Superconformal technicolor: Models and phenomenology

Aleksandr Azatov,[1] Jamison Galloway,[1] and Markus A. Luty[2]

[1]*Dipartimento di Fisica, Università di Roma "La Sapienza" and INFN Sezione di Roma, I-00185 Rome, Italy*

[2]*Physics Department, University of California Davis, Davis, California 95616, USA*

(Received 28 September 2011; published 23 January 2012)

In supersymmetric theories with a strong conformal sector, soft supersymmetry breaking naturally gives rise to confinement and chiral symmetry breaking in the strong sector at the TeV scale. We construct and analyze models where such a sector dynamically breaks electroweak symmetry, and take the first steps in studying their phenomenology. We consider two scenarios, one where the strong dynamics induces vacuum expectation values for elementary Higgs fields, and another where the strong dynamics is solely responsible for electroweak symmetry breaking. In both cases there is no fine-tuning required to explain the absence of a Higgs boson below the LEP bound, solving the supersymmetry naturalness problem. Quark and lepton masses arise from conventional Yukawa couplings to elementary Higgs bosons, so there are no additional flavor-changing effects associated with the strong dynamics. A good precision electroweak fit can be obtained because the strong sector is an $SU(2)$ gauge theory with one weak doublet, and has adjustable parameters that control the violation of custodial symmetry. In addition to the standard supersymmetry signals, these models predict production of multiple heavy standard model particles (t, W, Z, and b) from decays of resonances in the strong sector. The strong sector has no approximate parity symmetry, so WW scattering is unitarized by states that can decay to WWW as well as WW.

DOI: 10.1103/PhysRevD.85.015018 PACS numbers: 12.60.Nz, 11.30.Pb

I. INTRODUCTION

Supersymmetry (SUSY) gives a compelling solution to the electroweak hierarchy problem, and provides a sensible framework for speculations about physics above the TeV scale. It is for this reason that so much of the theoretical and experimental effort in physics beyond the standard model is devoted to SUSY. However, if SUSY is the solution of the hierarchy problem it generically predicts a standard-model-like Higgs boson with mass below m_Z, which is ruled out. In the minimal supersymmetric standard model (MSSM), this can be avoided only by radiative corrections that introduce fine-tuning at the percent level. It is possible to avoid this tuning by extending the MSSM, either to raise the Higgs mass [1] or to give it new decays that are less constrained by experiment [2], but the models must be carefully constructed to have these features.

Technicolor also gives a compelling solution to the hierarchy problem, but it is generally considered less plausible than SUSY mainly because of problems with flavor and precision electroweak tests. The traditional approach to incorporating flavor into technicolor theories involves extending the gauge group of technicolor to include the flavor symmetries, which are then broken above the TeV scale [3]. There are daunting obstacles to constructing realistic models of this kind, and there is no realistic example in the literature. Furthermore, any such model must have large numbers of technicolors and/or techniflavors, and therefore is expected to give large corrections to the precision electroweak parameters S and T that are incompatible with data. The prospects are much better if the couplings responsible for quark and lepton masses arise from the exchange of heavy scalars [4]. This is potentially natural in supersymmetric models, where SUSY is broken above the TeV scale. In this case, the higher-dimension operators that generate quark and lepton masses can be generated from exchange of Higgs scalars, which can incorporate minimal flavor violation and do not require extending the technicolor gauge sector. The pioneering attempts in this direction [5] cannot accommodate the large observed value of the top quark mass, but realistic models have recently been constructed [6] in the context of conformal technicolor [7]. These are explicit UV complete models with a minimal technicolor sector at a TeV, that do not conflict with precision electroweak and flavor constraints.

In this paper, we combine SUSY and conformal technicolor in a more direct way in an attempt to address the shortcomings of both. (For recent closely related work, see Ref. [8].) A companion paper [9] describes the main ideas and results in a succinct fashion, while this paper gives a full discussion. This paper is written to be self-contained, and can be read on its own.

We assume that the visible sector consists of the MSSM plus a strong sector. SUSY is assumed to be broken at the TeV scale in both the MSSM and the strong sector, as is natural in many theories of SUSY breaking (e.g. gravity mediation). The idea (already used in Refs. [6,10]) is that in the strong sector, conformal invariance is broken softly by SUSY breaking mass terms, giving rise to strong nonsupersymmetric dynamics at the TeV scale. Since all scalars get massive from SUSY breaking while fermions have chiral symmetries that forbid their masses, it is very plausible that the strongly interacting fermions confine and break chiral symmetries, as in QCD. This dynamics can play a role in electroweak symmetry breaking. This is the conformal technicolor mechanism [7] in the context of SUSY, so we refer to it as "superconformal technicolor."[1]

The presence of both SUSY and strong dynamics at the TeV scale opens up many interesting phenomenological possibilities, and this paper only initiates the exploration of these ideas. We will construct an explicit model of the strong sector that realizes this idea, which we argue can dynamically break electroweak symmetry. We then investigate two different limiting regimes of the same model that illustrate two phenomenologically distinct scenarios for electroweak symmetry breaking. The model has a strong conformal sector based on an $SU(2)$ gauge group with 4 flavors, which has a strongly interacting conformal fixed point [12]. Additional fields and interactions are required to stabilize runaway directions in the presence of SUSY breaking. The additional interactions and the SUSY breaking terms explicitly break the $SU(8)$ global symmetry of the theory down to $SU(2)_L \times SU(2)_R$, which is weakly gauged in the usual way so that chiral symmetry breaking in the strong sector breaks electroweak symmetry, as in technicolor.

The MSSM Higgs fields couple to the strong sector via superpotential couplings of the form

$$W = \lambda_u H_u \mathcal{O}_d + \lambda_d H_d \mathcal{O}_u, \tag{1.1}$$

where $\mathcal{O}_{u,d}$ are operators in the strong sector with the same electroweak quantum numbers as $H_{u,d}$. The two different regimes of the model referred to above correspond to different choices of $\lambda_{u,d}$.

In the model we construct the operators $\mathcal{O}_{u,d}$ have scaling dimension $3/2$, so the couplings $\lambda_{u,d}$ are relevant couplings that get strong at some scale. This scale cannot be too far from the TeV scale, otherwise they are not important for electroweak symmetry breaking. This amounts to a coincidence of scales, and the problem of explaining this coincidence is similar to the "μ problem" of the MSSM. In both cases we must explain why a relevant supersymmetric coupling is important near the scale of SUSY breaking. Perhaps the simplest and most elegant solution to the μ problem is the Giudice-Masiero mechanism [13], and we review an extension of this mechanism [6] that explains why the couplings Eq. (1.1) are important at the TeV scale.

A. Induced electroweak symmetry breaking

We first consider the case where the couplings Eq. (1.1) are perturbative at the TeV scale. In this case, the Higgs fields $H_{u,d}$ are ordinary perturbative degrees of freedom below the TeV scale. The strong sector dynamically breaks electroweak symmetry with an order parameter f that we assume is somewhat below the value required to explain the W and Z masses, e.g. $f \simeq 100$ GeV. The heavy hadrons of the strong sector are expected to have masses of order $4\pi f \sim$ TeV [14], and the $SU(2)_L \times SU(2)_R$ chiral symmetry of this theory is nonlinearly realized below this scale. The couplings Eq. (1.1) then generate a tadpole for $H_{u,d}$ in the effective potential. This induces a vacuum expectation value (VEV) for $H_{u,d}$ even if $m^2_{H_{u,d}} > 0$, which we assume to be the case. (In standard SUSY scenarios $m^2_{H_{u,d}} > 0$ at high scales and renormalization group running results in $m^2_{H_u} < 0$ at the TeV scale, but more general boundary conditions at high scales can lead to $m^2_{H_{u,d}} > 0$ at the TeV scale.) If we neglect the quartic terms in the potential for $H_{u,d}$, the masses of the physical Higgs bosons are simply eigenvalues of the quadratic terms in the effective potential, while the size of the VEV is determined by the coefficient of the tadpole. The Higgs mass therefore depends directly on the SUSY breaking masses, similar to a slepton or squark mass. The Higgs masses can easily be larger than the LEP bound with no tuning in this scenario, giving a simple and robust solution to the SUSY Higgs mass problem.

In this scenario electroweak symmetry breaking is shared by the elementary Higgs bosons and the strong sector:

$$v^2 = v_u^2 + v_d^2 + f^2, \tag{1.2}$$

where $v = 246$ GeV. For example, for $f \simeq 100$ GeV we have $\sqrt{v_u^2 + v_d^2} = 225$ GeV. Because the electroweak symmetry breaking VEV is dominantly in the elementary Higgs fields, quark and lepton masses can arise through conventional perturbative Yukawa couplings. This means that there is no additional flavor problem associated with the strong dynamics. Of course we still have the SUSY flavor problem, namely, the squark and slepton masses and A terms can be flavor dependent. We assume that this is addressed by one of the many possible mechanisms in the literature.

A good precision electroweak fit can be obtained in this model. The strong sector is based on a $SU(2)$ gauge theory with a single technidoublet, so the corrections are not enhanced by large N factors. The UV contribution to the S parameter is very uncertain because this theory is very different from QCD. The fact that the longitudinal modes of the W and Z are dominantly perturbative excitations reduces the IR contribution from the strong sector to the S parameter. The custodial symmetry breaking from $\lambda_u \gg \lambda_d$ gives a positive contribution to the T parameter that also helps with the fit. The conclusion is that we can get a good precision electroweak fit even if we assume that the UV

[1]This name has also been used in Ref. [11] for models that do not use the conformal technicolor mechanism to break electroweak symmetry.

contribution to the S parameter is large and given by the value extrapolated from QCD.

The collider phenomenology for this model includes all of the usual SUSY signals, together with additional signals arising from the strong sector. The strong sector has a relatively low scale $4\pi f \lesssim \text{TeV}$, which may make it more accessible than conventional technicolor.[2] The theory below the TeV scale has 3 additional CP odd states A_2^0 and $H_2^\pm$ that are heavier than the other Higgs fields and are dominantly pseudo-Nambu-Goldstone bosons (PNGBs) from the strong sector. These can be either singly produced, or pair produced from decays of heavy resonances in the strong sector. There are many possible signals, and we will only outline some of the possibilities in this paper.

B. Strong electroweak symmetry breaking

We next consider another possibility where there is no light Higgs below the TeV scale. SUSY breaking in the strong sector triggers electroweak symmetry breaking, as in conformal technicolor. The quark and lepton masses arise from couplings to the strong sector of the form

$$\Delta W \sim (y_u)_{ij} Q_i u_j^c \mathcal{O}_u + (y_d)_{ij} Q_i d_j^c \mathcal{O}_d + \cdots. \tag{1.3}$$

This can arise in the same model we construct for the previous scenario for a different choice of parameters. The couplings $\lambda_{u,d}$ in Eq. (1.1) are relevant operators that get strong at some scale Λ_*. If Λ_* is above the SUSY breaking scale, the elementary Higgs fields become part of the strong sector, and there is a dual description where the Yukawa couplings become couplings of the form Eq. (1.3). Below the scale Λ_*, the operators $\mathcal{O}_{u,d}$ have dimension $3/2$, so these operators behave like flavor-dependent interactions in "walking" technicolor.[3] Alternatively, the scale Λ_* may be naturally near the TeV scale, as discussed above. In this case we do not require large Yukawa couplings at high scales. In either case, the couplings Eq. (1.3) inherit the minimal flavor violating structure of the Yukawa couplings, so there is no flavor problem associated with the strong dynamics. Of course, the SUSY flavor problem must still be addressed by some mechanism.

The precision electroweak fit does not pose a problem for this model. There is a contribution to the T parameter from $\lambda_u \neq \lambda_d$. If this contribution is positive (as suggested by perturbation theory) we can get a good fit provided that the UV contribution to the S parameter from the strong sector is somewhat smaller (e.g. by a factor of 2) than the QCD estimate. We conclude that given our present state of knowledge precision electroweak data does not strongly constrain this model.

The collider signals include the standard missing energy SUSY signals, but not the SUSY Higgs signals. There are technicolor-like signals associated with the strong sector. One difference from conventional technicolor is that the strong sector generally has no approximate parity symmetry, so the resonances that unitarize WW scattering can decay to WWW as well as WW.

II. THE STRONG SUPERCONFORMAL SECTOR

In this section we describe the requirements for a successful model of the strong sector, and construct an explicit model as an existence proof. The main issue is preventing runaway directions due to soft SUSY breaking mass terms.

A. SUSY breaking in SUSY QCD

The main new feature of our framework is a strongly coupled superconformal sector. The simplest nontrivial 4D superconformal theory is $SU(N_c)$ SUSY QCD with N_f flavors in the conformal window $\frac{3}{2}N_c < N_f < 3N_c$ [12]. There is a dual description of these theories in terms of an $SU(\tilde{N}_c)$ gauge theory with $\tilde{N}_c = N_f - N_c$. The theories with $N_f \simeq 3N_c$ are weakly coupled, while the models with $N_f \simeq \frac{3}{2}N_c$ have a weakly coupled dual description. The models with $N_f \simeq 2N_c$ are have no weakly coupled description, and these are the simplest candidates for the strong sector of our model.

Conformal symmetry is broken softly by SUSY breaking terms in the strong sector. We begin by reviewing what is known about soft SUSY breaking for SUSY QCD at a conformal fixed point [17]. The effects of soft SUSY breaking terms are most readily understood if we view them as F and D components of superfield couplings and flavor gauge fields. We write the Lagrangian in superspace as

$$\begin{aligned}\mathcal{L} = {} & \int d^2\theta \tau \, \mathrm{tr}(W^\alpha W_\alpha) + \text{H.c.} \\ & + \int d^4\theta Z[Q_i^\dagger e^V (e^X)^i{}_j e^Y Q^j \\ & + \tilde{Q}_i^\dagger e^{-V^T} (e^{\tilde{X}})^i{}_j e^{-Y} \tilde{Q}].\end{aligned} \tag{2.1}$$

Here V and W_α are the $SU(N_c)$ gauge field and field strength, Q and $\tilde{Q}$ are the fundamental and antifundamental "quark" fields; τ is the holomorphic gauge coupling, Z is a real superfield wave function renormalization factor; X, $\tilde{X}$, and Y are background gauge superfields for the anomaly-free $SU(N_f) \times SU(N_f) \times U(1)$ flavor symmetry.

A flavor-universal mass-squared term can be parametrized by a D term for Z, and a gaugino mass can be parametrized by an F term for τ. The physical gauge coupling is the lowest component of a real superfield R that is a function of τ and Z [18], so these SUSY breaking

[2] Low-scale technicolor has been previously studied, motivated by large N technicolor theories [15]. However, as previously noted these theories have serious problems with the precision electroweak fit.

[3] The use of SUSY conformal fixed points to get walking behavior of flavor couplings has been previously considered in Ref. [16].

terms perturb R away from its fixed point value. Since the fixed point is IR attractive, the SUSY breaking perturbations scale away in the IR. On the other hand, D terms for the gauge superfields X, $\tilde{X}$, and Y are unsuppressed in the IR because the coupling of gauge fields in the IR is simply determined by group theory. Scalar mass-squared terms proportional to symmetry generators therefore scale in the IR just like in a free field theory. Detailed elaboration of these arguments can be found in Ref. [17].

This means that the only soft SUSY breaking in the strong sector that is naturally at the TeV scale is scalar mass-squared terms proportional to anomaly-free flavor generators. There are always directions in field space where the energy due to such mass-squared terms is negative. The ground state will then have a large VEV along such a direction, in which case conformal symmetry in the strong sector is broken well above the TeV scale.[4] For example, a soft mass proportional to "baryon number" $[B(Q) = -B(\tilde{Q}) = 1]$ will result in a runaway direction with either $Q \neq 0$, $\tilde{Q} = 0$ or $Q = 0$, $\tilde{Q} \neq 0$ depending on the sign of the mass-squared term.

Generalizing from SUSY QCD, we see that what we would like is a strong conformal theory with an anomaly-free flavor generator X such that all of the flat directions have the same sign of the X charge. A scalar mass-squared term proportional to X can then stabilize the vacuum at small field values. Note that this condition is never satisfied in theories with a charge conjugation invariance (such as SUSY QCD). In such theories the best we can hope for is that all flat directions have $X = 0$, in which case a more subtle analysis is required to determine whether the ground state is near the origin of field space.

We can lift dangerous flat directions by introducing additional perturbative couplings. For example, we can lift the $B \neq 0$ flat directions in the example above by gauging $U(1)_B$. However, as long as the $U(1)_B$ gauge coupling is weak, this will stabilize the VEV at a large value because the VEV goes to infinity as the $U(1)_B$ gauge coupling goes to zero. Such a model will have more than one scale, and will not give a strongly coupled model with a single scale that we are seeking.

B. A viable model

We now construct a working model in which the runaway directions in the strong sector are lifted. The detailed model will be described below, but we start by briefly outlining the basic mechanism. The strong sector is a $SU(2)$ gauge theory with 4 flavors, with superpotential couplings to elementary Higgs fields H and additional singlet fields S of the form

$$W \sim (\lambda_H H + \lambda_S S)\Psi\Psi. \tag{2.2}$$

The effect of these terms is that the flat directions of the strong sector are replaced by flat directions of the H and S fields, so the problem is now to lift these flat directions. The "meson" operator $\Psi\Psi$ has dimension $\frac{3}{2}$, so the λ couplings have dimension $+\frac{1}{2}$. We will want all of the λ couplings to become strong near the TeV scale where SUSY is broken in the strong sector. This is a coincidence problem precisely analogous to the "μ problem" of the MSSM. We will show below (in Sec. II D) that we can explain this coincidence using a generalization of the Giudice-Masiero mechanism for the μ term. Now the idea is that the couplings λ_S become strong at a scale Λ' somewhat above the weak scale, while the coupling λ_H is still weak. Below this scale, the theory quickly flows to a new fixed point where S is a strong operator. In this new conformal field theory , a universal positive soft mass for S is suppressed by a large anomalous dimension, but if the scale Λ' is not too far from the TeV scale this effect can be small, and there can be a positive soft mass at the TeV scale to stabilize the strong sector.

We now give a detailed description of the model. It is based on a strong $SU(2)_{\rm SC}$ gauge theory with 4 flavors, which has a strong conformal fixed point as discussed above. The anomaly-free global symmetry group is

$$SU(2)_1 \times SU(2)_2 \times SU(2)_3 \times SU(2)_4 \times U(1)_R. \tag{2.3}$$

The embedding of the electroweak gauge group in this global symmetry will be described below. The strongly interacting fields transform as

$$\begin{aligned} &\Psi_1 \sim (2,2,1,1,1)_{1/2}, && \Psi_2 \sim (2,1,2,1,1)_{1/2}, \\ &\Psi_3 \sim (2,1,1,2,1)_{1/2}, && \Psi_4 \sim (2,1,1,1,2)_{1/2}. \end{aligned} \tag{2.4}$$

The electroweak gauge group is embedded in the global symmetry by taking the $SU(2)_W \times U(1)_Y$ generators acting on the fields Ψ_i to be

$$\begin{aligned} T_a &= \frac{1}{2}\begin{pmatrix} \tau_a & & & \\ & 0 & & \\ & & 0 & \\ & & & 0 \end{pmatrix}, \\ Y &= \frac{1}{2}\begin{pmatrix} 0 & & & \\ & -\tau_3 & & \\ & & \tau_3 & \\ & & & -\tau_3 \end{pmatrix}. \end{aligned} \tag{2.5}$$

The fields $\Psi_{3,4}$ will not play a role in breaking electroweak symmetry. We could define e.g. $Y = \mathrm{diag}(0, -\tau_3, 0, 0)$, but then the model has physical states with fractional charge that we want to avoid.

[4]SUSY breaking may be communicated to the visible sector at a scale as low as 10 TeV. If we assume that the soft masses at 10 TeV are the same order of magnitude in the MSSM and the strong sector, and that the anomalous dimensions that suppress the soft terms in the strong sector are numerically small, we may get a viable model. We will not pursue this possibility here.

The fields transform as

$$\Psi_i \mapsto U\Psi_i V_i^T, \qquad i = 1, \ldots, 4, \tag{2.6}$$

where $U \in SU(2)_{\rm SC}$, $V_i \in SU(2)_i$. The $SU(2)_{\rm SC}$ gauge invariant holomorphic operators are the meson fields

$$M_{ij} = \Psi_i^T \epsilon \Psi_j. \tag{2.7}$$

These are 2×2 matrices, transforming under $SU(2)_i \times SU(2)_j$ as

$$M_{ij} \sim \begin{cases} (2, 2) & \text{for } i \neq j, \\ 1 & \text{for } i = j. \end{cases} \tag{2.8}$$

In addition to the techniquarks Eq. (2.4), the model contains $SU(2)_{\rm SC}$ singlet fields S_{ij} transforming under the global symmetries like the meson fields M_{ij} above. The theory has a superpotential

$$W = \sum_{i,j} \lambda_{ij} S_{ij} \Psi_i^T \epsilon \Psi_j. \tag{2.9}$$

The couplings λ_{ij} have dimension $\frac{1}{2}$, i.e. they are relevant couplings. We assume that there is no large hierarchy between the λ_{ij}, so they all get strong at roughly the same scale Λ_*.

Seiberg duality tells us that below the scale Λ_* the theory flows to a new strong fixed point. In the "electric" description presented here, this fixed point is one where the couplings λ_{ij} flow to strong fixed point values. The dual "magnetic" description has gauge group $SU(2)_{\widetilde{\rm SC}}$ and dual quark fields $\tilde{\Psi}_i \sim (2, 2)$ under $SU(2)_{\widetilde{\rm SC}} \times SU(2)_i$, as well as the meson fields M_{ij} as separate degrees of freedom. This theory has a superpotential

$$\tilde{W} = \sum_{i,j} (\lambda_{ij} S_{ij} M_{ij} + M_{ij} \tilde{\Psi}_i \tilde{\Psi}_j). \tag{2.10}$$

The first term arises from Eq. (2.9) and the second is dynamically generated. In this description the singlets get a mass with the meson fields, and we can integrate them out to get a $SU(2)_{\widetilde{\rm SC}}$ gauge theory with 8 flavors and no superpotential. This is precisely the argument used to show that the dual of a Seiberg dual is the original theory, except that the couplings λ_{ij} are here allowed to violate the flavor symmetries. This theory has a strongly coupled IR attractive fixed point, which shows that the theory flows to a new fixed point below the scale where the couplings λ_{ij} become strong.

The theory below the scale Λ_* is pure SUSY QCD, in which universal scalar mass-squared terms are suppressed. However, above the scale Λ_* universal soft mass-squared terms for S are not suppressed, and are therefore unsuppressed at the scale Λ_*. If the scale Λ_* is not too far above the TeV scale, these soft mass terms can break SUSY near the TeV scale in the strong sector. The effects of a universal scalar mass-squared term in the dual description of the strong sector below the scale Λ_* are discussed in an Appendix.

In addition to scalar mass-squared terms, we can have A terms for the superpotential couplings Eq. (2.10). In superspace these can be parametrized by terms

$$\Delta\mathcal{L} = \int d^4\theta (A\theta^2 + \text{H.c.}) S^\dagger S \tag{2.11}$$

which are not suppressed by the strong dynamics above the scale Λ_*. For $\Lambda_* \sim$ TeV these can also be important at the TeV scale.

Having $\Lambda_* \sim$ TeV requires a coincidence of scales between the supersymmetric relevant couplings λ_{ij} and the SUSY breaking scale. As discussed above, this is similar to the μ problem, and we will present an explanation of it using a generalization of the Giudice-Masiero mechanism below.

We have thus succeeded in constructing a strong superconformal theory where all flat directions are lifted by soft SUSY breaking. The conformal symmetry is therefore broken by the soft SUSY breaking in the strong sector at the scale $M_{\rm SUSY}$. SUSY breaking gives mass to all scalars, but unbroken chiral symmetries mean that technifermions are still massless. It is therefore very plausible that this theory confines and spontaneously breaks the chiral symmetries, like QCD or technicolor.

We now discuss the symmetry breaking and vacuum alignment in this model. A useful starting point is to choose the couplings λ_{ij} and the soft SUSY breaking terms to respect the full $SU(8)$ global symmetry of the $SU(2)_{\rm SC}$ gauge theory. We do this by assuming universal couplings λ_{ij} and a universal positive mass-squared for the singlets in the UV. The $U(1)_R$ symmetry is broken by A terms of the same form as the superpotential Eq. (2.10). In the dual description the dual techniquarks have no superpotential interactions. (When we include Yukawa couplings they will have perturbative superpotential couplings with ordinary quark and lepton superfields.) The techniscalars all get masses, but masses for the technifermions are forbidden by the $SU(8)$ chiral symmetry. A technigaugino mass is allowed because $U(1)_R$ is broken. We expect that the strong nonsupersymmetric gauge dynamics generates a fermion condensate

$$\langle \Psi^A \Psi^B \rangle = -\langle \Psi^B \Psi^A \rangle, \tag{2.12}$$

where A, B are $SU(8)$ indices. This spontaneously breaks $SU(8) \to Sp(8)$, giving rise to 27 Nambu-Goldstone bosons (NGBs).

Now we turn on additional terms that explicitly break the $SU(8)$ global symmetry down to

$$SU(2)_L \times SU(2)_R \times U(1)_{\tilde{Y}}, \tag{2.13}$$

with generators

$$T_{La} = \frac{1}{2}\begin{pmatrix} \tau_a & & & \\ & 0 & & \\ & & 0 & \\ & & & 0 \end{pmatrix},$$
$$T_{Ra} = \frac{1}{2}\begin{pmatrix} 0 & & & \\ & -\tau_a^T & & \\ & & 0 & \\ & & & 0 \end{pmatrix}, \tag{2.14}$$

and

$$\tilde{Y} = \frac{1}{2}\begin{pmatrix} 0 & & & \\ & 0 & & \\ & & \tau_3 & \\ & & & -\tau_3 \end{pmatrix}. \tag{2.15}$$

This explicit breaking is accomplished by nonuniversal couplings λ_{ij}, and nonuniversal soft masses for the S_{ij} and the Ψ_i. We assume that this breaking is maximal, so that there is no larger approximate global symmetry. This assumption is made just for simplicity, and it is also natural in this framework to have additional approximate global symmetries leading to pseudo-Nambu-Goldstone bosons that can have interesting phenomenological implications.

This explicit $SU(8)$ breaking determines the alignment of the fermion condensate. We assume that

$$\langle\Psi^A\Psi^B\rangle = \begin{pmatrix} 0 & a1_2 & & \\ -a1_2 & 0 & & \\ & & 0 & b1_2 \\ & & -b1_2 & 0 \end{pmatrix}, \tag{2.16}$$

which breaks

$$SU(2)_L \times SU(2)_R \to SU(2)_{\rm diag}, \tag{2.17}$$

and preserves $U(1)_{\tilde{Y}}$. This breaks electroweak symmetry in the desired pattern, with no pseudo-Nambu-Goldstone bosons.

We now describe how this theory generates masses for quarks and leptons. Note that S_{12} has the electroweak quantum numbers of 2 Higgs doublets. We can therefore write conventional Yukawa couplings

$$\Delta W = y_u(Qu^c)(S_{12})_u + y_d(Qd^c)(S_{12})_d + y_e(Le^c)(S_{12})_d. \tag{2.18}$$

Above the scale where the couplings λ_{ij} become strong, S_{12} is a conventional weakly coupled field with dimension 1, so the Yukawa couplings run as in the MSSM. Below the scale where the couplings λ_{ij} become strong, we use the dual description where we integrate out S_{ij} and the meson fields M_{ij}, and we obtain the superpotential

$$\Delta W = \frac{1}{\lambda_{12}}[(y_u)_{ij}Q_i u_j^c(\tilde{\Psi}_1\tilde{\Psi}_2)_u + (y_d)_{ij}Q_i d_j^c(\tilde{\Psi}_1\tilde{\Psi}_2)_d + (y_e)_{ij}L_i e_j^c(\tilde{\Psi}_1\tilde{\Psi}_2)_d]. \tag{2.19}$$

Note that these interactions have minimal flavor violating structure inherited from the Yukawa couplings Eq. (2.18). The operators $\tilde{\Psi}\tilde{\Psi}$ have dimension $\frac{3}{2}$ in the new fixed point, so we have, e.g.,

$$m_t \sim y_t(\Lambda_*)v\left(\frac{\rm TeV}{\Lambda_*}\right)^{1/2}. \tag{2.20}$$

We see that the quark masses have a mild suppression even if $\Lambda_* > \rm TeV$.

C. A model with a light Higgs

As we have described it, this models has no light Higgs field below the SUSY breaking scale. Since S_{12} contains the MSSM Higgs fields, it is easy to modify the theory to have a light Higgs: we simply choose the coupling λ_{12} to be smaller than the others. We assume that the other couplings λ_{ij} have the same order of magnitude, and get strong at a single scale $\Lambda_* \gtrsim \rm TeV$.

In the electric description of the theory, the strong Yukawa couplings λ_{ij} approach a strong fixed point, while λ_{12} remains weak. In the dual magnetic description the strong λ_{ij} turn into mass terms of order Λ_*, while λ_{12} is a smaller mass term. After integrating out the masses of order Λ_*, the dual superpotential is

$$\tilde{W} = \lambda_{12}S_{12}M_{12} + M_{12}\tilde{\Psi}_1\tilde{\Psi}_2. \tag{2.21}$$

In this description there is an additional light $SU(2)_{\rm SC}$ singlet field M_{12}, but it has a strong superpotential coupling to the dual techniquarks, and should be viewed as part of the strong sector. In either description, assuming that λ_{12} is small at the SUSY breaking scale, it will give rise to a weak coupling of the elementary Higgs fields in S_{12} to the strong dynamics. This strong dynamics can still have the symmetry structure described above, and it is equally plausible that it is spontaneously broken in the same pattern. This is all we need for the low-energy dynamics we are trying to achieve.

D. Coincidence problem

We now discuss the coincidence between the SUSY breaking scale and the scale where the couplings λ_{ij} become strong. We describe how this can happen in an extension of the Giudice-Masiero mechanism [6]. We assume that SUSY is broken in a hidden sector at high scales, and is communicated to the visible sector by higher-dimension operators. The hidden sector contains a gauge singlet superfield X with $\langle F_X\rangle \neq 0$, and higher-dimension interactions that connect the hidden and the visible sector are suppressed by a scale M. We then write all possible higher-dimension operators coupling X to the visible sector fields, e.g.,

$$\Delta\mathcal{L}_{\rm eff} \sim \int d^2\theta \frac{1}{M} X W^\alpha W_\alpha + \text{H.c.} + \int d^4\theta \left[\frac{1}{M}(X + X^\dagger) Q^\dagger Q + \frac{1}{M^2} X^\dagger X Q^\dagger Q\right] + \int d^4\theta \left[\frac{1}{M} X^\dagger H_u H_d + \frac{1}{M^2} X^\dagger X H_u H_d + \text{H.c.}\right]. \tag{2.22}$$

These terms generate, respectively, gaugino masses, A terms, scalar mass terms, the μ term, and the $B\mu$ term, all of order

$$M_{\rm SUSY} \sim \frac{\langle F_X \rangle}{M}. \tag{2.23}$$

Note also that the soft terms in the MSSM and the strong sector are generated at the same scale in this mechanism. One well-motivated choice is to take M of order the Planck scale, in which case one must also take into account supergravity corrections, but they do not change this result [13]. The main shortcoming of this mechanism is that it does not address the SUSY flavor problem, which is why the soft masses are flavor diagonal. On the other hand, models that address the SUSY flavor problem require significant complications to solve the μ problem, and it is not obvious which is preferred.

In the model above, the couplings λ_{ij} have mass dimension $\frac{1}{2}$, so the problem is to naturally generate $\lambda_{ij} \sim M_{\rm SUSY}^{1/2}$. This occurs naturally if the hidden sector contains a field Y with

$$\langle Y \rangle \sim \langle F_X \rangle^{1/2}, \qquad \langle F_Y \rangle \langle F_X \rangle^{1/2} M_{\rm SUSY}. \tag{2.24}$$

The couplings λ_{ij} can then be generated by

$$\Delta W = \frac{c_{ij}}{M^{1/2}} Y S_{ij} \Psi_i \Psi_j. \tag{2.25}$$

The second condition in Eq. (2.24) is required to ensure that this does not generate large A terms. For example, Ref. [6] shows that a hidden sector with superpotential

$$W = \kappa X + \frac{1}{M} Y^4 \tag{2.26}$$

has the desired features, even if supergravity effects are included. In this model κ sets the scale of the VEVs. The fact that Y and not X couples to the operator $S_{ij}\Psi_i\Psi_j$ can be enforced by symmetries, e.g. discrete R symmetries. This requires only a modest generalization of the hidden sector, and we believe it is natural in the aesthetic as well as the technical sense.

E. Discrete symmetries

We now discuss the discrete symmetries of the strong sector described above. Because the theory is based on a $SU(2)$ gauge group, there is no spacetime parity symmetry. CP is still a good symmetry (assuming that the soft SUSY breaking parameters are real). As discussed above, the theory has a $SU(8)$ flavor group that is explicitly broken down to $SU(2)_L \times SU(2)_R \times U(1)_{\tilde{Y}}$. The $SU(8)$ symmetry includes transformations that interchange the techniquarks charged under $SU(2)_L$ and $SU(2)_R$, but these are broken by (for example) different soft masses for the L and R techniscalars. The scale of confinement and chiral symmetry breaking is given by these same SUSY breaking masses (assuming there is no hierarchy among them), so in general there is no approximate symmetry that interchanges $SU(2)_L$ and $SU(2)_R$.

This means that the hadronic states of the strong sector are classified by their quantum numbers under the custodial $SU(2)$ (''isospin'') and CP only. This has phenomenological implications for the heavy resonances at the TeV scale. The 3 Nambu-Goldstone bosons π that arise from the symmetry breaking pattern $SU(2)_L \times SU(2)_R \to SU(2)$ have scattering amplitudes that grow with energy, and on general grounds we expect this to be unitarized by strong resonances at the TeV scale. Because there is no parity symmetry, these resonances can decay to $\pi\pi\pi$ as well as $\pi\pi$. When we couple this theory to the standard model, the longitudinal W will have an admixture of the π fields, and so the strong resonances can decay to WWW as well as WW. This can provide an interesting signal of this class of models that distinguish it from conventional technicolor models.

The absence of a parity symmetry is very general in the class of theories we are considering. In any gauge theory, scalars belonging to different irreducible multiplets will in general have different masses, and there will be no discrete symmetry interchanging them. In a non-SUSY technicolor theory, the only relevant terms that can break symmetries of this kind are mass terms. Mass terms for $SU(2)_L$ and $SU(2)_R$ fermions are allowed only in theories based on the $Sp(2N_c)$ strong gauge groups [including $SU(2)$]. A non-SUSY example without parity is therefore minimal conformal technicolor based on an $SU(2)$ strong gauge group with fermion mass terms at the TeV scale [19].

III. INDUCED ELECTROWEAK SYMMETRY BREAKING

We now consider the effective theory below the scale of confinement and chiral symmetry breaking in the strong sector. This theory controls the most prominent features of the phenomenology of these models, and depends only on a few qualitative features of the strong sector. We start with the case where the elementary Higgs fields are weakly coupled to the strong sector and are therefore present as light fields in the effective theory.

A. Low-energy effective theory of the strong sector

We first enumerate the assumptions about the strong sector that define the low-energy theory that describes the phenomenology. We assume that the strong sector has a $SU(2)_L \times SU(2)_R$ global symmetry that is spontaneously broken down to $SU(2)_V$ with order parameter f. The $SU(2)_L \times SU(2)_R$ symmetry is then weakly gauged by $SU(2)_W \times U(1)_Y$ in the standard way (see previous section), so that the electroweak gauge group is broken down to $U(1)_{\rm EM}$ with an approximate custodial symmmetry. The low-energy theory of the strong sector then has 3 Nambu-Goldstone bosons with decay constant f. The effective theory breaks down at the scale $\Lambda \sim 4\pi f$, which we identify with the scale of confinement and chiral symmetry breaking in the strong sector [14]. We assume that f is somewhat smaller than what is required to explain the W and Z masses, e.g. $f \simeq 100$ GeV. In this case, the scale $\Lambda \sim$ TeV is still larger than the W and Z masses, so it makes sense to describe electroweak symmetry breaking within the effective theory below the scale Λ.

We assume that the strong sector is coupled to the Higgs fields of the MSSM by Yukawa couplings of the form

$$\Delta \mathcal{L} = \lambda_u H_u \Omega_u^\dagger + \lambda_d H_d \Omega_d^\dagger, \tag{3.1}$$

where $\Omega_{u,d}$ are scalar operators with the same electroweak quantum numbers as $H_{u,d}$. To keep track of the custodial symmetry in the strong sector, we define the 2×2 matrices

$$\Omega = \begin{pmatrix} \Omega_d & \Omega_u \end{pmatrix}, \tag{3.2}$$

transforming as

$$\Omega \mapsto L \Omega R^\dagger. \tag{3.3}$$

We assume that Ω is an order parameter for electroweak symmetry breaking, i.e.

$$\langle \Omega \rangle \propto 1_2. \tag{3.4}$$

Similarly, we define

$$\mathcal{H} = \begin{pmatrix} H_d & H_u \end{pmatrix}, \qquad \lambda = \begin{pmatrix} \lambda_d & 0 \\ 0 & \lambda_u \end{pmatrix}, \tag{3.5}$$

transforming as

$$\mathcal{H} \mapsto L \mathcal{H} \tilde{R}^\dagger, \qquad \lambda \mapsto \tilde{R} \lambda R^\dagger, \tag{3.6}$$

where $\tilde{R}$ is a $SU(2)_{\tilde{R}}$ transformation. Gauged $U(1)_Y$ transformations correspond to

$$R = \tilde{R} = e^{-i\theta\tau_3/2}. \tag{3.7}$$

In particular, the spurion λ is gauge invariant. This implies that

$$\mathcal{H}\lambda \mapsto L(\mathcal{H}\lambda)R^\dagger, \qquad \lambda^\dagger \lambda \mapsto R(\lambda^\dagger \lambda) R^\dagger \tag{3.8}$$

are spurions that can break custodial symmetry of the strong sector in the effective theory.

The $SU(2)_L \times SU(2)_R$ symmetry is nonlinearly realized by fields $\Sigma(x) \in SU(2)$ transforming as

$$\Sigma = e^{2i\Pi/f} \mapsto L \Sigma R^\dagger. \tag{3.9}$$

The kinetic term and leading interaction term for these fields are contained in the effective coupling

$$\Delta \mathcal{L}_{\rm eff} = \frac{f^2}{4} \operatorname{tr}(D^\mu \Sigma^\dagger D_\mu \Sigma) + \text{H.c.} \tag{3.10}$$

To define the terms arising from the couplings Eq. (3.1) to the elementary Higgs fields we define the normalization of the couplings $\lambda_{u,d}$. As discussed in the previous section, these are relevant interactions above the scale Λ, and are therefore naturally viewed as dimensionful. In order to discuss their effects in the low-energy theory, we find it most convenient to make them dimensionless by multiplying by appropriate powers of Λ. This is a measure of the dimensionless strength of these couplings at the scale Λ where we match onto the low-energy effective theory. We then scale these couplings so that $\lambda_{u,d} \sim 4\pi$ corresponds to strong coupling at the scale Λ. This is the normalization appropriate to dimensionless Yukawa couplings.

We now consider the terms with no derivatives, i.e. the potential terms. Expanding in powers of the elementary Higgs fields, we have

$$V_{\rm eff} = \frac{\Lambda^4}{16\pi^2}\left[\frac{c_1}{\Lambda} \operatorname{tr}(\mathcal{H}\lambda\Sigma^\dagger) + \text{H.c.} + \mathcal{O}((\mathcal{H}\lambda/\Lambda)^2)\right]. \tag{3.11}$$

The size of these terms can be understood from the fact that they become strong at the scale Λ in the limit $\mathcal{H} \to f$, $\lambda \to 4\pi$. This implies that the dimensionless couplings in Eq. (3.11) are order 1.

We focus on the predictive scenario where $\mathcal{H}\lambda/\Lambda \ll 1$. The expansion is then in powers of

$$\epsilon = \frac{v\lambda}{\Lambda} = \frac{1}{\Lambda}\begin{pmatrix} \lambda_u v_u & 0 \\ 0 & \lambda_d v_d \end{pmatrix}. \tag{3.12}$$

In order to stabilize the Higgs VEV at this value, we need the soft masses for the Higgs fields to satisfy

$$m_H^2 \gg \frac{\lambda^2}{16\pi^2}\Lambda^2. \tag{3.13}$$

We assume that $m_{H_u}^2, m_{H_d}^2 > 0$ so that the VEVs for the Higgs fields are induced by the linear term in Eq. (3.11). Neglecting quartic terms and the $B\mu$ terms in the Higgs potential, minimizing the potential gives

$$m_H^2 \sim \frac{\lambda}{4\pi}\frac{f}{v}\Lambda^2 \sim \epsilon \frac{f^2}{v^2}\Lambda. \tag{3.14}$$

This is consistent with Eq. (3.13) provided $\epsilon \ll 1$. The parameter space of this scenario will be explored in detail below, including the boundary of the region where the expansion is under theoretical control. An example of a viable choice of parameters to keep in mind is

$$f = 100\,\text{GeV}, \quad \tan\beta = 10, \quad m_h = 120\,\text{GeV}, \tag{3.15}$$

which corresponds to $v_u = 224$ GeV, $v_d = 22$ GeV, and $\lambda_u/4\pi \sim 0.03$.

B. The scalar sector

We now consider the scalar sector of the effective theory, including all mixing effects. This sector depends on 6 couplings: the soft masses $m^2_{H_u}$, $m^2_{H_d}$, $B\mu$, the scale f, and the effective couplings in Eq. (3.11)

$$\kappa_{u,d} = \frac{c_1 \Lambda^3}{16\pi^2} \lambda_{u,d}. \tag{3.16}$$

We can redefine the fields to make $\kappa_{u,d} > 0$. The sign of $B\mu$ is then physically meaningful. Because the VEV v is measured, the scalar sector has 5 parameters, which we can take to be, e.g.,

$$\tan\beta, f, m^2_{H_u}, m^2_{H_d}, B\mu. \tag{3.17}$$

We parametrize the scalar fields as

$$H_u = \begin{pmatrix} H_u^+ \\ \frac{1}{\sqrt{2}}(v_u + h_u^0 - iA_u^0) \end{pmatrix}, \quad H_d = \begin{pmatrix} \frac{1}{\sqrt{2}}(v_d + h_d^0 + iA_d^0) \\ H_d^- \end{pmatrix}, \tag{3.18}$$

and

$$\Pi = \frac{1}{\sqrt{2}} \begin{pmatrix} \pi^0/\sqrt{2} & \pi^+ \\ \pi^- & -\pi^0/\sqrt{2} \end{pmatrix}. \tag{3.19}$$

We define fields perpendicular to the eaten Goldstones by

$$\begin{pmatrix} A_d^0 \\ A_u^0 \\ \pi^0 \end{pmatrix} = U \begin{pmatrix} A_h^0 \\ A_\pi^0 \\ G^0 \end{pmatrix}, \qquad \begin{pmatrix} H_d^\pm \\ H_u^\pm \\ \pi^\pm \end{pmatrix} = U \begin{pmatrix} H_h^\pm \\ H_\pi^\pm \\ G^\pm \end{pmatrix}, \tag{3.20}$$

where

$$U = \begin{pmatrix} s_\beta & -c_\gamma c_\beta & -s_\gamma c_\beta \\ c_\beta & c_\gamma s_\beta & s_\gamma s_\beta \\ 0 & s_\gamma & -c_\gamma \end{pmatrix}, \tag{3.21}$$

with

$$\tan\beta = \frac{v_u}{v_d}, \qquad \tan\gamma = \frac{v_h}{f}, \qquad v_h = \sqrt{v_u^2 + v_d^2}. \tag{3.22}$$

The Goldstone modes G^0, $G^\pm$ are massless eigenstates orthogonal to the other modes, so we have 2×2 mass matrices for the *CP* even, *CP* odd neutral, and *CP* odd charged scalars. For the *CP* even scalars, the mass matrix is

$$M^2_{h_u^0, h_u^0} = m^2_{H_u} + 2m_Z^2(s_\beta^2 - \tfrac{1}{4})s_\gamma^2, \tag{3.23}$$

$$M^2_{h_u^0, h_d^0} = -B\mu - m_Z^2 s_\beta c_\beta s_\gamma^2, \tag{3.24}$$

$$M^2_{h_d^0, h_d^0} = m^2_{H_d} + 2m_Z^2(c_\beta^2 - \tfrac{1}{4})s_\gamma^2. \tag{3.25}$$

For the *CP* odd neutral scalars, we have

$$M^2_{A_h, A_h} = m^2_{H_u} c_\beta^2 + m^2_{H_d} s_\beta^2 + 2B\mu s_\beta c_\beta - \tfrac{1}{2} m_Z^2 (s_\beta^2 - c_\beta^2)^2 s_\gamma^2, \tag{3.26}$$

$$M^2_{A_h, A_\pi} = \frac{1}{c_\gamma}[(m^2_{H_u} - m^2_{H_d}) s_\beta c_\beta + B\mu(s_\beta^2 - c_\beta^2) - m_Z^2 (s_\beta^2 - c_\beta^2) s_\gamma^2)], \tag{3.27}$$

$$M^2_{A_\pi, A_\pi} = \frac{1}{c_\gamma^2}\left[m^2_{H_u} s_\beta^2 + m^2_{H_d} c_\beta^2 - 2B\mu s_\beta c_\beta + \frac{1}{2} m_Z^2 (s_\beta^2 - c_\beta^2) s_\gamma^2 \right]. \tag{3.28}$$

For the charged scalars we have

$$M^2_{H_h^\pm, H_\pi^\mp} = M^2_{A_h, A_h}, \tag{3.29}$$

$$M^2_{H_h^\pm, H_\pi^\mp} = \frac{1}{c_\gamma}[(m^2_{H_u} - m^2_{H_d}) s_\beta c_\beta + B\mu(s_\beta^2 - c_\beta^2) + m_Z^2 s_\beta c_\beta (s_\beta^2 - c_\beta^2) s_\gamma^2)], \tag{3.30}$$

$$M^2_{H_\pi^\pm, H_\pi^\mp} = M^2_{A_\pi, A_\pi}. \tag{3.31}$$

We define the mass eigenstates by

$$\begin{pmatrix} h^0 \\ H^0 \end{pmatrix} = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} h_u \\ h_d \end{pmatrix}, \tag{3.32}$$

$$\begin{pmatrix} A_1^0 \\ A_2^0 \end{pmatrix} = \begin{pmatrix} \cos\alpha_A & \sin\alpha_A \\ -\sin\alpha_A & \cos\alpha_A \end{pmatrix} \begin{pmatrix} A_h^0 \\ A_\pi^0 \end{pmatrix}, \tag{3.33}$$

$$\begin{pmatrix} H_1^\pm \\ H_2^\pm \end{pmatrix} = \begin{pmatrix} \cos\alpha_H & \sin\alpha_H \\ -\sin\alpha_H & \cos\alpha_H \end{pmatrix} \begin{pmatrix} H_h^\pm \\ H_\pi^\pm \end{pmatrix}, \tag{3.34}$$

where

$$\tan 2\alpha = \frac{2M^2_{h_u^0 h_d^0}}{M^2_{h_u^0 h_u^0} - M^2_{h_d^0 h_d^0}}, \tag{3.35}$$

etc.

We can understand the qualitative features of the scalar spectrum by considering a simplified limit where $B\mu = 0$ and we neglect the quartic interactions which give rise to the terms proportional to m_Z^2 in the mass matrices. In this limit, $h^0_{u,d}$ are mass eigenstates with mass $m_{H_{u,d}}$, and the masses of the *CP*-odd scalars are (for $f \ll v$)

$$m^2_{A_1^0} = m_{H_1^\pm} = \frac{m^2_{H_u} m^2_{H_d}}{m^2_{H_u} s_\beta^2 + m^2_{H_d} c_\beta^2}, \tag{3.36}$$

$$m^2_{A_2^0} = m_{H_2^\pm} = \frac{1}{c_\gamma^2}(m^2_{H_u} s_\beta^2 + m^2_{H_d} c_\beta^2), \tag{3.37}$$

with mixing angle

$$\alpha_{A,H} = -\frac{m_{H_u}^2 - m_{H_d}^2}{m_{H_u}^2 s_\beta^2 + m_{H_d}^2 c_\beta^2} s_\beta c_\beta c_\gamma \sim \frac{f}{v}. \quad (3.38)$$

We see that for $c\gamma = f/v \ll 1$ the CP odd mass eigenstates A_1^0 and $H_1^\pm$ are dominantly elementary Higgs particles. The states $A_2^0, H_2^\pm$ have masses $\sim v/f$ times larger and are dominantly PNGBs from the strong sector. The mixing between these two sets of states is of order f/v. Using $c\gamma \sim f/v$ and the equation for v [see Eq. (3.14)], we see that the condition that the heavy fields have masses below the scale Λ is equivalent to the condition $\epsilon \ll 1$.

Some spectra including the full potential effects are illustrated in Figs. 1 and 2. The low-energy expansion breaks down when the heavy scalars have mass of order Λ, indicated by the upper grey shaded region. For light charged Higgs scalars, there is a constraint from $b \to s\gamma$ that is indicated by the lower pink shaded region (see e.g. [20]). Here we have neglected possible destructive interference from Higgsino contributions that may weaken the bound. We see that this constraint prefers somewhat heavier h^0 masses, but does not rule out much of the parameter space.

The couplings of these fields to standard model states are straightforward to work out using the formulas above. The qualitative features are that the new heavy states A_2^0 and $H_2^\pm$ mix with the light Higgs fields at order f/v. These fields will therefore couple most strongly to the heaviest standard model particles, but with a strength suppressed by $\mathcal{O}(f/v)$ compared to the lighter MSSM Higgs fields with the same quantum numbers.

C. Precision electroweak fit

We now discuss the precision fit for the case of induced electroweak symmetry breaking. The only couplings of the strong sector to the MSSM are via electroweak gauge couplings and Higgs couplings. The most important electroweak corrections are therefore the oblique corrections parametrized by the electroweak parameters S and T, and the corrections to the $Z\bar{b}b$ vertex.

We begin with the S parameter. The physics above the confinement scale Λ in the strong sector gives rise to a UV contribution to the S parameter that can be parametrized by the effective Lagrangian coupling

$$\Delta\mathcal{L}_{\text{eff}} = \frac{gg'}{16\pi} S_{\text{UV}} \operatorname{tr}(\Sigma^\dagger W_{\mu\nu}^3 \Sigma B^{\mu\nu}). \quad (3.39)$$

The first point to make is that the strong sector need not have either a large number of technicolors N_{TC} or technidoublets N_{TD}, which would enhance the S parameter. Traditional technicolor models generally require both N_{TC} and N_{TD} to be large to be embedded into extended technicolor. Since the quark and lepton masses arise from elementary Higgs fields, there is no reason for these parameters to be large. For example, the theory in Sec. II has $N_{\text{TC}} = 2$ and $N_{\text{TD}} = 1$.

The size of the UV contribution to the S parameter is very uncertain. Naive dimensional analysis (NDA) [14] tells us that

$$S_{\text{UV}} \sim \frac{1}{\pi}. \quad (3.40)$$

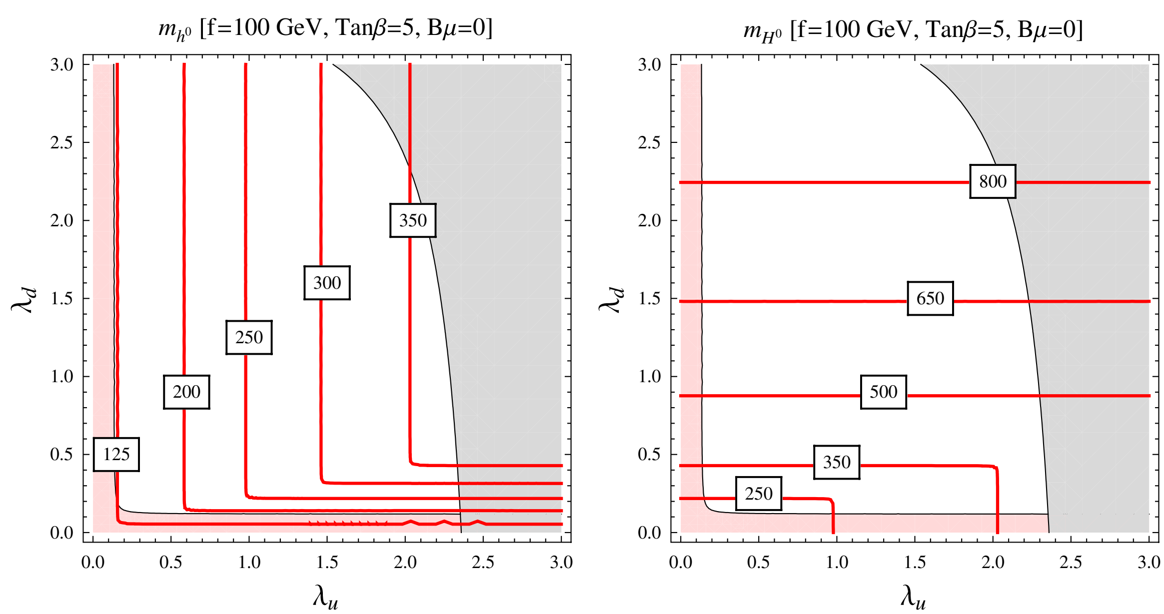

FIG. 1 (color online). Left panel: Masses (in GeV) for the light CP even Higgs h^0. Right panel: Masses for the heavy CP even Higgs H^0. The model has $f = 100$ GeV, $\tan\beta = 5$, and $B\mu = 0$, so all masses are a function of $\lambda_{u,d}$ normalized so that $c_1 = 1$ in Eq. (3.11). The upper grey shaded region is where the perturbative expansion breaks down, and the lower pink region is where the charged Higgs contribution to $b \to s\gamma$ comes into tension with experiment.

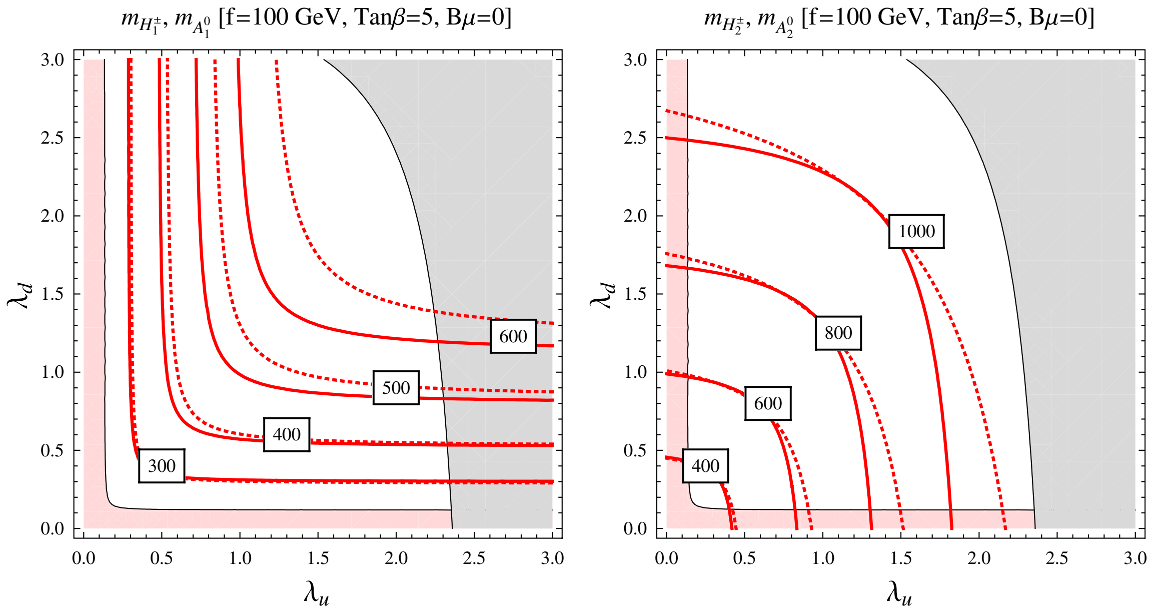

FIG. 2 (color online). Left panel: Masses (in GeV) for light *CP* odd Higgs particles. Solid lines denote A_1^0, dotted lines denote $H_1^\pm$. Right panel: Likewise for A_2^0 and $H_2^\pm$. The shaded regions are as in Fig. 1.

This is the same estimate as in technicolor theories, even though $f < v$ in this theory. There is no suppression by powers of f/v because we are in the regime where $\Lambda \sim 4\pi f \gg m_W$. In the effective theory below the scale Λ, S is a dimensionless quantity that is independent of the scale f. In terms of a resonance saturation picture, $S_{\rm UV} \sim f^2/m_\rho^2$ where m_ρ is the resonance mass; since $m_\rho \sim f$, the result is independent of f.

The S parameter in traditional technicolor theories can be estimated by scaling from QCD. Using large-N_c scaling, one obtains [21]

$$S_{\rm UV}({\rm QCD}) \simeq 0.25 \frac{N_{\rm TC}}{3} N_{\rm TD}. \tag{3.41}$$

Note that this is consistent with the NDA estimate Eq. (3.40). But Eq. (3.41) is better than an order of magnitude estimate only if the spectrum and couplings at the strong scale Λ are similar to QCD. However, the present theory is supersymmetric and conformal above the scale Λ, and there is no reason to believe that this is the case. In fact, it has been argued that theories that are conformal above the scale Λ have a significantly reduced S parameter [22]. There is also some support for a smaller S parameter from lattice simulations. A recent lattice simulation with $N_c = 3$, $N_f = 6$ found that the S parameter *per electroweak doublet* is reduced compared to QCD by a factor between 0.3 and 0.6 [23]. This theory is not conformal, but this at least emphasizes the large uncertainty in the S parameter from strongly coupled electroweak symmetry breaking sectors.

Our theoretical understanding of the S parameter in strongly coupled theories is very poor. For example, there is no rigorous theoretical understanding of even the sign of the S parameter in QCD, where many rigorous inequalities are known [24]. Data tells us that $S > 0$ in QCD, and Weinberg sum rules relate this to basic features of the hadron spectrum. In QCD, the S parameter can be well approximated by the contributions from the ρ and a_1 vector resonances, and the positivity of S follows from the fact that $m_{a_1} > m_\rho$. However, the present theory has no parity symmetry and there is no symmetry distinction between the analogs of the ρ and a_1. If vector meson dominance holds in the present theory, the sign of S will depend on whether the couplings of the lightest resonance are more like the ρ or the a_1. The breaking of parity symmetry depends on the SUSY breaking masses, so the UV contribution to S will change by $\mathcal{O}(100\%)$ as these parameters are varied. It is very plausible that there are choices of parameters where it is significantly reduced, perhaps even negative. On the other hand, 5D anti-de Sitter models can be interpreted as "holographic" descriptions of large-N conformal field theories, and in these theories S is positive whenever it is calculable [25]. In perturbation theory, S is generally positive unless special representations and couplings are chosen [26]. Perhaps these are hints that nature prefers $S > 0$.

In this paper, we will use the QCD value for the UV contribution to the S parameter as a benchmark, allowing us to make plots and gauge the impact of precision electroweak data on this model. As argued above, this is a conservative benchmark. We will see that we can get a good precision electroweak fit even with these assumptions, which means that precision electroweak data is not a strong constraint on this class of models.

There is an additional contribution to the S parameter coming from states below the scale Λ, the Nambu-Goldstone bosons in the strong sector and the elementary Higgs fields. These mix at order f/v, but given the large uncertainties in UV contribution, we will give the result neglecting these effects. For large $\tan\beta$, electroweak symmetry breaking is dominated by H_u, while H_d is decoupled, and we obtain

$$S_{\rm IR} \simeq \frac{1}{12\pi}\left[\ln\frac{m_h^2}{m_{h,\rm ref}^2} + \ln\frac{\Lambda^2}{m_\pi^2}\right]. \qquad (3.42)$$

The first term is the standard model Higgs contribution, while the second is the contribution from the composite pseudo-Nambu-Goldstone bosons in the strong sector. The first contribution is suppressed for light Higgs masses as usual, while the second is suppressed compared to conventional technicolor theories because the π fields are heavy. This means that the IR contribution to the S parameter is significantly reduced compared to ordinary technicolor.

We now turn to the T parameter. The couplings $\lambda_{u,d}$ in Eq. (3.1) violate custodial $SU(2)$ for $\lambda_u v_u \neq \lambda_d v_d$, so the T parameter depends on adjustable parameters. This can help give a good precision electroweak fit, as we will see.

In order to contribute to the T parameter, we need a spurion transforming as an isospin 2 representation of custodial $SU(2)$. The spurions $\lambda^\dagger\lambda$ and $\mathcal{H}\lambda$ are both isospin 1 [see Eq. (3.6)], so the leading contribution to the T parameter is quadratic in these spurions. The spurion $\lambda^\dagger\lambda$ always comes from diagrams with a loop of elementary Higgs fields, so we have

$$\mathcal{L}_{\rm eff} \sim \frac{\Lambda^4}{16\pi^2}\mathcal{F}\left(\frac{D_\mu}{\Lambda}, \frac{\lambda^\dagger\lambda}{16\pi^2}, \frac{\mathcal{H}\lambda}{\Lambda}\right), \qquad (3.43)$$

where $\mathcal{F}$ is an order-1 function of dimensionless arguments. From this we see that the largest contribution to the T parameter from the couplings $\lambda_{u,d}$ comes from couplings such as

$$\Delta\mathcal{L}_{\rm eff} = \frac{c_T}{16\pi^2}[\mathrm{tr}(\mathcal{H}\lambda D_\mu\Sigma^\dagger)]^2, \qquad (3.44)$$

where $c_T \sim 1$. This gives

$$\Delta m_W^2 = \Delta m_{W^\pm}^2 - \Delta m_{W_3}^2 \sim \frac{g^2 f^2}{4}(\epsilon_u - \epsilon_d)^2, \qquad (3.45)$$

or

$$\Delta T_{\rm UV} = \alpha^{-1}\frac{\Delta m_W^2}{m_W^2} \sim \alpha^{-1}(\epsilon_u - \epsilon_d)^2, \qquad (3.46)$$

where the expansion parameters $\epsilon_{u,d}$ are defined in Eq. (3.12). For the values used above, we find $\Delta T \sim 0.3$, which is just the right size to get a good precision electroweak fit (see below).

There is another UV contribution to the T parameter in the strong sector coming from $U(1)_Y$ loops that is of order $\Delta T \sim \pm 1/4\pi$. This should be regarded as an additional uncertainty on the size of the T parameter in these models. This contribution is sufficiently small that it does not affect our conclusions below.

There are also IR contributions to the T parameter from states below the scale Λ. The largest contribution comes from the light Higgs. For large $\tan\beta$ this is mainly the excitation from H_u and we have simply

$$\Delta T_{\rm IR} = -\frac{3}{16\pi\cos^2\theta_W}\ln\frac{m_h^2}{m_{h,\rm ref}^2}. \qquad (3.47)$$

The mass eigenstates $(A_1^0, H_1^\pm)$ and $(A_2^0, H_2^\pm)$ form approximately degenerate custodial $SU(2)$ multiplets, and we will neglect their contribution to the T parameter. Note that there is already a large uncertainty in the T parameter because we only know the order of magnitude of the effective coupling c_T in Eq. (3.44).

To give some idea of the prospects for a precision electroweak fit, we plot these estimates in Fig. 3. We assume that the UV contribution to the S parameter is given by the QCD value Eq. (3.41) and the UV contribution to the T parameter is given by the right-hand side of Eq. (3.46). We assume that the UV contribution to the T parameter is positive, as suggested by perturbation theory. With these assumptions, the plot shows the values of S and T for light Higgs masses of 120 and 350 GeV. For each Higgs mass there is a line of values corresponding to different values of custodial $SU(2)$ violation from the couplings $\lambda_{u,d}$. The curves are not entirely in the T

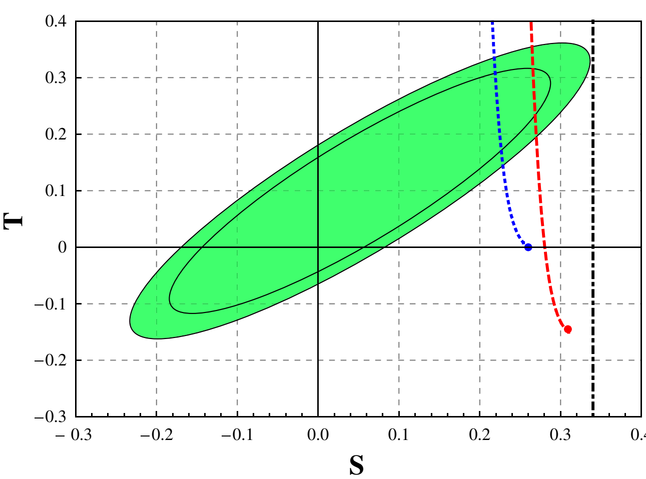

FIG. 3 (color online). Electroweak fit for $f = 100$ GeV, $\tan\beta = 5$, $B\mu = 0$. The inner (outer) ellipse is the 95% (99%) confidence level allowed region for a reference Higgs mass of 120 GeV [28]. The dotted blue (dashed red) line corresponds to a light Higgs mass of 120 (350) GeV in the model of Sec. III. The dot-dashed black line corresponds to the model of Sec. IV. As discussed in the text, there are large uncertainties in these curves; in particular, it is plausible that the S parameter is significantly smaller. The assumptions that go into these curves are described in the text.

direction because the masses of the heavy PNGB fields depend on these couplings, so changing these couplings gives a contribution to S as well as T. For values of the light Higgs mass above 350 GeV the expansion is not under theoretical control because λ_u becomes too large. The net result is that a positive contribution to the T parameter can give a good precision electroweak fit under these assumptions, in the region where the theory is under theoretical control. There is a large theoretical uncertainty in the predictions for S and T, so the plots cannot be taken too literally, and our conclusion is that precision electroweak data does not strongly constrain these models given our present knowledge. In fact, the only way that precision electroweak can rule out these models is if either the S parameter is much larger than expected, or the UV contributions to the T parameter are negative. Neither of these is expected.

Finally, we consider $Z \to \bar{b}b$. the strong sector couples weakly to the elementary Higgs fields, which have the Yukawa couplings to the top and bottom quarks. This means that any correction to $g_{Z\bar{b}b}$ from the strong sector must be suppressed by y_t^2 as well as $\lambda_{u,d}^2$. We write the third generation Yukawa couplings as

$$\Delta \mathcal{L} = Q_L^T \epsilon \mathcal{H} y Q_R^c + \text{H.c.}, \tag{3.48}$$

where $\mathcal{H}$ is defined in Eq. (3.5) and

$$Q_L = \begin{pmatrix} t_L \\ b_L \end{pmatrix}, \quad Q_R^c = \begin{pmatrix} b_R^c \\ t_R^c \end{pmatrix}, \quad y = \begin{pmatrix} y_b & 0 \\ 0 & y_t \end{pmatrix}. \tag{3.49}$$

The leading correction to $Z \to \bar{b}b$ comes from effective interactions of the form

$$\Delta \mathcal{L}_{\text{eff}} \sim \frac{1}{(4\pi)^4} Q_L^\dagger \bar{\sigma}^\mu Q_L \, \text{tr}(iD_\mu \Sigma \lambda y^\dagger y \lambda^\dagger \Sigma^\dagger). \tag{3.50}$$

This gives a correction

$$\frac{\Delta g_{Z\bar{b}b}}{g_{Z\bar{b}b}} \sim \frac{y_t^2}{16\pi^2} \frac{\lambda_u^2}{16\pi^2} \sim \frac{y_t^2}{16\pi^2} \left(\frac{m_h}{4\pi v}\right)^4 \left(\frac{v}{f}\right)^6. \tag{3.51}$$

The standard model agrees with the measured value at the level of 0.25%, which gives the constraint (for $m_h \simeq$ 120 GeV)

$$v < 5.6 f. \tag{3.52}$$

This is easily satisfied given the other constraints we have already considered above.

D. Collider phenomenology

We now discuss the collider phenomenology of this model, focusing on the LHC. This theory has SUSY broken at the TeV scale, so it has the standard SUSY signals resulting from pair production of strongly interacting superpartners followed by cascade decays. This work focuses on electroweak symmetry breaking, and does not prefer any particular pattern of masses for the MSSM superpartners.

In addition to the standard SUSY signals, this model extends the MSSM Higgs sector with a custodial $SU(2)$ triplet of PNGBs, which mix with the CP odd Higgs fields of the MSSM. The heavy mass eigenstates A_2^0 and $H_2^\pm$ are dominantly from the strong sector, with $\mathcal{O}(f/v)$ mixing with the light MSSM Higgs fields. The A_2^0 can be directly produced via gluon-gluon fusion through a top quark loop, with a cross section of order f^2/v^2 times the standard model cross section. For $m_{A_2^0} = 500$ GeV this cross section is of order 10 fb at the LHC. The A_2^0 has potential decay modes $A_2^0 \to h^0 Z$ and $A_2^0 \to A_1^0 h^0$ followed by either $A_1^0 \to \bar{t}t$ or Zh^0. As we have seen above, we can get a good precision electroweak fit for large values of the h^0 mass, so we can have either $h^0 \to \bar{b}b$ or WW/ZZ. There are many possible final states to investigate, but the common feature is a high multiplicity of heavy standard model particles.

We can also produce heavy hadrons from the strong sector. These are expected to be at the scale $4\pi f \sim$ TeV. They can be produced via vector boson fusion (for resonances of spin 0, 1, or 2), or by mixing with the W and Z (for spin 1). NDA tells us that the couplings of such a resonance ρ are

$$\mathcal{L}_{\text{eff}} \sim (\partial \rho)^2 + \Lambda^2 \rho^2 + \frac{g}{4\pi} \Lambda^2 \rho W + \frac{g^2}{4\pi} \Lambda \rho W W + \cdots. \tag{3.53}$$

This is the same coupling as in traditional technicolor theories, but with a reduced strong scale Λ. The mixing of spin-1 resonances with the W and Z is therefore of order $g/4\pi$, so we have production of neutral spin-1 resonances with a cross section suppressed by $g^2/16\pi^2$ compared to a sequential Z' of the same mass. Production via vector boson fusion is also possible.

These heavy resonances will generally decay to 2-body final states involving strong particles, i.e. they will pair produce A_2^0 and $H_2^\pm$. The decays of the A_2^0 have been discussed above. The dominant decays of the heavy charged Higgs fields are expected to be $H_2^\pm \to W^\pm h^0$ and $H_2^\pm \to A_1^0 W^\pm$. The light charged Higgs fields can decay via $H_1^+ \to \bar{b}t$ or $W^+ h^0$. We see that this opens up even more final states with even higher multiplicity of heavy standard model particles.

It should be clear from this discussion that the phenomenology is very rich and exciting. We will leave detailed investigation of LHC signals to future work.

IV. STRONG ELECTROWEAK SYMMETRY BREAKING

We now consider another scenario for electroweak symmetry breaking where there are no elementary Higgs fields below the TeV scale. The theory at the TeV scale consists of the MSSM *without* the Higgs fields, plus a strong conformal sector. SUSY breaking at the TeV scale gives masses to the MSSM superpartners, and triggers confinement and chiral symmetry breaking in the strong sector, breaking electroweak symmetry. Quark and lepton masses arise from interactions between the strong sector and the quarks and leptons.

As described above, this scenario is very similar to conformal technicolor. The main difficulties in constructing a realistic model of conformal technicolor are constructing a mechanism to generate the top quark mass without flavor-changing neutral currents, and the precision electroweak tests. The presence of SUSY broken at the TeV scale greatly alleviates both of these problems, as we will discuss below. The absence of a light Higgs of course means that the SUSY Higgs mass problem is absent, which is the main motivation for this model.

A. Flavor

We first discuss the origin of the quark and lepton masses. The strong sector is assumed to contain chiral superfield operators $\mathcal{O}_{u,d}$ with the quantum numbers of the MSSM Higgs fields. These have Yukawa-type couplings with the quark and leptons superfields that generate fermion masses. In any interacting conformal theory the operators $\mathcal{O}_{u,d}$ have dimension $d > 1$, so the Yukawa interactions are irrelevant interactions. (In the model described in Sec. II, $d = \frac{3}{2}$.) The general danger in conformal technicolor is that $\mathcal{O}^{\dagger}_{u,d}\mathcal{O}_{u,d}$ has dimension <4, so that there is a relevant singlet operator. But this operator is not invariant under SUSY, and is therefore protected from large UV contributions. This is just a restatement of the well-known fact that scalar mass terms are forbidden by SUSY, even for fields with $d = 1$.

The Yukawa coupling responsible for the top quark mass gets strong at a scale Λ_t that is quite low, even for small values of d. (For $d = \frac{3}{2}$, $\Lambda_t \sim 600$ TeV.) At or below the scale Λ_t we need a theory that generates these interactions without generating additional interactions that lead to large flavor-changing neutral currents. These can be generated by exchange of elementary scalars with the quantum numbers of Higgs doublets [5]. These scalar fields have ordinary Yukawa couplings with quarks and leptons, and therefore have minimal flavor violation. (Of course, because the theory is supersymmetric at the TeV scale we still have to address the SUSY flavor problem associated with squark and slepton masses and A terms.) For $\Lambda_t \gg$ TeV, getting a sufficiently large top mass requires that these scalars have large couplings to the top quark, the strong sector, or both [6].

An alternative is to have $\Lambda_t \sim$ TeV. This is very natural in the present class of models: the elementary Higgs scalars can have positive mass-squared terms of order the TeV scale, and generate the required couplings at this scale. The couplings of the elementary Higgs fields to the strong sector are generally relevant interactions, and so one must explain why these interactions are important at the SUSY breaking scale. This is similar to the problem of explaining why the μ term of the MSSM is of order the SUSY breaking scale, and in the model of Sec. II we give a solution based on a generalization of the Giudice-Masiero mechanism. If we normalize the Higgs coupling to the strong sector at the TeV scale like a dimensionless Yukawa coupling $y_{\rm TC}$, we have

$$m_t \sim y_t y_{\rm TC} v. \tag{4.1}$$

We see that this requires neither the top quark Yukawa coupling nor the coupling of the Higgs to the strong sector to be strong.

B. Precision electroweak fit

We now turn to the precision electroweak fit. Many of the comments made in Sec. III C apply to this case as well, so we will be brief.

We begin with the S parameter. The strong sector need not have large N, and so the contributions to the S parameter from this sector are not large to begin with. In addition, there are good reasons to think that the UV contribution to the S parameter may be significantly reduced compared to the QCD value. This is suggested by recent lattice calculations [23], and there are theoretical arguments that this occurs in theories that are conformal above the TeV scale [22]. The IR contribution to the S parameter is as in technicolor:

$$S_{\rm IR} = \frac{1}{12\pi} \ln\frac{\Lambda^2}{m^2_{h,\rm ref}}, \tag{4.2}$$

where $\Lambda \sim 4\pi v \sim 3$ TeV.

We now discuss the T parameter. The couplings of the elementary scalars to the strong sector that generate quark and lepton masses in general violate custodial symmetry, and give an additional contribution to the T parameter. We assume that this contribution is positive (as suggested by perturbation theory), in which case it can help with the precision electroweak fit. There is no limit to how large this contribution can be, since the couplings of the Higgs fields to the strong sector can naturally be strong at the TeV scale. This requires a reduced value for the top quark Yukawa coupling; see Eq. (4.1). On the other hand, it is natural for custodial symmetry to be an approximate symmetry of this sector, so these contributions to the T parameter need not be large.

The upshot is that the T parameter is an adjustable parameter in this model. This is illustrated in Fig. 3. Here we have simply assumed the QCD value for the UV contribution to the S parameter together with an arbitrary positive T contribution. To get a good precision electroweak fit, the UV contribution to S must be reduced compared to the QCD value, but a factor of 2 is more than sufficient. This is clearly within the uncertainties (see the discussion in Sec. III C), and we conclude that precision electroweak is not a strong constraint on these models given our present state of knowledge.

Finally, we discuss $Z \to \bar{b}b$. In this model, the strong sector couples directly to the top and bottom quarks, so the leading correction to $Z \to \bar{b}b$ comes from effective interactions of the form

$$\Delta \mathcal{L}_{\rm eff} \sim \frac{1}{\Lambda^2}\,{\rm tr}(D_\mu \Sigma y y^\dagger \Sigma^\dagger) Q_L^\dagger \sigma^\mu Q_L + {\rm H.c.}, \qquad (4.3)$$

where y is defined in Eq. (3.49). This gives

$$\frac{\Delta g_{Z\bar{b}b}}{g_{Z\bar{b}b}} \sim \frac{y_t^2}{16\pi^2}. \qquad (4.4)$$

The standard model agrees with the measured value at the level of 0.25%, and this contribution is about the same size. We conclude that this correction is at the level of the measured precision, but there is no direct conflict.

C. Phenomenology

Below the scale $\Lambda \sim 4\pi v \sim 3$ TeV the light states in this model include the usual MSSM superpartners, minus the Higgs and Higgsino fields. The absence of the Higgsino fields simplifies the chargino and neutralino sectors of the theory. In particular, the lightest neutralino is a mixture of the Bino and the Wino. Their mixing is suppressed because the Higgs fields are heavy, so the only neutralino thermal dark matter candidate is a light Bino, requiring slepton masses right near the experimental limits [27]. There are of course many other possibilities for dark matter in supersymmetric theories.

We now turn to the LHC phenomenology of this model. In addition to the standard SUSY signals, this theory has a strong electroweak symmetry breaking sector at the TeV scale. The minimal model has a strong sector with a $SU(2)_L \times SU(2)_R$ symmetry broken down to the diagonal $SU(2)$. Nonminimal symmetry breaking patterns with additional PNGBs are also possible, but are not discussed here. An important difference from traditional technicolor models is that the strong sector generally does not have an approximate parity symmetry that interchanges $SU(2)_L \times SU(2)_R$. This arises because the technisquarks charged under $SU(2)_L$ and $SU(2)_R$ need not have the same masses. Since these masses determine the confinement scale, this breaking of parity is unsuppressed at this scale. This implies that the resonances that unitarize WW scattering can generally decay to WWW as well as WW.

V. CONCLUSIONS

This work has begun the exploration of models in which SUSY breaking triggers confinement and chiral symmetry breaking in a strong sector at the TeV scale. This is very generic in SUSY gauge theories with a strong conformal fixed point, since soft SUSY breaking in the strong sector also breaks conformal invariance softly. This generates masses for all scalars in the strong sector, while fermion masses are generally protected by chiral symmetries. Since the gauge coupling is strong at all scales, this very plausibly leads to confinement and chiral symmetry breaking at the SUSY breaking scale.

We have considered models in which the strong dynamics breaks electroweak symmetry, in two different limits. In one limit the strong sector induces large VEVs in elementary Higgs fields, while in the other the strong dynamics is solely responsible for electroweak symmetry breaking. Both of these scenarios can have a good precision electroweak fit thanks to an adjustable T parameter arising from the elementary Higgs couplings to the strong sector. Both have no problems generating the large top quark mass without additional flavor-changing interactions. Both scenarios share the usual SUSY flavor problem with the MSSM, which may be solved using one of the many mechanisms in the literature. The important point is that the presence of the strong dynamics does not give rise to any additional flavor problem. Unlike the MSSM, gauge coupling unification is no longer a prediction of the models described here, since the strong sector affects the evolution of the $SU(2)_W \times U(1)_Y$ gauge couplings but not $SU(3)_C$. Unification can be accommodated with additional matter fields, which however have no other apparent motivation in this framework. The phenomenology of these scenarios is rich and deserves further study. We also believe that further theoretical investigation of the combination of SUSY breaking and strong dynamics will be fruitful.

ACKNOWLEDGMENTS

We thank R. Contino, R. Kitano, T. Okui, and J. Terning for discussions. We also thank J. Serra for participation at early stages of this work, and M. Schmaltz for emphasizing the importance of constraints on $Z \to \bar{b}b$. This work was supported by DOE Grant No. DE-FG02-91-ER40674.

APPENDIX: SINGLET SOFT MASSES

We now discuss the effect of a universal soft SUSY breaking mass for the singlets S_{ij} in the model of Sec. II B. The terms in the UV Lagrangian involving S can be written

$$\mathcal{L} = \int d^4\theta Z_S S_{ij}^\dagger S_{ij} + \left(\int d^2\theta \lambda_{ij} S_{ij} \Psi_i \Psi_j + {\rm H.c.} \right). \quad ({\rm A1})$$

The universal soft mass can be parametrized by a nonzero D component for Z_S:

$$Z_S \sim 1 + D_S \theta^4, \qquad ({\rm A2})$$

where $D_S \sim M_{\rm SUSY}^2 \ll \Lambda_*^2$. We can think of Z_S as a gauge field for a $U(1)_S$ gauge symmetry under which

$$S_{ij} \mapsto e^{i\Omega} S_{ij}, \quad \lambda \mapsto e^{-i\Omega}\lambda, \quad Z_S \mapsto e^{i(\Omega-\Omega^\dagger)} Z_S, \tag{A3}$$

where Ω is a chiral superfield gauge transformation parameter. The fact that $\lambda \neq 0$ breaks the $U(1)$ gauge symmetry explicitly, but this breaking is soft in the UV theory. Another important symmetry is a $U(1)_R$ symmetry with charges

$$R(\Psi) = \tfrac{1}{2}, \qquad R(S) = 1. \tag{A4}$$

Now consider this theory below the scale Λ_* where the couplings λ become strong. The question is then how does the spurion Z_S appear in the low-energy effective theory? The low-energy degrees of freedom are the dual techniquarks $\tilde{\Psi}$ which carry no $U(1)_S$ charge. The dependence on Z_S is therefore via the $U(1)_S$ gauge invariant quantities

$$\xi = \frac{\lambda^\dagger \lambda}{Z_S}, \tag{A5}$$

$$S_\alpha = \bar{D}^2 D_\alpha \ln Z_S. \tag{A6}$$

ξ is proportional to the physically normalized superpotential coupling strength, while S_α is the $U(1)_S$ gauge field strength. These contain SUSY breaking

$$\xi \sim \lambda^2(1 + \theta^4 D_S), \tag{A7}$$

$$S_\alpha \sim \theta_\alpha D_S, \tag{A8}$$

and therefore parametrize the SUSY breaking arising from the S soft mass in the low-energy theory. For example, the effective theory contains the terms

$$\Delta \mathcal{L}_{\rm eff} \sim \int d^4\theta \xi \tilde{\Psi}^\dagger \tilde{\Psi}. \tag{A9}$$

This gives a universal soft mass for the dual techniquarks. Since the operator $\tilde{\Psi}^\dagger \tilde{\Psi}$ has dimension >2, this operator becomes important at a scale parametrically below $M_{\rm SUSY}$.

There can be terms in the effective Lagrangian proportional to strong operators that are not singlets, which are not required to be irrelevant operators. These all involve the spurion S_α since ξ is a singlet under all symmetries. It is easily checked that there are no allowed F terms involving S_α allowed by $U(1)_R$ symmetry. We can systematically enumerate all D terms involving S_α. An example is

$$\int d^4\theta S^\alpha \mathcal{O}_\alpha = D_S \times \bar{D}^2 D^\alpha \mathcal{O}_\alpha|_{\theta=0}. \tag{A10}$$

Unitarity requires $\dim(\mathcal{O}_\alpha) > \frac{3}{2}$, so the operator on the right-hand side must have dimension $> \frac{3}{2} + \frac{3}{2} = 3$. Since the theory is strongly coupled, we expect this inequality to be violated by $\mathcal{O}(1)$. Matching at the scale Λ_* and running down, we see that dimensionless strength of this SUSY breaking is

$$\delta \ll \left(\frac{D_S}{\Lambda_*^2}\right)^2 \left(\frac{E}{\Lambda_*}\right)^{-2}. \tag{A11}$$

This gets strong at a scale

$$E \ll \frac{M_{\rm SUSY}^2}{\Lambda_*} \ll M_{\rm SUSY}. \tag{A12}$$

Similarly, we have

$$\int d^4\theta D^\alpha S_\alpha \mathcal{O} = D_S \times D^2 \bar{D}^2 \mathcal{O}|_{\theta=0} \Rightarrow \dim > 3, \tag{A13}$$

$$\int d^4\theta S^\alpha S_\alpha \mathcal{O} = D_S^2 \times \bar{D}^2 \mathcal{O}|_{\theta=0} \Rightarrow \dim > 2, \tag{A14}$$

$$\int d^4\theta S^\alpha (S^\dagger)^{\dot\alpha} \mathcal{O}_{\alpha\dot\alpha} = D_S^2 \times D^\alpha \bar{D}^{\dot\alpha} \mathcal{O}_{\alpha\dot\alpha}|_{\theta=0} \Rightarrow \dim > 4, \tag{A15}$$

$$\int d^4\theta S^\alpha S_\alpha (S^\dagger)^{\dot\alpha} \mathcal{O}_{\dot\alpha} = D_S^3 \times \bar{D}^{\dot\alpha} \mathcal{O}_{\dot\alpha}|_{\theta=0} \Rightarrow \dim > 2, \tag{A16}$$

$$\int d^4\theta |S^\alpha S_\alpha|^2 \mathcal{O} = D_S^4 \mathcal{O}|_{\theta=0} \Rightarrow \dim > 2. \tag{A17}$$

In Eqs. (A13)–(A17) we used the unitarity constraint on the dimension of operators, while in Eq. (A17) we used the fact that $\mathcal{O}$ is a $R = 0$ operator, and therefore the operator $\int d^4\theta \mathcal{O}$ is an allowed term in the Lagrangian, so $\mathcal{O}$ must have dimension >2. All of these terms become important at scales parametrically below $M_{\rm SUSY}$. Terms with additional derivatives are even more suppressed. We conclude that all possible SUSY breaking terms in the low-energy theory are suppressed compared to $M_{\rm SUSY}$.

[1] For an entry into the literature, see A. Birkedal, Z. Chacko, and Y. Nomura, Phys. Rev. D **71**, 015006 (2005); A. Maloney, A. Pierce, and J. G. Wacker, J. High Energy Phys. 06 (2006) 034; S. Chang, C. Kilic, and R. Mahbubani, Phys. Rev. D **71**, 015003 (2005).

[2] For a review, see S. Chang, R. Dermisek, J. F. Gunion, and N. Weiner, Annu. Rev. Nucl. Part. Sci. **58**, 75 (2008).

[3] For a review, see C. T. Hill and E. H. Simmons, Phys. Rep. **381**, 235 (2003).

[4] E. H. Simmons, Nucl. Phys. **B312**, 253 (1989); C. D. Carone and H. Georgi, Phys. Rev. D **49**, 1427 (1994).

[5] S. Samuel, Nucl. Phys. **B347**, 625 (1990); M. Dine, A. Kagan, and S. Samuel, Phys. Lett. B **243**, 250 (1990).

[6] J. A. Evans, J. Galloway, M. A. Luty, and R. A. Tacchi, J. High Energy Phys. 04 (2011) 003.

[7] M. A. Luty and T. Okui, J. High Energy Phys. 09 (2006) 070; M. A. Luty, J. High Energy Phys. 04 (2009) 050.

[8] A. Kagan, 2008 KITP workshop on Physics of the Large Hadron Collider, http://online.itp.ucsb.edu/online/lhc08/kagan/.

[9] A. Azatov, J. Galloway, and M. A. Luty, arXiv:1106.3346 [Phys. Rev. Lett. (to be published)].

[10] Y. Cai, M. A. Luty, and D. E. Kaplan, arXiv:0909.5499.

[11] M. Antola, S. Di Chiara, F. Sannino, and K. Tuominen, Eur. Phys. J. C **71**, 1784 (2011); arXiv:1009.1624.

[12] N. Seiberg, Nucl. Phys. **B435**, 129 (1995).

[13] G. F. Giudice and A. Masiero, Phys. Lett. B **206**, 480 (1988).

[14] H. Georgi, Phys. Lett. B **298**, 187 (1993).

[15] See e.g. E. Eichten and K. Lane, Phys. Lett. B **669**, 235 (2008).

[16] H. Murayama, arXiv:hep-ph/0307293.

[17] M. A. Luty and R. Rattazzi, J. High Energy Phys. 11 (1999) 001.

[18] V. A. Novikov, M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, Phys. Lett. **166B**, 329 (1986); M. A. Shifman and A. I. Vainshtein, Nucl. Phys. **B359**, 571 (1991); N. Arkani-Hamed, G. F. Giudice, M. A. Luty, and R. Rattazzi, Phys. Rev. D **58**, 115005 (1998).

[19] J. Galloway, J. A. Evans, M. A. Luty, and R. A. Tacchi, J. High Energy Phys. 10 (2010) 086.

[20] A. W. El Kaffas, P. Osland, and O. M. Ogreid, Phys. Rev. D **76**, 095001 (2007).

[21] M. Golden and L. Randall, Nucl. Phys. **B361**, 3 (1991); B. Holdom and J. Terning, Phys. Lett. B **247**, 88 (1990); M. E. Peskin and T. Takeuchi, Phys. Rev. Lett. **65**, 964 (1990); Phys. Rev. D **46**, 381 (1992).

[22] R. Sundrum and S. D. H. Hsu, Nucl. Phys. **B391**, 127 (1993).

[23] T. Appelquist *et al.* (LSD Collaboration), Phys. Rev. Lett. **106**, 231601 (2011).

[24] For a review, see S. Nussinov and M. A. Lampert, Phys. Rep. **362**, 193 (2002).

[25] H. Davoudiasl, J. L. Hewett, B. Lillie, and T. G. Rizzo, Phys. Rev. D **70**, 015006 (2004); G. Burdman and Y. Nomura, Phys. Rev. D **69**, 115013 (2004); R. Barbieri, A. Pomarol, R. Rattazzi, and A. Strumia, Nucl. Phys. **B703**, 127 (2004); K. Agashe, C. Csaki, C. Grojean, and M. Reece, J. High Energy Phys. 12 (2007) 003.

[26] See e.g. M. J. Dugan and L. Randall, Phys. Lett. B **264**, 154 (1991); E. Gates and J. Terning, Phys. Rev. Lett. **67**, 1840 (1991).

[27] K. J. Bae, R. Dermisek, H. D. Kim, and I.-W. Kim, J. Cosmol. Astropart. Phys. 08 (2007) 014; K. Hsieh, Phys. Rev. D **77**, 015004 (2008).

[28] H. Flacher, M. Goebel, J. Haller, A. Hocker, K. Monig, and J. Stelzer, Eur. Phys. J. C **60**, 543 (2009).