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Nucleons at finite temperature and low density in Faddeev equation approach

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We study some properties of nucleons with a simplified version of a Faddeev equation at finite temperature and baryon chemical potential in the framework of the Nambu–Jona-Lasinio model. By taking diquark-quark bubble summation, we constructed the nucleon propagator and calculated the dynamical masses of the diquark and the nucleon in the pole approximation. We show that diquarks can survive as resonant states in the chiral symmetry restored phase at high temperature, and that nucleons are restricted in the chiral symmetry broken phase at low temperature. The phase diagram of the strongly interacting matter is then given.

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While quantum chromodynamics (QCD) which is the fundamental theory for strong interaction, works well in the perturbative region, it is still difficult to extend it to the nonperturbative region and to study hadron properties where quarks and gluons are confined. As the most important three-quark system, even though some properties of a nucleon have been studied in the global color symmetry model [1–4], in the spirit of the Dyson-Schwinger equation approach of QCD [5], because the collective quantization in the case of nonlocal interaction has not yet been settled down well [6], it has long been a challenging topic to understand the nucleon structure and its evolution in a hot and dense medium in QCD directly. Furthermore, although some properties of the nucleon have been described with the quark-diquark model and even the Faddeev equation approach with the direct help of the Dyson-Schwinger equation approach [7,8], the temperature and baryon density effects have not yet been investigated either. In this paper, we discuss then nucleons in a QCD model, the Nambu–Jona-Lasinio (NJL) model [9] at quark level [10–12], at finite temperature and density, and focus on how the chiral phase transition affects the property of a nucleon.

Similar to the description of mesons as quark-antiquark bound states in the framework of the Bethe-Salpeter equation, we take a simple picture and regard a baryon as a composite of three valence quarks, which can describe some important features of baryons such as the mass spectrum, the electromagnetic form factors, and the quark distribution functions [7,13,14]. The relativistic Faddeev equation is usually taken to describe the bound states of three-quark systems where baryons are considered as poles of their corresponding Green's functions [15–19]. In general, the Faddeev equation is a complicated inhomogeneous integral equation. If one concentrates only on the static properties of baryons, the free term can be neglected, and the equation turns into a homogenous one. In this paper, we implement a simplified version of the Faddeev equation, namely, the quark-diquark model. According to QCD, the two quarks in the color-$\bar{3}$ channel can form a diquark since the interaction between them is attractive. If the attraction is strong enough, it makes the Fermi sea of quarks unstable at high density, and the diquarks will then condense and the system will go into a color-superconducting phase, and the gap can be of the order of 100 MeV [20]. To simplify calculations, we will concentrate on the temperature effect and take into account only the case at low baryon density where quarks can pair but not form a condensate. Diquarks are not observable particles because they are not color singlets. They can then exist only inside baryons if the density of the system is not high enough. The idea of the quark-diquark model for baryons is the following: consider first the scattering of two quarks in the color-$\bar{3}$ channel, which can be described by the Bethe-Salpeter equation, and then couple it to the third quark after a suitable projection onto color, flavor, and spin spaces of baryons. This procedure simplifies the three-body problem to an effective two-body problem. Eichmann and his collaborators have taken the full Poincáre covariant structure of the three-quark amplitude in the Faddeev equation approach [8] and found that the resulting current-mass evolution of the nucleon mass agrees well with the lattice data and deviates only by 5% from the quark-diquark result obtained in previous studies.

Two approaches are usually taken in the practical calculation to study baryon properties in the vacuum in the framework of the relativistic Faddeev equation. One implements the Dyson-Schwinger equation, and the other works in the framework of the NJL model. Both methods can guarantee the covariance and Ward-Takahashi identity in the case of electromagnetic current [21]. Here we adopt simply the NJL model at quark level to discuss the static properties of nucleons and the phase diagram at finite

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temperature and baryon density. The NJL model shares the same global symmetries with QCD and incorporates the mechanism of spontaneous chiral symmetry breaking. Within this model, one can obtain the hadronic mass spectrum and the static properties of mesons remarkably well. In particular, one can recover the Goldstone mode, and some important low-energy properties of current algebra such as the Goldberger-Treiman and Gell-Mann-Oakes-Renner relations [12]. In the NJL model, the four-fermion point interaction simplifies the calculation dramatically in comparison with directly treating the relativistic three-body Faddeev equation.

Generally, both the scalar and the axial vector diquark channel contribute to the binding energy of baryons. Since the attraction in the scalar channel is sufficient to bind three quarks into a baryon, it plays the central role. In the chiral symmetry broken phase, the exchanged quark is heavy [22], we can then apply the static approximation to the quark-diquark model, and therefore neglect the momentum dependence of the quark exchange kernel [18]. Under this approximation, the propagator of the exchanged quark between the quark-diquark bubbles is only a function of the constituent quark mass, the Faddeev equation turns into a separable one and can be reduced to an effective Bethe-Salpeter equation.

Most work on the Faddeev equation discusses the properties of baryons in vacuum, since the equation at finite temperature and density is more difficult to deal with. Until now, only a few papers discussed the properties of baryons at finite temperature and/or density [23–27]. , Here we adopt a revised version of the pole approximation to the diquark propagator and explicitly include the chemical potential in it, which is different from the work in Ref. [25].

In this paper, we apply the Hartree approximation to the quark propagator and random phase approximation to the diquark[28–30]. The two-flavor NJL model at quark level is defined through the Lagrangian density [26,29]

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu\partial_\mu - m_0 + \mu\gamma_0)\psi + G_s[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\tau\psi)^2] + G_d\sum_a(\bar{\psi}\gamma_5 C\tau_2\beta^a\bar{\psi}^T)(\psi^T C^{-1}\gamma_5\tau_2\beta^a\psi), \tag{1}$$

where m_0 is the current quark mass; G_s and G_d are, respectively, the coupling constants in the scalar and pseudoscalar meson channel and the scalar diquark channel with dimension $(\mathrm{GeV})^{-2}$; the Pauli operators $\tau = (\tau_1, \tau_2, \tau_3)$ and the quark chemical potential $\mu = \mathrm{diag}(\mu_u, \mu_d) = \mathrm{diag}(\mu_B/3, \mu_B/3)$ are matrices in flavor space where μ_B is the baryon chemical potential; the matrices $\beta^a = \sqrt{3/2}\lambda^a$ for $a = 2, 5, 7$ project the system onto the $\bar{3}$ channel; the quark fields ψ and $\bar{\psi}$ are defined in flavor, color, and Dirac space; and $C = i\gamma_2\gamma_0$ is the charge conjugation matrix. The system has the symmetry $U_B(1) \otimes SU_I(2) \otimes SU_A(2)$, corresponding to baryon number symmetry, isospin symmetry, and chiral symmetry. The last term in Eq. (1) denotes the scalar diquark channel which can be obtained from the original NJL model with a Fierz transformation. We have neglected here the axial diquark channel.

The quarks obtain the dynamic mass M_q from the spontaneous breaking of chiral symmetry. In the mean field approximation, the contribution is from the quark loop which reads (see, for example, Refs. [10,28])

$$M_q = m_0 - 2G_s\langle\bar{\psi}\psi\rangle = m_0 + 2iG_s\int\frac{d^4p}{(2\pi)^4}\mathrm{Tr}S(p), \tag{2}$$

where the trace is taken in flavor, color, and Dirac space. At finite temperature, $p_0 = i\omega_n = i(2n+1)\pi T (n = 0, \pm1, \pm2, \ldots)$ where ω_n is the Matsubara frequency of quark. The $\int\frac{d^4p}{(2\pi)^4}$ should correspondingly be replaced with $i\pi T\sum_n(2n+1)\int\frac{d^3\mathbf{p}}{(2\pi)^3}$. Furthermore, at finite chemical potential, it has been known that there is no diquark condensate if the baryon chemical potential is not high enough; the mean field quark propagator can be expressed as

$$S(p) = \frac{1}{\gamma^\mu p_\mu - M_q + \gamma_0\mu}.$$

For nonzero current quark mass m_0, the order parameter for chiral phase transition, namely, the chiral condensate $\langle\bar{\psi}\psi\rangle$ or dynamic quark mass $M_q = m_0 - 2G_s\langle\bar{\psi}\psi\rangle$, cannot reach zero; the critical temperature T_χ at a fixed chemical potential is then defined by the maximum change of $M_q(T)$.

Since the model is not renormalizable, one needs to resort to a regularization scheme in order to make the momentum integral in the gap equation [Eq. (2)] finite. The three parameters m_0, G_s, and G_d in the Lagrangian and the momentum cutoff Λ in Eq. (2) are then usually determined by fitting the vacuum values of the pion mass m_π, pion decay constant f_π, nucleon mass M_n, and the chiral condensate $\langle\bar{\psi}\psi\rangle$. With the values $m_\pi = 135$ MeV, $f_\pi = 92.4$ MeV, $M_n = 939$ MeV, and $\langle\bar{\psi}\psi\rangle = -(242.4\ \mathrm{MeV})^3$ for each flavor being fitted, one has $m_0 = 5.47$ MeV, $G_s = 8.68\ \mathrm{GeV}^{-2}$, $G_d = 0.87G_s$, and $\Lambda = 0.569$ GeV [11]. In the following numerical calculations, we take such a set of parameters. In the usual quark-diquark model, the constituent quark mass in vacuum M_q is larger than one third of the nucleon mass in order to fix the nucleon mass $M_n = 939$ MeV at zero temperature and zero baryon chemical potential with a remarkable binding energy. As a consequence, we get the constituent quark mass as $M_q = 499$ MeV in the present case.

In the random phase approximation, the diquark propagator D can be expressed as (see, for example, Ref. [29])

$$D(k) = \frac{2iG_d}{1 - 2G_d\Pi_d(k)}, \tag{3}$$

with the diquark polarization function,

$$\Pi_d(k) = i\int \frac{d^4p}{(2\pi)^4}\mathrm{Tr}[\gamma_5 C\tau_2\beta^a S(-p)^T\gamma_5 C\tau_2\beta^a S(k+p)], \tag{4}$$

where $k_0 = i\omega_m = 2im\pi T(m = 0, \pm 1, \pm 2, \ldots)$, and ω_m are the Matsubara frequencies for diquark. The integral has the same meaning as that in Eq. (2). After the Matsubara summation over n (corresponding to the p_0 in the integral), we can obtain the Matsubara frequency ω_m (corresponding to the k_0) dependent diquark propagator $D(k)$.

In order to determine the diquark mass, we need to analytically continue the Matsubara frequency ω_m to the real variable $i\omega_m \to k_0$. The diquark mass $\bar{M}_d$ is defined as the pole of the propagator at $k^2 = \bar{M}_d^2$. At finite temperature and density, there is no more Lorentz invariance, Π_d depends separately on k_0^2 and $\mathbf{k}^2$, and $\bar{M}_d$ is determined by the real part of the diquark polarization function Π_d. In our case, we do thus not consider the decay width of diquark, which is related with the imaginary part of the diquark polarization function at high temperature and density [28]. In the following, we neglect the symbol, Re, in front of the diquark polarization function for simplification. The pole condition can be simply written as

$$1 - 2G_d\Pi_d(k_0 = \bar{M}_d, \mathbf{k} = 0) = 0. \tag{5}$$

The diquark polarization function Π_d at $\mathbf{k} = 0$ and finite temperature and baryon chemical potential can be explicitly written as

$$\Pi_d(k_0, \mathbf{k} = 0) = -12\int \frac{d^3\mathbf{p}}{(2\pi)^3}\bigg[\frac{2f(E_p + \mu_B/3) - 1}{k_0 + 2(E_p + \mu_B/3)} - \frac{2f(E_p - \mu_B/3) - 1}{k_0 - 2(E_p - \mu_B/3)}\bigg], \tag{6}$$

with the quark energy $E_p = \sqrt{M_q^2 + \mathbf{p}^2}$ and Fermi-Dirac distribution $f(x) = 1/(e^{x/T} + 1)$.

With increasing temperature, the diquarks will finally melt in the hot medium, the dissociation temperature T_d is defined as the temperature at which the pole equation (5) starts to have no solution.

While in our calculation we did not consider color condensate at low temperature and high baryon density, the critical temperature T_c for the color superconductivity to appear can be determined by the Thouless criterion [29–31],

$$1 - 2G_d\Pi_d(k_0 = 0, \mathbf{k} = 0) = 0. \tag{7}$$

Now we investigate nucleons in the NJL model. A nucleon constructed by three quarks can be described by the three-body Faddeev equation [15,16,19]. In the static approximation [18], the Faddeev equation is reduced to an effective Bethe-Salpeter equation constructed by a quark and a diquark, and the quark propagator between the quark—diquark bubbles becomes a function of the constituent quark mass M_q. In this case, we can use again the random phase approximation to derive the nucleon propagator [32]

$$D_n(q) = \frac{3/M_q}{1 - (3/M_q)\Pi_n(q)}, \tag{8}$$

where Π_n is the nucleon polarization function,

$$\Pi_n(q) = -\int \frac{d^4k}{(2\pi)^4}D(k)S(q-k), \tag{9}$$

where $q_0 = i\omega_l = i(2l+1)\pi T(l = 0, \pm 1, \pm 2, \ldots)$ where ω_l is the Matsubara frequency of the nucleon, and the four momentum integral is, in fact, the integral over the three momentum and the summation over the Matsubara frequencies.

After the analytic continuation to the real variable $i\omega_l \to q_0$, the nucleon mass $\bar{M}_n$ is given by the real part of the pole position of the propagator [in Eq. (8)] at zero momentum,

$$\det(1 - (3/M_q)\Pi_n(q_0 = \bar{M}_n, \mathbf{q} = 0)) = 0, \tag{10}$$

where the determinant is taken in Dirac space. Similar to the diquark dissociation temperature T_d, the nucleon dissociation temperature T_n in a hot and dense medium is determined by the disappearance of the solution of this pole equation.

To further simplify the numerical calculation, we take in the nucleon polarization [in Eq. (9)] the pole approximation for the diquark propagator, i.e.,

$$\begin{aligned} D(k) &= \frac{-ig_{Dqq}^2}{(k_0 + 2\mu_B/3)^2 - ((\bar{M}_d + 2\mu_B/3)^2 + \mathbf{k}^2)} \\ &= \frac{-ig_{Dqq}^2}{(k_0 + 2\mu_B/3)^2 - (M_d^2 + \mathbf{k}^2)}, \end{aligned} \tag{11}$$

where g_{Dqq} is the diquark-quark-quark coupling constant defined in the pole approximation,

$$g_{Dqq}^{-2} = \frac{\partial\Pi_d(k_0 + 2\mu_B/3, \mathbf{k} = 0)}{\partial(k_0 + 2\mu_B/3)^2}\bigg|_{k_0 + 2\mu_B/3 = M_d}. \tag{12}$$

Note that at the finite chemical potential, the diquark pole mass $\bar{M}_d$ defined by the real part of the pole position of the diquark propagator is not exactly the dynamical mass M_d defined through the Lagrangian density. The two quantities satisfy the relation $M_d = \bar{M}_d + 2\mu_B/3$. Similarly, there is $M_n = \bar{M}_n + \mu_B$ for the nucleon pole mass $\bar{M}_n$ and the dynamical mass M_n.

Making use of the pole approximation for Eq. (11), we can explicitly express the nucleon polarization function at $\mathbf{q} = 0$ as a function of temperature and baryon chemical potential,

$$\Pi_n(q_0) = \frac{g_{Dqq}^2}{4}\int \frac{d^3\mathbf{k}}{(2\pi)^3}\bigg[\frac{M_q}{E_k}(A - B) - \gamma_0(A + B)\bigg] \tag{13}$$

with definitions,

$$A(q_0, \mathbf{k}) = \frac{1-2f(-E_k+\mu_B/3)}{\epsilon_k^2-(E_k-(\mu_B+q_0))^2} - \frac{1}{2\epsilon_k}\frac{\coth((\epsilon_k-2\mu_B/3)/(2T))}{\epsilon_k+E_k-(\mu_B+q_0)} + \frac{1}{2\epsilon_k}\frac{\coth((\epsilon_k+2\mu_B/3)/(2T))}{\epsilon_k-E_k+(\mu_B+q_0)},$$
$$B(q_0, \mathbf{k}) = \frac{1-2f(E_k+\mu_B/3)}{\epsilon_k^2-(E_k+(\mu_B+q_0))^2} + \frac{1}{2\epsilon_k}\frac{\coth((\epsilon_k-2\mu_B/3)/(2T))}{\epsilon_k-E_k-(\mu_B+q_0)} - \frac{1}{2\epsilon_k}\frac{\coth((\epsilon_k+2\mu_B/3)/(2T))}{\epsilon_k+E_k+(\mu_B+q_0)}, \tag{14}$$

in which the diquark energy $\epsilon_k = \sqrt{M_d^2 + \mathbf{k}^2}$.

We now show our numerical results on the quark, diquark, and nucleon dynamic masses and the corresponding phase diagram. The obtained pure temperature effect on M_q, M_d, $M_d + M_q$, and M_n is displayed in the upper panel of Fig. 1. The figure manifests apparently that, with increasing temperature, the dynamically broken chiral symmetry is gradually restored with the quark mass decreasing gradually against the increasing of temperature as a manifestation. For a nonzero current quark mass m_0, the chiral symmetry can not be fully restored, and the critical temperature T_χ is defined through the maximum change of the order parameter M_q. With the parameters chosen in our calculation, it is determined as $T_\chi = 282$ MeV. At this point, the diquark mass changes its temperature dependence from a decreasing to an increasing function, similar to the behavior of mesons in the original NJL model [28] even with the Polyakov-loop improvement [33,34]. From the definition for the diquark and nucleon dissociation temperature in the hot medium, namely, the disappearance of the corresponding pole, we have the diquark dissociation temperature $T_d = 412\ \text{MeV} > T_\chi$ and the nucleon dissociation temperature $T_n = 253\ \text{MeV} < T_\chi$. This means that nucleons can survive only in the chiral symmetry broken phase but diquarks can exist not only in the chiral symmetry broken phase but also in the chiral symmetry restored phase. This result is qualitatively in agreement with the physics picture: a three-body system is easier to be melted than a two-body system. The higher dissociation temperature for diquarks can be understood clearly from the results shown in Fig. 1, since the figure demonstrates apparently that the binding energy $2M_q - M_d$ for diquarks is quite large but the binding energy $M_d + M_q - M_n$ for nucleons is small. Therefore, diquarks are tightly bound states of quarks but nucleons are relatively weakly bound states. It is necessary to note that the diquarks become resonant states in the chiral symmetry restored phase where the binding energy becomes negative $2M_q - M_d < 0$. Such an evolution process provides more evidence, such that the quark matter at the temperature above but near the critical one (corresponding to the T_n here) is in the strongly correlated state [35–38].

In the case of nonzero chemical potential, for instance $\mu_B = 600$ MeV, the obtained temperature dependence of the nucleon and its compositions are shown in the lower

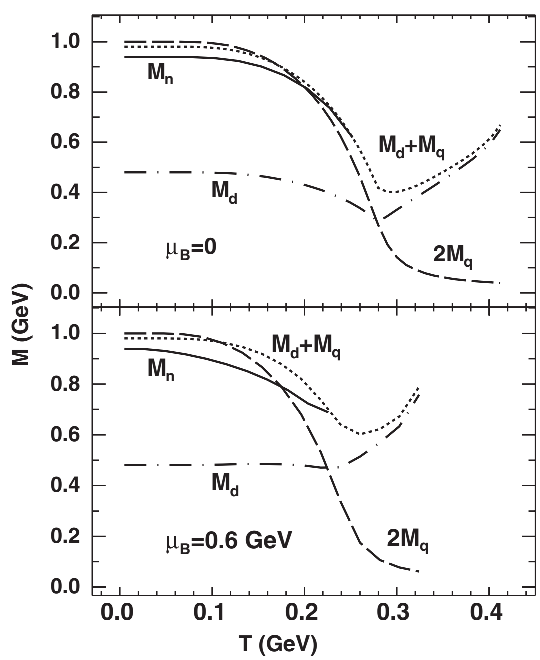

FIG. 1. Calculated temperature dependence of the quark, diquark, and nucleon dynamic masses M_q (multiplied by a factor 2), M_d, and M_n. $2M_q$, M_d, $M_d + M_q$, and M_n are, respectively, displayed by dashed, dot-dashed, dotted, and solid lines. The upper and lower panels correspond to $\mu_B = 0$ and $\mu_B = 0.6$ GeV, respectively.

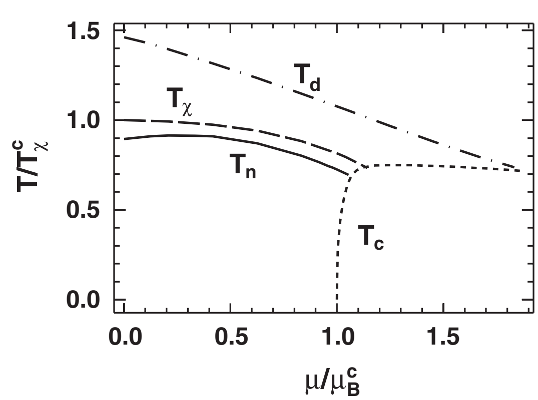

FIG. 2. Calculated phase diagram in the scaled temperature and chemical potential plane. The chiral phase transition temperature T_χ, diquark and nucleon dissociation temperatures T_d and T_n, and color superconductivity transition temperature T_c are, respectively, displayed by dashed, dot-dashed, solid, and dotted lines. T_χ^c is the T_χ at $\mu_B = 0$ and μ_B^c is the color superconductivity transition chemical potential at $T = 0$.

panel of Fig. 1. From the figure, one can notice evidently that the density effect reduces the chiral phase transition temperature T_χ and the dissociation temperatures T_d and T_n. However, the relation among them $T_d > T_\chi > T_n$ is always satisfied. In our calculation of nucleon mass, we adopted the static approximation for the quark propagator between the diquark-quark bubbles. From Fig. 1, one can see easily that, in the temperature region where nucleons can survive, we have $M_q > 400$ MeV, and the requirement for the static approximation is reasonably satisfied.

With the above results, we can get the phase diagram of the strongly interacting matter at finite temperature and chemical potential. The obtained phase diagram is illustrated in Fig. 2 where the temperature is scaled by the critical temperature $T_\chi^c = 282$ MeV of the chiral phase transition at $\mu_B = 0$ and the chemical potential is scaled by the critical chemical potential $\mu_B^c = 720$ MeV for the color superconductivity to appear at $T = 0$. The phase boundary for the chiral symmetry restoration and those of the diquark dissociation and nucleon dissociation are displayed in the figure as dashed, dot-dashed, and solid lines (marked with T_χ, T_d, T_n), respectively. Recalling Fig. 1, one can recognize that, at low baryon density and below the dissociation temperatures T_n, there exists definitely positive binding energy $M_q + M_d - M_n$. The temperature dependence of the binding energy at several values of the chemical potential is displayed in Fig. 3 for intuitiveness. Such a positive binding energy means that nucleons can survive as the bound states of a quark and a diquark. The strongly interacting matter at low temperature and/or low chemical potential is then nuclear matter. As the baryon density and the temperature get larger, the nucleons will melt due to the positivity of the binding energy being violated before the chiral symmetry is restored, but the diquarks can survive (as resonant states, as mentioned above) at temperatures much higher than the critical one of the chiral phase transition temperature. It hints that as the baryon density $\mu/\mu_B^c > 1$, the color-superconducting phase emerges. Since we have not introduced explicitly the diquark condensate at high baryon chemical potential, the phase transition line of color superconductivity is calculated through the Thouless criterion [29–31]. The obtained boundary is shown as the dotted line (marked with T_c) in Fig. 2. The temperature relation $T_d > T_c$ means that the thermal excitations of the diquark condensates at T_c are diquarks at moderate chemical potential instead of quarks at high chemical potential. Only at the higher temperature T_d do the diquarks melt and the system becomes a pure quark system [39]. This indicates

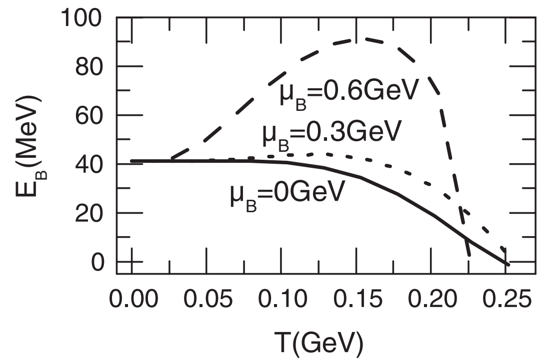

FIG. 3. Calculated temperature dependence of the binding energy $M_q + M_d - M_n$ at several values of the baryon chemical potential.

that the color superconductivity is in the phase of Bose-Einstein condensation at moderate baryon density and of Bardeen-Cooper-Schrieffer condensation at extremely high baryon density.

In summary, we have investigated some properties of nucleons and the phase diagram of the strongly interacting matter in the framework of the NJL model at finite temperature and baryon density. With the Faddeev equation approach, we constructed nucleons by the summation of diquark-quark bubbles and obtained the nucleon propagator under the static approximation. With the diquark and nucleon dissociation temperatures being defined as that at which the corresponding poles disappear, our calculation shows that diquarks can survive beyond the chiral phase transition; nucleons are dissociated as the chiral symmetry has not yet been completely restored. The phase diagram of the strongly interacting matter is then given. In addition, we found that the diquark condensate is in the Bose-Einstein condensation phase at moderate baryon density. By the way, it is necessary to mention that our calculation shows that the binding energy of the nucleon (as a bound state of a quark and a diquark) at the finite chemical potential changes nonmonotonously with respect to temperature. It may be interesting to explore the mechanism of such nonmonotonousness.

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