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Chiral particle decay of heavy-light mesons in a relativistic potential model

Takayuki Matsuki[1,*] and Koichi Seo[2,†]

[1]*Tokyo Kasei University, 1-18-1 Kaga, Itabashi, Tokyo 173-8602, Japan*
[2]*Gifu City Women's College, 7-1 Hito-ichiba Kitamachi, Gifu 501-0192, Japan*
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Partial decay widths of the heavy-light mesons, D, D_s, B, and B_s, emitting one chiral particle (π or K) are evaluated in the framework of a relativistic potential model. Decay amplitudes are calculated by keeping the Lorentz invariance as far as possible and use has been made of the Lorentz-boosted relativistic wave functions of the heavy-light mesons. One of predictions of our calculation is very narrow widths of a few keV for yet undiscovered $B_s(0^+, 1^+)$ mesons corresponding to ${}^{2S+1}L_J = {}^3P_0$ and "${}^3P_1$" assuming their masses to be 5617 and 5682 MeV, respectively, as calculated in our former paper. In the course of our calculation, new sum rules are discovered on the decay widths in the limit of $m_Q \to \infty$. Among these rules, $\Gamma(D_{s0}^*(2317) \to D_s + \pi) = \Gamma(D_{s1}(2460) \to D_s^* + \pi)$ and $\Gamma(B_{s0}^*(5615) \to B_s + \pi) = \Gamma(B_{s1}(5679) \to B_s^* + \pi)$ are predicted to hold with a good accuracy.

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I. INTRODUCTION

We have been trying to explain the mass spectrum of the heavy-light mesons, including the famous $D_{s0}(2317)$ and $D_{s1}^*(2460)$, by a relativistic potential model with a linear potential and the Coulombic potential. In our model besides the current quark mass m_q ($m_{u,d} \sim 10$ MeV) the constant term b is introduced in the scalar potential. They are independent parameters in general. However, if we restrict our computation up to the first order in $1/m_Q$, the current quark mass m_q and the constant b appear only in the combination $m_q + b$. Hence we set $b = 0$ and call m_q in place of $m_q + b$. In this model, we have successfully reproduced many of the experimental mass spectra of D, D_s, B, and B_s with a fairly good accuracy including radially excited states [1].

To confirm the validity of our relativistic potential model, we have calculated the semileptonic weak form factors (Isgur-Wise functions) for the process $\bar{B} \to D^{(*)}\ell\bar{\nu}$ in Ref. [2] to obtain reasonable results compared with the experiments. In the present paper [3], we calculate the decay processes of the heavy-light mesons with one chiral particle emission. Although the decay rates of these processes have been examined by Goity and Roberts [4] and by Di Pierro and Eichten [5] in a relativistic potential model, we would like to see the effects of using our relativistic wave functions as well as Lorentz covariant amplitudes that have been neglected in their papers. The authors of Refs. [4,5] have used wave functions of the heavy-light mesons in the rest frame to calculate transition amplitudes. That is, they have neglected not only the relativistic effects of the wave functions but also the recoil effects of the heavy-light mesons. We claim that this recoil effect is essential to calculate the relativistic transition amplitude because the plane wave function of the emitted particle is replaced by another phase factor, which involves the heavy quark mass and the velocity of the heavy-light meson. The plane wave, which, for instance, is used in a hydrogen atom, is inserted without any criticism, which may lead to the erroneous conclusion. In the course of our calculation, new sum rules on the decay widths are discovered, which hold among the different decay processes in the heavy quark limit, i.e., $m_Q \to \infty$. The breaking of the sum rules is of the order of $(M_i - M_f)/m_Q$, which is at most 10% or so with the initial and final heavy-light meson masses, M_i and M_f.

In Sec. II, the wave function of the heavy-light meson in the moving frame is related to the one in the rest frame by regarding the heavy quark to be at rest in the heavy-light meson. Then we derive the basic formula to express the transition amplitude in terms of the wave functions of the heavy-light meson in the rest frame. The plane wave of the emitted particle moving in the z direction, $\exp(-ikz)$, used in other works [4,5] is found to be replaced by another phase factor, $\exp(-2im_Q Vz)$, with V being the heavy-light meson velocity in the Breit frame. In the limit of $m_Q \to \infty$, the heavy quark mass m_Q coincides with the heavy-light meson mass and these two phase factors become equal to each other.

Interaction between light quarks and chiral particles (π or K) is assumed *à la* Georgi-Manohar [6]. The matrix elements of axial-vector currents are dissolved into several form factors, which are expressed in terms of the radial wave functions.

In Sec. III, we numerically evaluate the matrix elements of axial-vector currents. Except for the axial-vector coupling constant, we adopt the parameters of the model which have been determined in the previous paper to fit with the experimental mass spectra. The axial-vector coupling constant is adjusted so that the theoretical decay widths of $D^{*+}(2010)$ coincide with the experimental

*matsuki@tokyo-kasei.ac.jp
†seo@gifu-cwc.ac.jp

values, whose derivation is still contentious and is discussed in Sec. V. We predict the decay widths of the various decay modes, most of which have not yet been experimentally observed. The total widths of some heavy-light mesons are experimentally known, which are compared with our theoretical results. Because $D_s(1^-)$, $D_s(0^+)$, and $D_s(1^+)$ are not allowed kinematically to emit one K meson, assuming the mixing of π^0 and η we have the narrow decay widths of these particles. Likewise, we predict very narrow decay widths of $B_s(0^+)$ and $B_s(1^+)$, which are not yet observed and hence we have used our theoretical values of their masses, 5615 MeV and 5679 MeV, respectively.

In Sec. IV, new sum rules on the decay widths are presented in the limit of $m_Q \to \infty$. In this limit the spin of the heavy quark is decoupled from the orbital angular momentum and the spin of the light quark, and the heavy quark is no more than a spectator in the decay process. The radial wave functions of J^P and $(J+1)^P$ coincide with each other and their masses become degenerate. We say that these degenerate particles belong to a spin multiplet. The decay widths of particles in the same multiplet become equal if we sum over the final states in another multiplet.

Section V is devoted to the conclusions and discussion. In Appendix A, the matrix elements of the axial-vector currents are described by the polarization vectors/tensors in the rest frame of each heavy-light meson. In Appendix B, the properties of the polarization vectors are given as well as the explicit forms of the angular-spin part of the wave function. In Appendix C, the transition amplitudes are expressed in terms of the radial wave functions and the spherical Bessel function, $f(x) \equiv \sin x/x$.

II. LORENTZ INVARIANT EVALUATION OF TRANSITION AMPLITUDES

The wave function of a heavy-light meson with a finite momentum is defined as

$$\begin{aligned}\langle 0|q_i^c(\vec{x},t)Q_j(\vec{y},t)|P\rangle &= \langle 0|q_i^c(\vec{x}-\vec{X}_\xi,0)\\ &\quad\times Q_j(\vec{y}-\vec{X}_\xi,0)|P\rangle e^{-iP\cdot X_\xi}\\ &= \psi_{ij}^{(\xi)}(\vec{x}-\vec{y};P)e^{-iP\cdot X_\xi},\end{aligned} \tag{1}$$

where $X_\xi(=\xi x + (1-\xi)y)$ denotes the position of the heavy-light meson and ξ is a free parameter. If we set $\xi = 0$ or 1, then X_ξ coincides with the position of the heavy quark or light quark, respectively.

We assume chiral interaction of pseudoscalar mesons (π and K) with light quarks. In the present paper we only compute one pseudoscalar particle emission from heavy-light mesons, and then the relevant interaction Lagrangian is as follows:

$$\mathcal{L}_{\rm int} = \frac{g}{\sqrt{2}f_\pi}\bar{q}_i\gamma_\mu\gamma_5 q_j\partial^\mu\phi_{ij}, \tag{2}$$

where g is a dimensionless coupling constant and f_π is the pion decay constant. Here the flavor SU(3) symmetry is assumed and ϕ_{ij}'s represent the octet meson fields, that is,

$$(\phi_{ij}) = \sqrt{2}\begin{pmatrix} \frac{1}{\sqrt{2}}\phi_3 + \frac{1}{\sqrt{6}}\phi_8 & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\phi_3 + \frac{1}{\sqrt{6}}\phi_8 & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\phi_8 \end{pmatrix}. \tag{3}$$

The mixing of π^0 and η is taken into account with a small parameter ϵ as follows:

$$\begin{pmatrix}\pi^0\\ \eta\end{pmatrix} = \frac{1}{\sqrt{1+\epsilon^2}}\begin{pmatrix}1 & \epsilon\\ -\epsilon & 1\end{pmatrix}\begin{pmatrix}\phi_3\\ \phi_8\end{pmatrix}. \tag{4}$$

The transition amplitude for a heavy-light meson with a momentum P into another heavy-light meson with P' and one pseudoscalar particle with k is transformed by inserting the number operator of the heavy quark and by neglecting the sea quark effects as follows:

$$\begin{aligned}\mathcal{M}_{P\to P'} &= \int d^4x\langle P',k|\mathcal{L}_{\rm int}(x)|P\rangle = \int d^4x\langle P',k|\mathcal{L}_{\rm int}(x)\int d^3y Q_\ell^\dagger(y)Q_\ell(y)|P\rangle\\ &\approx \int d^4x\int d^3y O_{ij}\langle P'|Q_\ell^\dagger(y)q_i^{c\dagger}(x)|0\rangle\langle 0|q_j^c(x)Q_\ell(y)|P\rangle e^{ik\cdot x}\\ &= \int d^4x\int d^3y\,{\rm tr}[\psi_{i\ell}^{\prime(\xi)\dagger}(\vec{x}-\vec{y};P')O_{ij}\psi_{j\ell}^{(\xi)}(\vec{x}-\vec{y};P)]e^{ik\cdot x-i(P-P')\cdot X_\xi}\\ &= (2\pi)^4\delta^4(P-P'-k)\int d^3z\,{\rm tr}[\psi_{i\ell}^{\prime(0)\dagger}(\vec{z};P')O_{ij}\psi_{j\ell}^{(0)}(\vec{z};P)]e^{-i\vec{k}\cdot\vec{z}}\end{aligned} \tag{5}$$

$$= (2\pi)^4\delta^4(P-P'-k)\int d^3z\,{\rm tr}[\psi_{i\ell}^{\prime(1)\dagger}(\vec{z};P')O_{ij}\psi_{j\ell}^{(1)}(\vec{z};P)]. \tag{6}$$

In Ref. [2] the following approximate relation has been obtained between the wave function with a finite momentum ($P_3 = \gamma MV$) and the one at rest:

$$\psi^{(0)}_{\alpha\beta}(\vec{x}; P) \approx G_{\alpha\gamma} G_{\beta\delta} \psi_{\gamma\delta}(x_\perp, \gamma z; M) e^{i(M-m_Q)V\gamma z}, \quad (7)$$

and the following relation is derived likewise:

$$\psi^{(1)}_{\alpha\beta}(\vec{x}; P) \approx G_{\alpha\gamma} G_{\beta\delta} \psi_{\gamma\delta}(x_\perp, \gamma z; M) e^{-im_Q V\gamma z}, \quad (8)$$

where G is the boost operator from the rest frame to the moving frame and $\psi(x; M)$ is the wave function in the rest frame. The Lorentz transformation from the rest frame to the moving frame produces the time difference between two constituent quarks, and this time difference is compensated by the free propagation of the heavy quark, which gives rise to the phase factors in Eqs. (7) and (8).

If we evaluate the transition amplitude in the Breit frame, where the initial heavy-light meson is moving to the z direction with a velocity V and the final heavy-light meson is moving in the opposite direction with the same velocity V, and insert Eqs. (7) and (8) into Eqs. (5) and (6), we obtain the following unique expression:

$$\begin{aligned} \mathcal{M}_{P\to P'} &\approx (2\pi)^4 \delta^4(P - P' - k) \int d^3x\, \mathrm{tr}[(G^{-1})^T \psi'^{\dagger}_{i\ell}(x_\perp, \gamma z; M') G^{-1} O_{ij} G \psi_{j\ell}(x_\perp, \gamma z; M) G^T] e^{-2im_Q\gamma V z} \\ &= (2\pi)^4 \delta^4(P - P' - k)\gamma^{-1} \int d^3x\, \mathrm{tr}[(G^{-1})^T \psi'^{\dagger}_{i\ell}(\vec{x}; M') G^{-1} O_{ij} G \psi_{j\ell}(\vec{x}; M) G^T] e^{-2im_Q V z}. \end{aligned} \quad (9)$$

Here two points should be noted. First this is independent of ξ as it should be. Next the phase factor $e^{-2im_Q Vz}$ has appeared in place of the wave function of the emitted pseudoscalar particle, e^{-ikz}, that is absorbed into the delta function to preserve the four-momentum conservation in either Eq. (5) or (6). This is the result of taking into account the heavy-meson's recoil effect.

In general the matrix elements of the axial-vector current between the various spin states of the heavy-light mesons have the following tensor structures:

$$\frac{\langle 0^-|j_{5\mu}|1^-\rangle}{\sqrt{M_2M_1}} = (1+\omega)\varepsilon_{1\mu}\xi^{(k)}_{A1}(\omega) + (\varepsilon_1\cdot v_2)\{(v_1+v_2)_\mu \xi^{(k)}_{A2}(\omega) + (v_1-v_2)_\mu \xi^{(k)}_{A3}(\omega)\} \qquad (k=1,10), \quad (10)$$

$$\frac{\langle 0^-|j_{5\mu}|0^+\rangle}{i\sqrt{M_2M_1}} = (v_1+v_2)_\mu \xi^{(2)}_{A1}(\omega) + (v_1-v_2)_\mu \xi^{(2)}_{A2}(\omega), \quad (11)$$

$$\frac{\langle 1^-|j_{5\mu}|0^+\rangle}{\sqrt{M_2M_1}} = \epsilon_{\mu\nu\rho\sigma} v_1^\nu v_2^\rho \varepsilon_2^{*\sigma} \xi^{(3)}_{A}(\omega), \quad (12)$$

$$\frac{\langle 0^-|j_{5\mu}|1^+\rangle}{\sqrt{M_2M_1}} = \epsilon_{\mu\nu\rho\sigma} v_1^\nu v_2^\rho \varepsilon_1^{\sigma} \xi^{(k)}_{A}(\omega) \qquad (k=4,5), \quad (13)$$

$$\begin{aligned} \frac{\langle 1^-|j_{5\mu}|1^+\rangle}{i\sqrt{M_2M_1}} &= (\varepsilon_2^*\cdot\varepsilon_1)(v_1+v_2)_\mu \xi^{(k)}_{A1}(\omega) + (\varepsilon_2^*\cdot\varepsilon_1)(v_1-v_2)_\mu \xi^{(k)}_{A2}(\omega) + (\varepsilon_2^*\cdot v_1)\varepsilon_{1\mu}\xi^{(k)}_{A3}(\omega) \\ &\quad + (\varepsilon_1\cdot v_2)\varepsilon^*_{2\mu}\xi^{(k)}_{A4}(\omega) \qquad (k=6,7), \end{aligned} \quad (14)$$

$$\frac{\langle 0^-|j_{5\mu}|2^+\rangle}{i\sqrt{M_2M_1}} = \varepsilon_{1\mu\nu} v_2^\nu \xi^{(8)}_{A1}(\omega) + \varepsilon_{1\alpha\beta} v_2^\alpha v_2^\beta \{v_{1\mu}\xi^{(8)}_{A2}(\omega) + v_{2\mu}\xi^{(8)}_{A3}(\omega)\}, \quad (15)$$

$$\begin{aligned} \frac{\langle 1^-|j_{5\mu}|2^+\rangle}{\sqrt{M_2M_1}} &= \epsilon_{\mu\nu\rho\sigma}[\varepsilon_1^{\nu\alpha} v_{2\alpha}\varepsilon_2^{*\rho}\{(v_1+v_2)^\sigma \xi^{(9)}_{A1}(\omega) + (v_1-v_2)^\sigma \xi^{(9)}_{A2}(\omega)\} + v_1^\rho v_2^\sigma \{\varepsilon_1^{\nu\alpha}\varepsilon^*_{2\alpha}\xi^{(9)}_{A3}(\omega) \\ &\quad + \varepsilon_1^{\nu\alpha} v_{2\alpha}(\varepsilon_2^*\cdot v_1)\xi^{(9)}_{A4}(\omega) + \varepsilon_{1\alpha\beta} v_2^\alpha v_2^\beta \varepsilon_2^{*\nu}\xi^{(9)}_{A5}(\omega)\}] + \epsilon_{\alpha\beta\gamma\delta}\varepsilon_1^{\alpha\alpha'} v_{2\alpha'}\varepsilon_2^{*\beta} v_1^\gamma v_2^\delta \{(v_1+v_2)_\mu \xi^{(9)}_{A6}(\omega) \\ &\quad + (v_1-v_2)_\mu \xi^{(9)}_{A7}(\omega)\}, \end{aligned} \quad (16)$$

$$\frac{\langle 1^-|j_{5\mu}|1^-\rangle}{i\sqrt{M_2M_1}} = \epsilon_{\mu\nu\rho\sigma}[\varepsilon_1^\nu\varepsilon_2^{*\rho}\{(v_1+v_2)^\sigma\xi_{A1}^{(11)}(\omega)+(v_1-v_2)^\sigma\xi_{A2}^{(11)}(\omega)\}+v_1^\rho v_2^\sigma\{\varepsilon_1^\nu(\varepsilon_2^*\cdot v_1)\xi_{A3}^{(11)}(\omega) + \varepsilon_2^{*\nu}(\varepsilon_1\cdot v_2)\xi_{A4}^{(11)}(\omega)\}] + \epsilon_{\alpha\beta\gamma\delta}\varepsilon_1^\alpha\varepsilon_2^{*\beta}v_1^\gamma v_2^\delta\{(v_1+v_2)_\mu\xi_{A5}^{(11)}(\omega)+(v_1-v_2)_\mu\xi_{A6}^{(11)}(\omega)\}, \tag{17}$$

where the initial and final quantities have subindices 1 and 2, respectively, $\varepsilon_{i\mu}$ and $\varepsilon_{i\mu\nu}$ are polarization vector and tensor, v_1 and v_2 are velocity vectors defined by $P_1 = M_1v_1$ and $P_2 = M_2v_2$, which in the Breit frame have components, $v_1^0 = v_2^0 = \gamma$, $v_1^3 = -v_2^3 = \gamma V$, and $\omega = v_1\cdot v_2 = P_1\cdot P_2/(M_1M_2)$, which in the Breit frame is related to V by

$$\omega = \gamma^2(1+V^2) = \frac{1+V^2}{1-V^2}. \tag{18}$$

In our model the 1^+ mass-eigenstates appearing in Eqs. (13) and (14) are characterized by the quantum number of $j = L + s_q$, and $^3P_1$ and $^1P_1$ dominant states are denoted by "$^3P_1$" and "$^1P_1$", respectively. This is the reason why we have two kinds of form factors in these equations, i.e., $k = 4, 5$ and $k = 6, 7$. There are also two 1^- states with $k = 1, 10$ in Eq. (10) corresponding to $^3S_1$ and $^3D_1$ states, respectively. Each component of the matrix elements of the axial-vector current in the Breit frame is written down with the polarization vectors/tensors in each rest frame of the initial and final mesons in Appendix A.

In order to obtain explicit forms of the form factors, ξ_A's, we need to calculate the matrix elements by inserting the explicit forms of the wave functions. According to Eq. (9) the matrix elements of the axial-vector current are evaluated in terms of the wave functions in the rest frame of the heavy-light mesons, $\psi_i(\vec r, M)$, as follows:

$$\frac{\langle\psi_2|j_5^\mu|\psi_1\rangle}{2\sqrt{M_1M_2}} \approx \gamma^{-1}\int d^3x\frac{1}{4\pi r^2}\frac{1}{2}\,\mathrm{tr}\left[y_2^*(u_2, i(\vec n\cdot\vec\sigma)v_2)G^{-1}(\rho_1,\sigma_1,\sigma_2,\sigma_3)G\begin{pmatrix}u_1\\ -i(\vec n\cdot\vec\sigma)v_1\end{pmatrix}y_1\right]e^{iqz}, \tag{19}$$

$$\psi_i(\vec r; M) = \begin{pmatrix}u_i(r)\\ -i(\vec n\cdot\vec\sigma)v_i(r)\end{pmatrix}y_i, \tag{20}$$

where $q = -2m_QV$, $\sigma_i \equiv \sigma_i\otimes 1_{2\times2}$, and $\vec n = \vec r/r$. By substitution of the explicit expression for G, the matrix elements of each component of the axial-vector current are obtained as follows:

$$\frac{\langle\psi_2|j_5^0|\psi_1\rangle}{2\sqrt{M_1M_2}} \approx -i\gamma^{-1}\int d^3x\frac{1}{4\pi r^2}(u_2v_1 - v_2u_1)\frac{1}{2}\,\mathrm{tr}[y_2^*(\vec n\cdot\vec\sigma)y_1]e^{iqz}, \tag{21}$$

$$\frac{\langle\psi_2|j_5^3|\psi_1\rangle}{2\sqrt{M_1M_2}} \approx \gamma^{-1}\int d^3x\frac{1}{4\pi r^2}u_2u_1\frac{1}{2}\,\mathrm{tr}[y_2^*\sigma_3y_1]e^{iqz} + \gamma^{-1}\int d^3x\frac{1}{4\pi r^2}v_2v_1\frac{1}{2}\,\mathrm{tr}[y_2^*(\vec n\cdot\vec\sigma)\sigma_3(\vec n\cdot\vec\sigma)y_1]e^{iqz}, \tag{22}$$

$$\frac{\langle\psi_2|j_5^i|\psi_1\rangle}{2\sqrt{M_1M_2}} \approx \int d^3x\frac{1}{4\pi r^2}u_2u_1\frac{1}{2}\,\mathrm{tr}[y_2^*\sigma_iy_1]e^{iqz} + \int d^3x\frac{1}{4\pi r^2}v_2v_1\frac{1}{2}\,\mathrm{tr}[y_2^*(\vec n\cdot\vec\sigma)\sigma_i(\vec n\cdot\vec\sigma)y_1]e^{iqz} + \int d^3x\frac{1}{4\pi r^2}\epsilon_{3ij}V\frac{1}{2}\,\mathrm{tr}[y_2^*\{v_2u_1(\vec n\cdot\vec\sigma)\sigma_j - u_2v_1\sigma_j(\vec n\cdot\vec\sigma)\}y_1]e^{iqz}\qquad (i=1,2). \tag{23}$$

The angular-spin part of the wave functions, $y_i(\Omega)$, are given in Appendix B.

The form factors, ξ_A's, are expressed in terms of the radial wave functions u's and v's in Appendix C, where we have adopted only the terms up to the first order in V. The formula for the decay width,

$$\Gamma = \frac{k_R}{8\pi M_1^2}\frac{1}{2j_1+1}\sum_{\mathrm{pol}}|\langle P_2, k|\mathcal{L}_{\mathrm{int}}(0)|P_1\rangle|^2, \tag{24}$$

is also evaluated in terms of ξ_A's in Appendix C, where k_R is the momentum of the chiral particle in the rest frame of the parent heavy-light meson.

If the mass difference of the parent heavy-light meson with strangeness and daughter nonstrange meson is smaller than the kaon mass, then the parent heavy-light meson with strangeness decays into a heavy-light meson with strangeness and a neutral pion through π^0-η mixing. The mixing parameter ϵ has been related to the current quark masses by Gasser and Leutwyler [7] as follows:

$$\epsilon = \frac{\sqrt3}{4}\frac{m_d - m_u}{m_s - (m_u+m_d)/2} = 1.00\times10^{-2}. \tag{25}$$

As a result, the formula of the decay width is multiplied by the factor $\epsilon^2(= 1.00\times10^{-4})$, in addition to the factor 2/3

TABLE I. Values of the parameters.

α_s	a (GeV^{-1})	$b + m_u (= m_d)$ (MeV/c^2)	$b + m_s$ (MeV/c^2)	m_c (MeV/c^2)	m_b (MeV/c^2)	g^2
0.259(D) 0.392(B)	1.937	86	168	1023	4634	0.608

[the square of the coefficient of ϕ_8 in the octet meson matrix (3)].

III. NUMERICAL RESULTS

In the leading order of $1/m_Q$ expansion, the radial wave functions $u(r)$ and $v(r)$ are determined by the following Dirac equation:

$$\begin{pmatrix} m_q + S + V & -\frac{\partial}{\partial r} + \frac{k}{r} \\ \frac{\partial}{\partial r} + \frac{k}{r} & -m_q - S + V \end{pmatrix} \begin{pmatrix} u(r) \\ v(r) \end{pmatrix} = E \begin{pmatrix} u(r) \\ v(r) \end{pmatrix}, \tag{26}$$

where S and V are confining scalar and Coulombic potentials parametrized by $S(r) = b + \frac{r}{a^2}$ and $V(r) = \frac{4\alpha_s}{3r}$. k is the eigenvalue of the operator $-\beta_q(\vec{L} \cdot \vec{\sigma}_q + 1)$. The states with the total angular momentum j and $j + 1$ have the same value of k, and they have the same radial wave function and the same eigenvalue E. For example, S-wave states 0^- and 1^- with $k = -1$ are degenerate. Taking into account the asymptotic behaviors at infinity and at the origin, $u(r)$ and $v(r)$ are approximated by the following forms:

$$u(r) = r^\gamma \exp\{-r^2/a^2 - (b + m_q)r\} \sum_{i=0}^{7} a_i^{(u)} r^i, \tag{27}$$

$$v(r) = r^\gamma \exp\{-r^2/a^2 - (b + m_q)r\} \sum_{i=0}^{7} a_i^{(v)} r^i, \tag{28}$$

where $\gamma = \sqrt{k^2 - (\frac{4\alpha_s}{3})^2}$. The values of the parameters used in the numerical calculation are listed in Table I, which are determined so as to reproduce the mass spectra of the heavy-light mesons in the order of $1/m_Q$. (See Ref. [1].) In the present calculation the constant b and the light quark mass appear only in the linear combination, and the values of this combination are given in the table.

Here different values of the strong coupling α_s are used for D/D_s and B/B_s mesons. This parameter set is not a unique solution, but the results are not so changed even if we take another set of values given in Ref. [2]. The axial-vector coupling constant g is determined so that the calculated decay widths of $D^{*+}(2010)$ agree with the experimental values. The value of g^2 is taken as 0.608 with a 27% uncertainty. The errors of prediction are caused by the uncertainty of our parameters besides g^2, and only the central values of the calculated widths are shown in Tables II, III, IV, and V. k_R is the momentum of the emitted chiral particle in the rest frame of the parent heavy-light meson. The observed widths in our tables are taken from Ref. [8], and experimental values depending on model assumptions are omitted.

The narrow width of $D^*(1^-)$ is due to the smallness of the Q value. The width of $D_1(2420)$ is smaller than that of $D_1(2430)$ by an order of magnitude. The values of k of the former and the latter are assumed to be -2 and 1, respectively, and their angular momenta neglecting the heavy quark spin denoted as j_ℓ are $\frac{3}{2}$ and $\frac{1}{2}$, respectively. By the parity and the angular momentum conservation, 1^+ is able to decay into 1^- and one pseudoscalar particle with an orbital angular momentum of $L = 0$ or 2. Since j_ℓ of the daughter meson $D^*(1^-)$ is $\frac{1}{2}$, $D_1(2420)(k = -2)$ emits a pseudoscalar particle with $L = 2$ only, while $D_1(2430) \times (k = 1)$ emits one with $L = 0$. This is the widely accepted explanation of the difference of the decay widths of $D_1(2420)$ and $D_1(2430)$. As for the time component of the axial-vector current, this argument is true and the ratio of the matrix elements is almost 0.001. As a matter of fact, the space component parallel to the momentum of the emitted particle in the Breit frame is not so suppressed, and the ratio of the widths amounts to 10.

D_s^*, $D_{s0}^*(2317)$, and $D_{s1}(2460)$ are kinematically forbidden to emit a strange particle, and their decays proceed through the π^0-η mixing. The mixing parameter ϵ is assumed to be 0.01, and these decay widths are smaller than those of the heavy mesons allowed to decay with an emission of a strange particle by 4 orders of magnitude.

$^3P_0(0^+)$, "$^3P_1$"(1^+), and $^3D_1(1^+)$ states of the bottom meson are not yet established, and their masses are taken from our paper [1]. The only observed decay width of $B_2^*(5747)$ is consistent with our prediction.

$^3P_0(0^+)$, "$^3P_1$"(1^+), and $^3D_1(1^+)$ states of the B_s meson are not yet found, and their masses are also taken from our paper [1]. As is the D_s meson, the narrow widths of $^3P_0(0^+)$ and "$^3P_1$"(1^+) states of the B_s meson are explained by kinematics. The predicted mass values of these particles are below the threshold of kaonic decay, and they decay only through the π^0-η mixing.

IV. NEW SUM RULES ON THE DECAY WIDTHS IN THE LIMIT OF $m_Q \to \infty$

In the limit of $m_Q \to \infty$, the spin of the heavy quark is decoupled from the orbital angular momentum and the spin of the light quark, and the radial wave function is solved for each eigenvalue of k. The states with J^P and $(J + 1)^P$ take the same value of k, and they belong to the same spin

TABLE II. Numerical evaluation of the decay widths of excited D mesons. Here "$^3P_1$" and "$^1P_1$" are mixed states of pure $^3P_1$ and $^1P_1$ states. See Ref. [1] for details.

Initial state ($^{2S+1}L_J$)	Final state	k_R (MeV/c)	$\Gamma_{\rm th}$ (MeV)	$\Gamma_{\rm exp}$ (MeV)
D^{*0}	$D^{\pm}\pi^{\mp}$			Not allowed
($^3S_1$)	$D^0\pi^0$	43.1	0.042	<1.3
$D^{*\pm}$	$D^0\pi^{\pm}$	39.5	Input	0.065 ± 0.018
($^3S_1$)	$D^{\pm}\pi^0$	38.2	Input	0.029 ± 0.08
$D_0^*(2400)^0$	$D^{\pm}\pi^{\mp}$	414.2	0.99×10^2	
	$D^0\pi^0$	419.4	0.50×10^2	
($^3P_0$)	$D\pi$ (sum)		2.5×10^2	
	All			261 ± 50
$D_0^*(2400)^{\pm}$	$D^0\pi^{\pm}$	461.3	1.2×10^2	
	$D^{\pm}\pi^0$	458.5	0.61×10^2	
($^3P_0$)	$D\pi$ (sum)		1.8×10^2	
	All			$283 \pm 24 \pm 34$
$D_0^*(2400)$	$D^*\pi$		0	Not seen
$D_1(2430)$	$D\pi$		0	
$D_1(2430)^0$	$D^{*\pm}\pi^{\mp}$	358.7	0.76×10^2	
	$D^{*0}\pi^0$	363.1	0.39×10^2	
("$^3P_1$")	$D^*\pi$ (sum)		1.9×10^2	
	All			$384^{+107}_{-75} \pm 74$
$D_1(2420)$	$D\pi$		0	Not seen
$D_1(2420)^0$	$D^{*\pm}\pi^{\mp}$	354.5	2.7	
	$D^{*0}\pi^0$	358.9	1.5	
("$^1P_1$")	$D^*\pi$ (sum)		6.9	
	All			20.4 ± 1.7
$D_1(2420)^{\pm}$	$D^{*0}\pi^{\pm}$	358.4	2.9	
	$D^{*\pm}\pi^0$	357.0	1.4	
("$^1P_1$")	$D^*\pi$ (sum)		4.3	
	All			25 ± 6
$D_2^*(2460)^0$	$D^{\pm}\pi^{\mp}$	505.5	6.8	
	$D^0\pi^0$	510.2	3.6	
($^3P_2$)	$D^{*\pm}\pi^{\mp}$	389.2	2.6	
	$D^{*0}\pi^0$	393.4	1.4	
	$D\pi + D^*\pi$ (sum)		24	
	All			43 ± 4
$D_2^*(2460)^{\pm}$	$D^0\pi^{\pm}$	508.4	7.1	
	$D^{\pm}\pi^0$	505.6	3.4	
($^3P_2$)	$D^{*0}\pi^{\pm}$	391.2	2.7	
	$D^{*\pm}\pi^0$	389.7	1.3	
	$D\pi + D^*\pi$ (sum)		15	
	All			37 ± 6
$D(2760)^0$	$D^{\pm}\pi^{\mp}$	739.6	4.8	
	$D^0\pi^0$	743.6	2.4	
($^3D_1$)	$D^{*\pm}\pi^{\mp}$	638.9	0.79	
	$D^{*0}\pi^0$	642.1	0.41	
	$D\pi + D^*\pi$ (sum)		14	
	All			60.9 ± 5.1
$D(2760)^{\pm}$	$D^0\pi^{\pm}$	742.9	4.9	
	$D^{\pm}\pi^0$	740.3	2.4	
($^3D_1$)	$D^{*0}\pi^{\pm}$	641.3	0.81	
	$D^{*\pm}\pi^0$	639.6	0.39	
	$D\pi + D^*\pi$ (sum)		8.5	
	All			No exp

TABLE III. Numerical evaluation of the decay widths of excited D_s mesons.

Initial state ($^{2S+1}L_J$)	Final state	k_R (MeV/c)	Γ_{th} (MeV)	Γ_{exp} (MeV)
$D_s^{*\pm}(^3S_1)$	$D_s^{\pm}\pi^0$	47.8	8.0×10^{-6}	<0.11
$D_{s0}^*(2317)^{\pm}$	$D_s^{\pm}\pi^0$	297.7	3.8×10^{-3}	<3.8
($^3P_0$)	$D_s^{*\pm}\pi^0$	148.0	0	
$D_{s1}(2460)^{\pm}$	$D_s^{\pm}\pi^0$	424.8	0	
("$^3P_1$")	$D_s^{*\pm}\pi^0$	297.3	3.9×10^{-3}	<1.7
$D_{s1}(2536)^{\pm}$	$D^0K^{\pm}$	390.4	0	Not seen
	$D^{\pm}K^0$	385.6	0	Not seen
("$^1P_1$")	$D^{*0}K^{\pm}$	165.3	0.066	
	$D^{*\pm}K^0$	159.4	0.055	
	D^*K (sum)		0.12	
	All			<2.3
$D_{s2}(2573)^{\pm}$	$D^0K^{\pm}$	433.9	3.4	
	$D^{\pm}K^0$	429.4	3.3	
($^3P_2$)	$D^{*0}K^{\pm}$	242.6	0.27	
	$D^{*\pm}K^0$	238.3	0.24	
	$DK + D^*K$ (sum)		7.2	
	All			20 ± 5
$D_s(2818)^{\pm}$	$D^0K^{\pm}$	673.2	6.6	
	$D^{\pm}K^0$	669.8	6.4	
($^3D_1$)	$D^{*0}K^{\pm}$	547.3	1.1	
Not observed	$D^{*\pm}K^0$	544.8	1.0	
	$DK + D^*K$ (sum)		15	

TABLE IV. Numerical evaluation of the decay widths of excited B mesons.

Initial state ($^{2S+1}L_J$)	Final state	k_R (MeV/c)	Γ_{th} (MeV)	Γ_{exp} (MeV)
$B_0^*(5590)^0$	$B^{\pm}\pi^{\mp}$	269.9	35	
($^3P_0$)	$B^0\pi^0$	271.8	17	
Not observed	$B\pi$ (sum)		87	No exp
$B_1(5646)^0$	$B^{*\pm}\pi^{\mp}$	280.7	37	
("$^3P_1$")	$B^{*0}\pi^0$	282.8	19	
Not observed	$B^*\pi$ (sum)		93	No exp
$B_1(5721)^0$	$B^{*\pm}\pi^{\mp}$	360.0	7.5	
("$^1P_1$")	$B^{*0}\pi^0$	361.7	3.8	
	$B^*\pi$ (sum)		19	
	All			No exp
$B_2^*(5747)^0$	$B^{\pm}\pi^{\mp}$	424.4	6.4	
	$B^0\pi^0$	425.5	3.2	
($^3P_2$)	$B^{*\pm}\pi^{\mp}$	379.5	5.7	
	$B^{*0}\pi^0$	381.1	2.9	
	$B\pi + B^*\pi$ (sum)		30	
	All			$22.7^{+3.8+3.2}_{+3.2-10.2}$
$B(5985)^0$	$B^{\pm}\pi^{\mp}$	651.0	5.8	
	$B^0\pi^0$	651.6	2.9	
($^3D_1$) Not observed	$B^{*\pm}\pi^{\mp}$	609.4	2.3	
	$B^{*0}\pi^0$	610.3	1.1	
	$B\pi + B^*\pi$ (sum)		20	No exp

TABLE V. Numerical evaluation of the decay widths of excited B_s mesons.

Initial state ($^{2S+1}L_J$)	Final state	k_R (MeV/c)	$\Gamma_{\rm th}$ (MeV)	$\Gamma_{\rm exp}$ (MeV)
$B^*_{s0}(5615)^0$	$B^0_s\pi^0$	204.2	1.6×10^{-3}	No exp
($^3P_0$) Not observed	$B^{*0}_s\pi^0$		0	
$B_{s1}(5679)^0$	$B^0_s\pi^0$		0	
(“$^3P_1$”) Not observed	$B^{*0}_s\pi^0$	224.0	1.9×10^{-3}	No exp
$B_{s1}(5830)^0$	$B^\pm K^\mp$	229.2	0	
	B^0K^0	230.5	0	
(“$^1P_1$”)	$B^{*\pm}K^\mp$	93.6	0.011	
	$B^{*0}K^0$	98.3	0.014	
	B^*K (sum)		0.036	
	All			No exp
$B^*_{s2}(5840)^0$	$B^\pm K^\mp$	250.6	0.56	
	B^0K^0	251.8	0.58	
($^3P_2$)	$B^{*\pm}K^\mp$	135.3	0.041	
	$B^{*0}K^0$	138.6	0.046	
	$BK+B^*K$ (sum)		1.8	
	All			No exp
$B_s(6025)^0$	$B^\pm K^\mp$	523.1	10	
	B^0K^0	523.5	10	
($^3D_1$)	$B^{*\pm}K^\mp$	465.9	3.5	
Not observed	$B^{*0}K^0$	466.8	3.5	
	$BK+B^*K$ (sum)		41	No exp

multiplet. They are mixed by the spin rotation of the heavy quark, $\mathcal{R}$. If we denote the transformation of the state as

$$\mathcal{R}\,|P,k,p\rangle = |P,k,q\rangle, \quad (29)$$

then

$$\mathcal{R}\Big(\sum_p |P,k,p\rangle\langle P,k,p|\Big)\mathcal{R}^\dagger = \sum_q |P,k,q\rangle\langle P,k,q|, \quad (30)$$

where P stands for the four-momentum of the heavy-light meson, k is the eigenvalue to discriminate the spin multiplet, and p and q label the polarization of the total spin. The sum is taken over all states belonging to the same spin multiplet. Because the axial-vector current for the light quark and hence the interaction given by Eq. (2) are invariant under $\mathcal{R}$, we obtain the following relation among the transition amplitudes:

$$\begin{aligned}\sum_{p'}|\langle P',k',p'|\mathcal{L}_{\rm int}|P,k,p\rangle|^2 &= \sum_{p'}|\langle P',k',p'|\mathcal{R}^\dagger\mathcal{R}\mathcal{L}_{\rm int}\mathcal{R}^\dagger\mathcal{R}|P,k,p\rangle|^2\\ &= \langle P,k,p|\mathcal{R}^\dagger\mathcal{L}_{\rm int}\mathcal{R}\Big(\sum_{p'}|P',k',p'\rangle\langle P',k',p'|\Big)\mathcal{R}^\dagger\mathcal{L}_{\rm int}\mathcal{R}|P,k,p\rangle\\ &= \langle P,k,q|\mathcal{L}_{\rm int}\sum_{q'}|P',k',q'\rangle\langle P',k',q'|\mathcal{L}_{\rm int}|P,k,q\rangle = \sum_{q'}|\langle P',k',q'|\mathcal{L}_{\rm int}|P,k,q\rangle|^2. \quad (31)\end{aligned}$$

Neglecting the mass difference of the mesons in the same multiplet, we obtain the following relation on the decay widths:

$$\sum_{p'}\Gamma(|P,k,p\rangle \to |P',k',p'\rangle + \pi/K) = \sum_{p'}\Gamma(|P,k,q\rangle \to |P',k',p'\rangle + \pi/K), \quad (32)$$

where the sum is taken over all states belonging to the same spin multiplet and p and q are in the same spin multiplet also,

$$\frac{1}{2j+1}\sum_{p(j),p'}\Gamma(|P,k,p(j)\rangle \to |P',k',p'\rangle + \pi/K) = \frac{1}{2(j+1)+1}\sum_{p(j+1),p'}\Gamma(|P,k,p(j+1)\rangle \to |P',k',p'\rangle + \pi/K). \quad (33)$$

We are able to explicitly confirm these sum rules in the case of the initial states $(0^+,1^+)$ with $k=1$ and $(1^+,2^+)$

with $k = -2$ and the final states $(0^-, 1^-)$ with $k = -1$ by using the expressions of the decay widths given in Appendix C.

As a matter of fact, there exists mass difference between the mesons belonging to the same spin multiplet and the sum rules are violated more or less by the kinematical phase factor. Among the various decay modes the following sum rules are not largely affected by the kinematical phase factor because of relatively small mass difference between initial and final heavy-light mesons, and these rules should be confirmed by experiments:

$$\begin{aligned}\Gamma(D^*_{s0}(0^+,2317)&\to D_s(0^-,1968)+\pi)\\&=\Gamma(D_{s1}(1^+,2460)\to D^*_s(1^-,2112)+\pi),\\\Gamma(B^*_{s0}(0^+,5615)&\to B_s(0^-,5366)+\pi)\\&=\Gamma(B_{s1}(1^+,5679)\to B^*_s(1^-,5415)+\pi).\end{aligned} \tag{34}$$

Here the decay modes $0^+ \to 1^- + \pi$ and $1^+ \to 0^- + \pi$ are forbidden and the sum rules are simply reduced to the above equations.

V. CONCLUSIONS AND DISCUSSION

The wave function of the heavy-light meson in the moving frame is approximately related to the one in the rest frame. Then the transition amplitudes of the excited states of the heavy-light meson to the lower states by emitting a chiral particle are expressed in terms of the wave functions of the heavy-light meson in the rest frame. The plane wave of the emitted particle, e^{-ikz}, has been inserted without an explanation by all authors in the preceding works, but we have found that a phase factor e^{-2im_QVz} should be inserted instead of e^{-ikz}. The boost operator affects the matrix elements of the components of the axial-vector currents perpendicular to the momentum of the emitted chiral particle. However these components are irrelevant in our calculation because the matrix elements of the axial-vector current are contracted with the momentum of the emitted chiral particle.

We have used the wave functions of the heavy-light meson obtained by the relativistic potential model in Ref. [1]. Because ${}^3P_0(0^+)$, "${}^3P_1$"(1^+), and ${}^3D_1(1^+)$ states of the B and B_s mesons have not yet been observed, we have used our predicted mass values for these particles [1].

The partial decay widths of the excited heavy-light meson emitting one chiral particle are numerically evaluated, and the predicted values are consistent with the experimentally observed total decay widths of excited heavy-light mesons.

One of notable predictions is the narrow decay widths of undiscovered $B_s(0^+, 1^+)$ mesons. Our predicted mass values of these are below the threshold of the kaonic decay, and they decay through the π^0-η mixing with narrow decay widths of a few keV. Our predictions await the experimental confirmation.

The parameters used in our calculation are not much different from those used by Di Pierro and Eichten [5], but the plain wave of the emitted particle (e^{-ikz}) in the transition amplitudes is replaced by another phase factor e^{-2im_QVz} due to the recoil effect. This replacement makes the decay widths large in general, because the oscillatory cancellation due to this phase factor becomes weak.

There are numerous studies to compute the decay widths using the Schwinger-Dyson amplitudes, for example, Ref. [9] and references therein. The coupling constant $\hat{g}$ appearing in these articles and our axial-vector coupling constant g appearing in Eq. (2) are related to each other in the case of $D^* \to D + \pi$ or the kinematically forbidden case of $B^* \to B + \pi$ as follows:

$$\begin{aligned}&k^\mu\langle 0^-\pi|j_{5\mu}|1^-\rangle = \langle H\pi|H^*\rangle \text{ (in their notation)},\\&\text{left-hand side} \approx \frac{2g\sqrt{M_1M_2}}{f_\pi}(k^0\eta^{(1)}_{A1} - k^3\eta^{(1)}_{A2})\epsilon_3\\&\qquad = -\frac{2g\sqrt{M_1M_2}}{f_\pi}k^3\eta^{(1)}_{A2}\epsilon_3, \qquad (\eta^{(1)}_{A1}=0),\\&\text{right-hand side} = -\frac{2\hat{g}\sqrt{M_1M_2}}{f_\pi}k^3\epsilon_3, \qquad g\eta^{(1)}_{A2} \approx \hat{g}.\end{aligned} \tag{35}$$

$\eta^{(1)}$'s are defined in Eqs. (C1) and (C2) in Appendix C, which are the overlapping integrals of the initial and final wave functions. Coupling constants thus determined may be used in the study of dissociation processes $\pi + J/\psi \to D + \bar{D}$ or $\pi + \Upsilon \to B + \bar{B}$ by exchanging D^* or B^*. We have obtained $g\eta^{(1)}_{A2} = \sqrt{0.608} \times 0.74 = 0.577$ while they gave the value $\hat{g} = 0.53$ using the leptonic decay of the heavy-light system [9]. Agreement of these values is not surprising because the authors of Ref. [9] also have fitted couplings with the experimental data.

In the course of the calculation, new sum rules are found to hold in the limit of $m_Q \to \infty$. Among these rules, $\Gamma(D^*_{s0}(2317) \to D_s + \pi) = \Gamma(D_{s1}(2460) \to D^*_s + \pi)$ and $\Gamma(B^*_{s0}(5615) \to B_s + \pi) = \Gamma(B_{s1}(5679) \to B^*_s + \pi)$ are supposed to hold with a good accuracy and should be verified by future experiments.

The radiative decays of the heavy-light mesons are under study in the same formalism as the present one and the results will be published in the near future.

APPENDIX A: TENSOR STRUCTURES OF THE MATRIX ELEMENTS OF THE AXIAL-VECTOR CURRENT

The matrix elements of the axial-vector current are expressed by the polarization vectors/tensors in the rest frame of each heavy-light meson as follows:

$$\frac{\langle 0^-|j_5^\mu|1^-\rangle}{2\sqrt{M_2M_1}}=\gamma^2(\gamma V\varepsilon^3\{\xi_{A1}^{(k)}+2\xi_{A2}^{(k)}\},\ \varepsilon^1\xi_{A1}^{(k)},\varepsilon^2\xi_{A1}^{(k)},\gamma\varepsilon^3\{\xi_{A1}^{(k)}+2V^2\xi_{A3}^{(k)}\}),\quad (k=1,10) \tag{A1}$$

$$\frac{\langle 0^-|j_5^\mu|0^+\rangle}{2i\sqrt{M_2M_1}} = \gamma(\xi_{A1}^{(2)},0,0,V\xi_{A2}^{(2)}), \tag{A2}$$

$$\frac{\langle 1^-|j_5^\mu|0^+\rangle}{2\sqrt{M_2M_1}} = \gamma^2V(0,\varepsilon^{*2},-\varepsilon^{*1},0)\xi_A^{(3)}, \tag{A3}$$

$$\frac{\langle 0^-|j_5^\mu|1^+\rangle}{2\sqrt{M_2M_1}} = \gamma^2V(0,\varepsilon^2,-\varepsilon^1,0)\xi_A^{(k)} \qquad (k=4,5), \tag{A4}$$

$$\begin{aligned}\frac{\langle 1^-|j_5^0|1^+\rangle}{2i\sqrt{M_2M_1}} &= -\gamma[(\omega\varepsilon_1^3\varepsilon_2^{*3}+\varepsilon_1^i\varepsilon_2^{*i})\xi_{A1}^{(k)} + \gamma^2V^2\varepsilon_1^3\varepsilon_2^{*3}\{\xi_{A3}^{(k)}+\xi_{A4}^{(k)}\}],\\ \frac{\langle 1^-|j_5^3|1^+\rangle}{2i\sqrt{M_2M_1}} &= -\gamma V[(\omega\varepsilon_1^3\varepsilon_2^{*3}+\varepsilon_1^i\varepsilon_2^{*i})\xi_{A2}^{(k)} + \gamma^2\varepsilon_1^3\varepsilon_2^{*3}\{\xi_{A3}^{(k)}-\xi_{A4}^{(k)}\}],\\ \frac{\langle 1^-|j_5^1|1^+\rangle}{2i\sqrt{M_2M_1}} &= \gamma^2V\{-\varepsilon_1^1\varepsilon_2^{*3}\xi_{A3}^{(k)}+\varepsilon_1^3\varepsilon_2^{*1}\xi_{A4}^{(k)}\}, \qquad (k=6,7)\end{aligned} \tag{A5}$$

$$\frac{\langle 0^-|j_5^\mu|2^+\rangle}{2i\sqrt{M_2M_1}}=\gamma^2V(\gamma V\varepsilon^{33}[\xi_{A1}^{(8)}+2\gamma^2\{\xi_{A2}^{(8)}+\xi_{A3}^{(8)}\}],\ \varepsilon^{13}\xi_{A1}^{(8)},\varepsilon^{23}\xi_{A1}^{(8)},\gamma\varepsilon^{33}[\xi_{A1}^{(8)}+2\gamma^2V^2\{\xi_{A2}^{(8)}-\xi_{A3}^{(8)}\}]), \tag{A6}$$

$$\begin{aligned}\frac{\langle 1^-|j_5^0|2^+\rangle}{2\sqrt{M_2M_1}} &= 2\gamma^3V^2(\varepsilon_1^{13}\varepsilon_2^{*2}-\varepsilon_1^{23}\varepsilon^{*1})\{\xi_{A2}^{(9)}-2\gamma^2\xi_{A6}^{(9)}\},\\ \frac{\langle 1^-|j_5^3|2^+\rangle}{2\sqrt{M_2M_1}} &= 2\gamma^3V(\varepsilon_1^{13}\varepsilon_2^{*2}-\varepsilon_1^{23}\varepsilon^{*1})\{\xi_{A1}^{(9)}-2\gamma^2V^2\xi_{A7}^{(9)}\},\\ \frac{\langle 1^-|j_5^1|2^+\rangle}{2\sqrt{M_2M_1}} &= 2\gamma^4V\{(\varepsilon_1^{23}\varepsilon_2^{*3}-\varepsilon_1^{33}\varepsilon_2^{*2})\xi_{A1}^{(9)} + V^2(\varepsilon_1^{23}\varepsilon_2^{*3}+\varepsilon_1^{33}\varepsilon_2^{*2})\xi_{A2}^{(9)}\}\\ &\quad -\gamma^2V[(\omega\varepsilon_1^{23}\varepsilon_2^{*3}+\varepsilon_1^{2j}\varepsilon_2^{*j})\xi_{A3}^{(9)} + 4\gamma^4V^2\{\varepsilon_1^{23}\varepsilon_2^{*3}\xi_{A4}^{(9)}-\varepsilon_1^{33}\varepsilon_2^{*2}\xi_{A5}^{(9)}\}],\end{aligned} \tag{A7}$$

$$\begin{aligned}\frac{\langle 1^-|j_5^0|1^-\rangle}{2i\sqrt{M_2M_1}} &= \gamma V(\varepsilon_1^1\varepsilon_2^{*2}-\varepsilon_1^2\varepsilon_2^{*1})\{\xi_{A2}^{(11)}-2\gamma^2\xi_{A5}^{(11)}\},\\ \frac{\langle 1^-|j_5^3|1^-\rangle}{2i\sqrt{M_2M_1}} &= \gamma(\varepsilon_1^1\varepsilon_2^{*2}-\varepsilon_1^2\varepsilon_2^{*1})\{\xi_{A1}^{(11)}-2\gamma^2V^2\xi_{A6}^{(11)}\},\\ \frac{\langle 1^-|j_5^1|1^-\rangle}{2i\sqrt{M_2M_1}} &= \gamma^2\{(\varepsilon_1^2\varepsilon_2^{*3}-\varepsilon_1^3\varepsilon_2^{*2})\xi_{A1}^{(11)} + V^2(\varepsilon_1^2\varepsilon_2^{*3}+\varepsilon_1^3\varepsilon_2^{*2})\xi_{A2}^{(11)}\}\\ &\quad -2\gamma^4V^2\{\varepsilon_1^2\varepsilon_2^{*3}\xi_{A3}^{(11)}-\varepsilon_1^3\varepsilon_2^{*2}\xi_{A4}^{(11)}\},\end{aligned} \tag{A8}$$

where the repeated roman indices should be understood as contraction with respect to the spatial components perpendicular to the momentum, that is, $\varepsilon_1^i\varepsilon_2^{*i} \equiv \sum_{i=1,2}\varepsilon_1^i\varepsilon_2^{*i}$. The lower indices 1 and 2 of M and the polarization vectors/tensors stand for the initial and the final one, respectively.

APPENDIX B: THE ANGULAR-SPIN PART OF THE WAVE FUNCTIONS

Polarization vectors and tensors of mesons satisfy the following orthonormal conditions and completeness conditions:

$$(\vec{\varepsilon}^{(p)*}\cdot\vec{\varepsilon}^{(q)}) = \delta_{pq} \qquad (p,q=1,2,3), \tag{B1}$$

$$\sum_p \varepsilon_i^{(p)*}\varepsilon_j^{(p)} = \delta_{ij}, \tag{B2}$$

$$\sum_{j,k}\varepsilon_{jk}^{(p)*}\varepsilon_{jk}^{(q)} = \delta_{pq} \qquad (p,q=1,2,3,4,5), \tag{B3}$$

$$\sum_p \varepsilon_{jk}^{(p)}\varepsilon_{j'k'}^{(p)*} = \frac{1}{2}(\delta_{jj'}\delta_{kk'}+\delta_{jk'}\delta_{kj'}-\frac{2}{3}\delta_{jk}\delta_{j'k'}). \tag{B4}$$

The angular-spin part of the wave functions of various spins takes the expression with the polarization vectors or tensors as listed in Table VI.

They satisfy the orthonormal conditions,

$$\int\frac{d\Omega}{4\pi}\frac{1}{2}\,\mathrm{tr}[y^{(p)*}y^{(q)}] = \delta_{pq}, \tag{B5}$$

where p and q represent both the polarization state and the spin state.

APPENDIX C: THE DECAY WIDTHS IN TERMS OF THE RADIAL WAVE FUNCTIONS

The transition amplitudes are expressed by the radial wave functions and the spherical Bessel function, $f(x) \equiv \sin x/x$, as follows.

TABLE VI. The angular-spin part of wave functions of low lying states.

$^{2S+1}L_J$	k	$y^{(p)}$
$^1S_0$	-1	1
$^3S_1$	-1	$\vec{\sigma}\cdot\vec{\varepsilon}^{(p)}$
$^3P_0$	1	$\vec{n}\cdot\vec{\sigma}$
"$^3P_1$"	1	$(\vec{n}\cdot\vec{\sigma})(\vec{\sigma}\cdot\vec{\varepsilon}^{(p)})$
"$^1P_1$"	-2	$\frac{1}{\sqrt{2}}\{3\vec{n}\cdot\vec{\varepsilon}^{(p)}-(\vec{n}\cdot\vec{\sigma})(\vec{\sigma}\cdot\vec{\varepsilon}^{(p)})\}$
$^3P_2$	-2	$\sqrt{3}n_j\sigma_k\varepsilon_{jk}$
$^3D_1$	2	$\frac{1}{\sqrt{2}}\{3(\vec{n}\cdot\vec{\varepsilon}^{(p)})(\vec{n}\cdot\vec{\sigma})-(\vec{\sigma}\cdot\vec{\varepsilon}^{(p)})\}$

(1) $1^-(^3S_1)\rightarrow 0^-(^1S_0)$:

$$\eta_{A1}^{(1)}\equiv V(\xi_{A1}^{(1)}+2\xi_{A2}^{(1)}) = -\int drf'(qr)(u_2v_1-v_2u_1)+O(V^3), \tag{C1}$$

$$\eta_{A2}^{(1)}\equiv \xi_{A1}^{(1)}+2V^2\xi_{A3}^{(1)} = \int dr[f(qr)u_2u_1-\{f(qr)+2f''(qr)\}v_2v_1] + O(V^2), \tag{C2}$$

$$\xi_{A1}^{(1)} = \int dr[f(qr)u_2u_1+f''(qr)v_2v_1 - Vf'(qr)(u_2v_1+v_2u_1)]+O(V^2), \tag{C3}$$

$$\Gamma=\frac{(g\zeta)^2k_RM_2}{6\pi M_1f_\pi^2}\{\omega_B\eta_{A1}^{(1)}-k_B\eta_{A2}^{(1)}\}^2. \tag{C4}$$

(2) $0^+(^3P_0)\rightarrow 0^-(^1S_0)$:

$$\xi_{A1}^{(2)} = -\int drf(qr)(u_2v_1-v_2u_1)+O(V^2), \tag{C5}$$

$$\eta_{A}^{(2)}\equiv V\xi_{A2}^{(2)} = -\int drf'(qr)(u_2u_1+v_2v_1)+O(V^3), \tag{C6}$$

$$\Gamma=\frac{(g\zeta)^2M_2k_R}{2\pi f_\pi^2M_1}\{\omega_B\xi_{A1}^{(2)}-k_B\eta_{A}^{(2)}\}^2. \tag{C7}$$

(3) $0^+(^3P_0)\rightarrow 1^-(^3S_1)$:

$$V\xi_{A}^{(3)} = \int dr\{-f'(qr)(u_2u_1-v_2v_1) - Vf(qr)u_2v_1+Vf''(qr)v_2u_1\}+O(V^3), \tag{C8}$$

$$\Gamma=0. \tag{C9}$$

(4) $1^+($"$^3P_1$"$)\rightarrow 0^-(^1S_0)$

$$V\xi_{A}^{(4)} = \int dr\{-f'(qr)(u_2u_1-v_2v_1) - Vf(qr)u_2v_1+Vf''(qr)v_2u_1\}+O(V^3), \tag{C10}$$

$$\Gamma=0. \tag{C11}$$

(5) $1^+($"$^1P_1$"$)\rightarrow 0^-(^1S_0)$:

$$V\xi_{A}^{(5)} = \frac{1}{\sqrt{2}}\int dr\Big[f'(qr)(u_2u_1-v_2v_1)-\frac{V}{2}\{f(qr) + 3f''(qr)\}u_2v_1+\frac{V}{2}\{3f(qr) + f''(qr)\}v_2u_1\Big]+O(V^3), \tag{C12}$$

$$\Gamma=0. \tag{C13}$$

(6) $1^+($"$^3P_1$"$)\rightarrow 1^-(^3S_1)$:

$$\xi_{A1}^{(6)} = \int drf(qr)(u_2v_1-v_2u_1)+O(V^2), \tag{C14}$$

$$\eta_{A}^{(6)}\equiv V\xi_{A2}^{(6)} = \int drf'(qr)(u_2u_1+v_2v_1)+O(V^3), \tag{C15}$$

$$V\xi_{A3}^{(6)}=V\xi_{A4}^{(6)} = \int dr\{-f'(qr)(u_2u_1-v_2v_1) - Vf(qr)u_2v_1+Vf''(qr)v_2u_1\}+O(V^3), \tag{C16}$$

$$\Gamma=\frac{(g\zeta)^2k_RM_2}{2\pi M_1f_\pi^2}\{\omega_B\xi_{A1}^{(6)}-k_B\eta_{A}^{(6)}\}^2. \tag{C17}$$

(7) $1^+(``{}^1P_1") \to 1^-({}^3S_1)$:

$$\xi_{A1}^{(7)} = \frac{1}{2\sqrt{2}} \int dr\{f(qr) + 3f''(qr)\}(u_2 v_1 - v_2 u_1) + O(V^4), \tag{C18}$$

$$\eta_{A1}^{(7)} \equiv \xi_{A1}^{(7)} + V^2\{\xi_{A3}^{(7)} + \xi_{A4}^{(7)}\} = -2\xi_{A1}^{(7)} + O(V^4), \tag{C19}$$

$$\eta_{A2}^{(7)} \equiv V\xi_{A2}^{(7)} = \frac{1}{\sqrt{2}} \int dr[-f'(qr)u_2 u_1 + \{2f'(qr) + 3f'''(qr)\}v_2 v_1] + O(V^3), \tag{C20}$$

$$\eta_{A3}^{(7)} \equiv V\{\xi_{A2}^{(7)} + \xi_{A3}^{(7)} - \xi_{A4}^{(7)}\} = -2\eta_{A2}^{(7)} + O(V^3), \tag{C21}$$

$$V\xi_{A3}^{(7)} = \frac{1}{\sqrt{2}} \int dr\Big[f'(qr)u_2 u_1 + \{2f'(qr) + 3f'''(qr)\}v_2 v_1 - \frac{V}{2}\{f(qr) + 3f''(qr)\}u_2 v_1 - \frac{V}{2}\{3f(qr) + 5f''(qr)\}v_2 u_1\Big] + O(V^3), \tag{C22}$$

$$V\xi_{A4}^{(7)} = \frac{1}{\sqrt{2}} \int dr[-2f'(qr)u_2 u_1 - \{f'(qr) + 3f'''(qr)\}v_2 v_1 + V\{f(qr) + 3f''(qr)\}u_2 v_1 + 2Vf''(qr)v_2 u_1] + O(V^3), \tag{C23}$$

$$\Gamma = \frac{(g\zeta)^2 k_R M_2}{2\pi M_1 f_\pi^2} \frac{1}{3}[\{\omega_B \xi_{A1}^{(7)} - k_B \eta_{A2}^{(7)}\}^2 \times 2 + \{\omega_B \eta_{A1}^{(7)} - k_B \eta_{A3}^{(7)}\}^2] = \frac{(g\zeta)^2 k_R M_2}{\pi M_1 f_\pi^2}\{\omega_B \xi_{A1}^{(7)} - k_B \eta_{A2}^{(7)}\}^2. \tag{C24}$$

(8) $2^+({}^3P_2) \to 0^-({}^1S_0)$:

$$\eta_{A1}^{(8)} \equiv V^2\{\xi_{A1}^{(8)} + 2\xi_{A2}^{(6)} + 2\xi_{A3}^{(6)}\} = \frac{\sqrt{3}}{2} \int dr\{f(qr) + 3f''(qr)\}(u_2 v_1 - v_2 u_1) + O(V^4), \tag{C25}$$

$$\eta_{A2}^{(8)} \equiv V[\xi_{A1}^{(8)} + 2V^2\{\xi_{A2}^{(6)} - \xi_{A3}^{(6)}\}] = \sqrt{3} \int dr[-f'(qr)u_2 u_1 + \{2f'(qr) + 3f'''(qr)\}v_2 v_1] + O(V^3), \tag{C26}$$

$$V\xi_{A1}^{(8)} = \sqrt{3} \int dr[-f'(qr)u_2 u_1 - \{f'(qr) + 2f'''(qr)\}v_2 v_1 + \frac{V}{2}\{f(qr) + 3f''(qr)\}(u_2 v_1 + v_2 u_1)] + O(V^3), \tag{C27}$$

$$\Gamma = \frac{(g\zeta)^2 k_R M_2}{15\pi M_1 f_\pi^2}\{\omega_B \eta_{A1}^{(8)} - k_B \eta_{A2}^{(8)}\}^2. \tag{C28}$$

(9) $2^+({}^3P_2) \to 1^-({}^3S_1)$:

$$\eta_{A1}^{(9)} \equiv 2V^2\{\xi_{A2}^{(9)} - 2\xi_{A6}^{(9)}\} = -\frac{\sqrt{3}}{2} \int dr\{f(qr) + 3f''(qr)\}(u_2 v_1 - v_2 u_1) + O(V^4), \tag{C29}$$

$$\eta_{A2}^{(9)} \equiv 2V\{\xi_{A1}^{(9)} - 2V^2\xi_{A7}^{(9)}\} = \sqrt{3} \int dr[f'(qr)u_2 u_1 - \{2f'(qr) + 3f'''(qr)\}v_2 v_1] + O(V^3), \tag{C30}$$

$$-V\xi_{A3}^{(9)} = \sqrt{3} \int dr[-\{f'(qr) + f'''(qr)\}v_2 v_1 + V\{f(qr) + f''(qr)\}v_2 u_1] + O(V^3), \tag{C31}$$

$$\eta_{A3}^{(9)} \equiv 2V[-\xi_{A1}^{(9)} + V^2\{\xi_{A2}^{(9)} + 2\xi_{A5}^{(9)}\}]$$
$$= \sqrt{3}\int dr[-f'(qr)u_2u_1 - \{f'(qr) + 2f'''(qr)\}v_2v_1 + \frac{V}{2}\{f(qr) + 3f''(qr)\}(u_2v_1 + v_2u_1)] + O(V^3), \quad \text{(C32)}$$

$$\Gamma = \frac{(g\zeta)^2 k_R M_2}{10\pi M_1 f_\pi^2}\{\omega_B\eta_{A1}^{(9)} - k_B\eta_{A2}^{(9)}\}^2. \quad \text{(C33)}$$

(10) $1^-(^3D_1) \to 0^-(^1S_0)$:

$$\eta_{A1}^{(10)} \equiv V(\xi_{A1}^{(10)} + 2\xi_{A2}^{(10)}) = -\sqrt{2}\int drf'(qr)(u_2v_1 - v_2u_1) + O(V^3), \quad \text{(C34)}$$

$$\eta_{A2}^{(10)} \equiv \xi_{A1}^{(10)} + 2V^2\xi_{A3}^{(10)} = \frac{1}{\sqrt{2}}\int dr[-\{f(qr) + 3f''(qr)\}u_2u_1 + \{f(qr) - f''(qr)\}v_2v_1] + O(V^2), \quad \text{(C35)}$$

$$\Gamma = \frac{(g\zeta)^2 k_R M_2}{6\pi M_1 f_\pi^2}\{\omega_B\eta_{A1}^{(10)} - k_B\eta_{A2}^{(10)}\}^2. \quad \text{(C36)}$$

(11) $1^-(^3D_1) \to 1^-(^3S_1)$:

$$\eta_{A1}^{(11)} \equiv V(\xi_{A2}^{(11)} + 2\xi_{A5}^{(11)}) = \frac{1}{\sqrt{2}}\int drf'(qr)(u_2v_1 - v_2u_1) + O(V^3), \quad \text{(C37)}$$

$$\eta_{A2}^{(11)} \equiv \xi_{A1}^{(11)} + 2V^2\xi_{A6}^{(11)} = \frac{1}{2\sqrt{2}}\int dr[\{f(qr) + 3f''(qr)\}u_2u_1 + \{-f(qr) + f''(qr)\}v_2v_1] + O(V^2), \quad \text{(C38)}$$

$$\Gamma = \frac{(g\zeta)^2 k_R M_2}{3\pi M_1 f_\pi^2}\{\omega_B\eta_{A1}^{(11)} - k_B\eta_{A2}^{(11)}\}^2. \quad \text{(C39)}$$

As defined in Sec. II, $q = -2m_QV$. V is the velocity of the heavy-light meson in the Breit frame. ω_B and k_B denote the energy and momentum of the emitted chiral particle in the Breit frame, respectively. ζ, which appears in the final expression of Γ, is $\frac{1}{\sqrt{2}}$ times the coefficient in front of the chiral field in the matrix Eq. (3).

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