_u}^2 + 2U_L^\dagger A_D^\dagger A_D U_L, \tag{64}$$

$$16\pi^2\mu\frac{d}{d\mu}(U_E m_E^2 U_E^\dagger) \supset 2U_E Y_e Y_e^\dagger m_E^2 U_E^\dagger + 2U_E m_E^2 Y_e Y_e^\dagger U_E^\dagger + 4U_E Y_e m_L^2 Y_e^\dagger U_E^\dagger + 4(U_E Y_e Y_e^\dagger U_E^\dagger)m_{H_d}^2 + 4U_E A_e A_e^\dagger U_E^\dagger. \tag{65}$$

We first focus on the differences among the diagonal components of different flavors. From Eq. (61), the difference between the components $(U_{Qu}^\dagger m_Q^2 U_{Qu})_{ii}$ and $(U_{Qu}^\dagger m_Q^2 U_{Qu})_{jj}$ that arises through RG evolutions is given by $(i > j)$

$$\Delta\{(U_{Qu}^\dagger m_Q^2 U_{Qu})_{ii} - (U_{Qu}^\dagger m_Q^2 U_{Qu})_{jj}\} \sim 2(\alpha_i)^2(\beta_i)^2\frac{1}{16\pi^2}\int d(\ln\mu)(m_Q^2 + m_U^2 + m_{H_u}^2 + M_u^2) + 2(\alpha_i)^2(\gamma_3)^2\frac{1}{16\pi^2} \times \int d(\ln\mu)(m_Q^2 + m_D^2 + m_{H_d}^2 + M_d^2), \tag{66}$$

where we neglected the terms proportional to $(\alpha_j)^2$ because they are smaller than those proportional to $(\alpha_i)^2$. Similarly, the difference between $(U_L^\dagger m_L^2 U_L)_{ii}$ and $(U_L^\dagger m_L^2 U_L)_{jj}$ is given, from (64), by

$$\Delta\{(U_L^\dagger m_L^2 U_L)_{ii} - (U_L^\dagger m_L^2 U_L)_{jj}\} \sim 2(\delta_i)^2(\epsilon_i)^2\frac{1}{16\pi^2}\int d(\ln\mu)(m_L^2 + m_E^2 + m_{H_d}^2 + M_e^2) + 2(\delta_i)^2(\zeta_3)^2\frac{1}{16\pi^2} \times \int d(\ln\mu)(m_L^2 + m_N^2 + m_{H_u}^2 + M_D^2). \tag{67}$$

On the other hand, the differences among the diagonal components of the SU(2) singlet soft mass terms follow different formulas. From Eqs. (62), (63), and (65), we have

$$\Delta\{(U_U^\dagger m_U^2 U_U)_{ii} - (U_U^\dagger m_U^2 U_U)_{jj}\} \sim 4(\beta_i)^2(\alpha_i)^2\frac{1}{16\pi^2}\int d(\ln\mu)(m_U^2 + m_Q^2 + m_{H_u}^2 + M_u^2), \tag{68}$$

$$\Delta\{(U_D^\dagger m_D^2 U_D)_{ii} - (U_D^\dagger m_D^2 U_D)_{jj}\} \sim 4(\gamma_i)^2(\alpha_i)^2\frac{1}{16\pi^2}\int d(\ln\mu)(m_D^2 + m_Q^2 + m_{H_d}^2 + M_d^2), \tag{69}$$

$$\Delta\{(U_E^\dagger m_E^2 U_E)_{ii} - (U_E^\dagger m_E^2 U_E)_{jj}\} \sim 4(\epsilon_i)^2(\delta_i)^2\frac{1}{16\pi^2}\int d(\ln\mu)(m_E^2 + m_L^2 + m_{H_d}^2 + M_e^2). \tag{70}$$

We next study the off-diagonal components. In (61), terms $2U_{Qu}^\dagger Y_d^\dagger m_D^2 Y_d U_{Qu}$, $2(U_{Qu}^\dagger Y_d^\dagger Y_d U_{Qu})m_{H_d}^2$, and $2U_{Qu}^\dagger A_d^\dagger A_d U_{Qu}$ generate off-diagonal components, whose magnitudes are given by $(i \neq j)$

$$\Delta(U_{Qu}^{\dagger}m_Q^2U_{Qu})_{ij} \sim 2\alpha_i(\gamma_3)^2\alpha_j\frac{1}{16\pi^2}\int d(\ln\mu)(m_D^2 + m_{H_d}^2 + M_d^2). \tag{71}$$

Similarly, we have

$$\Delta(U_L^{\dagger}m_L^2U_L)_{ij} \sim 2\delta_i(\zeta_3)^2\delta_j\frac{1}{16\pi^2}\int d(\ln\mu)(m_N^2 + m_{H_u}^2 + M_D^2). \tag{72}$$

On the other hand, RG contributions to the off-diagonal components of $(U_U^{\dagger}m_U^2U_U)$ arise from those of $(U_{Q_u}^{\dagger}m_Q^2U_{Q_u})$ and $(U_UA_uU_{Qu})$ via terms $4U_UY_um_Q^2Y_u^{\dagger}U_U^{\dagger}$, $4U_UA_uA_u^{\dagger}U_U^{\dagger}$ in (62). From Eqs. (58), (62), and (71), we obtain the following estimate on the magnitudes of the off-diagonal components ($i \neq j$):

$$\Delta(U_U^{\dagger}m_U^2U_U)_{ij} \sim 8\beta_i(\alpha_i)^2(\gamma_3)^2(\alpha_j)^2\beta_j\left(\frac{1}{16\pi^2}\right)^2\int d(\ln\mu)\int d(\ln\mu')(m_D^2 + m_{H_d}^2 + M_d^2) + 16\beta_i(\alpha_i)^2(\gamma_3)^2(\alpha_j)^2 \times \beta_j\left(\frac{1}{16\pi^2}\right)^2\int d(\ln\mu)\left(M_u\int d(\ln\mu')M_d\right). \tag{73}$$

In the same way, we obtain the following estimates on the RG contributions to the off-diagonal components of $(U_D^{\dagger}m_D^2U_D)$, $(U_E^{\dagger}m_E^2U_E)$:

$$\Delta(U_D^{\dagger}m_D^2U_D)_{ij} \sim 8\gamma_i(\alpha_i)^2(\beta_3)^2(\alpha_j)^2\gamma_j\left(\frac{1}{16\pi^2}\right)^2\int d(\ln\mu)\int d(\ln\mu')(m_U^2 + m_{H_u}^2 + M_u^2) + 16\gamma_i(\alpha_i)^2(\beta_3)^2(\alpha_j)^2\gamma_j \times \left(\frac{1}{16\pi^2}\right)^2\int d(\ln\mu)\left(M_d\int d(\ln\mu')M_u\right), \tag{74}$$

$$\Delta(U_E^{\dagger}m_E^2U_E)_{ij} \sim 8\epsilon_i(\delta_i)^2(\zeta_3)^2(\delta_j)^2\epsilon_j\left(\frac{1}{16\pi^2}\right)^2\int d(\ln\mu)\int d(\ln\mu')(m_N^2 + m_{H_u}^2 + M_D^2) + 16\epsilon_i(\delta_i)^2(\zeta_3)^2(\delta_j)^2 \times \epsilon_j\left(\frac{1}{16\pi^2}\right)^2\int d(\ln\mu)\left(M_e\int d(\ln\mu')M_D\right). \tag{75}$$

Finally, we briefly discuss whether this model gives a realistic mass spectrum consistent with the bounds on flavor-violating processes. For cases without messenger fields, i.e. when gaugino mediation is the only source of soft SUSY breaking masses, the paper [4] showed that mass spectra exist below the TeV scale that satisfy all experimental bounds. However, ~ 0.1 suppression on the term $(A_e)_{21}$ relative to its natural scale ($\sim \epsilon_2\delta_1M_{\rm grav}$) is required to evade the bound on $\mu \to e\gamma$ branching ratio. Other soft SUSY breaking terms are less constrained.

If there are one to several messenger pairs, the resultant mass spectra are more likely to evade the experimental bounds because gauge mediation contributes solely to flavor-universal soft SUSY breaking terms.

IV. SIGNATURES OF THE MODEL

In the previous section, we saw that the bulk matter RS model combined with the 5D MSSM predicts a unique flavor structure of gravity mediation contributions to flavor-violating soft terms. We discuss the ways to observe this structure through future collider experiments.

Focus on the flavor compositions of SUSY matter particle mass eigenstates. Because of flavor-violating soft mass terms $(m_*^2)_{ij}$ and flavor-violating A terms $(A_*)_{ij}$, SUSY particles of different flavors mix in one mass eigenstate, whose flavor composition reflects the relative size of the flavor-violating terms. Since sparticles of different flavors decay into different SM particles (plus the lightest or the next-to-lightest SUSY particle), one can measure the flavor composition by detecting the decay products of that mass eigenstate, counting the event numbers of different decay modes and calculating their ratios. These ratios are connected to the structure of flavor-violating soft SUSY breaking terms and make it possible to experimentally test the predictions of the bulk matter RS model.

Below we formulate the relation between flavor-violating terms and sparticle flavor mixings. In the first section, we interpret the predictions of the bulk matter RS model in terms of the flavor-mixing ratios of sparticle mass eigenstates. In the next section, we look into the predictions of models other than the bulk matter RS model and discuss whether or not it is possible to distinguish different models.

Consider the situation where sparticle a with soft SUSY breaking mass m_a^2 mixes with sparticle b with soft mass m_b^2 through mixing term Δm^2. The mass matrix in the basis of (a, b) is given by

$$\begin{pmatrix} m_a^2 & \Delta m^2 \\ \Delta m^2 & m_b^2 \end{pmatrix}.$$

The mass eigenstates are derived by diagonalizing the matrix above. If $|m_a^2 - m_b^2| \gg 2|\Delta m^2|$ holds, the mixing ratios of a and b in the two mass eigenstates are approximately given by

$$|m_a^2 - m_b^2|:|\Delta m^2|, \qquad |\Delta m^2|:|m_a^2 - m_b^2|.$$

A. Predictions of the bulk matter RS model

The bulk matter RS model predicts a nontrivial structure of flavor-violating soft SUSY breaking terms, given by Eqs. (41), (45)–(47), and (66)–(75). This structure can be translated into the flavor composition of each SUSY particle mass eigenstate. One subtlety is that the flavor-violating terms contain two different SUSY breaking mass scales, namely, the IR-scale-suppressed gravity mediation scale, $M_{\rm grav}$, and the gauge mediation scale, $M_{\rm gauge}$; flavor-violating gravity mediation contributions depend solely on $M_{\rm grav}$, whereas RG contributions are proportional to the net soft SUSY breaking mass scale that depends both on $M_{\rm grav}$ and $M_{\rm gauge}$. The relative size of these scales affects the predictions on the flavor compositions. We consider three cases with $M_{\rm grav} \gtrsim M_{\rm gauge}$, $M_{\rm grav} < M_{\rm gauge}$, and $M_{\rm grav} \ll M_{\rm gauge}$, whose precise definitions will be given each time. These cases lead to different predictions.

1. Case with $M_{\rm grav} \gtrsim M_{\rm gauge}$

In this case, flavor-universal soft SUSY breaking masses, m_*^2, and gaugino masses, M_{**}, are of the same magnitude as the gravity mediation scale, $M_{\rm grav}$. The differences among the diagonal components of different flavors come from the gravity mediation contributions (41) and the RG contributions (66)–(70). Since we now have $m_*^2 \sim M_{\rm grav}^2$, $M_{**} \sim M_{\rm grav}$, Eq. (41) surpasses Eqs. (66)–(70). Hence we may make the following approximations for $i > j$ in any flavor basis:

$$(m_Q^2)_{ii} - (m_Q^2)_{jj} \sim \alpha_i^2 M_{\rm grav}^2 \tag{76}$$

and similar formulas with (U, β), (D, γ), (L, δ), and (E, ϵ) replacing (Q, α) in the above formula.

In a similar manner, in any flavor basis, the A terms are approximated by

$$\begin{aligned} (A_u)_{ij} &\supset \beta_i \alpha_j M_{\rm grav}, \\ (A_d)_{ij} &\supset \gamma_i \alpha_j M_{\rm grav}, \\ (A_e)_{ij} &\supset \epsilon_i \delta_j M_{\rm grav}, \end{aligned} \tag{77}$$

and the off-diagonal components of soft SUSY breaking mass terms are by $(i \neq j)$

$$(m_Q^2)_{ij} \sim \alpha_i \alpha_j M_{\rm grav}^2 \tag{78}$$

and similar formulas with (U, β), (D, γ), (L, δ), and (E, ϵ) replacing (Q, α) in the above formula.

Sparticle Q_i mixes with sparticle Q_j $(j \neq i)$ through the term $(m_Q^2)_{ij}$ and with U_k or D_k $(k \neq i)$ through the A terms and the VEVs of the Higgs bosons. In this way, there appears a mass eigenstate that consists mainly of Q_i and partly of Q_j and U_k or D_k, which we hereafter call the almost Q_i mass eigenstate. From Eqs. (76) and (78), the mixing ratio of Q_j in the almost Q_i mass eigenstate is given by

$$\frac{|(m_Q^2)_{ij}|}{|(m_Q^2)_{ii} - (m_Q^2)_{jj}|} \simeq \frac{\alpha_i \alpha_j M_{\rm grav}^2}{(\alpha_i)^2 M_{\rm grav}^2} \sim \frac{\alpha_j}{\alpha_i} \tag{79}$$

for $i > j$, and by

$$\frac{|(m_Q^2)_{ij}|}{|(m_Q^2)_{ii} - (m_Q^2)_{jj}|} \simeq \frac{\alpha_i \alpha_j M_{\rm grav}^2}{(\alpha_j)^2 M_{\rm grav}^2} \sim \frac{\alpha_i}{\alpha_j} \tag{80}$$

for $i < j$. On the other hand, the mixing ratio of U_j in the up-sector of the almost Q_i mass eigenstate is given by $(i \neq j)$

$$\frac{v_u |(A_u)_{ji}|}{|m_Q^2 - m_U^2|} \sim \frac{v_u \beta_j \alpha_i M_{\rm grav}}{M_{\rm SUSY}^2} \sim \beta_j \alpha_i \frac{v_u}{M_{\rm SUSY}}, \tag{81}$$

where we used the fact that the difference between the flavor-universal masses of the SU(2) doublet and the singlet squarks is of the same magnitude as the soft SUSY breaking mass scale itself, denoted by $M_{\rm SUSY}$.

The mixing ratios in other mass eigenstates follow similar formulas. There is a subtlety about the ratio of L_j in the almost L_i mass eigenstate because we have $3\delta_1 \sim \delta_2 \sim \delta_3$ and the approximation used to derive Eqs. (79)–(81) is no longer valid. Actually, the mixing ratio of L_j in the almost L_i mass eigenstate is $O(1)$ for any i, j.

2. Case with $M_{\rm grav} \ll M_{\rm gauge}$

In this section, we concentrate on the case where the ratio $M_{\rm grav}/M_{\rm gauge}$ is so small that the RG contributions to flavor-violating soft SUSY breaking terms, Eqs. (58)–(60) and (66)–(75), are of the same magnitude as or larger than the gravity mediation contributions, Eqs. (41) and (45)–(47).

In these cases, the mixing ratio of Q_j in the almost Q_i mass eigenstate is given, from Eqs. (66) and (71), by $(i > j)$

$$\begin{aligned} \frac{|(m_Q^2)_{ij}|}{|(m_Q^2)_{ii} - (m_Q^2)_{jj}|} &\sim \frac{\alpha_i (\gamma_3)^2 \alpha_j}{(\alpha_i)^2 (\beta_i)^2 + (\alpha_i)^2 (\gamma_3)^2} \\ &\sim \frac{\alpha_j}{\alpha_i} \frac{(\gamma_3)^2}{(\beta_i)^2 + (\gamma_3)^2}, \end{aligned} \tag{82}$$

in the flavor basis where Y_u is diagonalized. Here we used the fact that the integrands of the right-hand side of (66) and that of (71) are of the same magnitude. On the other hand, from Eq. (58), the mixing ratio of U_j in the up-sector of the almost Q_i mass eigenstate is given by $(i \neq j)$

$$\frac{v_u |(A_u)_{ji}|}{|m_Q^2 - m_U^2|} \sim 2\beta_j (\alpha_j)^2 (\gamma_3)^2 \alpha_i \frac{v_u}{M_{\rm gauge}} \tag{83}$$

in the Y_u-diagonal basis. Here we approximated the difference of flavor-conserving masses of SU(2) doublet squarks and singlet up-type squarks by $M_{\rm gauge}$. The mixing ratio of D_j in the down-sector of the almost Q_i mass eigenstate in the Y_d-diagonal basis takes a similar expression. The same discussion applies to the mixings in the almost L_i mass eigenstate.

The mixing ratios in the almost U_i mass eigenstate follow different formulas. From Eqs. (68) and (73), the ratio of U_j is given by $(i > j)$

$$\frac{|(m_U^2)_{ij}|}{|(m_U^2)_{ii} - (m_U^2)_{jj}|} \sim \frac{24\beta_i(\alpha_i)^2(\gamma_3)^2(\alpha_j)^2\beta_j}{4(\beta_i)^2(\alpha_i)^2} \sim 6(\gamma_3)^2(\alpha_j)^2\frac{\beta_j}{\beta_i} \tag{84}$$

in the Y_u-diagonal basis. On the other hand, from (58), the ratio of the up-sector of Q_j in the almost U_i mass eigenstate is given by $(i \neq j)$

$$\frac{v_u|(A_u)_{ij}|}{|m_Q^2 - m_U^2|} \sim 2\beta_i(\alpha_i)^2(\gamma_3)^2\alpha_j\frac{v_u}{M_{\rm gauge}} \tag{85}$$

in the Y_u-diagonal basis. The same discussion applies to the mixings in the almost D_i mass eigenstate and the almost E_i mass eigenstate.

3. Case with $M_{\rm grav} < M_{\rm gauge}$ but not with $M_{\rm grav} \ll M_{\rm gauge}$

Consider the case where $M_{\rm grav}$ is slightly smaller than $M_{\rm gauge}$. Then gravity mediation contributions surpass RG contributions for some of the flavor-violating soft SUSY breaking terms, and the opposite holds for the other terms. In these cases, the mixing ratios of sparticle mass eigenstates generally depend on the unknown ratio $M_{\rm grav}/M_{\rm gauge}$ and the model loses its predictive power.

However, certain mixing ratios are more likely to reflect the gravity mediation contributions. For example, if $M_{\rm grav} \gtrsim \delta_3 M_{\rm gauge}$, as to terms $(m_E^2)_{ii} - (m_E^2)_{jj}$ and $(m_E)^2_{ij}$, the gravity mediation contributions described by (41) are larger than the RG contributions, (70) and (75). Then the mixing ratio of E_j in the almost E_i mass eigenstate is the same as in the case with $M_{\rm grav} \gtrsim M_{\rm gauge}$. Focusing on such mixing ratios, it is still possible to observe the signatures of the model.

B. Comparison with other models

To test the predictions of the bulk matter RS model, we must check whether they contain signatures distinguishable from other models. As an example, we investigate two types of models; one is minimal flavor violation, in which RG contributions of the Yukawa couplings are the only source of flavor-violating soft SUSY breaking terms. The other is 4D gravity mediation, in which gravity mediation contributes uniformly to all flavor-violating terms. We will compare the predictions of these models with the bulk matter RS model and discuss the ways to distinguish them.

1. Minimal flavor violation

The MFV scenario leads to the same result as in Sec. IVA 2, i.e. the bulk matter RS model with $M_{\rm grav} \ll M_{\rm gauge}$. This is because the argument in Sec. IVA 2 holds irrespective of gravity mediation contributions. We thus conclude that it is impossible to experimentally distinguish the bulk matter RS model from the MFV scenario when we have $M_{\rm grav} \ll M_{\rm gauge}$, as in Sec. IVA 2.

In contrast, if $M_{\rm grav} \gtrsim M_{\rm gauge}$, the MFV scenario and the bulk matter RS model have distinctively different predictions on the mixing ratios in the almost U_i, D_i, and E_i mass eigenstates with $i = 1, 2$. This is seen by comparing (79)–(81) (Q relaced by U, D, E) with (84) and (85); the flavor mixings in these mass eigenstates are suppressed at least by $(\alpha_2)^2$ or $(\delta_2)^2$ in "minimal flavor violation" compared to the bulk matter RS model. Therefore it is possible to discriminate the two models by observing the flavor compositions of "almost 1st or 2nd generation SU (2) singlet sparticle mass eigenstates."

2. 4D gravity mediation

Here we discuss the case where 4D theory description is valid even at the Planck scale, or all matter superfields are confined on the same 4D brane. Then the gravity mediation contributions are of the same magnitude irrespective of flavors. Of particular interest is the situation where the gravity mediation contributions surpass the flavor-violating RG contributions, which is the case when $M_{\rm grav}$ is only slightly smaller than $M_{\rm gauge}$. In this situation, the differences between diagonal components of soft SUSY breaking masses $(m_*^2)_{ii} - (m_*^2)_{jj}$, and off-diagonal components $(m_*^2)_{ij}$, in any flavor basis are of the same magnitude. Then the mixing ratios in sparticle mass eigenstates are all $O(1)$. It is easy to distinguish this model from the bulk matter RS model, where the mixing ratios of recessive flavors are suppressed by the geometrical factors.

V. EXPERIMENTAL STUDIES

In the previous section, we saw that the bulk matter RS model has a unique prediction on the flavor compositions of sparticle mass eigenstates that may be distinguishable from other models. In this section, we study how to measure the predicted mixing ratios through collider experiments. We focus on the case where $M_{\rm grav} \simeq M_{\rm gauge}$ holds or $M_{\rm grav}$ is slightly smaller than $M_{\rm gauge}$, and put emphasis on distinguishing the bulk matter RS model from the MFV scenario.

The basic strategy is to create a specific mass eigenstate (s) of SUSY matter particles, detect its decay products, and count the numbers of events of different decay modes. The branching fractions of different modes reflect the flavor composition of that mass eigenstate. There are, however, three challenges for this study.

First, we have to detect small flavor components of sparticle mass eigenstates, which means that we have to observe *rare* decay events in collider experiments. For this purpose, the probability of misidentifying the decay products of the dominant mode as those of a rare mode must be negligibly small. For example, the stau components of the almost smuon mass eigenstates are detectable because the SM tau from the stau components, when we focus on its hadronic decay, leaves a signal different from muon events. However, it is impossible to observe the smuon components of the almost stau mass eigenstates because the SM tau from the dominant stau components may decay into a SM muon, which mimics the smuon component signal.

Second, we have to extract the decay products of a *specific* mass eigenstate in order to compare the data with the predictions of the bulk matter RS model. It is thus required to produce only specific mass eigenstates at a collider. This is achieved by lepton colliders, such as the ILC [16] and the CLIC [17], where the center-of-mass energy of a process is fixed. For example, the flavor-mixing ratios in the almost SU(2) doublet smuon mass eigenstate and in the almost SU(2) singlet smuon mass eigenstate are predicted to be different. To confirm this prediction, we must produce one of the two eigenstates selectively. If the latter is lighter than the former, we take the center-of-mass energy between their thresholds so that only the latter is created on shell. We then measure the mixing ratios of the latter eigenstate through its decay products. In conclusion, lepton colliders are essential when studying the flavor compositions of sparticle mass eigenstates.

Finally, we have to focus on the almost SU(2) singlet mass eigenstates in order to discriminate the bulk matter RS model from the MFV scenario. This is understood by comparing the predictions of the bulk matter RS model, (79)–(81), with those of the MFV scenario, (82)–(85). Remember that we have

$$\gamma_3 \sim \tan\beta \frac{m_b}{v}, \qquad \beta_3 \sim 1$$

and we do not know the magnitude of ζ_3. Hence it can be the case that the mixing ratios of Q_j in the almost Q_i mass eigenstate, and those of L_j in the almost L_i mass eigenstate are of the same magnitudes for the bulk matter RS model and the MFV scenario. In contrast, the mixing ratios of U_j, D_j, and E_j in the almost U_j, D_j, and E_j mass eigenstates are of different magnitudes for the two models because the mixing ratios in the MFV scenario, (82)–(85), are suppressed by the factors $(\alpha_1)^2$, $(\alpha_2)^2$, $(\delta_1)^2$, or $(\delta_2)^2$ compared to those in the bulk matter RS model, (79)–(81). We further notice that the almost 3rd generation sparticle mass eigenstates are not suitable for our study because the 3rd generation sparticles have significant left-right mixing terms due to their large Yukawa couplings. We conclude that observing the rare decays of the almost SU(2) singlet 1st and 2nd generation mass eigenstates is the only way to distinguish the bulk matter RS model and the MFV scenario.

Taking these points into account, we discuss three types of experiments that are feasible at future lepton colliders. The first type of experiment deals with the rare decay of the almost SU(2) singlet smuon mass eigenstate into a SM tau, which reflects the mixing of the singlet smuon with the stau. Another type of experiment deals with the rare decay of the almost SU(2) singlet smuon mass eigenstate into the SM electron or that of the almost SU(2) singlet selectron into the SM muon, which reflects the mixing of the singlet smuon and the selectron. The other type of experiment deals with the rare decay of the almost SU(2) singlet scharm mass eigenstate into the SM top, which reflects the mixing of the singlet scharm with stop. For a concrete discussion, we make assumptions on the SUSY particle mass spectrum in Sec. V A. We then look into the three types of experiments in Secs. V B, V C, and V D.

The almost lighter stau/stop mass eigenstates are schematically denoted by $\tilde{\tau}_1/\tilde{t}_1$, and the almost singlet selectron/smuon/scharm mass eigenstates are by $\tilde{e}_R/\tilde{\mu}_R/\tilde{\mu}_R$.

It is impossible to do these experiments at hadron colliders. This is fundamentally because we need to create the almost SU(2) singlet mass eigenstates exclusively, without creating their the almost SU(2) doublet counterparts, in order to discriminate the bulk matter RS model from the MFV scenario. Hadron colliders would necessarily create both eigenstates, and the decay products of the latter would contaminate the signals that allow us to distinguish the two scenarios. It is true that almost SU(2) singlet eigenstates are normally lighter than their almost SU(2) doublet counterparts, and hence the production cross sections of the latter are lower even at hadron colliders. However, since the two scenarios predict only the orders of magnitudes of the branching ratios of rare events, even small contamination from the latter would make it difficult to test the predictions.

A. Assumptions on the mass spectrum

In this section, we make plausible assumptions on the SUSY particle mass spectrum that are consistent with the bulk matter RS model combined with the 5D MSSM.

We assume that squarks are heavier than sleptons and a gluino is heavier than a wino and bino because of their SU (3) charges. Also, a wino is assumed heavier than a bino due to its SU(2) charge. SU(2) doublet squarks are heavier than singlet squarks, and doublet sleptons are heavier than singlet sleptons. Since gauge superfields are flat in the bulk, i.e. they have no y dependence, they obtain large soft SUSY breaking masses through contact terms on the IR brane. Therefore gluinos tend to be heavier than squarks. Winos and binos are heavier than sleptons but lighter than squarks.

The μ term is assumed larger than wino and bino masses, as is normally the case.

We do not specify the mass order among doublet and singlet squarks and sleptons because gravity mediation contributions may distort the mass spectrum. However, we expect that the masses of the 1st and 2nd generation SUSY particles are almost degenerate because their Yukawa couplings as well as their overlap with the IR brane are small.

A gravitino is always the lightest SUSY particle (LSP) because its mass is given by $\sim\text{TeV}\times e^{-kR\pi}$. The next-to-lightest SUSY particle (NLSP) is an almost SU(2) singlet selectron, an almost singlet smuon, or almost the lighter stau mass eigenstate. The lifetime of the NLSP satisfies

$$ct_{\text{NLSP}}\simeq 48\pi\frac{|\langle F_{\tilde{X}}\rangle|^2}{m_{\text{NLSP}}^5}\simeq 48\pi\frac{M_{\text{grav}}^2(M_5e^{-kR\pi})^2}{(m_{\text{NLSP}})^5}$$
$$\simeq(1.2\times 10^{-26})\,\text{m}\times\left(\frac{M_{\text{grav}}}{\text{GeV}}\right)^2\left(\frac{M_5e^{-kR\pi}}{\text{GeV}}\right)^2\left(\frac{300\,\text{GeV}}{m_{\text{NLSP}}}\right)^5.$$

We assume that the lifetime is long enough that the NLSP reaches the inner detector before it decays.

The order of the sparticle soft SUSY breaking masses is summarized as

$$\tilde{H}_u, \tilde{H}_d > \tilde{g} > \tilde{q}_L > \tilde{q}_R > \chi_1^{\pm},$$
$$\chi_2^0(\fallingdotseq\tilde{W}) > \chi_1^0(\fallingdotseq\tilde{B}) > \tilde{l}_L > \tilde{l}_R > \psi_{3/2}.$$

B. Type I—Smuon rare decay with staulike NLSP

Consider the case where almost the lighter stau mass eigenstate ($\tilde{\tau}_1$) is the NLSP. Tune the center-of-mass energy of the lepton collider between the thresholds of almost singlet selectron/smuon mass eigenstates ($\tilde{e}_R/\tilde{\mu}_R$), the almost SU(2) doublet selectron/smuon, and almost the heavier stau mass eigenstates. Then $\tilde{e}_R$, $\tilde{\mu}_R$, and $\tilde{\tau}_1$ are produced on shell, while other sparticle mass eigenstates are not.

The signal for $\tilde{e}_R$ or $\tilde{\mu}_R$ pair production is a pair of long-lived charged massive particles, which are NLSP $\tilde{\tau}_1$'s, plus two pairs of hard SM leptons. Note that, since the masses of $\tilde{e}_R$ and $\tilde{\mu}_R$ are almost degenerate, we cannot detect SM leptons emitted when the heavier one decays into the lighter one. Normally, we have two SM muons or electrons plus two SM taus in these events (we call this the "main" mode), e.g.

$$ee\to\tilde{\mu}_R\tilde{\mu}_R\to\mu\tau\tilde{\tau}_1\mu\tau\tilde{\tau}_1 \tag{86}$$

for the smuon production. However, due to the small stau components in $\tilde{e}_R/\tilde{\mu}_R$, we may also have one SM muon or electron plus three SM taus in these events (we call this the "rare" mode), e.g.

$$ee\to\tilde{\mu}_R\tilde{\mu}_R\to\tau\tau\tilde{\tau}_1\mu\tau\tilde{\tau}_1 \tag{87}$$

for the smuon production. Requiring hadronic decay of SM taus and taking advantage of the tau vertexing, one can reduce the probability of misidentifying a main mode event as a rare mode event to a negligible level.

The branching ratio of the rare mode is proportional to the square of the mixing ratio. The stau component in $\tilde{\mu}_R$ plays a dominant role because the stau component in $\tilde{e}_R$ is much more suppressed. From Eq. (79) with (Q, α) replaced by (E, ϵ), and from Eq. (81) with $(Q, U, \alpha, \beta, v_u)$ replaced by $(L, E, \delta, \epsilon, v_d)$, the bulk matter RS model predicts that the branching ratio of the rare mode is given by

$$\text{Br}(\tilde{\mu}_R\to\tau\tau\tilde{\tau}_1)\sim\left(\frac{\epsilon_2}{\epsilon_3}\right)^2+\left(\epsilon_2\delta_3\frac{v_d}{M_{\text{SUSY}}}\right)^2$$
$$\sim\left(\frac{m_\mu}{m_\tau}\right)^2+\left(\frac{m_\mu}{M_{\text{SUSY}}}\right)^2, \tag{88}$$

where the first term comes from the mixing with the singlet stau and the second from the mixing with the doublet stau. If $\tilde{e}_R$ is lighter than $\tilde{\mu}_R$, the branching ratio is reduced by $1/2$ compared to the opposite case because $1/2$ of $\tilde{\mu}_R$'s decay into $\tilde{e}_R$'s. However, this does not affect the order estimate above. Since we have $M_{\text{SUSY}}\gg m_\tau$, the second term is negligible and the branching ratio becomes

$$\text{Br}\,(\tilde{\mu}_R\to\tau\tau\tilde{\tau}_1)\sim 0.004. \tag{89}$$

Note that the prediction above may change by $O(0.1)$–$O(10)$ because we only know the magnitudes of the higher-dimensional couplings for soft SUSY breaking terms.

On the other hand, the MFV scenario predicts that the branching ratio of the rare mode is given by

$$\text{Br}(\tilde{\mu}_R\to\tau\tau\tilde{\tau}_1)\sim\left\{6(\zeta_3)^2(\delta_2)^2\frac{\epsilon_2}{\epsilon_3}\right\}^2$$
$$+\left\{2(\zeta_3)^2(\delta_2)^2\epsilon_2\delta_3\frac{v_d}{M_{\text{SUSY}}}\right\}^2$$
$$\sim(\zeta_3\delta_2)^4\times 0.1, \tag{90}$$

where we used Eqs. (84) and (85) with $(Q, U, \alpha, \beta, \gamma, v_u)$ replaced by $(L, E, \delta, \epsilon, \zeta, v_d)$. Although we cannot determine the magnitude of $\zeta_3\delta_2$, we expect it to be smaller than 0.1; from Eq. (72), we have the following flavor-mixing term for the SU(2) doublet smuon and selectron:

$$(m_L^2)_{12}\sim\delta_1(\zeta_3)^2\delta_2M_{\text{SUSY}}^2\sim\tfrac{1}{3}(\zeta_3\delta_2)^2M_{\text{SUSY}}^2.$$

For example, with $M_{\text{SUSY}}=500$ GeV, $m_{\tilde{l}_L}^2=500$ GeV, $M_{\tilde{B}}=M_{\tilde{W}}=750$ GeV, $\mu=1000$ GeV, $\tan\beta=10$ and vanishing A terms, taking $\zeta_3\delta_2=0.1$ would saturate the current bound on the $\mu\to e\gamma$ branching ratio, $\text{Br}(\mu\to e\gamma)\leq 1.2\times 10^{-11}$ [18]. Hence the branching ratio of the rare mode in the MFV scenario satisfies

$$\text{Br}\,(\tilde{\mu}_R\to\tau\tau\tilde{\tau}_1)\lesssim 10^{-5}, \tag{91}$$

which may get smaller if the bound on $\text{Br}(\mu\to e\gamma)$ improves. We conclude that the branching ratio of the rare mode is distinctively smaller in the MFV scenario than in the bulk matter RS model.

C. Type II—NLSP selectron rare decay into muon, or NLSP smuon rare decay into electron/tau

Consider the case where the almost singlet smuon or the almost singlet selectron2 mass eigenstate ($\tilde{\mu}_R$ or $\tilde{e}_R$) is the NLSP and is long lived. Tune the center-of-mass energy slightly above the threshold of $\tilde{\mu}_R/\tilde{e}_R$ so that they are produced with a low β (Lorentz velocity). Slow long-lived sleptons may be trapped in the inner detector. According to the paper [19], taking $\beta \lesssim 0.2$ is sufficient to trap 600 GeV or lighter sleptons in the inner detector. We analyze the decay products of these sleptons to study their flavor compositions.

First study the case where $\tilde{e}_R$ is lighter than $\tilde{\mu}_R$ and is the long-lived NLSP. $\tilde{e}_R$ mainly decays into a SM electron and a gravitino (main mode). However, due to its smuon component, it also decays into a SM muon and a gravitino (rare mode). Hence we expect to observe rare mode events where one of the sparticle pair produced by the collider decays into a SM muon and the other into a SM electron with large vertex separation due to the longevity of $\tilde{e}_R$.

The bulk matter RS model predicts that the branching ratio of the rare mode is given by

$$\begin{aligned} \mathrm{Br}(\tilde{e}_R \to \mu\psi_{3/2}) &\sim \left(\frac{\epsilon_1}{\epsilon_2}\right)^2 + \left(\epsilon_1\delta_2\frac{v_d}{M_{\mathrm{SUSY}}}\right)^2 \\ &\sim \left(3\frac{m_e}{m_\mu}\right)^2 + \left(3\frac{m_e}{M_{\mathrm{SUSY}}}\right)^2 \\ &\sim 0.0002, \end{aligned} \tag{92}$$

where we neglected the second terms of the right-hand sides because we have

$$3\frac{m_e}{m_\mu} \gg 3\frac{m_e}{M_{\mathrm{SUSY}}}$$

in realistic SUSY models.

On the other hand, the MFV scenario predicts that the branching ratio of the rare mode is given by

$$\begin{aligned} \mathrm{Br}(\tilde{e}_R \to \mu\psi_{3/2}) &\sim \left\{6(\zeta_3)^2(\delta_1)^2\frac{\epsilon_1}{\epsilon_2}\right\}^2 \\ &\quad + \left\{2(\zeta_3)^2(\delta_1)^2\epsilon_2\delta_1\frac{v_d}{M_{\mathrm{SUSY}}}\right\}^2 \\ &\sim (\zeta_3\delta_1)^4 \times 0.03. \end{aligned} \tag{93}$$

Again, the bound on the $\mu \to e\gamma$ branching ratio gives a severe constraint on the value of $\zeta_3\delta_1$, and the branching ratio satisfies

$$\mathrm{Br}\,(\tilde{e}_R \to \mu\psi_{3/2}) < 10^{-6} \tag{94}$$

for realistic mass spectra.

$\tilde{e}_R$ also decays into a SM tau and a gravitino but the branching ratio is suppressed by the factor $(m_e/m_\tau)^2$ and is thus negligibly small.

Next consider the case where $\tilde{\mu}_R$ is lighter than $\tilde{e}_R$ and is the long-lived NLSP. $\tilde{\mu}_R$ mainly decays into a SM muon and a gravitino (main mode), but also into a SM electron and a gravitino, or into a SM tau and a gravitino (rare modes). The branching ratio of the rare mode where the sparticle pair decays into a muon and an electron and two gravitinos is the same as Eqs. (92) and (93). The branching ratio of the rare mode where the sparticle pair decays into a tau and a muon and two gravitinos is the same as Eqs. (89) and (90).

D. Type III—Scharm rare decay into SM top

1. Scharm is lighter than stop

Consider the case where $\tilde{c}_R$ is lighter than $\tilde{t}_1$. Tune the center-of-mass energy between the thresholds of $\tilde{c}_R$ and $\tilde{t}_1$. Then $\tilde{c}_R$, $\tilde{u}_R$, $\tilde{s}_R$, and $\tilde{d}_R$, whose masses are almost degenerate, are produced on shell, while other squark mass eigenstates are not. $\tilde{c}_R/\tilde{u}_R/\tilde{s}_R/\tilde{d}_R$ mainly decay into SM charm/up/strange/down and the lightest neutralino χ_1^0, which is bino like (the main mode). χ_1^0 promptly decays into several SM leptons and NLSP, e.g. we have

$$\begin{aligned} ee &\to \tilde{c}_R\tilde{c}_R \to c\chi_1^0 c\chi_1^0 \\ &\to (c\text{ jet})\mathrm{NLSP}(c\text{ jet})\mathrm{NLSP}(\mathrm{SM\ leptons}) \end{aligned} \tag{95}$$

for scharm pair production. Because of the small stop components, they also decay into SM top and χ_1^0 with a tiny branching ratio (the rare mode), e.g. we have

$$\begin{aligned} ee &\to \tilde{c}_R\tilde{c}_R \to t\chi_1^0 c\chi_1^0 \\ &\to (\text{top decay products})\mathrm{NLSP}(c\text{ jet})\mathrm{NLSP}(\mathrm{SM\ leptons}) \end{aligned} \tag{96}$$

for scharm pair production. The signal of the main mode is two hard jets, two long-lived charged massive particles, and several SM leptons. On the other hand, the signal of the rare mode is, when the SM top decays hadronically, four hard jets, two long-lived charged massive particles, and several SM leptons. We see that the probability of misidentifying a main mode event as a rare mode event is negligibly small if we require the hadronic top decay in rare mode events.

Of the four eigenstates, $\tilde{c}_R$ dominantly contributes to rare mode events because the stop components in the other eigenstates are more suppressed than in $\tilde{c}_R$. One can confirm this by requiring c tagging for one of the jets in rare mode events.

From (80) with (Q, α) replaced by (U, β) and (81), the bulk matter RS model predicts that the branching ratio of the rare mode is given by

$$\begin{aligned} \mathrm{Br}(\tilde{c}_R \to t\chi_1^0) &\sim \left(\frac{\beta_2}{\beta_3}\right)^2 + \left(\beta_2\alpha_3\frac{v_u}{M_{\mathrm{SUSY}}}\right)^2 \\ &\sim \left(\frac{1}{\lambda^2}\frac{m_c}{m_t}\right)^2 + \left(\frac{1}{\lambda^2}\frac{m_c}{m_t}\frac{m_t}{M_{\mathrm{SUSY}}}\right)^2 \\ &\sim 0.02, \end{aligned} \tag{97}$$

where we neglected the second term because we have

$$m_t < M_{\rm SUSY}$$

in realistic SUSY models.

From Eqs. (84) and (85), the MFV scenario predicts that the branching ratio of the rare mode is given by

$$\begin{aligned}\mathrm{Br}(\tilde{c}_R \to t\chi_1^0) \sim & \left\{6(\gamma_3)^2(\alpha_2)^2\frac{\beta_2}{\beta_3}\right\}^2 \\ & + \left\{2(\gamma_3)^2(\alpha_2)^2\beta_2\alpha_3\frac{v_u}{M_{\rm SUSY}}\right\}^2 \\ & \lesssim 5\times 10^{-6}, \end{aligned} \tag{98}$$

where we used the fact that $\gamma_3 \leq 1$. Comparing Eq. (96) with (95), we notice that the bulk matter RS model and the MFV scenario have distinctively different predictions.

2. Stop is lighter

Consider the case where $\tilde{t}_1$ is lighter than $\tilde{c}_R$. $\tilde{c}_R$ is still lighter than the almost SU(2) doublet squark mass eigenstates (including $\tilde{t}_1$, $\tilde{b}_1$). Tune the center-of-mass energy between the thresholds of $\tilde{c}_R$ and the almost doublet squark eigenstates. Then $\tilde{c}_R$, $\tilde{u}_R$, $\tilde{s}_R$, $\tilde{d}_R$, $\tilde{b}_1$, and $\tilde{t}_1$ are produced on shell, while the other squark mass eigenstates are not.

We want to extract the signals of rare mode events where one of the pair of $\tilde{c}_R$, $\tilde{u}_R$, $\tilde{s}_R$, or $\tilde{d}_R$'s decays into a SM top and a neutralino. However, these events are contaminated by the events where one of the pair of $\tilde{b}_1$'s decays into a SM top and a chargino; the chargino decays into a NLSP and SM leptons, but one charged lepton is misdetected. The b jet from the other $\tilde{b}_1$ is mis-b tagged. There is also a contamination from the events where one of the pair of $\tilde{t}_1$'s decays into a SM bottom and a chargino, or into a SM charm and a neutralino due to the scharm component in $\tilde{t}_1$. We take advantage of kinematical properties to reject these contaminations.

Tune the center-of-mass energy close to the threshold of $\tilde{c}_R$, $\tilde{u}_R$, $\tilde{s}_R$, and $\tilde{d}_R$ so that they are produced almost at rest. Suppose that one observed three hard jets from the hadronic decay of the SM top (t), another hard jet (j) and several SM leptons plus two NLSPs. Further assume that the 3-momenta of t and j are reconstructed successfully. We want to know whether this event comes from the decay of $\tilde{c}_R$, $\tilde{u}_R$, $\tilde{s}_R$, $\tilde{d}_R$ or from $\tilde{t}_1$, $\tilde{b}_1$. In the former case, the 3-momenta satisfy

$$|\vec{p}_j| + \sqrt{|-\vec{p}_j|^2 + m_\chi^2} \simeq m_{\tilde{c}_R}, \tag{99}$$

$$\sqrt{|\vec{p}_t|^2 + m_t^2} + \sqrt{|-\vec{p}_t|^2 + m_\chi^2} \simeq m_{\tilde{c}_R}, \tag{100}$$

where $\vec{p}_j$ and $\vec{p}_t$, respectively, denote the 3-momenta of j and t, and m_χ the mass of a bino-like neutralino. In the latter case, however, the above equations hold for specific situations where j and t go in special directions against the initial $\tilde{t}_1$'s or $\tilde{b}_1$'s because the $\tilde{t}_1$, $\tilde{b}_1$'s are boosted. To summarize, the rare mode signals of $\tilde{c}_R$, $\tilde{u}_R$, $\tilde{s}_R$, and $\tilde{d}_R$ can be extracted through the discriminants (99) and (100).

The branching ratios of the rare mode in the bulk matter RS model and the MFV scenario are the same as in Eqs. (97) and (98).

E. Cross sections

We calculate the cross sections of the rare modes given above and discuss their accessibility at collider experiments.

First focus on the type I experiment, where one of the pairs of almost singlet smuon or selectron mass eigenstates decays into two SM taus and a NLSP in case the NLSP is staulike. The center-of-mass energy is tuned above the threshold of $\tilde{\mu}_R$, namely, we take

$$\sqrt{s} = 2m_{\tilde{\mu}_R} + 100\ \mathrm{GeV}.$$

In Fig. 1, we plot the mass of $\tilde{\mu}_R$ vs the cross section of the rare mode at a e^+e^- collider. We take the branching ratio of the rare mode as

$$\mathrm{Br}(\tilde{\mu}_R \to \tau\tau\tilde{\tau}_1) = \left(\frac{m_\mu}{m_\tau}\right)^2$$

based on Eq. (88). We assume that $\tilde{\mu}_R$ is slightly lighter than $\tilde{e}_R$ so that one-half of $\tilde{e}_R$'s decay into $\tilde{\mu}_R$ and contribute to the rare mode. In calculating the $\tilde{e}_R$ pair production cross section, the bino mass is assumed to be $1.5m_{\tilde{\mu}_R}$. Also shown is the total cross section of $\tilde{\mu}_R$, $\tilde{e}_R$ production events.

Next focus on type II, where one of the pairs of almost singlet selectron mass eigenstates decays into a SM muon and a gravitino if it is the NLSP, or one of the pairs of almost singlet smuon mass eigenstates decays into a SM electron/tau and a gravitino if it is the NLSP. We tune the center-of-mass energy slightly above the threshold of $\tilde{e}_R/\tilde{\mu}_R$ so that their velocities are low enough to trap them inside the inner detector. For simplicity, we take

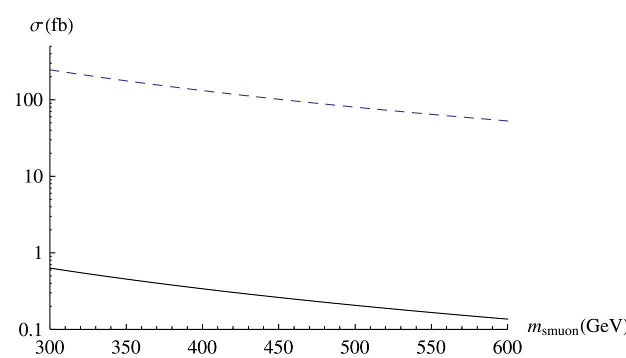

FIG. 1 (color online). Mass of $\tilde{\mu}_R$ vs the cross section of the rare mode where a $\tilde{\mu}_R$ is produced and decays into two SM taus and a NLSP stau (straight line). The center-of-mass energy is taken as $\sqrt{s} = 2m_{\tilde{\mu}_R} + 100$ GeV. The total cross section of $\tilde{\mu}_R$ or $\tilde{e}_R$ production process is also shown (dashed line).

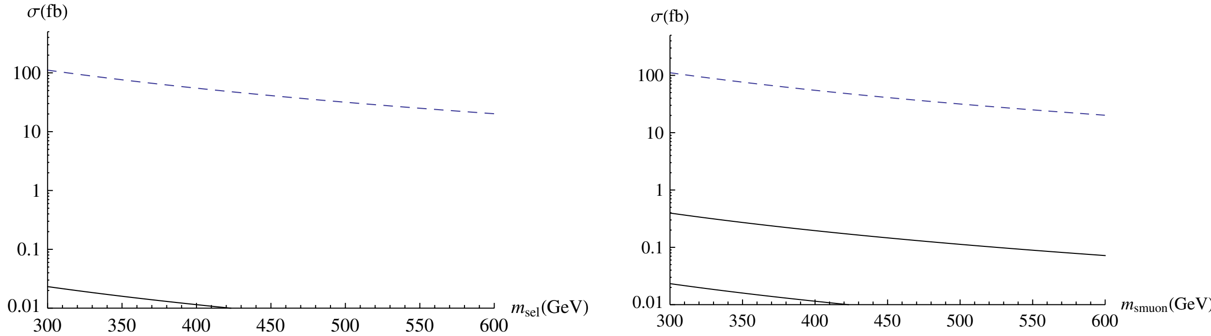

FIG. 2 (color online). Left panel: Mass of $\tilde{e}_R$ vs the cross section of the rare mode where a NLSP $\tilde{e}_R$ is produced and decays into a SM muon and a gravitino (straight line). Right panel: Mass of $\tilde{\mu}_R$ vs the cross section of a rare mode where a NLSP $\tilde{\mu}_R$ is produced and decays into a SM electron and a gravitino (bottom straight line), and the other rare mode where a NLSP $\tilde{\mu}_R$ is produced and decays into a SM tau and a gravitino (top straight line). The total cross sections of $\tilde{e}_R$ (left panel) and $\tilde{\mu}_R$ (right panel) production processes are also shown (dashed lines).

$$\sqrt{s} = 2m_{\tilde{e}_R/\tilde{\mu}_R} + 20 \text{ GeV}.$$

In Fig. 2, we plot the mass of $\tilde{e}_R$ vs the cross section of the rare mode if it is the NLSP, and the mass of $\tilde{\mu}_R$ vs the cross section of the rare mode if it is the NLSP. We take the branching ratios of the rare modes as

$$\mathrm{Br}(\tilde{e}_R \to \mu \psi_{3/2}) = \mathrm{Br}(\tilde{\mu}_R \to e \psi_{3/2}) = \left(\frac{1}{3}\frac{m_e}{m_\mu}\right)^2,$$

$$\mathrm{Br}(\tilde{\mu}_R \to \tau \psi_{3/2}) = \left(\frac{m_\mu}{m_\tau}\right)^2,$$

based on (88) and (92). Also shown are the total cross sections of $\tilde{e}_R$ and $\tilde{\mu}_R$ production processes.

Finally focus on type III, where one of the pairs of $\tilde{c}_R$, $\tilde{u}_R$, $\tilde{s}_R$, and $\tilde{d}_R$'s decays into a SM top and a neutralino. The center-of-mass energy is tuned above the threshold of $\tilde{c}_R$. First we take

$$\sqrt{s} = 2m_{\tilde{c}_R} + 100 \text{ GeV},$$

so that the cross section is nearly maximized. Second we take

$$\sqrt{s} = 2m_{\tilde{c}_R} + 10 \text{ GeV},$$

so that $\tilde{c}_R$'s are produced almost at rest and the rare mode events are kinematically distinguishable from $\tilde{t}_1$, $\tilde{b}_1$ production events in case $\tilde{t}_1$, $\tilde{b}_1$ are lighter. In Fig. 3, we plot the mass of $\tilde{c}_R$ vs the cross section of the rare mode for both cases. The branching ratio of the rare mode is taken as

$$\mathrm{Br}\,(\tilde{c}_R \to t\chi_1^0) = \left(\frac{1}{\lambda^2}\frac{m_c}{m_t}\right)^2$$

for both cases, based on (97). Also shown is the total cross section of $\tilde{c}_R$, $\tilde{u}_R$, $\tilde{s}_R$, and $\tilde{d}_R$ production processes.

Note that we can in principle reject all background events when observing the signals of the rare modes. Therefore, detecting several signals is sufficient to confirm the bulk matter RS model. From Figs. 1 and 2, we find that one can study the stau component in $\tilde{\mu}_R$ at the ILC with the integrated luminosity of $\sim$100 fb^{-1}. However, studying the smuon component in $\tilde{e}_R$ or the selectron component in $\tilde{\mu}_R$ requires $\sim$1000 fb^{-1} integrated luminosity. From

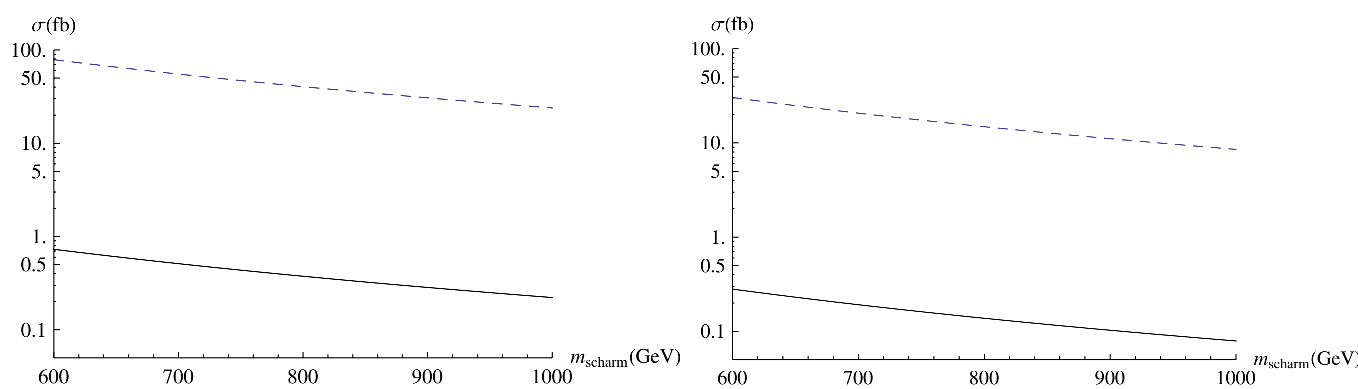

FIG. 3 (color online). Mass of $\tilde{c}_R$ vs the cross section of the rare mode where a $\tilde{c}_R$ is produced and decays into a SM top and a neutralino (straight lines). The center-of-mass energy is taken as $\sqrt{s} = 2m_{\tilde{c}_R} + 100$ GeV on the left and $\sqrt{s} = 2m_{\tilde{c}_R} + 10$ GeV on the right. The total cross section of $\tilde{c}_R$, $\tilde{u}_R$, $\tilde{s}_R$, or $\tilde{d}_R$ production processes is also shown (dashed lines).

Fig. 3, we see that $\sim$1000 fb^{-1} integrated luminosity is sufficient to study the stop component in $\tilde{c}_R$.

VI. SUMMARY AND OUTLOOK

We discussed observing signals of the bulk matter RS model, especially when the IR scale is far above the TeV scale. We saw that this is possible in the case of the minimal supersymmteric extension of the bulk matter RS model where the warped spacetime solely explains the hierarchy of the Yukawa couplings, while SUSY solves the gauge hierarchy problem. There, gravity mediation contributions to soft SUSY breaking terms reflect the 5D disposition of superfields. Hence flavor-violating soft SUSY breaking matter mass terms that arise from gravity mediation exhibit a flavor structure unique to the bulk matter RS model. RG running of the Yukawa coupling constants also contributes to the flavor-violating terms, but its contributions and the gravity mediation contributions are distinguishable if the mass scale of gauge mediation is not much larger than that of gravity mediation. Then the latter contributions can be extracted by investigating the 1st and 2nd generation SU(2) singlet SUSY particles, where the former are further suppressed by the small Yukawa coupling constants. We focused on the flavor compositions of SUSY particle mass eigenstates, which reflect the relative size of flavor-violating soft SUSY breaking terms. We enumerated three modes of collider experiments where one can measure the compositions by observing rare decays of SUSY particles. Predictions on their branching ratios were made based on the bulk matter RS model and were compared with those of the minimal flavor violation scenario. These predictions will be confirmed or rejected by a future lepton collider whose center-of-mass energy is tuned appropriately.

A lesson of this study is that if new physics at the TeV scale contains a flavor-violating sector other than the Yukawa couplings, it is possible to observe signatures of models that explain the Yukawa coupling hierarchy through the flavor structure of the new sector. In the case of this paper, MSSM contains gravity-mediation-origined soft mass terms, which provide a new source of flavor violation. Gravity mediation and the Yukawa couplings are independent in the original MSSM, but have a correlation if the bulk matter RS model is the origin of the Yukawa coupling hierarchy. Hence we can predict the flavor structure of gravity mediation contributions (up to their orders of magnitudes) from the data on SM, and eventually confirm or reject the bulk matter RS model through a detailed study on SUSY matter particles. This study can be extended to any new physics scenario at the TeV scale as long as it couples to matter fields and may violate flavor. In any case, the SU(2) singlet muon and charm and their new physics partners play a pivotal role; SU(2) singlets receive less flavor-violating quantum corrections from the SM Yukawa couplings, and thus new flavor-violating terms are easy to extract. Since the 1st and 2nd generation particles have only small Yukawa couplings, we expect that their new SU(2) singlet partners almost do not mix with SU(2) doublets. A muon has a much larger Yukawa coupling than an electron and is much more sensitive to the origin of the Yukawa coupling hierarchy. The partner of a charm can have a large flavor-violating mixing with a top, the only quark whose flavor can be identified with virtually no misidentification rate.

ACKNOWLEDGMENTS

T. Y. is grateful to Yasuhiro Okada (KEK), Nobuchika Okada (The University of Alabama), and Keisuke Fujii (KEK) for useful advice on writing this paper. This work is in part supported by a grant of the Japan Society for the Promotion of Science, No. 23-3599.

[1] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 3370 (1999).

[2] Y. Grossman and M. Neubert, Phys. Lett. B **474**, 361 (2000); T. Gherghetta and A. Pomarol, Nucl. Phys. **B586**, 141 (2000); S. J. Huber and Q. Shafi, Phys. Lett. B **498**, 256 (2001); G. Burdman, Phys. Rev. D **66**, 076003 (2002); S. J. Huber, Nucl. Phys. **B666**, 269 (2003).

[3] C. Csaki, A. Falkowski, and A. Weiler, J. High Energy Phys. 09 (2008) 008.

[4] N. Okada and T. Yamada, Phys. Rev. D **84**, 035005 (2011).

[5] A. H. Chamseddine, R. Arnowitt, and P. Nath, Phys. Rev. Lett. **49**, 970 (1982); R. Barbieri, S. Ferrara, and C. A. Savoy, Phys. Lett. **119B**, 343 (1982); L. E. Ibanez, Phys. Lett. **118B**, 73 (1982); L. J. Hall, J. D. Lykken, and S. Weinberg, Phys. Rev. D **27**, 2359 (1983); N. Ohta, Prog. Theor. Phys. **70**, 542 (1983).

[6] E. A. Mirabelli and M. E. Peskin, Phys. Rev. D **58**, 065002 (1998); D. E. Kaplan, G. D. Kribs, and M. Schmaltz, Phys. Rev. D **62**, 035010 (2000); Z. Chacko, M. A. Luty, A. E. Nelson, and E. Ponton, J. High Energy Phys. 01 (2000) 003; M. Schmaltz and W. Skiba, Phys. Rev. D **62**, 095005 (2000); **62**, 095004 (2000); Z. Chacko and E. Ponton, J. High Energy Phys. 11 (2003) 024; M. McGarrie and D. C. Thompson, Phys. Rev. D **82**, 125034 (2010); M. McGarrie, J. High Energy Phys. 09 (2011) 138.

[7] M. Dine and W. Fischler, Phys. Lett. **110B**, 227 (1982); C. R. Nappi and B. A. Ovrut, Phys. Lett. **113B**, 175 (1982); L. Alvarez-GaumLe, M. Claudson, and M. B. Wise, Nucl. Phys. **B207**, 96 (1982); M. Dine and A. E.

Nelson, Phys. Rev. D **48**, 1277 (1993); M. Dine, A. E. Nelson, and Y. Shirman, Phys. Rev. D **51**, 1362 (1995); M. Dine, A. E. Nelson, Y. Nir, and Y. Shirman, Phys. Rev. D **53**, 2658 (1996).

[8] L. Randall and R. Sundrum, Nucl. Phys. **B557**, 79 (1999); G. F. Giudice, M. A. Luty, H. Murayama, and R. Rattazzi, J. High Energy Phys. 12 (1998) 027.

[9] Y. Nomura, M. Papucci, and D. Stolarski, J. High Energy Phys. 07 (2008) 055.

[10] Y. Nomura, M. Papucci, and D. Stolarski, Phys. Rev. D **77**, 075006 (2008).

[11] T. Gherghetta and A. Pomarol, Nucl. Phys. **B586**, 141 (2000); N. Arkani-Hamed, T. Gregoire, and J. Wacker, J. High Energy Phys. 03 (2002) 055; D. Marti and A. Pomarol, Phys. Rev. D **64**, 105025 (2001).

[12] P. Minkowski, Phys. Lett. **67B**, 421 (1977); T. Yanagida, in *Proceedings of the Workshop on the Unified Theory and the Baryon Number in the Universe*, edited by O. Sawada and A. Sugamoto (KEK, Tsukuba, Japan, 1979), p. 95; M. Gell-Mann, P. Ramond, and R. Slansky, *Supergravity*, edited by P. van Nieuwenhuizen *et al.* (North-Holland, Amsterdam, 1979), p. 315; S. L. Glashow, *The Future of Elementary Particle Physics, in Proceedings of the 1979 Carg'ese Summer Institute on Quarks and Leptons*, edited by M. LLevy *et al.* (Plenum Press, New York, 1980), p. 687; R. N. Mohapatra and G. SenjanoviLc, Phys. Rev. Lett. **44**, 912 (1980).

[13] S. R. Juarez, S. F. Herrera, P. Kielanowski, and G. Mora, Phys. Rev. D **66**, 116007 (2002).

[14] P. H. Chankowski and Z. Pluciennik, Phys. Lett. B **316**, 312 (1993).

[15] K. Nakamura *et al.* (Particle Data Group), J. Phys. G **37**, 075021 (2010).

[16] J. Brau *et al.* (ILC Collaboration), arXiv:0712.1950; G. Aarons *et al.* (ILC Collaboration), arXiv:0709.1893.

[17] M. Battaglia, arXiv:hep-ph/0103338.

[18] M. L. Brooks *et al.* (MEGA Collaboration), Phys. Rev. Lett. **83**, 1521 (1999).

[19] J. L. Feng and B. T. Smith, Phys. Rev. D **71**, 015004 (2005); **71**, 019904(E) (2005).