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$\eta - \eta'$ mixing: From electromagnetic transitions to weak decays of charm and beauty hadrons

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It has been realized for a long time that knowing the η and η' wave functions in terms of quark and gluon components probes our understanding of nonperturbative QCD dynamics. Great effort has been given to this challenge, yet no clear picture has emerged even with the most recent KLOE data. We point out which measurements would be most helpful in arriving at a more definite conclusion. A better knowledge of these wave functions will significantly help to disentangle the weight of different decay subprocesses in semileptonic decays of D^+, D_s^+, and B^+ mesons. The resulting insights will be instrumental in treating even nonleptonic B transitions involving η and η' and their CP asymmetries; thus they can sharpen the case for or against new physics intervening there.

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I. INTRODUCTION

The question of $\eta - \eta'$ mixing,[1] i.e., how their wave functions are composed of $SU(3)_{fl}$ singlet and octet $\bar{q}q$ components, goes back to the beginning of the quark model era [1–9]. With the advent of QCD it became even more involved, since QCD brought with it more dynamical degrees of freedom, namely, gluons, which can form a second class of $SU(3)_{fl}$ singlets. Determining $\eta - \eta'$ mixing is thus an intriguing element in understanding QCD's nonperturbative dynamics. Lattice QCD's attempts to establish theoretical control over this mixing are still in their infancy [10,11]. Showing that there is a purely gluonic component in the η and/or η' wave functions would establish for the first time that gluons, which have been introduced to mediate the strong interactions and whose presence as independent degrees of freedom has been demonstrated as progenitors of jets in "hard" collisions, play an independent role also in hadronic spectroscopy. In Sec. II we introduce basic notions relevant for $\eta - \eta'$ mixing, while in Sec. III we review the somewhat ambivalent findings from several phenomenological studies. Armed with this knowledge we discuss weak D and B decays producing η and η' mesons in Sec. IV and what the observed rates can tell us about the underlying quark level transitions; we comment briefly on how the structure of the η and η' wave functions affect CP asymmetries in the channels $B_d \to \eta' K_S$ and $B_d \to \eta K_S$. Finally in Sec. V we present a summary and outlook.

[1]The term "mixing" is often used when oscillations, e.g., $B^0 - \bar{B}^0$ are involved; however with oscillations one has a nontrivial time evolution, but not for $\eta - \eta'$ mixing.

II. $\eta - \eta'$ MIXING

Based on approximate QCD flavor $SU(3)_{fl}$ symmetry, the mixing of the η and η' mesons can be described in two different bases:

(1) The $SU(3)_{fl}$ singlet and octet components $|\eta_0\rangle = \frac{1}{\sqrt{3}}|u\bar{u} + d\bar{d} + s\bar{s}\rangle$ and $|\eta_8\rangle = \frac{1}{\sqrt{6}}|u\bar{u} + d\bar{d} - 2s\bar{s}\rangle$, respectively:

$$\begin{pmatrix} |\eta\rangle \\ |\eta'\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta_P & -\sin\theta_P \\ \sin\theta_P & \cos\theta_P \end{pmatrix} \begin{pmatrix} |\eta_8\rangle \\ |\eta_0\rangle \end{pmatrix}. \qquad (2.1)$$

(2) The quark-flavor basis with $|\eta_q\rangle = \frac{1}{\sqrt{2}}|u\bar{u} + d\bar{d}\rangle$ and $|\eta_s\rangle = |s\bar{s}\rangle$:

$$\begin{pmatrix} |\eta\rangle \\ |\eta'\rangle \end{pmatrix} = \begin{pmatrix} \cos\phi_P & -\sin\phi_P \\ \sin\phi_P & \cos\phi_P \end{pmatrix} \begin{pmatrix} |\eta_q\rangle \\ |\eta_s\rangle \end{pmatrix}. \qquad (2.2)$$

As long as state mixing is regarded, one may freely transform from one basis to the other; the two parametrizations are related through

$$\theta_P = \phi_P - \arctan\sqrt{2} \simeq \phi_P - 54.7°. \qquad (2.3)$$

In the $SU(3)_{fl}$ symmetry limit, $\theta_P = 0$, and ϕ_P takes the so-called "ideal" value $\phi_P = \arctan\sqrt{2} \simeq 54.7°$.

Just for orientation: the quadratic (linear) Gell-Mann Okubo (GMO) mass formula points to $\theta_P \simeq -10°$, $\phi_P \simeq 44.7°$ ($\theta_P \simeq -23°$, $\phi_P \simeq 31.7°$).

The mixing schemes have been analyzed in the context of chiral perturbation theory. On lattice, it is not an easy task to study η and η', as experienced in the last decade of attempts. The RBC-UKQCD Collaboration has reported a pioneering calculation of the η and η' masses and mixing angle of $\theta_P = -14.1(2.8)°$ using $N_f = 2 + 1$ flavor domain wall ensembles on an Iwasaki gauge action [10]. Their results show small octet-singlet mixing, consistent with the quadratic GMO within the large statistical errors. Masses and mixing angle of the η and η' have also been calculated by the Hadron Spectrum Collaboration [11], using lattice QCD with unphysically heavy light (up, down) quarks and a single lattice spacing: their estimate value is $\phi_P = 42(1)°$. The large value of the mixing angle ϕ_P in the pseudoscalar sector, with respect to other ones [e.g. the vector mesons $|\omega> \simeq |\eta_q>$ and $\phi \simeq |\eta_s>$, with a mixing angle $\phi_V = (3.4 \pm 0.2)°$ [12]] is expected, because of the additional mixing induced by the axial $U(1)$ anomaly (see [13] and references therein).

In the 1990s the possibility of a single angle description being inadequate started to be considered. Several papers [13–21], based on theoretical studies as well as on comparison with data, pointed out that the pattern of $SU(3)_{fl}$ breaking requires a description in terms of two angles. Phenomenological analyses have often involved weak decay constants $f^a_{\eta^{(\prime)}}$, defined by the relation $<0|A^a_\mu|\eta^{(\prime)}(p)> = if^a_{\eta^{(\prime)}} p_\mu$. In the octet-singlet basis $a = 8$, 0 and $A^{8,0}_\mu$ are the octet and singlet axial-vector currents; in the quark-flavor basis, $a = q$, s, and $A^{q,s}_\mu$ are the nonstrange and strange axial-vector currents. Because of $SU(3)_{fl}$ breaking, the mixing of the decay constants does not necessarily follow the same pattern as the state mixing (see e.g. [20,21]). For completeness, we report here the most general parametrizations involving two independent axial-vector currents and two different physical states:

(1)

$$\begin{pmatrix} f^8_\eta & f^0_\eta \\ f^8_{\eta'} & f^0_{\eta'} \end{pmatrix} = \begin{pmatrix} f_8 \cos\theta_8 & -f_0 \sin\theta_0 \\ f_8 \sin\theta_8 & f_0 \cos\theta_0 \end{pmatrix}. \tag{2.4}$$

(2)

$$\begin{pmatrix} f^q_\eta & f^s_\eta \\ f^q_{\eta'} & f^s_{\eta'} \end{pmatrix} = \begin{pmatrix} f_q \cos\phi_q & -f_s \sin\phi_s \\ f_q \sin\phi_q & f_s \cos\phi_s \end{pmatrix}. \tag{2.5}$$

We observe that in Eq. (2.4) as in Eq. (2.1) the angles are chosen in such a way that $\theta_P = \theta_8 = \theta_0 = 0$ corresponds to the $SU(3)_{fl}$ symmetric world. As before any expression in one scheme can be translated into the other one in a straightforward mathematical way. However different dynamical implementations of $SU(3)_{fl}$ breaking suggest a different ansatz; for example, it has been suggested that attributing $SU(3)_{fl}$ breaking to Okubo-Zweig-Iizuka (OZI) violating contributions leads to $\phi_q \simeq \phi_s$, recovering a description in terms effectively of a single angle in the quark-flavor basis [13,19]. As it is well known, the OZI rule leads to a suppression of strong interaction processes where the final states can only be reached through quark-antiquark annihilation. In the octet-singlet basis, instead, the differences in θ may be sizable, and most analyses find the range $\theta_8 - \theta_0 \approx [-19°, -12°]$ (see [13,16,18,19,21] and references therein). In this respect, the quark-flavor basis plays a privileged role; we will use such a basis in the following, assuming a single mixing angle $\phi_P = \phi_q = \phi_s$ that corresponds to Eq. (2.2). We can see from Eq. (2.5) that under this assumption the decay constants follow the same pattern of particle state mixing.

The plot thickens further still in QCD, for one can form an $SU(3)_{fl}$ singlet not only from quark-antiquark combinations, but also from pure gluon configurations with the simplest one being a gg combination. Since in general all components compatible with the quantum numbers of a state can appear in that state's wave function, there is no *a priori* reason why the η and η' wave functions could not contain such configurations. On general grounds they will contain also $c\bar{c}$ (or $b\bar{b}$) components, but probably on a significantly smaller level, since the mass scale for gluonic excitations is presumably lower than the J/ψ mass; therefore we will ignore $c\bar{c}$ (and $b\bar{b}$) admixtures in our subsequent analysis. Using the quark-flavor basis, we write down [5]

$$\begin{aligned} |\eta'\rangle &\simeq X_{\eta'}|\eta_q\rangle + Y_{\eta'}|\eta_s\rangle + Z_{\eta'}|gg\rangle, \\ |\eta\rangle &\simeq X_\eta|\eta_q\rangle + Y_\eta|\eta_s\rangle + Z_\eta|gg\rangle. \end{aligned} \tag{2.6}$$

One would expect the heavier η' to contain a higher dose of gluonic components than the η, which is also mainly an $SU(3)_{fl}$ octet. Setting Z_η to zero is presumably a pragmatically sound approximation. In [22] the authors use a number of parametrization schemes to analyze J/ψ and ψ' decays into vector and pseudoscalar mesons; in most cases they find a value for the gluonic content of η compatible with zero, with an exact numeric value of $Z^2_\eta/Z^2_{\eta'}$ that is strongly model dependent and ranges from 10^{-11} to 0.08. Reference [22] also presents a framework, based on old perturbation theory, that allows a much higher gluonic content in η, that is $Z^2_\eta/Z^2_{\eta'} \approx 1$. This result is inconsistent with the analysis of the same data made in [23], where $Z_\eta = 0$ is assumed.

In the following, we use the approximation $Z_\eta = 0$, $Z_{\eta'} \neq 0$ and we parametrize the two orthonormal states in terms of ϕ_P plus an additional mixing angle ϕ_G:

$$\begin{aligned} |\eta'\rangle &\simeq \cos\phi_G \sin\phi_P|\eta_q\rangle + \cos\phi_G \cos\phi_P|\eta_s\rangle + \sin\phi_G|gg\rangle, \\ |\eta\rangle &\simeq \cos\phi_P|\eta_q\rangle - \sin\phi_P|\eta_s\rangle. \end{aligned} \tag{2.7}$$

As already mentioned it is unlikely that lattice simulations of QCD will determine the η and η' wave functions in the *near* future. Phenomenological studies are thus our only recourse. Several such analyses have been undertaken recently: while their findings are not inconsistent, their messages are ambivalent, as we will discuss in the next section.

III. PHENOMENOLOGICAL STUDIES OF $\eta - \eta'$ MIXING

There are three classes of electromagnetic and strong transitions that can provide information on the mixing angles and the gluonic content:

(i) Radiative vector and pseudoscalar meson decays:

$$\psi', \psi, \phi \to \gamma\eta' \text{ vs } \gamma\eta, \qquad \rho, \omega \to \gamma\eta, \qquad \eta' \to \gamma\omega, \gamma\rho. \tag{3.1}$$

(ii) Decays into two photons or production in $\gamma\gamma$ collisions:

$$\eta' \to \gamma\gamma \text{ vs } \eta \to \gamma\gamma, \tag{3.2}$$

$$\gamma\gamma \to \eta \text{ vs } \gamma\gamma \to \eta'. \tag{3.3}$$

(iii) Decays of ψ into PV final states with the vector meson acting as a "flavor filter":

$$\psi \to \rho/\omega/\phi + \eta \text{ vs } \eta'. \tag{3.4}$$

A. Present status

Recent papers on the glue content of the η' by KLOE [24] and Li *et al.* [22] have motivated other studies of a range of different processes [23,25,26]. Escribano, Nadal [25,27] and Thomas [23] have analyzed all processes of Eqs. (3.1), (3.2), (3.3), and (3.4). The old and new analyses from KLOE [24,26] and Escribano and Nadal [25] refer to those processes of Eq. (3.1) whose dynamical scale is below 1.02 GeV (that is, including ϕ, but excluding ψ and ψ' decays), while Li *et al.* [22] have analyzed the ones above 1.02 GeV. In Table I we have summarized the results of [23–25], based on radiative decays of vector/pseudoscalar mesons below 1.02 GeV. The KLOE analysis also includes constraints from $\pi^0/\eta' \to \gamma\gamma$, according to the prescription of Ref. [28]. These results are obtained by including vector-pseudoscalar wave function overlaps, assuming the $\eta^{(\prime)}$ to be a pure $q\bar{q}$ state, i.e. $Z^2_{\eta^{(\prime)}} = 0$, and the dependence of the decay widths on the mixing angle as in [25]. We see that the different analyses yield very consistent values for the mixing angle, namely $\phi_P \simeq 42°$, which happens to be close to the value suggested by the *quadratic* GMO mass formula. Including the latest data from KLOE [24] and SND [29] does not cause a significant shift.

TABLE I. Fit values for the $\eta - \eta'$ mixing angle as inferred by different authors from radiative decays of vector/pseudoscalar mesons below 1.02 GeV, assuming $Z^2_{\eta'} = 0$. Only the KLOE analysis includes also constraints from $\eta' \to \gamma\gamma$. I labels the results from the analysis without including the latest data on $\phi \to \eta'\gamma$ (KLOE) and $(\rho, \omega, \phi) \to \eta\gamma$ (SND), while II indicates the same analyses performed including them.

Analysis	ϕ_P (ansatz $Z^2_{\eta'} \equiv 0$)
KLOE	$(41.3 \pm 0.3_{\rm stat} \pm 0.7_{\rm sys})°$
Escribano I	$(41.5 \pm 1.2)°$
Escribano II	$(42.7 \pm 0.7)°$
Thomas I	$(41.3 \pm 0.8)°$
Thomas II	$(41.7 \pm 0.5)°$
Thomas I with form factors	$(41.9 \pm 1.1)°$
Thomas II with form factors	$(42.8 \pm 0.8)°$

TABLE II. Fits allowing for a gluonium component using radiative decays of vector/pseudoscalar mesons below 1.02 GeV. Only the KLOE analysis includes also constraints from $\eta' \to \gamma\gamma$. I again labels the results from analyses without including the latest data on $\phi \to \eta'\gamma$ (KLOE) and $(\rho, \omega, \phi) \to \eta\gamma$ (SND), while II indicates the same analysis performed including them.

Analysis	ϕ_P	$Z^2_{\eta'}$
KLOE	$(39.7 \pm 0.7)°$	0.14 ± 0.04
Escribano I	$(41.4 \pm 1.3)°$	0.04 ± 0.09
Escribano II	$(42.6 \pm 1.1)°$	0.01 ± 0.07
Thomas I	$(41.3 \pm 0.9)°$	0.04 ± 0.06
Thomas II	$(41.7 \pm 0.5)°$	0.04 ± 0.04
Thomas I with form factors	$(41.9 \pm 1.1)°$	0.10 ± 0.06
Thomas II with form factors	$(41.9 \pm 0.7)°$	0.10 ± 0.04

In Table II results from the same studies are listed, now allowing for a gluonic component in η', i.e. $Z^2_{\eta'} \neq 0$. The different analyses again yield consistent values for the mixing angle with $\phi_P \simeq 42°$ with only KLOE finding a somewhat smaller number. As before the latest data from KLOE and SND do not cause a significant shift. Yet while the numbers given for the size of a gluonic component are not truly inconsistent considering the stated uncertainties, they seem to carry an ambivalent message: while the first and last studies—listed as "KLOE" and "Thomas with form factors"—point to a significant gluonic component, the others do not. We understand some of the differences. As explained around Eqs. (2.4) and (2.5) we think that assuming the mixing of the decay constants to follow the same pattern as state mixing is an oversimplification. Only Thomas has gone beyond this assumption, and when he includes the form factors he finds some intriguing evidence for a gluonic contribution.

The form factors included in "Thomas" are phenomenological Gaussians, whose aim is to introduce a momentum dependence for exclusive processes. In order to understand why the findings from "KLOE" and "Escribano/Thomas I-II" for the gluonic content in Table II are as different as they appear (for neither analysis allows for different form factors), we can offer one comment, though: only "KLOE" includes $\eta' \to \gamma\gamma$, and that observable pushes up the value of $Z^2_{\eta'}$, as pointed out by Thomas.

In fact, the above theoretical discussion has prompted the KLOE Collaboration to perform another fit [26], updated by using the branching ratio values from PDG 2008 [30], the more recent KLOE results on the ω meson [31] and using a larger number of free parameters, as suggested by [23,25]. The fit has been performed in the two cases: imposing the gluonium content to be zero that resulted in $\phi_P = (41.4 \pm 0.5)^\circ$, or allowing it free, giving $\phi_P = (40.4 \pm 0.6)^\circ$. New KLOE results confirm the gluonium content of η' at the 3σ level with $Z^2_{\eta'} = 0.115 \pm 0.036$, in contrast with "Escribano/Thomas I-II" values in Table I. Therefore, the actual difference between "Escribano/Thomas I-II" and KLOE values appears due to the inclusion in the latter of $\eta' \to \gamma\gamma$.

The comparison presented above pointed out that decays into two photons can play a key role in the mixing parameters determination. They can be exploited also looking at the inverse processes, namely, the production in $\gamma\gamma$ collisions.

The L3 Collaboration at LEP has published [32] the measurement of the radiative width $\Gamma(\eta' \to \gamma\gamma)$ produced via the collision of virtual photons, in the reaction $e^+e^- \to e^+e^-\gamma^*\gamma^*$, $\gamma^*\gamma^* \to \eta'$, $\eta' \to \pi^+\pi^-\gamma$, using data collected at center-of-mass energies $\sqrt{s} \simeq 91$ GeV. They compare the photon-meson transition form factor with a model by Anisovich *et al.* [33] that allows a variable admixture of gluonic content, from 0% to 15%. The central values of L3 data points favor a low gluonium content, but the whole interval is allowed within the large errors.

Before L3, the same $e^+e^- \to e^+e^-\eta'$ reaction had been performed at lower energy e^+e^- colliders, by using various η' decay channels (see the references in [32]). Let us review some old measurements of the radiative widths $\Gamma(\eta^{(\prime)} \to \gamma\gamma)$ used to evaluate the mixing angles. These estimates did not consider the possibility of gluonic content and refer to the octet-singlet basis and the single angle approximation, whose limits have been discussed in Sec. II. To facilitate the comparison, we have quoted the results in the flavor basis, using the relation (2.3). The observation of η meson production from $\gamma\gamma$ fusion has been reported in a 1983 Rapid Communication by the Crystal Ball Collaboration; the given mixing angle reads $\phi_P = 37.1^\circ \pm 3.6^\circ$ [34]. In 1988 they published the radiative widths for π^0, η, and η' and determined mixing angles from the experimental averages, finding $\phi_P = 32.3^\circ \pm 1.2^\circ$ [35]. Two years later both the MD-1 [36] and the ASP Collaborations [37] presented the measurement of the η, $\eta' \to \gamma\gamma$ widths, with results in agreement within the errors. The ASP Collaboration calculated the pseudoscalar mixing angle $\phi_P = 34.9^\circ \pm 2.2^\circ$ [37]. While these values are compatible among them, they appear to fall significantly below those in Tables I and II.

A new surge of experimental data and updated analyses is strongly needed. The *BABAR* Collaboration has led the way presenting recent studies on the $\gamma\gamma^* \to \eta^{(\prime)}$ transition form factors in the momentum transfer range from 4 to 40 GeV^2 [38]. They compare measured values of the $\eta^{(\prime)}$ form factors with theoretical predictions and data for the π^0 form factor by using the description of $\eta - \eta'$ mixing in the quark-flavor basis (2.2). They assume no gluonic admixture and a mixing angle $\phi_P = 41^\circ$. The dependence on the transfer momentum of the form factor for the $|\eta_s\rangle$ state is different from the QCD prediction [39] of the asymptotic distribution amplitudes; data points are systematically below the theoretical curve. Because of the strong sensitivity of the result for the $|\eta_s\rangle$ state to mixing parameters, an admixture of the two-gluon component in the $\eta^{(\prime)}$ meson cannot be excluded as a possible origin of this discrepancy.

A new investigation is being performed by KLOE, from the analysis of off-peak data, with integrated luminosity $L = 240$ pb^{-1}, already on tape, devoted to the measurement of the $\gamma\gamma \to \eta$ rate. The off-peak analysis, at $\sqrt{s} = 1$ GeV instead of $\sqrt{s} = 1.02$ GeV, allows one to reduce the main background, coming from resonant contributions $\phi \to \eta\gamma$. After the full selection, the data set consists of 600 $\gamma\gamma \to \eta$ with $\eta \to \pi^+\pi^-\pi^0$ and 900 $\gamma\gamma \to \eta$ with $\eta \to \pi^0\pi^0\pi^0$; the cross section $\sigma(\gamma\gamma \to \eta)$ at 1 GeV is under evaluation [40]. The upgraded KLOE detector (KLOE-2) will be suited for taking data also at energies away from the ϕ mass. Taggers designed to detect the outcoming e^+e^- are being inserted into the KLOE detector, to provide a better background rejection without going off peak and to allow precision measurements of the $\gamma\gamma$ cross section. There is a proposal to increase the DAΦNE energy up to $\sqrt{s} \simeq 2.5$ GeV; however, a run at $\sqrt{s} \simeq 1.4$ GeV is already enough to measure the η' decay width [41].

Starting in September 2009, the Crystal Ball at MAMI has undertaken a huge upgrade, with an increase of the MAMI beam energy and the construction and assemblage of a new tagging device; one reason of the upgrade is a measure of the $\eta' \to \gamma\gamma$ branching ratio [42].

The quoted measurements of the width are obtained with the QED process $e^+e^- \to e^+e^-\gamma^*\gamma^* \to e^+e^-\eta$. The 2010 PDG average is taken from such experiments and gives $\Gamma(\eta \to \gamma\gamma) = 0.510 \pm 0.026$ KeV. The error on the average is 5%, while the errors in individual experiments range from 8% to 25%. There is a different type of measurement of $\Gamma(\eta \to \gamma\gamma)$, not included in the 2010 PDG average, based on the Primakoff effect, where η's are produced by the interaction of a real photon with a virtual photon in the Coulomb field of the nucleus. In 1974 at Cornell a measurement based on the Primakoff effect gave $\Gamma(\eta \to \gamma\gamma) = 0.324 \pm 0.046$ KeV, a value 4σ away from the QED results [43]. Recently, a reanalysis of the Primakoff experiment, with a different modeling of the nuclear background, brought the value of the width in line with direct measurements, precisely to $\Gamma(\eta \to \gamma\gamma) = 0.476 \pm 0.062$ KeV [44]. Extraction of the Primakoff amplitude from the data is very delicate, because this amplitude interferes with hadronic amplitudes due to vector meson (ρ and ω) and axial-vector meson b_1 exchanges.

Increasing the energy may help, since at very high energies the growth of the Coulomb peak must dominate over the Regge behavior of the strong amplitude. After more than 30 years from the Cornell experiment, a new experiment to measure the $\Gamma(\eta \to \gamma\gamma)$ decay width via the Primakoff effect has been proposed and approved at Jefferson Laboratory, using a 11.5 GeV tagged photon beam on two light targets, proton and ^{4}He [45]. The targets have been chosen with the aim of minimizing the nuclear incoherent background and enabling a good separation of the Primakoff production mechanism from the nuclear coherent background. They estimate to reach a 3% accuracy in the measurement of the η width that would yield less than 1° of uncertainty on the $\eta - \eta'$ mixing angle.

As it is well known, all η meson possible strong decays are forbidden in lowest order by C, CP invariance and G-parity conservation. First order electromagnetic η decays are forbidden as well, or occur at a suppressed rate because of involving an anomaly. The first allowed decay is therefore the second-order electromagnetic transition $\eta \to \gamma\gamma$. The decay $\eta \to 3\pi$ violates isospin symmetry and it is mainly due to the isospin breaking part of the QCD Lagrangian, since contributions from the electromagnetic interaction are strongly suppressed by chiral symmetry [46]. The main interest of this decay resides in the fact that, in principle, it offers a way to determine the mass difference of the up-down quarks. The absolute value of the partial decay width for $\eta \to 3\pi$ is experimentally obtained via normalization to $\eta \to \gamma\gamma$; therefore, a change in one decay width has influence on the other [30].

A few comments are in order for the analysis of $\psi \to PV$. It was pioneered by Mark III in 1985, when they inferred from their data $Z^2_{\eta'} = 0.35 \pm 0.18$ [47]. They assumed that such decays proceed via singly disconnected diagrams (SOZI) with their strong quark line correlations and ignored doubly disconnected diagrams (DOZI). In Figs. 1(a) and 1(b) we show examples of SOZI and DOZI diagrams. Motivated by the measurement of $\psi \to \gamma\omega\phi$, which showed the relevance of DOZI-suppressed processes in ψ decays, they performed a new analysis [48], including DOZI contributions and any additional component as gluonium or radial excitation. The new analysis did not show evidence for non-$\bar{q}q$ components in the η and η' wave functions.

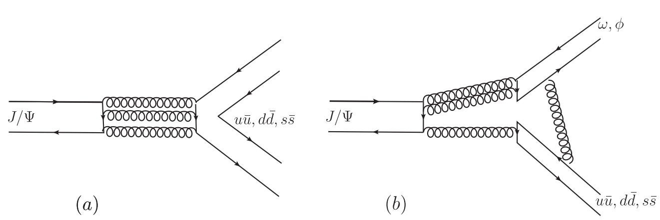

FIG. 1. (a) SOZI, and (b) DOZI diagrams contributing to $\psi \to PV$ decays.

In 2007 Thomas [23]—following the approach of Seiden *et al.* [49]—investigated the strong $\psi \to PV$ transitions; he concluded that DOZI contributions are significant, and that any gluonium components should play a role similar to that of DOZI contributions. From such an analysis he finds that the fit favors a small gluonic component in the η', with no great significance. Without form factors, Thomas finds $\phi_P = (45 \pm 4)^\circ$ and $\phi_G = (33 \pm 13)^\circ$ [i.e. $Z^2_{\eta'} = (0.30 \pm 0.21)$], whereas with form factors $\phi_P = (46^{+4}_{-5})^\circ$ and $\phi_G = (44 \pm 9)^\circ$ [i.e. $Z^2_{\eta'} = (0.48 \pm 0.16)$]. Another phenomenological analysis of $\psi \to PV$, without form factors, by Escribano [27], finds $\phi_P = (40.7 \pm 2.3)^\circ$ in the hypothesis of no gluonium and, allowing for it, $\phi_P = (44.6 \pm 4.4)^\circ$ with $Z^2_{\eta'} = (0.29^{+0.28}_{-0.26})$.

The remaining decays of the list (3.1), (3.3), and (3.4) are charmonium decays into $\gamma\eta^{(\prime)}$. BESII data have better precision than previous measurements; according to the hypothesis of no gluonic contribution, SU(3) flavor symmetry and the exact OZI rule, they extract an angle in the octet-singlet scheme. Their value, translated in the flavor scheme according to the relation (2.3), reads $\phi_P = (32.62 \pm 0.81)^\circ$ [50], a quite low value compared to other determinations. The extraction of the mixing angle in [50] has been performed in a very symmetric—and therefore simplified—scheme; we observe that just by introducing a dependence on a strange/nonstrange factor, the author in [23] finds for the same processes and PDG averaged data (including BESII results) values of the mixing angle in line with determinations from other processes. If there is any charmonium component in the $\eta^{(\prime)}$, we expect the decays of ψ and ψ' into $\gamma\eta^{(\prime)}$ to be dominated by the magnetic dipole transition of charmonium. In that case, it is possible to estimate that the amplitudes of the charmonium components of the $\eta^{(\prime)}$ are negligible, being less that 5% [23].

More recent measurements of $\gamma\eta^{(\prime)}$ branching fractions have been reported by CLEO-c [51]. The last update of the $\psi \to \gamma\eta'$ branching fraction has been given by BESIII [52] and reads $\mathcal{B}(\psi \to \gamma\eta^{(\prime)}) = (4.84 \pm 0.03(\mathrm{stat}) \pm 0.24(\mathrm{sys})) \times 10^{-3}$, which is consistent with the BESII value within 1.5σ and with the CLEO value within 1.4σ. The $\psi' \to \gamma\eta^{(\prime)}$ decays have also been observed by BESIII [53], but no new mixing angle estimate has been reported by the Collaboration. As far as $\Upsilon(1S) \to \gamma\eta^{(\prime)}$ is concerned, only upper limits are available for the branching ratios from CLEO III [54].

Since all extractions of the mixing angle involve some nontrivial theory assumptions, it is not totally surprising to find different compositions of the wave functions, yet it is still frustrating. The best short- or midterm prospects for improvement lie in obtaining constraints from more data of even greater variety.

Let us now provide some estimates of how much future data can reduce most of the uncertainties discussed here.

B. Improving the constraints of the $\eta - \eta'$ wave functions

The determination of mixing angles and gluonium content is based on measurements. The significance of such constraints depends on the experimental uncertainties. Therefore we analyze which experimental inputs will best improve our knowledge of the $\eta - \eta'$ wave functions. We start with the PDG 2010 values [55]:

(i) The stated $\phi \to \eta'\gamma$ partial width is mainly due to the KLOE measurement in [24]; the error is dominated by systematics due to the secondary η' branching ratio. The $\phi \to \eta\gamma$ branching ratio has been accurately measured by CMD-2 and SND [55].

(ii) The $\eta' \to \omega\gamma$ partial width of (0.0053 ± 0.0005) MeV with a relative error of 9% comes from the overall PDG 2010 fit. The relevant experiment was performed in 1977 and was based on 68 events [56]. The KLOE-2 Collaboration [41] could measure the branching ratio $\mathcal{B}(\eta' \to \omega\gamma)$ more accurately by collecting at least 20 fb^{-1} of data; the limiting factor then comes from the uncertainty in the total η' width, $\Gamma_{\eta'}$, since it is the partial width that matters.

(iii) The $\eta' \to \rho\gamma$ partial width inferred from the PDG 2010 fit is (0.0568 ± 0.0030) MeV; the absolute branching ratio measurement was performed in 1969 by Rittenberg [57] based on 298 events. The PDG fit value is slightly lower than the directly measured one. Again the error is dominated by the uncertainty in $\Gamma_{\eta'}$.

(iv) The latest values on $\rho \to \eta\gamma$ and $\omega \to \eta\gamma$ partial widths are obtained in [58], based on SND data on $e^+e^- \to \eta\gamma$: their accuracy is quite comparable to that of the PDG 2010 fit values.

In Table III we sketch different experimental scenarios. Starting from the present status as given by PDG 2010 we analyze the impact various conceivable improvements in the experimental constraints would have on the determination of the mixing angle ϕ_P and the size of $Z^2_{\eta'}$, the gluonic component in the η' wave function. We have chosen the radiative processes that are common to analyses [23–25] discussed in Sec. III A.

In column I we list the uncertainties in the experimental input values as stated in PDG 2010. In column II we indicate the improvement that could be achieved by studying $\eta' \to \omega\gamma$ with a sample of 20 fb^{-1} of $e^+e^- \to \phi$ events, that KLOE-2 anticipates to acquire in the next few years [41]. We assume a selection efficiency of order 20% in the analysis of $\phi \to \eta'\gamma$ with $\eta' \to \omega\gamma$ and neglect background subtraction. We observe that the limiting factor is provided from the uncertainty in the total η' width. In column III we indicate the improvement that could be achieved by reducing the uncertainty on $\eta' \to \rho\gamma$ of one-half respect to the present scenario; such improvement is also possible after a few years of running of KLOE-2 [41]. In columns IV and V we indicate the sensitivity to an improvement in the determination of the partial widths for $\phi \to \eta^{(\prime)}\gamma$ and for all the partial widths, respectively. Among possible secondary decays of $\phi \to \eta'\gamma$, there are both decays $\eta' \to \rho\gamma$ and $\eta' \to \omega\gamma$, whose errors are dominated by the uncertainty on $\Gamma_{\eta'}$. However, the former is more convenient to measure, e.g. at KLOE, since it has a branching ratio of almost an order of magnitude larger; also the total ρ decay width Γ_ρ is much larger, partially including and obscuring, from an experimental point of view, the total ω decay width Γ_ω. Since the partial widths of processes containing η' and the total width $\Gamma_{\eta'}$ are correlated, in column VI we evaluate the impact of the reduction of the uncertainty on $\Gamma_{\eta'}$. We assume a future $\Gamma_{\eta'}$ measurement with 1.4% uncertainty, which is within the possibility of KLOE-2 [26]. Such a measurement allows the determination of a nonzero gluonium content at 5σ, as shown in column VI. The crucial quantities to consider are not the central values for ϕ_P and $Z^2_{\eta'}$, since they are likely to shift, but their uncertainties. We conclude it is most important to reduce the uncertainty in the partial width for $\eta' \to \rho\gamma$; i.e., one has to measure both $\mathcal{B}(\eta' \to \rho\gamma)$ and $\Gamma_{\eta'}$ more accurately.

The situation concerning the η' full width is somewhat curious at present: PDG 2010 lists as its best value

TABLE III. I: widths from PDG 2010 fits; II: errors on $\eta' \to \omega\gamma$ reduced; III: errors on $\eta' \to \rho\gamma$ reduced; IV: errors on $\phi \to \eta^{(\prime)}\gamma$ reduced; V: reducing the uncertainties for all partial widths; VI: all recalculated in the hypothesis of 1.4% for the η' full width.

Processes	$(\delta\Gamma/\Gamma)_I$	$(\delta\Gamma/\Gamma)_{II}$	$(\delta\Gamma/\Gamma)_{III}$	$(\delta\Gamma/\Gamma)_{IV}$	$(\delta\Gamma/\Gamma)_V$	$(\delta\Gamma/\Gamma)_{VI}$
$\phi \to \eta'\gamma$	3.5%	3.5%	3.5%	1.7%	1.7%	1%
$\phi \to \eta\gamma$	2%	2%	2%	1%	1%	2%
$\eta' \to \omega\gamma$	9%	4.5%	9%	9%	4.5%	1.7%
$\eta' \to \rho\gamma$	5%	5%	2.5%	5%	2.5%	1.7%
$\rho \to \eta\gamma$	7%	7%	7%	7%	3.4%	7%
$\omega \to \eta\gamma$	9%	9%	9%	9%	4.5%	9%
ϕ_P	$(40.6 \pm 0.9)^\circ$	$(40.1^{+0.8}_{-1.0})^\circ$	$(40.7 \pm 0.7)^\circ$	$(40.6^{+0.5}_{-0.6})^\circ$	$(40.4 \pm 0.5)^\circ$	$(40.1 \pm 0.3)^\circ$
$Z^2_{\eta'}$	(0.09 ± 0.05)	(0.13 ± 0.05)	(0.08 ± 0.04)	(0.09 ± 0.03)	(0.10 ± 0.03)	(0.13 ± 0.02)

$\Gamma_{\eta'} = (0.194 \pm 0.009)$ MeV—with the error including a scale factor of 1.2—resulting from an overall fit. Direct measurements from 1979 [59] and 1996 [60] on the other hand yield the average $\Gamma_{\eta'} = (0.30 \pm 0.09)$ MeV, which would lead to $\phi_P = (42.7^{+1.0}_{-1.7})^\circ$ and $Z^2_{\eta'} = (0.00 \pm 0.13)$. Recently a new measurement has been performed at the COSY-11 facility: $\Gamma_{\eta'} = 0.226 \pm 0.017(\text{stat}) \pm 0.014(\text{syst})$ MeV; the value of the width was established directly from the measurement of the mass distribution of the η' meson, determined with a very high resolution [61]. The present average world value (2011 PDG partial update) contains this last measurement and gives $\Gamma_{\eta'} = (0.199 \pm 0.009)$; in the global fit to the η' partial widths the correlations among the partial widths do not change significantly.

Let us observe that the total width $\Gamma_{\eta'}$ extracted by PDG and the value of the partial width $\Gamma(\eta' \to \gamma\gamma)$ are strongly correlated, which may create difficulties when the total and the partial width are used at the same time, as in the present case of the mixing angle extraction. Moreover, the branching ratios of the η' meson decay channels are generally known with a relative precision of more than an order of magnitude better than the present accuracy with which $\Gamma_{\eta'}$ is extracted.

IV. WEAK DECAYS OF CHARM AND BEAUTY HADRONS

After many years of strenuous efforts to obtain the η and η' wave functions with nontrivial bounds why should one not declare "victory" and go on to something else? There are three reasons:

(i) Professional pride—not to be belittled in Italy and Bavaria.

(ii) Lattice QCD simulations have just entered the adult period.

(iii) Yet there is the most topical reason, namely, that knowing reliably the η and η' wave functions are an important input for our understanding of several *weak* decays of beauty and charm hadrons. Most crucially we need it for predicting *CP* asymmetries involving η and η' in the final states and to understand whether a deviation from standard model (SM) predictions can be seen as a signal of physics beyond the SM [62–65].

The SuperB and Super KEK B factories approved in Italy and in Japan, respectively, will produce crucial statistics needed for $B_{(s)} \to \eta/\eta' X$ and $D_{(s)} \to \eta/\eta' X$. There is a good chance that LHCb will likewise and much sooner.

A. Semileptonic modes

Since one expects semileptonic transitions to be driven by SM dynamics only (or at least to a high degree of accuracy), their detailed studies teach us lessons on how nonperturbative hadronization transforms quark level transitions. We will analyze here what semileptonic D and B decays can tell us about the η and η' wave functions and maybe more importantly, how our knowledge of those can help us to better understand the decay mechanisms.

Before going into a more detailed discussion, a few general points should be mentioned. The transitions $D_s^+ \to \eta^{(\prime)} l^+ \nu$, $D^+ \to \eta^{(\prime)} l^+ \nu$, and $B^+ \to \eta^{(\prime)} l^+ \nu$ proceed on greatly different time scales, since they are driven by weak interactions on the Cabibbo-allowed, Cabibbo-suppressed, and Kobayashi-Maskawa–suppressed levels, respectively. Yet they can provide us with highly complementary information in the sense that they produce the $\eta^{(\prime)}$ via their $s\bar{s}$, $d\bar{d}$, and $u\bar{u}$ components, respectively. In addition, as explained below, $\eta^{(\prime)}$ could be excited via a gg component.

1. $D_{(s)} \to \eta^{(\prime)} l \nu$

According to the heavy quark expansion the so-called spectator diagrams (see e.g. Fig. 2) provide the leading contribution to semileptonic as well as nonleptonic charm decays [66].

Data on semileptonic decays need to improve greatly before they can constrain the physics related to the mixing with the gluonic component. In 1995 CLEO extracted the branching fraction $\mathcal{B}(D_s^+ \to \eta^{(\prime)} e^+ \nu)$ from ratios to hadronic decays of the D_s^+ [67]. In 2009 CLEO-c presented the first absolute measurement of the branching fraction of $\mathcal{B}(D_s^+ \to \eta^{(\prime)} e^+ \nu)$ [68]; the ratio

$$\frac{\mathcal{B}(D_s^+ \to \eta' e^+ \nu)}{\mathcal{B}(D_s^+ \to \eta e^+ \nu)}\bigg|_{\text{CLEO-c}} = 0.36 \pm 0.14 \tag{4.1}$$

is in agreement with the previous CLEO results [67]. In semileptonic D_s decays the final state hadron has to be produced off an $s\bar{s}$ configuration; if η and η' are pure $q\bar{q}$ states, i.e. $Z^2_{\eta'} = Z^2_\eta = 0$, then one finds in the quark-flavor basis

$$\frac{\Gamma(D_s^+ \to \eta' e^+ \nu)}{\Gamma(D_s^+ \to \eta e^+ \nu)} = R_D \cot^2\phi \tag{4.2}$$

with the quantity R_D given by the relative phase space and the ratio of the η and η' form factors integrated over the

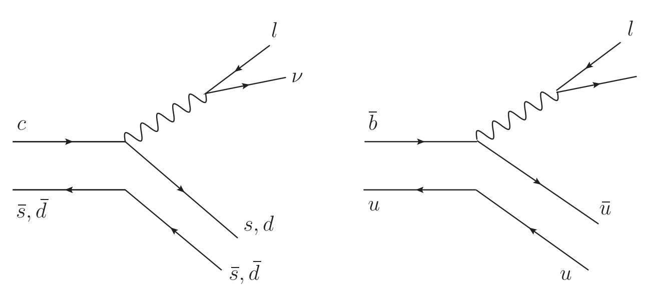

FIG. 2. Spectator diagrams for $D_{(s)} \to \eta^{(\prime)} l \nu$ and $B^+ \to \eta^{(\prime)} l \nu$ decays.

appropriate range in q^2. To calculate the explicit form of R_D one has to model the q^2 dependence of the form factors, but the factorization of the mixing angle dependence can help to devise tests of the mixing angle itself (see e.g. [69]). From the previous CLEO results [67], using η and η' as pure $q\bar{q}$ states and a pole ansatz for the form factors Feldmann, Kroll, and Stech inferred $\phi_P = (41.3 \pm 5.3)°$ [14]; it agrees even better than one might have expected with the values given above as extracted from weak and electromagnetic transitions. Their value is consistent with the new CLEO data within the errors.

Gronau and Rosner in a very recent paper [70] gave a similar number for $\Gamma(D_s^+ \to \eta' l^+ \nu)/\Gamma(D_s^+ \to \eta l^+ \nu)$ (among other predictions) applying a very simple model, where R_D is inferred from kinematic factors in the quark level; again, η and η' are described as pure $q\bar{q}$ states.

The transition form factors encode complex hadronic dynamics and momentum dependence: in [71] they have been expressed through the light-cone wave functions of the initial and final mesons. An allowed range for $Z_\eta^2/Z_{\eta'}^2$ is given; at the point $Z_\eta^2 = 0$, the angle ϕ_P is estimated to be $\phi_P = (37.7 \pm 2.6)°$ and the simple factorized relation holds [71]

$$\frac{\Gamma(D_s^+ \to \eta' e^+ \nu)}{\Gamma(D_s^+ \to \eta e^+ \nu)} = R_D \cot^2\phi_P \cos^2\phi_G, \tag{4.3}$$

where ϕ_G has been defined in Eq. (2.7). In [71] the value $R_D = 0.28$ is estimated by neglecting the nontrivial dependence on the constituent quark transition form factor, that is a conventional approximation in literature, while $R_D = 0.23$ is estimated by assuming a simple monopole q^2 dependence. We observe that the mixing angle extracted from (4.3) is strongly dependent on the value of R_D; in order to provide a rough estimation of the theoretical error we consider an averaged R_D, that is, $R_D = 0.255 \pm 0.050$. By using the experimental ratio of branching fractions (4.1), we estimate $Z_{\eta'}^2 = 0.16 \pm 0.33_{\rm exp} \pm 0.23_{\rm th}$, that is $\phi_G = (23.3 \pm 25.8_{\rm exp} \pm 18.0_{\rm th})°$, where the theoretical error refers to the errors on R_D and ϕ_P added in quadrature. The experimental error dominates over the rough estimate of the theoretical error and it prevents any conclusion on the gluonic content of the $\eta - \eta'$ system.

For the Cabibbo-suppressed transitions one finds in the same framework:

$$\frac{\Gamma(D^+ \to \eta' e^+ \nu)}{\Gamma(D^+ \to \eta e^+ \nu)} = \tilde{R}_D \tan^2\phi_P. \tag{4.4}$$

In 2008 CLEO-c reported its first measurement of $\Gamma(D^+ \to \eta e^+ \nu)$ and an upper bound on $\Gamma(D^+ \to \eta' e^+ \nu)$ [72]. Two years later, the same collaboration presented the first observation of $D^+ \to \eta' e^+ \nu$, with branching fraction $\mathcal{B}(D^+ \to \eta' e^+ \nu) = (2.16 \pm 0.53 \pm 0.07) \times 10^{-4}$, and an improved $\mathcal{B}(D^+ \to \eta e^+ \nu) = (11.4 \pm 0.9 \pm 0.4) \times 10^{-4}$ [73]. By using the above data and the reasonable assumption $R_D \simeq \tilde{R}_D$, we estimate from Eq. (4.4) the value $\phi_P = (41 \pm 4_{\rm exp} \pm 3_{\rm th})°$.

By including a nonzero gluon contribution, we can parametrize the D^+ ratio as in (4.3). However, with the available recent data, the estimate of the angle ϕ_P can be made independently of ϕ_G by taking the ratio

$$\frac{\Gamma(D_s^+ \to \eta' e^+ \nu)/\Gamma(D_s^+ \to \eta e^+ \nu)}{\Gamma(D^+ \to \eta' e^+ \nu)/\Gamma(D^+ \to \eta e^+ \nu)} \simeq \cot^4\phi_P. \tag{4.5}$$

The left side is given by the recent experimental data quoted before, and we get $\phi_P = (40 \pm 3)°$.

Yet this is not the final word on the experimental or theoretical side. A few years down the line we can expect BESIII to obtain an even larger sample allowing a more accurate measurement with errors on the angle ϕ_P going down to about 2%.

The theoretical situation is more complex. While the spectator diagram generates the leading contribution, for a precision study we cannot ignore nonleading ones. The so-called "weak annihilation" (WA) process contributes even to semileptonic meson decays [66,74], as can be illustrated most directly for D_s^+ and D_s; see Fig. 3. An analysis based on inclusive semileptonic D decays, which considers both the widths and the lepton energy moments, shows no clear evidence of WA effects [75]. While WA might affect the corresponding inclusive semileptonic width only moderately, it should impact the exclusive channels $D_s^+ \to \eta' l^+ \nu$ and $D^+ \to \eta' l^+ \nu$ on the Cabibbo- favored and suppressed levels via the η''s gluonic component. The strength of the effect depends on two factors, namely, the size of the gg component in the η' wave function and on how much gg radiation one can expect in semileptonic D_s^+, D^+, and B^+ decays. Lastly, since the main effect might come from the interference with the spectator amplitude, it can *a priori* enhance or reduce those rates. Simple relations such as (4.2) do not necessarily hold any longer.

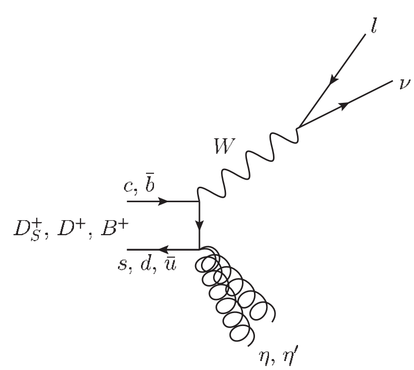

FIG. 3. Valence quarks $c/\bar{s}/\bar{d}$ (as well as $\bar{b}/u$) emitting two gluons which generate η/η' via the gluonic component of the wave functions.

2. $B^+ \to \eta^{(\prime)} l \nu$

In $B^+ \to \eta^{(\prime)} l \nu$ decays one encounters a situation analogous to that for $D^+ \to \eta^{(\prime)} l \nu$ except that their rates are suppressed by $|V_{ub}/V_{cb}|^2$ rather than $|V_{cd}/V_{cs}|^2$ and that the range in q^2 is much larger. In passing we just want to mention that one needs to understand their rates to determine $|V_{ub}/V_{cb}|^2$ from $\Gamma(B \to X_u l \nu)/\Gamma(B \to X_c l \nu)$ with the hoped-for accuracy of about 5% [76].

In the spectator ansatz one finds using the quark-flavor basis

$$\frac{\Gamma(B^+ \to \eta' l^+ \nu)}{\Gamma(B^+ \to \eta l^+ \nu)} = \tilde{R}_B \tan^2\phi \tag{4.6}$$

with the factor $\tilde{R}_B$ again describing the relative phase space (much more abundant than for D mesons) and the ratio of the integrated form factors. The semileptonic form factors $B \to \eta^{(\prime)}$ have been calculated in [77] from QCD sum rules on the light cone, to next-to-leading order in QCD. In frameworks based on QCD factorization the mesons Fock-state wave functions enter in the form of light-cone distribution amplitudes. Equation (4.6) keeps robust under the dynamical assumptions in [77]. Data on the ratio (4.6) have started to appear since a few years. The errors are still quite large, comparable in percentage to the ones analyzed in the previous section, and prevent definite conclusions on the glue mixing to be drawn.

In 2007 CLEO found first evidence for $B^+ \to \eta' l^+ \nu$ decay, with branching fraction $\mathcal{B}(B^+ \to \eta' l^+ \nu) = (2.66 \pm 0.80 \pm 0.56) \times 10^{-4}$. This year, also the *BABAR* Collaboration measured for the first time $\mathcal{B}(B^+ \to \eta' l^+ \nu) = (0.24 \pm 0.08_{\rm stat} \pm 0.03_{\rm syst}) \times 10^{-4}$ [78], superseding the 2008 upper limit [79]. The *BABAR* value has a significance of 3.0σ and it is an order of magnitude smaller than the CLEO result.

The same 2007 CLEO analysis also reported a new value of the branching fraction $\mathcal{B}(B^+ \to \eta l^+ \nu) = (0.44 \pm 0.23 \pm 0.11) \times 10^{-4}$ [80], improving previous 2003 values [81]. The result is similar to the newest one by *BABAR*: $\mathcal{B}(B^+ \to \eta l^+ \nu) = (0.36 \pm 0.05_{\rm stat} \pm 0.04_{\rm syst}) \times 10^{-4}$ [78]. By using *BABAR* data [78], the ratio (4.6) reads

$$\frac{\mathcal{B}(B^+ \to \eta' l^+ \nu)}{\mathcal{B}(B^+ \to \eta l^+ \nu)}\bigg|_{\rm BABAR} = 0.67 \pm 0.24_{\rm stat} \pm 0.11_{\rm syst}. \tag{4.7}$$

It is evident that the experimental situation is not yet satisfying, although the previous value does not exclude a large gluonic singlet contribution to the η' form factor.

The corresponding ratio involving the B_s mesons, that is $\mathcal{B}(B_s \to \eta' l^+ l^-)/\mathcal{B}(B_s \to \eta l^+ l^-)$, is also potentially informative on the $\eta^{(\prime)}$ gluonic content, although experimentally much more challenging. The results for the branching fractions of modes with two charged leptons in the standard model are of order 10^{-7}–10^{-8} [82], suggesting that they are within the reach of SuperB and Super KEK B factories.

B. Nonleptonic D and charmless B decays

Although estimates of the mixing angles may come from $b \to c$ dominated processes, such as $B_s^0 \to J/\psi \eta^{(\prime)}$ (see e.g. [23,69,83]), within the SM many charmless nonleptonic B decays receive significant or even leading contributions from loop processes, which represent quantum corrections. Thus they provide fertile hunting grounds for new physics, in particular, in their CP asymmetries. Yet to make a convincing case that an observed CP asymmetry is such that it could not be generated by SM forces alone, one has to be able to evaluate hadronic matrix elements. Such an undertaking is greatly helped by knowing the wave functions of the relevant particles.

Modes such as $B \to \eta^{(\prime)} K$ and $B_s \to \eta^{(\prime)} \phi$ seem particularly well suited in this respect. It should be noted that the branching ratio observed for $B \to \eta' K$ exceeds the original predictions considerably for reasons that have not been established yet. Those predictions had been based (among other assumptions) on identifying η' as a pure $q\bar{q}$ state. Allowing for a gluonic component opens the door to diverse decay mechanisms. For example, Kou and Sanda [84] suggested producing the η' meson via its gluonic component with the gluons being radiated off *different* quark lines; see Fig. 4. Having the gluon radiation being emitted from a single quark line might be a more favorable dynamical scenario (see e.g. [85,86]).

Recent branching ratio values are $\mathcal{B}(B^0 \to K^0 \eta) = (1.1 \pm 0.4) \times 10^{-6}$ and $\mathcal{B}(B^0 \to K^0 \eta') = (6.6 \pm 0.4) \times 10^{-5}$ [55]. The $B \to \eta^{(\prime)} K$ decays may proceed through tree diagrams $\bar{b} \to \bar{u} u \bar{s}$, but such contributions are color and Cabibbo-Kobayashi-Maskawa suppressed, and by one-loop $b \to s$ penguins. Although the same basic penguin mechanism is expected to drive both $B \to \eta^{(\prime)} K$ and $B \to \pi K$, the rate of the former is measured to be much larger. A possible distinctive contribution are flavor singlet amplitudes that are not allowed, if the final state contains only flavor nonsinglet states such as pions and kaons. In flavor

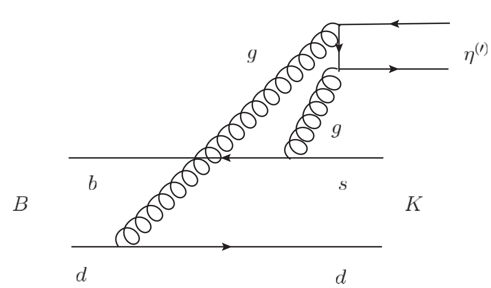

FIG. 4. $\eta^{(\prime)}$ produced via its gluonic component with the gluons being radiated off different quark lines. The transition $b \to s$ is penguin mediated.

singlet penguins two gluons couple to the η' violating the OZI rule and the amplitude can get contributions from the pure gluonic component of the η'.

The cases where two gluons are emitted by a single line ($b \to sgg$) together with spectator scattering and singlet weak annihilation have been explored in the context of QCD factorization (QCDF) [87]. In this approach the constructive interference between nonflavor singlet penguins seems already sufficient to enhance the $B \to \eta' K$ branching ratio, without the recourse to flavor singlet contributions; however, due to large hadronic uncertainties, a sizable gluonic contribution (up to 40%) to the $B \to \eta'$ form factor cannot be excluded.

In the perturbative QCD approach the impact of the gluonic component on the branching ratio—potentially important since it increases the branching ratios $B \to \eta' K$, while decreasing the $B \to K\eta$ one—has been estimated to be numerically very small [88]. The phenomenological importance of the η' gluonic content was instead emphasized in the context of soft collinear effective theory (SCET) [89].

Let us note that the previous exclusive analyses have been performed no later than 2006, when relevant new data, such as semileptonic $B \to \eta l \nu$ branching ratios, were not yet available. In semileptonic decays there is no enhancement in the B decays into η' mesons. The enhancement is also not observed in $D_s^+ \to K^+ \eta'$ relative to $D_s^+ \to K^+ \pi^0$. Recent data from *BABAR* for decays into K^* [90] favor an opposite pattern with respect to K, namely $\Gamma(B \to K^* \eta') < \Gamma(B \to K^* \eta)$. It would be interesting to check the impact of all recent experimental values on the different approaches. For instance in [89], the effort to fix the size for the gluonic contribution to the $B \to \eta'$ form factor, in a more constrained way with respect to [87], partly depends on fitting nonperturbative parameters to experimental data. We have to admit that a quantitative comparison with data is hampered by the theoretical uncertainties in nonleptonic decays.

The large measured branching ratio for $B^0 \to K_S \eta'$ by the *BABAR* and Belle Collaborations [91,92] greatly improves the usefulness of the decay mode for measuring CP asymmetry and to produce significant deviation from the SM prediction. The projected SuperB and Super KEK B factories will probe highly nontrivial ranges for new dynamics.

While it is true that the size of the time-dependent CP asymmetry established in $B_d \to \eta' K_S$ conforms well with the SM expectation, one cannot count on an intervention of new physics being numerically large there. Having a smallish deviation being significant implies good theoretical control over the SM prediction, which in turn requires good knowledge of the η' as well as η wave functions.

Finding CP asymmetries in $D \to \eta^{(\prime)}\pi$, $\eta^{(\prime)}\eta$, and $\eta\phi$ and interpreting them as signals of new dynamics has two experimental and theoretical advantages:

(i) The branching ratios are not very small.

(ii) The SM can produce only very tiny CP asymmetries. Even small asymmetries produce clear signatures for new physics, as long as one can control systematic uncertainties.

V. SUMMARY

In this paper we have described the status of ongoing investigations, starting from a review of the knowledge on the η and η' wave functions in terms of quark and gluon components as has been inferred mainly from radiative ϕ and ψ decays. The different determinations of the $\eta - \eta'$ mixing are generally consistent, but the message concerning the gluon content in the η' remains ambivalent. The semileptonic D^+, D_S^+, and B^+ decays can give other constraints to check the η' gluonium role. Moreover a sizable gluonium content could help to understand the unexpected high value of the branching ratio for $B \to \eta' K_S$ decay.

In conclusion: after many and difficult efforts to understand the $\eta - \eta'$ wave functions it might be seen as ‘‘smart’’ to call it a ‘‘victory’’ and move to another problem. We want to emphasize that it is a ‘‘noble’’ goal to improve our understanding of nonperturbative effects in QCD, in particular, when more ‘‘allies’’ from lattice QCD come to the battle line. Furthermore, and maybe even more important, it will help significantly to identify the footprints of new physics in CP asymmetries in B and D decays.

ACKNOWLEDGMENTS

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