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Travels on the squark-gluino mass plane

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Soft supersymmetry breaking appears in the weak-scale effective action but is usually generated at higher scales. For these models, the structure of the renormalization group evolution down to the electroweak scale leaves only part of the squark-gluino and slepton-gaugino mass planes accessible. Our observations divide these physical mass planes into three wedges: the first can be reached by all models of high-scale breaking; the second can only be populated by models with a low mediation scale; in the third, wedge squarks and gluinos would have to be described by an exotic theory. All usual benchmark points reside in the first wedge, even though an LHC discovery in the third wedge would arguably be the most exciting outcome.

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I. INTRODUCTION

Searches for supersymmetry (SUSY) are one of the most visible tasks of the LHC experiments [1,2]. To interpret the data, they have to rely on specific SUSY models determining the mass spectrum and the decay patterns. Limiting the Higgs sector to two doublets, a good starting point for such an interpretation, is the minimal supersymmetric standard model (MSSM) defined at the weak scale. However, for practical purposes one needs to significantly constrain its vast parameter space. After taking into account the strong constraints, for example, from flavor physics [3] and electric dipole moments [4] we are left with $\mathcal{O}(20)$ parameters which can be relevant for LHC searches or observations [5]. A further reduction of this parameter space is traditionally achieved in terms of simplest constrained realizations such as the CMSSM/mSUGRA [6,7], gauge mediation [8–10], or anomaly mediation [11] (see also [12] for an overview).

These models share two important features. First, by construction they have a small or even minimal number of free parameters to describe all the soft supersymmetry breaking terms. Second, the soft parameters in these models are determined at a high scale arising from an underlying theory of supersymmetry breaking and mediation. At the mediation scale M, the values for the soft parameters are initialized. For example, in gauge mediation $M = M_{\mathrm{mess}}$ is an effective mass of the messenger fields transmitting supersymmetry breaking to the standard model sector. In these models M_{mess} is typically taken to be in the range 10^5–10^{14} GeV. In gravity mediation models M is set by M_{Planck} which, with the additional assumption of grand unification, in CMSSM is traded down to M_{GUT} (where GUT refers to the grand unified theory). In order to make contact with the scale at which experiments operate the soft terms have to undergo renormalization group evolution down from the mediation scale M to the weak scale [12,13].

In this paper, we point out that all such high-scale models automatically impose severe restrictions on superpartner masses at collider energies. In the strongly as well as weakly interacting sfermion-gaugino mass planes as much as half of the available parameter space becomes inaccessible. For example in the squark-gluino case, squarks cannot become significantly lighter than gluinos. Similar relations hold between sleptons and electroweak gauginos in all high-scale models.

The details of these restrictions dominantly depend on one parameter, namely, the value of the mediation scale. As a result, each sfermion-gaugino mass plane can be divided into three wedge-shaped regions. One region can be reached by all the usual models of high-scale supersymmetry breaking. A second wedge can only be populated by models with a mediation scale $M < M_{\mathrm{GUT}}$, while sfermions and gauginos in the third wedge would have to originate from a theory which either does not have a high SUSY scale or a qualitatively different RG evolution. Thus, from measuring gaugino and sfermion masses we can draw powerful conclusions on the way supersymmetry is realized in nature.

Conversely, when searching for supersymmetry one should make as few assumptions as possible about the way supersymmetry breaking is realized. This definitely includes its high-scale origin. With the next round of SUSY searches at the LHC being imminent, new sets of benchmark points and test models will be defined to determine, optimize, and calibrate the search strategies. In order to minimize the bias of assumed specific models it may be useful to include also points which do not originate from high-scale models and which are distributed more democratically on the sfermion-gaugino planes accessible at collider energies. One way to obtain points not prejudiced toward high-scale models is to use the MSSM defined directly at the weak scale. A manageable incarnation of this idea is the so-called phenomenological MSSM or pMSSM [14]. Alternatively, one can use the simplified model approach for constructing test models based on kinematic considerations and a selection of a small number of allowed sparticle species [15]. To some degree, squark and gluino searches both by ATLAS [1] and CMS [2] are already following this route.

This paper is organized as follows: in the next section we will show explicitly how the renormalization group evolution from the high scale M to the weak scale restricts the accessible regions in the squark-gluino plane. In Sec. III we extend our discussion to binos, winos, and sleptons. In particular, we discuss the additional complications arising from electroweak symmetry breaking. In Sec. IV we investigate the distribution of benchmark points as well as a variety of test models. Finally, in Sec. V we summarize our findings and conclude.

II. SQUARK VS GLUINO MASS

The key to understanding the coverage of the squark-gluino mass plane is the renormalization group equation for these masses. It has been known for a long time [12,13] that the gaugino masses strongly impact the running of the sfermion masses to the weak scale. Starting from a high scale, they generate contributions to the soft sfermion masses even if the initial soft sfermion masses vanish.

To illustrate this structure, we can approximately solve the RG equations in SUSY-QCD. In the absence of Yukawa couplings we find schematically

$$\begin{aligned} m^2_{\tilde q}(Q) &\sim m^2_{\tilde q}(M) + A_{\tilde q}(Q) m^2_{\tilde q}(M) + A_{\tilde g}(Q) M^2_{\tilde g}(M), \\ M_{\tilde g}(Q) &\sim M_{\tilde g}(M) + B_{\tilde g}(Q) M_{\tilde g}(M), \end{aligned} \tag{2.1}$$

where $A_{\tilde q}(Q)$ and $A_{\tilde g}(Q)$ are Q-dependent functions specifying the one-loop RG corrections to the mass. Numerically, $A_{\tilde g}$ dominates. The running of the gluino mass does not include any squark mass terms on the right-hand side. The reason for this is that Majorana fermion masses are protected by the R symmetry, in analogy to the chiral symmetry for the Dirac masses of the standard model fermions. This feature persists for the entire MSSM and can be exploited, for example, to decouple all scalars from a high-scale SUSY model while keeping all gauginos light enough to ensure gauge coupling unification, dark matter, etc. [16].

Moving on to the full theory, for the first two generations we neglect the Yukawa couplings [3]. If the trilinear A terms are proportional to the Yukawa couplings, the same holds for the renormalization group contributions from them. Using this, the RG equations for the first generation sfermions read [12]

$$16\pi^2 \frac{d}{dt} m^2_{\tilde f} = -8 \sum_r C^{\tilde f}_r g^2_r |M_r|^2 + 2 Y_{\tilde f} g^2_1 S, \tag{2.2}$$

where M_r are the gaugino masses, $r = (1, 2, 3)$ the (bino, wino, gluino) labels, g_r the gauge couplings not in the GUT normalization for $U(1)$, and

$$\begin{aligned} S &:= \mathrm{Tr}(Y m^2) \\ &= \sum_{\text{generations}} (m^2_{\tilde Q_L} - 2m^2_{\tilde u_R} + m^2_{\tilde d_R} - m^2_{\tilde L_L} + m^2_{\tilde e_R}) \\ &\quad + m^2_{H_u} - m^2_{H_d}. \end{aligned} \tag{2.3}$$

The Casimir invariants and hypercharge assignments for the relevant fermions are

$$\begin{aligned} (C^{\tilde f}_1, C^{\tilde f}_2, C^{\tilde f}_3) &= (Y^2_{\tilde f}, \tfrac{3}{4}, \tfrac{4}{3}), \\ (Y_{\tilde Q L}, Y_{\tilde u R}, Y_{\tilde d R}, Y_{\tilde L L}, Y_{\tilde e R}) &= (\tfrac{1}{6}, -\tfrac{2}{3}, \tfrac{1}{3}, -\tfrac{1}{2}, 1). \end{aligned} \tag{2.4}$$

The gaugino masses, couplings, and scalar masses then evolve according to

$$\begin{aligned} &16\pi^2 \frac{d}{dt} M_r = -2 b_r g^2_r M_r, \qquad 16\pi^2 \frac{d}{dt} g^2_r = -2 b_r g^4_r, \\ &16\pi^2 \frac{d}{dt} S = -2 b_1 g^2_1 S, \qquad (b_1, b_2, b_3) = (-11, -1, 3). \end{aligned} \tag{2.5}$$

Comparing Eqs. (2.2) and (2.5) we indeed see that the gaugino masses contribute to the running of the sfermion masses but not vice versa.

Equation (2.2) can easily be integrated,

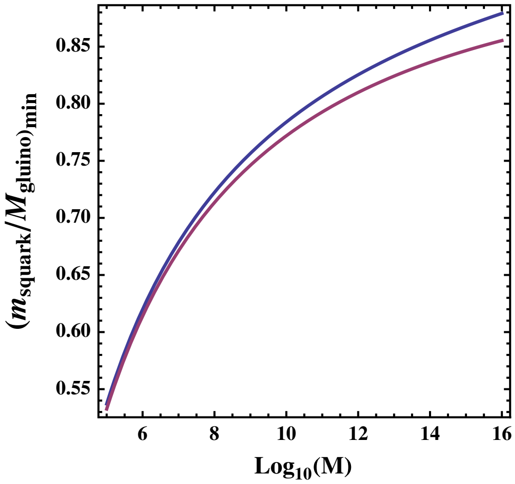
$$\begin{aligned} m^2_{\tilde f}(Q) = m^2_{\tilde f}(M) &+ \frac{Y_{\tilde f}}{b_1} \left[\frac{\alpha_1(Q)}{\alpha_1(M)} - 1 \right] S(M) \\ &+ \sum_{r=1}^{3} \frac{2C^{\tilde f}_r}{b_r} \left[1 - \frac{\alpha^2_r(M)}{\alpha^2_r(Q)} \right] M^2_r(Q). \end{aligned} \tag{2.6}$$

Because we will mainly be interested in sfermion masses smaller than the gaugino masses, the on-shell corrections to the gaugino masses are small, so we can identify the gaugino mass parameter M_r at the low scale with the corresponding pole (physical) mass. In addition, we can average over the light squark masses. The term proportional to the hypercharge and S then drops out and we find for the average squark mass

$$\begin{aligned} m^2_{\tilde q}(Q) &:= \frac{1}{4}[2m^2_{\tilde Q_L} + m^2_{\tilde u_L} + m^2_{\tilde d_R}](Q) \\ &= m^2_{\tilde q}(M) + \frac{1}{4} \sum_{\tilde f = 2\tilde Q_L, \tilde u_R, \tilde d_R} \sum_{r=1} \frac{2C^{\tilde f}_r}{b_r} \left[1 - \frac{\alpha^2_r(M)}{\alpha^2_r(Q)} \right] M^2_r(Q). \end{aligned} \tag{2.7}$$

Similar averaged expressions can be obtained for the sleptons.

At scales Q below M the $U(1)$ and $SU(2)$ running implies $\alpha(M)/\alpha(Q) > 1$, while for $SU(3)$ this ratio is less than one. Thus, all three terms in the r sum on the right-hand side of Eq. (2.7) are positive. Assuming that the initial soft sfermion mass terms are non-negative, i.e.,



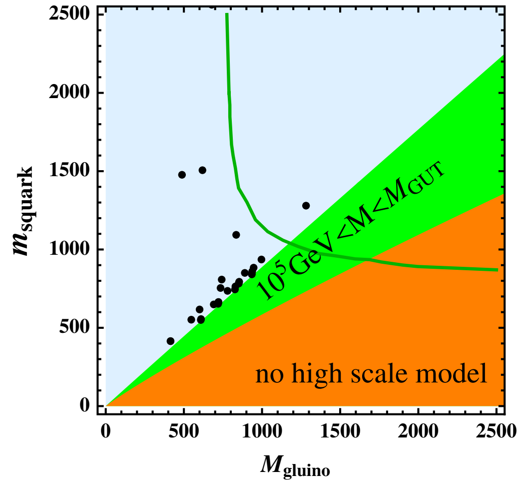


FIG. 1 (color online). Left: Minimal ratio $m_{\tilde{q}}/M_{\tilde{g}}$ as a function of the mediation scale. The blue curve (first from the top) assumes universal gaugino masses at the GUT scale, whereas for the red curve (second from the top) only $M_{\tilde{g}}(M_{\rm GUT})$ is nonzero. Right: Accessible regions in the $m_{\tilde{q}}$-$M_{\tilde{g}}$ plane, assuming gaugino mass unification. Their boundaries correspond to mediation scales $M = M_{\rm GUT} = 2 \times 10^{16}$ GeV and $M = 10^5$ GeV. The thick green line in the right panel shows the simplified model ATLAS exclusion with 1.04 $\rm fb^{-1}$ [20]. The dots show benchmark points from Refs. [1,2,18,19].

avoiding tachyonic sfermions at the high scale,[1] we obtain minimal sfermion mass values at the low scale Q as a function of the gaugino masses.

Our argument is most straightforward for the first generation squarks where electroweak symmetry breaking effects play no role. Given a fixed gluino mass, we find the lowest possible squark mass when the wino and bino masses vanish. The red curve (second from the top) in the left panel of Fig. 1 gives the minimal ratio of squark to gluino mass averaged over $\tilde{u}_{L,R}$ and $\tilde{d}_{L,R}$. If instead of very light weak gaugino masses we assume gaugino unification this mass ratio slightly increases, as can be seen from the blue curve (first from the top) in Fig. 1.

Different mediation scales, which we implicitly assume for any SUSY model, put restrictions on the achievable physical squark masses in terms of a lower limit on $m_{\tilde{q}}/M_{\tilde{g}}$. The constraints on the mass-ratio $m_{\tilde{q}}/M_{\tilde{g}}$ can be interpreted as region boundaries on the two-dimensional squark-gluino mass plane as shown on the right panel in Fig. 1.

Beyond our basic observation, we need to make a technical aside on the role of the low scale in Fig. 1. Equation (2.7) depends on the choice of the renormalization point Q defining the physical masses observable at the LHC. This dependence is logarithmic and therefore quite weak. In the left panel of Fig. 1 we simply choose $Q = 1$ TeV. In the right panel we included this dependence by evaluating $m_{\tilde{q}}(m_{\tilde{q}})$ and $M_{\tilde{g}}(M_{\tilde{g}})$. Therefore, the lines separating the three regions are not entirely straight.

Ignoring any high-scale physics features we start from phenomenological weak-scale SUSY models populating the entire squark-gluino mass plane. The more we then increase the scale of mediation, the stronger the constraints become and the smaller the area in the mass plane we can cover. Turning this argument around, the position of a low-energy supersymmetric model on the squark-gluino plane can be used to find an upper limit on the possible mediations scales or even make a statement about the absence of such a scale.

In the right panel of Fig. 1 we divide the full plane into three regions: region I (blue, top) can easily be reached by all known models of SUSY breaking, including gravity and gauge mediation. To illustrate this we have also indicated in Fig. 1 a set of SUSY benchmark points proposed and studied over the years in Refs. [1,2,18,19]. Region II (green, middle) corresponds to SUSY models where the breaking is mediated in the window 10^5 GeV $- M_{\rm GUT}$. It is not accessible to gravity mediation but provides a good home for gauge mediation. Finally, if SUSY should be discovered in region III (orange, bottom) its breaking would have to be described by an exotic theory. It would have to descend from a theory with no or little separation between the electroweak and the SUSY mediation scales, excluding anything similar to gauge and gravity mediation. These and other possibilities will be further discussed in Sec. V.

There are different ways to study region III. One way is to start from the so-called pMSSM [14] Lagrangian where all MSSM soft parameters are defined at the weak scale and no assumptions on the SUSY-breaking mechanism need to be made. For studies along these lines see [5]. Alternatively, we can utilize the simplified model approach [15], where one reduces the number of decay topologies and with it the parameter dependence of branching ratios to a level where only the masses of the particles appearing in

[1]For models with high-scale tachyons see [17].

the production and decay channels have to be tracked. The main difference between these two approaches is that for the weak-scale pMSSM several decay topologies can contribute to a given signature and that nontrivial branching ratios are included in the analysis. From our point of view, both approaches are well suited to avoid LHC searches based on a theory bias.

One example is a simplified model with light squarks and gluinos and a massless neutralino. The 95% confidence level exclusion contour for this model based on 1.04 fb^{-1} of ATLAS data from [20] is shown in Fig. 1. The considerable mismatch between these exclusion contours and their CMSSM counterparts [1,2] (where they are defined) is mostly due to different neutralino mass assumptions. We can, however, easily modify the simplified model by assuming a light rather than massless bino and locking its mass to an appropriately rescaled gluino mass, $m_{\tilde{\chi}_1^0} \sim M_1 = (\alpha_1/\alpha_3)M_3$. This would largely be equivalent to the high-scale motivated models where they are possible, while avoiding assumptions about the scale and the precise nature of SUSY mediation mechanisms which should be results of an analysis instead of assumptions.

Last but not least, it should be noted that the regions of the squark-gluino plane which lie outside the usual high-scale motivated region are particularly interesting from a phenomenological point of view. If a gluino (or other color octet) becomes significantly heavier than the color-triplet squark it is likely that we will reconstruct two hard decay jets, in addition to the well-understood softer QCD jet radiation [21]. The observation of, for example, four such hard jets would clearly point to the production of a pair of color octet particles [22]. The reconstruction of the effective mass is also easier if we see several hard decay products, so we can correlate it with the number of jets, to get a first global guess at the properties of the new particles [22]. Finally, longer on-shell decay chains with hard decay products are the basis of any kind of SUSY parameter analysis, which, for example, rely on the decay $\tilde{g} \to \tilde{b}_1 \to \tilde{\chi}_2^0 \to \tilde{\ell} \to \tilde{\chi}_1^0$ [5,23].

III. SLEPTON VS BINO/WINO MASS

Similarly to the squark-gluino mass plane discussed in Sec. II, we can also project SUSY models onto the electroweak slepton-gauginos mass planes. Again, the region attainable for models with a reasonably high mediation scale turns out to be wedge shaped.

Using Eq. (2.6) we can compute ratios of the left- and right-handed slepton masses to the bino and wino masses. Before doing that let us address the term $\propto S$ which is not positive definite. In the simplest and most commonly used models, like the CMSSM or gauge mediation with universal Higgs masses, $S = 0$ at the mediation scale and remains so at other scales in the one-loop approximation. Thus, the second term in Eq. (2.6) is absent. For more general models, including models with nonuniversal Higgs masses, the S term is generally nonzero. One way to address this issue would be to average over the charged sleptons similarly to what was done for squarks in the previous section. Instead we will choose to work with left- and right-handed sleptons separately and make use of the fact that the hypercharge has opposite sign for the left- and the right-handed species. Therefore, if the effect is to lower the sfermion to the gaugino mass ratio in one case, it will unavoidably increase it in the other. Thus, we will proceed with the determination of the minimal slepton to gaugino mass ratios derived from Eq. (2.6) *without* the S term. The caveat is that a nonzero S has the potential to lower either

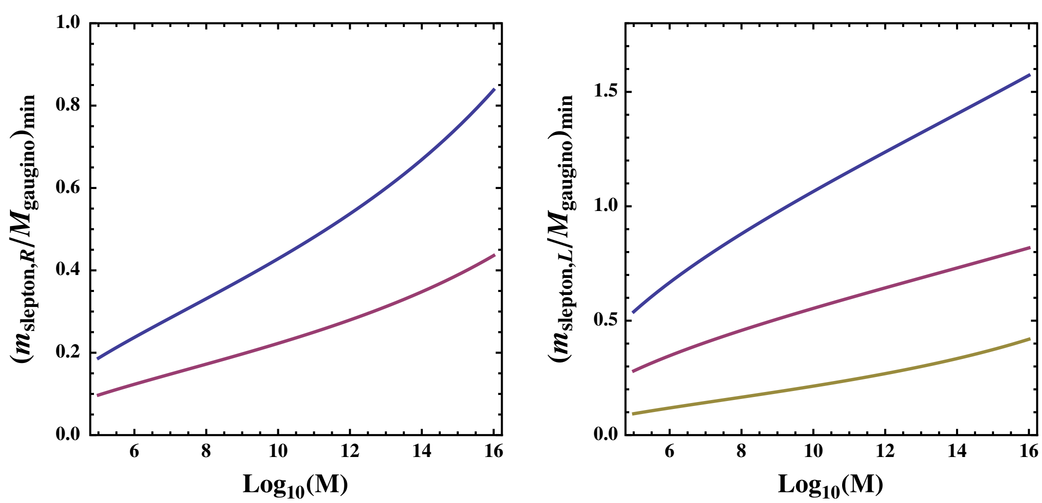


FIG. 2 (color online). Left: Minimal ratios $m_{\tilde{e}_R}/M_j$ for the bino (blue, first curve from the top) and wino (red, second curve from the top) as a function of the messenger scale. Whereas the bino curve is independent of all other masses, the wino curve assumes universal gaugino masses. Right: Minimal ratios $m_{\tilde{e}_L}/M_j$ for bino/wino [blue/red (first/second from the top)], assuming universal gaugino masses. For the yellow curve (third from the top) which shows $m_{\tilde{e}_L}/M_1$, only the bino mass is taken to be nonzero.

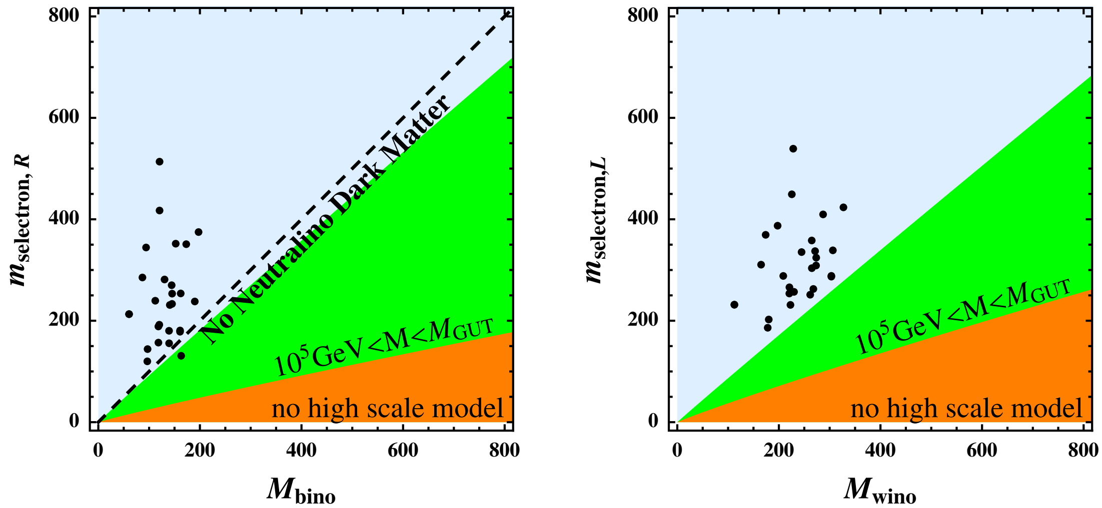


FIG. 3 (color online). Three regions in the sfermion-gaugino mass plane, for a bino or wino and left- and right-handed selectrons. The color coding is the same as in Fig. 1. We assume gaugino mass unification. The ''No Neutralino Dark Matter'' diagonal indicates where selectrons are lighter than the lightest neutralino. We also display benchmark points presented in [1,2,18,19]. In the left panel the dots indicate χ_1^0 and selectron masses, whereas in the right panel they correspond to $\chi_1^\pm$ and left handed selectron masses.

the right- or left-handed sfermion masses but never both. Hence, one of the minimal ratios cannot be lowered.

We show the minimum values for all four combinations of left and right handed sleptons compared to bino and wino in Fig. 2. The corresponding regions in the two-dimensional mass planes are shown in Fig. 3. Naively, one would think that the renormalization group running should be flatter than in the case of squarks, due to the smaller gauge couplings. However, the relative contribution of the gauginos to the sfermion masses in Eq. (2.6) is proportional to the relative change in the gauge coupling divided by the beta function coefficient which is of the same order of magnitude for all three gauge groups.

The reason for the significant difference between the ratios for the left- and right-handed selectrons is the chiral nature of the electroweak interactions and the gauge structure of the gauginos. Even if the initial supersymmetry breaking for the sfermions is chirality blind, left- and right-handed sfermions will be split during the renormalization group evolution. The typical example for such a mediation is gravity. In contrast, gauge mediation does have a chiral structure already at the messenger scale.

The main difference between the squark-gluino case and the slepton-gaugino results shown in Fig. 3 is the translation of the Lagrangian parameters into the masses of the physical states. While for the gluino we only have to take into account a moderately small correction to the on-shell mass scheme, the weak gauginos are generally mixed.

What we can study at the Lagrangian parameter level, however, are the different slepton masses. From Fig. 2 we already know that renormalization scale evolution separates left- and right-handed selectrons. For universal gaugino masses, the contribution to the left-handed sfermions is always bigger and therefore left-handed sfermions are heavier.

The two-dimensional slepton mass plane in Fig. 4 shows the ordering of the left- and right-handed masses as a function of the bino and wino masses assuming chiral degeneracy at the messenger scale. Gaugino mass unification, as often assumed in LHC searches, implies $M_2 \sim 2M_1$. This translates into a solid prediction $m_{\tilde{e}_L} > m_{\tilde{e}_R}$. However, for non-universal gaugino masses [24] this can be different. If the bino is significantly heavier than the wino, the right-handed

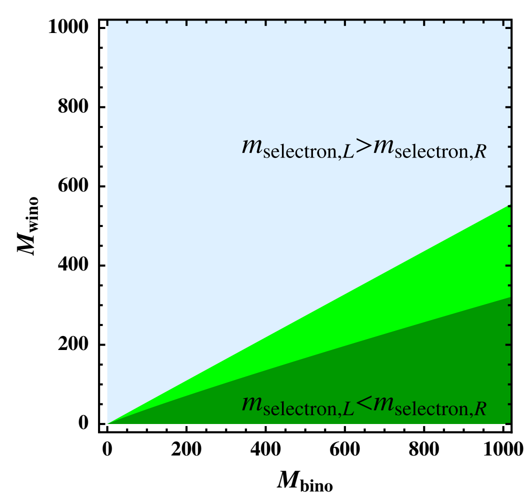


FIG. 4 (color online). Left- and-right handed selectron masses for a nonchiral input at the messenger scale, as well as $S = 0$. In the blue region labeled $m_{selectron,L} > m_{selectron,R}$ (dark green, labeled $m_{selectron,L} < m_{selectron,R}$) the left (right) handed selectrons are heavier for $M > 10^5$ GeV. In the light green region, which falls between the other two, the right-handed selectrons can be heavier for sufficiently large M.

sfermions could indeed be heavier. Therefore, in the same way that we should not unnecessarily assume the squark-gluino mass hierarchy as described in the previous section, LHC searches should not be based on the assumption that the lighter sleptons do not couple to the wino.

Our discussion so far has been in terms of bino and wino components of the electroweak gauginos, but as already noted, due to the effects of electroweak symmetry breaking the bino and the neutral wino are not the appropriate mass eigenstates. Their mass matrix is given by

$$M_{\tilde{\chi}^0} = \begin{pmatrix} M_1 & 0 & -c_\beta s_w m_Z & s_\beta s_w m_Z \\ 0 & M_2 & c_\beta c_w m_Z & -s_\beta c_w m_Z \\ -c_\beta s_w m_Z & c_\beta c_w m_Z & 0 & -\mu \\ s_\beta s_w m_Z & -s_\beta c_w m_Z & -\mu & 0 \end{pmatrix}, \tag{3.1}$$

where $s_w = \sin\theta_w$, $c_w = \cos\theta_w$, etc. This mass matrix is real and symmetric, so its eigenvalues are real. Accordingly, the mass matrix squared is positive definite and its smallest eigenvalue is smaller than any of its diagonal elements

$$\begin{aligned} \min m_{\tilde{\chi}^0} &< \min\left[\sqrt{M_1^2 + m_Z^2\frac{1-\cos(2\theta_w)}{2}}, \sqrt{M_2^2 + m_Z^2\frac{1+\cos(2\theta_w)}{2}}, \sqrt{\mu^2 + m_Z^2\frac{1\pm\cos(2\beta)}{2}}\right], \\ &< \min\left[\sqrt{M_1^2 + m_Z^2\frac{1-\cos(2\theta w)}{2}}, \sqrt{M_2^2 + m_Z^2\frac{1+\cos(2\theta w)}{2}}\right]. \end{aligned} \tag{3.2}$$

For $M_{1,2} \gg m_Z$ the smallest eigenvalue of the neutralino mass matrix is usually smaller than both M_1 and M_2. Therefore, the minimum curves for $m_{\tilde{e}}/M_1$ in Fig. 2 also set a lower limit on the ratio of selectron-slepton to the smallest neutralino mass.

Because of the wealth of additional parameters, the relevant question is if these bounds are saturated. In a first attempt we assume gaugino mass unification, which means the bino is roughly 6 times lighter than the gluino. Current LHC constraints imply $M_{\tilde{g}} > 750$ GeV, translating into $M_1 > 125$ GeV, so our original assumption $M_1 \gg m_Z$ is reasonable. For illustration purposes we also assume large μ, so we can consider the limit $m_Z \ll |M_1 \pm \mu|$, $|M_2 \pm \mu|$, and $M_{1,2} \ll \mu$. In this regime, the lightest neutralino is binolike and its mass is given

$$\begin{aligned} m_{\tilde{\chi}_1^0} &= M_1 - \frac{m_Z^2 s_w^2}{\mu^2 - M_1^2}[M_1 + \mu s_{2\beta}] \\ &= M_1\left[1 + \mathcal{O}\left(\frac{m_Z^2}{\mu M_1}\right)\right]. \end{aligned} \tag{3.3}$$

In this limit, the bound in the slepton-gaugino mass plane can indeed be saturated. Our expectations for the ratios

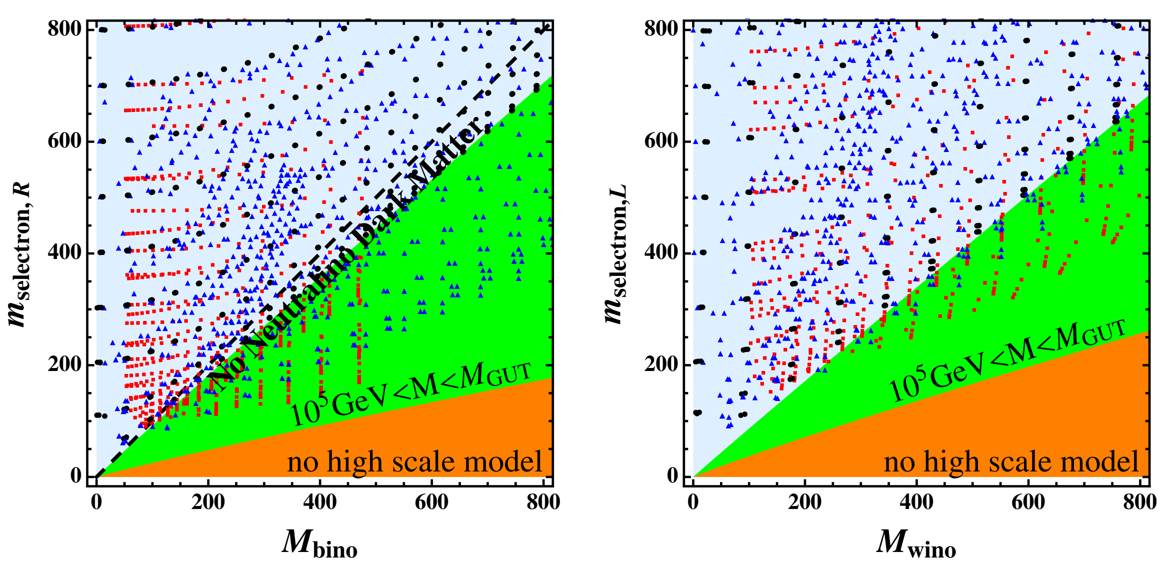


FIG. 5 (color online). The sfermion-gaugino mass plane, for a bino or wino and left and right selectrons and with the same color coding as Fig. 1. The dots represent scans over high-scale models, namely, the CMSSM (black dots), a low-scale CMSSM (blue triangles), and pure general gauge mediation (red squares) (see text for details).

between neutralino and bino masses are confirmed in the test models briefly discussed in the next section. Corresponding points are shown in the left panel of Fig. 5.

One might be curious to see how the SUSY benchmark points included in the squark-gluino plane in Fig. 1 are distributed on the electroweak mass planes. The black dots in the left panel of Fig. 3 denote values of the mass for $(m_{\tilde{\chi}_1^0}, m_{\tilde{e}_R})$ for those benchmark points. As expected, they lie in the high-scale region. The only point located in the green region corresponding to messenger scales below 10^{16} GeV is a gauge mediated point with a very low messenger scale of 80 TeV. We will continue the discussion of benchmark points and models in the next section.

In the left panel of Fig. 3 we also introduce a "No Neutralino Dark Matter" line. Below it, a binolike neutralino cannot be dark matter. It would decay into the lighter right-handed selectron which cannot be dark matter, as it is charged. This requirement is strongly correlated with a high mediation scale, i.e., once we require the bino to be the dark matter candidate we automatically constrain the available parameter space to the fraction accessible by high-scale SUSY breaking. Perhaps an obvious point to note is that if dark matter is not made of neutralinos all points below the dashed line remain perfectly viable.

If the lightest neutralino is binolike in the limit of large $|\mu|$, the second lightest neutralino is winolike. In this case we can interpret dark green region in the right-hand panel of Fig. 3 as the area for $\tilde{\chi}_2^0$ vs $m_{\tilde{e}_L}$ inaccessible to high-scale models. However, here we need to be careful with possible gaugino-Higgsino mixing effects.

We can apply the same argument as for neutralinos to the chargino sector with its mass matrix

$$M_{\tilde{\chi}^\pm} = \begin{pmatrix} 0 & X^T \\ X & 0 \end{pmatrix} \quad \text{with} \quad X = \begin{pmatrix} M_2 & \sqrt{2} s_\beta m_W \\ \sqrt{2} c_\beta m_W & \mu \end{pmatrix}. \tag{3.4}$$

Its eigenvalues are given by (each twice),

$$m^2_{\tilde{\chi}_j^\pm} = \tfrac{1}{2}\Big[|M_2|^2 + |\mu|^2 + 2m_W^2 \mp \sqrt{(|M_2|^2 + |\mu|^2 + 2m_W^2)^2 - 4|\mu M_2 - m_W^2 s_{2\beta}^2|^2}\Big]. \tag{3.5}$$

Again, we find that the smallest eigenvalue is bounded from above as

$$\begin{aligned} \min m_{\tilde{\chi}^\pm} &< \min\Big[\sqrt{M_2^2 + 2s_\beta^2 m_W^2}, \sqrt{M_2^2 + 2c_\beta^2 m_W^2}\Big] \\ &< \sqrt{M_2^2 + m_W^2}. \end{aligned} \tag{3.6}$$

For $m_W \ll M_2$ this smallest eigenvalue is typically below M_2. Therefore, the wino curves for $m_{\tilde{e}}/M_2$ in Fig. 2 can be interpreted as lower limits on the selectron to the lightest chargino mass ratio as a function of the messenger mass. Hence, the separation into three regions in the right-hand panel of Fig. 3 can be directly interpreted in terms of physical masses. Again, for illustration we have indicated the distribution of the benchmark points.

IV. BENCHMARK AND TEST MODELS

As shown in Sec. II, no supersymmetric model arising from a theory with a high scale and containing Majorana gauginos can cover the squark-gluino mass plane. As a result, large regions in the squark-gluino and slepton-gaugino mass planes are not populated by such high-scale models. For example, assuming gravity mediation at M_{GUT}, only roughly half of the parameter space corresponding to the light blue area (right panel, top) in Fig. 1 will be covered. For gauge mediation this effect is slightly more moderate as such models can enter into and (if the mediation scale is chosen suitably low) cover the green area (right panel, middle) in Fig. 1.

This shortcoming becomes particularly obvious when we study benchmark points provided by theorists to help guide the LHC experiments. Reference [25] lists a standard set of the benchmarks compiled for and used at the LHC [1,2,18,19]. These benchmarks are shown as black dots in Fig. 1. The first and most important requirement on benchmark points is to represent the available parameter space. The distribution we observe in Fig. 1 clearly shows that this is not the case, provided we consider the weak-scale MSSM the model the LHC looks for. All benchmark points populate the region of the squark-gluino mass plane which can be linked to high-scale SUSY breaking. In addition, reminiscent of the population of Scotland (or Canada), the vast majority of benchmark points in Fig. 1 live along the southern border which saturates the $m_{\tilde{q}}/M_{\tilde{g}}$ mass ratio, i.e., values of the squark mass where the renormalization group induced contribution shown in Eq. (2.6) dominates over the soft breaking scalar mass. One of the underlying reasons for this squeezed distribution is that most of the benchmark points are CMSSM points. In the CMSSM all sfermions have the same initial mass at the GUT scale characterized by the parameter m_0 which is typically chosen to be of the order of the electroweak scale. At the same time, the contributions arising from gauginos scale with their gauge couplings and gluino contributions are therefore dominant.

For the weakly interacting particles all but one benchmark point also lie in the upper region. Indeed, by construction, all those points lie even above the "No Neutralino Dark Matter" line. The benchmark points are now more spread out because the initial value of the universal CMSSM sfermion mass is comparable to electroweak gaugino contributions. Nevertheless, they still cover only a restricted area of parameter space.

To illustrate more generally (i.e., not just based on the limited set of benchmark points) how the sfermion-gaugino mass plane is populated by high-scale models we show in Fig. 5 a large set of parameter points scanning over a variety of test models:

(i) the CMSSM with $\tan\beta = 3, 10, 40$, and $A_0 = 0$;
(ii) the same initial soft parameters ($\tan\beta = 3, 10, 40$, $A_0 = 0$) but at lower $M = 2 \times 10^6, 2 \times 10^{10}$ GeV;
(iii) pure general gauge mediation with $M_{\rm mess} = 10^8, 10^{10}, 10^{14}$ GeV; see Refs. [19,25,26] for details.

Following our discussion in the previous section, we use for the x axis coordinates the masses of the lightest neutralino $m_{\tilde{\chi}_1^0}$ and the lightest chargino $m_{\tilde{\chi}_1^+}$. The (black dots) CMSSM points indeed cover the accessible parameter space and saturate the minimal ratios for $m_{\tilde{e}}/M_{1,2}$. The (blue triangles) low-scale "gravity mediation" points extend into the intermediate M wedge though they do not approach the lower end it. The pure general gauge mediation points marked in red (squares) extend further into this intermediate region. The same models have a qualitatively very similar behavior on the squark-gluino mass plane.

We also note that in the left panel the pure general gauge mediation points do not extend to arbitrarily high neutralino masses. This is special to this model which becomes nonperturbative for parameter values that correspond to large bino masses.

V. CONCLUSIONS

Many LHC searches for supersymmetry are conveniently interpreted on the squark-gluino mass plane. In this paper, we have argued that in the MSSM all sfermion-gaugino mass planes can be divided into three wedge-shaped regions: the first region with high squark masses is accessible to all types of SUSY models including those with a high mediation scale $M \gtrsim M_{\rm GUT}$. The second region with intermediate values of the sfermion to gaugino mass-ratio requires a mediation scale $M < M_{\rm GUT}$. Finally, the third region with the low sfermion to gaugino mass ratios cannot be accessed by any MSSM-type model with a mediation scale $M \gtrsim 10^5$ GeV. The models in this third wedge would have to be described by an exotic SUSY theory. Discovering SUSY in this region would be a particularly surprising and exciting outcome.

What does "exotic" mean in this context and how might such theories look? In general, any renormalization group evolution of scalar masses sufficiently different from the one considered here could result in a theory living in the third wedge. The renormalization group equations we employed are inherently MSSM equations. A non-MSSM matter content could therefore give an exotic theory. One well-understood example of this are models with Dirac gauginos [27]. In these theories, the Dirac gaugino masses simply do not determine the running the sfermion masses [28]. Of course, this is just one example of an exotic theory arising from a non-MSSM setup.

One of our technical assumptions was that the sfermion masses at the mediation should be nontachyonic. In principle, allowing such tachyons is a way to lower the physical sfermion to gaugino mass ratios below minimal values for high-scale models we have computed in this paper. The examples of these models examined in [17] often contain low lying color breaking vacua and while, in general, models of this type are not necessarily excluded they need to be carefully screened for dangerous instabilities.

A third large class of exotic theories are models without a significant separation between the electroweak scale and the scale at which the soft terms are generated. Practically, such models could be described by effective actions with soft terms defined at the collider scale, avoiding any renormalization group evolution. An even simpler approach to generate model points would be to use various versions of simplified models [15].

From an LHC perspective, the striking result of our study is that these fundamentally very different structures can be classified in terms of the standard scalar-gaugino mass planes and that their physics is essentially determined by one parameter, the mediation scale of SUSY breaking.

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