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Higgs boson decay into two photons through the W-boson loop: No decoupling in the $m_W \to 0$ limit

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We reanalyze the W-boson loop in the amplitude of the Higgs boson decay into two photons to show the absence of decoupling in the limit of massless W bosons, $m_W \to 0$. The Higgs coupling to longitudinal polarizations survives in this limit and generates a nonvanishing contribution in the $H \to \gamma\gamma$ decay. This shows that the recent claim of decoupling by R. Gastmans, S. L. Wu, and T. T. Wu is incorrect, and the old calculations for the two-photon decay well known in the literature are valid.

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The two-photon mode of the Higgs-particle decay is important in experimental searches. Therefore theoretical calculations of the $H \to \gamma\gamma$ rate received much attention. The H transition into two photons goes via loops of charged particles: leptons, quarks, and W bosons. In the standard model (SM), the Higgs coupling to other fields is proportional to the masses of the latter; the most massive particle has the strongest coupling. All these loops had been calculated long ago [1–4], by different methods, with totally consistent results.

Surprisingly, in two recent publications [5,6] the issue of $H \to \gamma\gamma$ was raised anew, as if the passage of time negates the knowledge of the past. Raymond Gastmans, Sau Lan Wu, and Tai Tsun Wu revisited the calculation of the W-boson loop in the $H \to \gamma\gamma$ decay claiming that the old results [1–4] were erroneous. Using the unitary gauge, they obtained a different $H \to \gamma\gamma$ decay rate not coinciding with that of [1–4]. Technically, Gastmans *et al.* identify dimensional regularization exploited in some previous calculations as a source of the alleged mishap.

The main argument of Gastmans *et al.* in favor of the statement of [5,6] is the requirement of decoupling: their amplitude vanishes in the limit $m_W/m_H \to 0$ while that of [1–4] does not vanish in this limit.

Superficially this argument might seem reasonable. Indeed, the above-mentioned decoupling works for the fermion loop in the limit $m_f/m_H \to 0$ because the Higgs coupling to fermions is proportional to m_f. Likewise, in the W-boson case the Higgs coupling to W^+W^- is proportional to m_W^2; thus why not expect vanishing of the W-loop contribution at $m_W = 0$?

Actually this vanishing *does not occur*. In this paper we will explain the *absence of decoupling* for the W-boson loop in the $m_W \to 0$ limit owing to some general features of the non-Abelian vector fields. Our argument will connect a residual nonvanishing constant in the $H \to \gamma\gamma$ amplitude at $m_W = 0$ with a Goldstone-particle loop well known in the literature (see e.g. [7]).

There is a crucial difference between, say, spin-$1/2$ and spin-1 particles with regard to the massless limit. Namely, the number of polarization states stays the same for spin-$1/2$ massive and massless particles, while for the massive spin-1 particle we have three polarization states in contradistinction with the massless spin-1 field, with two polarization states. In the massive case, in addition to two spatially transverse polarizations (intrinsic to the massless vector field), we have also the longitudinal polarization. Moreover, the amplitude of this polarization grows in the limit $m_W \to 0$. Indeed, the longitudinal polarization of the W boson with 4-momentum $k^\mu = (E, 0, 0, k)$ moving along the z axis has the form

$$\epsilon_L^\mu = \frac{1}{m_W}(k, 0, 0, E) = \frac{k^\mu}{m_W} + \frac{m_W}{E + k}(-1, 0, 0, 1). \qquad (1)$$

In the case of an Abelian vector field, the singular in m_W term $\sim k^\mu/m_W$ does not contribute due to convolution with the conserved current; as a result the longitudinal quanta decouple in electrodynamics of a massive photon in the $m_\gamma \to 0$ limit.[1]

For non-Abelian vector fields, the longitudinal quanta do *not* decouple if two or more such quanta are involved in the process under consideration. In terms of loops, it means that, starting from one-loop, longitudinal W bosons produce a nondecoupling contribution. This was first demonstrated as long ago as 1971 in Ref. [8].

In application to the W loop in the $H \to \gamma\gamma$ decay, we will show now that the longitudinal quanta do not decouple from the Higgs boson and, as a result, the W-boson loop does not vanish in the massless limit.

In the standard model the Higgs coupling to W bosons has the form

[1]Note, however, that even for Abelian vector (massive) fields, the longitudinal polarizations do not decouple from gravity.

$$\mathcal{L}_{HWW} = 2\frac{H}{v} m_W^2 W^\mu W_\mu^*, \qquad \frac{1}{v} = (G\sqrt{2})^{1/2}. \tag{2}$$

For our purposes the unitary gauge is sufficient for the description of the W vector field, the same gauge as was exploited in Refs. [5,6]. There is no need to invoke the ξ-gauge, ghost fields, etc. Only the physical degrees of freedom are relevant.

It is clear that the HWW coupling following from the Lagrangian (2) vanishes at $m_W = 0$ for the transverse polarizations. The transverse polarizations can be safely neglected in this limit. At the same time, the longitudinal polarization containing $1/m_W$ [as seen from Eq. (1)] remains coupled to H (with a nonvanishing coupling) in the limit $m_W \to 0$. Technically, we can substitute W_μ as follows:

$$W_\mu \to W_\mu^L = \frac{1}{m_W}\partial_\mu \phi, \qquad m_W \to 0, \tag{3}$$

where ϕ is a charged scalar field. Then,

$$\mathcal{L}_{HWW} \to 2\frac{H}{v}\partial^\mu \phi \partial_\mu \phi^* = \frac{H}{v}\partial^2(\phi\phi^*), \tag{4}$$

where we neglected the terms $\sim\partial^2\phi$ which is certainly perfectly legitimate on mass shell. Equation (4) means that the Higgs field interacts with the Goldstone fields $(\phi\phi^*)$ through the trace of the energy-momentum tensor of these Goldstone fields.

Furthermore, omitting total derivatives, we can write

$$\mathcal{L}_{HWW} \to \frac{\partial^2 H}{v}(\phi\phi^*) = -\frac{m_H^2}{v} H\phi\phi^*. \tag{5}$$

Summarizing, we have just demonstrated that at $m_W = 0$ the Higgs coupling to the longitudinally polarized W bosons is equivalent to the coupling to the massless scalar field. This is nothing else but an abbreviated proof of the equivalence theorem at the tree level. One can find a detailed derivation of the equivalence in Refs. [8–10]; see also [11] for application to the $H \to \gamma\gamma$ decay.

In the SM one can also readily establish directly from the Lagrangian that the coupling (5) is the source of the $H \to \gamma\gamma$ amplitude in the limit $m_W \to 0$. Indeed, at a fixed v and m_H, the latter limit corresponds to a vanishing $SU(2)$ coupling, $g \to 0$, so that the transversal W bosons fully decouple from the Higgs boson. The only remaining relevant dynamics is that of the scalar sector,[2] which in this limit contains the massive Higgs boson and three Goldstone bosons, two of which, ϕ^+ and ϕ^-, are charged and mediate the $H \to \gamma\gamma$ decay, due to the vertex described by Eq. (5).

[2]This behavior, first established in Ref. [8], is also known as the equivalence theorem [9,10].

The next step, calculation of matrix element $\langle\gamma\gamma|\phi\phi^*|0\rangle$ for transition of scalars to two photons, contains no ambiguity. For the massless scalars loop, we get [3,7,11]

$$\langle\gamma\gamma|\phi\phi^*|0\rangle = -\frac{2}{(k_1+k_2)^2}\cdot\frac{\alpha}{4\pi}(k_1^\mu e_1^\nu - k_1^\nu e_1^\mu) \times (k_{2\mu}e_{2\nu} - k_{2\nu}e_{2\mu}), \tag{6}$$

where $k_{1,2}$ and $e_{1,2}$ are the 4-momenta and polarization vectors of the photons, respectively. In the Higgs boson decay, with the coupling (4), $(k_1+k_2)^2 = m_H^2$ and, therefore,

$$\langle\gamma\gamma|\mathcal{L}_{HWW}|H\rangle = \frac{2}{v}\cdot\frac{\alpha}{4\pi}(k_1^\mu e_1^\nu - k_1^\nu e_1^\mu) \times (k_{2\mu}e_{2\nu} - k_{2\nu}e_{2\mu}). \tag{7}$$

At $m_W = 0$ this coincides with results of Refs. [1–3]. Thus, we demonstrate the origin of nondecoupling for the W-boson loop at $m_W/m_H \to 0$ in the most transparent way.

We would like add a few comments on the considerations in Refs. [5,6].

(a) Instead of the two-photon decay, let us consider first the decay $H \to W^+W^-$ in the limit $m_W \to 0$. This is a tree-level process and its unambiguous calculation is sufficient to verify that (i) the amplitude does not vanish at $m_W = 0$ and (ii) only longitudinal polarizations contribute in this limit, in full correspondence with Eq. (5). This is where the argumentation of the authors of [5,6] based on the requirement of decoupling is flawed. It is an important point since Gastmans *et al.* understand that their diagrammatic expressions are not well-defined *per se* in the unitary gauge.
Moreover, one can start with the diagrammatic expression for the W loop in the unitary gauge and apply the Ward identities for the singular in m_W part coming from the vector propagators to verify the equivalence to the scalar loop at the small mass limit. This route was not explored in Refs. [5,6].

(b) The authors of [5,6] suggest that an inaccurate use of dimensional regularization causes the difference between their results and results of [1–4]. While this regularization had been indeed used in Ref. [1], we did not invoke it at all in our old calculation [3]. And yet the results of [1,3] are in accord with each other. Moreover, the absence of decoupling is visible at the tree level as was noted in the previous comment. Obviously, one cannot then blame dimensional regularization of the one-loop integrals.

(c) The low-energy theorem we derived in [3] relates the $H\gamma\gamma$ amplitude in the opposite limit $m_W/m_H \to \infty$ to the β function of the corresponding particles. For the massive W bosons this β function was first found in 1965 by Vanyashin and Terentev [12]; actually, it was the first signal of the soon-to-be discovered asymptotic freedom. Their calculation is simple to verify by modern methods.[3] This gives an independent check of calculations in [1–3] and contradicts [5,6].

(d) To pinpoint the issue let us consider the integral

$$I_{\mu\nu} = \int d^4 l \frac{4l_\mu l_\nu - g_{\mu\nu} l^2}{(l^2 - B + i\epsilon)^3}, \qquad (8)$$

which appears in the calculations of [5,6]. The authors put this integral to zero. It is true indeed under spherically symmetric integration over l (after Euclidian continuation) which they have used. On the other hand, the integral (8) can be rewritten in the form

$$I_{\mu\nu} = \frac{1}{2}\int d^4 l \frac{\partial^2}{\partial l^\mu \partial l^\nu}\frac{1}{l^2 - B + i\epsilon} - g_{\mu\nu}\int d^4 l \frac{B}{(l^2 - B + i\epsilon)^3}. \qquad (9)$$

The second integral is well defined and gives just a number $\int d^4 l B(l^2 - B + i\epsilon)^{-3} = -i\pi^2/2$. The first integral of total derivatives which cancels this number in dimension-4 is the one which breaks gauge invariance. Indeed, it can be viewed as a second order of expansion in the constant gauge potential A_μ of the expression

$$\int d^4 l \frac{1}{(l + eA)^2 - B + i\epsilon}. \qquad (10)$$

Thus, to preserve gauge invariance we should put the integral of total derivative to zero. This is automatic in both dimensional regularization and the Pauli-Villars one, but we can also use it as a constraint which allows one to maintain the current conservation.

Once it is realized that the integral (8) is nonvanishing, $I_{\mu\nu} = g_{\mu\nu} i\pi^2/2$, it is simple to check that substituting this into the Gastmans-Wu-Wu calculations leads to reproducing of the standard result. It appears first in terms $1/m_W^2$, Eq. (3.40) of Ref. [6], where instead of vanishing one arrives at a nonvanishing at the large m_H term. The integral (8) also appears in Eq. (3.50) of Ref. [6] for nonsingular in m_W terms. The above substitution leads to the transversal result (3.52) without any kind of subtraction; no Dyson prescription is needed.

The generic issue of finite but undetermined loops was discussed earlier by Roman Jackiw [13]. We thank him for pointing this out to us.

In summary. We reconfirm the results of the previous calculations [1–4] of the $H \to \gamma\gamma$ decay amplitude, including the nondecoupling of the Higgs boson from the two-photon channel in the limit $m_W/m_H \to 0$ as found (and emphasized) in our old calculation [3]. We have explicitly demonstrated here that this behavior is due to the contribution of the longitudinal W bosons in the intermediate state and can be traced to the Goldstone modes of the scalar field within the context of the well-known equivalence theorem. We thus assert that the recent claim [5,6] of an error in previous calculations is incorrect.

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Note added.—After the arXiv posting of our text, two more papers [14,15] on the same subject were posted. The detailed analysis given in these papers identifies a culprit in the calculations of [5,6]: they do not maintain the electromagnetic gauge invariance. Once this invariance is supported, either by regulators of Pauli-Villars type [14], or by dimensional regularization [15], the correct result is unambiguous.

[3]The β-function coefficient is $b = 7$ for W bosons; see [3] for details. It worth noting that in [12] one can read off this coefficient from the large field strength limit. In the charge renormalization, Vanyashin and Terentev had $b = 20/3 = 7 - 1/3$. The extra $-1/3$ should be taken away; it comes from an auxiliary heavy scalar with the mass $m_W/\sqrt{\xi}$, $\xi \to 0$. The parameter ξ in [12] is just an inverse of ξ in the R_ξ gauge, where the ξ dependence is canceled by ghosts.

[1] J. R. Ellis, M. K. Gaillard, and D. V. Nanopoulos, Nucl. Phys. **B106**, 292 (1976).
[2] B. L. Ioffe and V. A. Khoze, Fiz. Elem. Chastits At. Yadra **9**, 118 (1978) [Sov. J. Part. Nucl. **9**, 50 (1978)].
[3] M. A. Shifman, A. I. Vainshtein, M. B. Voloshin, and V. I. Zakharov, Yad. Fiz. **30**, 1368 (1979) [Sov. J. Nucl. Phys. **30**, 711 (1979)].
[4] T. G. Rizzo, Phys. Rev. D **22**, 178 (1980).
[5] R. Gastmans, S. L. Wu, and T. T. Wu, arXiv:1108.5322.
[6] R. Gastmans, S. L. Wu, and T. T. Wu, arXiv:1108.5872.
[7] H. Leutwyler and M. A. Shifman, Phys. Lett. B **221**, 384 (1989).
[8] A. I. Vainshtein and I. B. Khriplovich, Yad. Fiz. **13**, 198 (1971) [Sov. J. Nucl. Phys. **13**, 111 (1971)].

[9] J. M. Cornwall, D. N. Levin, and G. Tiktopoulos, Phys. Rev. D **10**, 1145 (1974); **11**, 972(E) (1975); C. E. Vayonakis, Lett. Nuovo Cimento Soc. Ital. Fis. **17**, 383 (1976).

[10] M. S. Chanowitz and M. K. Gaillard, Nucl. Phys. **B261**, 379 (1985).

[11] J. G. Korner, K. Melnikov, and O. I. Yakovlev, Phys. Rev. D **53**, 3737 (1996).

[12] V. S. Vanyashin and M. V. Terentev, J. Exp. Theor. Phys. **48**, 565 (1965) [Sov. Phys. JETP **21**, 375 (1965)].

[13] R. Jackiw, Int. J. Mod. Phys. B **14**, 2011 (2000).

[14] D. Huang, Y. Tang, and Y.-L. Wu, arXiv:1109.4846.

[15] W. J. Marciano, C. Zhang, and S. Willenbrock, Phys. Rev. D **85**, 013002 (2012).