Unification and cogeneration of dark matter and baryonic matter

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In grand unified theories with gauge groups larger than $SU(5)$, the multiplets that contain the known quarks and leptons also contain fermions that are singlets under the standard model gauge group. Some of these could be the dark matter of the Universe. Grand unified theories can also have accidental $U(1)$ global symmetries (analogous to $B - L$ in minimal $SU(5)$) that can stabilize dark matter. These ideas are illustrated in an $SU(6)$ model.

I. INTRODUCTION

It seems a strange coincidence that the cosmic densities of dark matter and ordinary baryonic matter are of the same order of magnitude, given that in most theoretical scenarios they are generated by unrelated mechanisms involving different particles, forces, and parameters. This coincidence suggests that the dark matter and baryonic matter may have been "cogenerated" in the early Universe, i.e. that the dark matter is a product of the same processes that created the cosmic baryon asymmetry. There is a rapidly growing literature studying various ways that this might have happened [1–5].

The first papers to propose this possibility [1] were based on the idea that primordial asymmetries in baryon and lepton number (B, L) were partially converted into an asymmetry in some other global quantum number (call it X) by sphaleron processes [6] when the temperature of the Universe was above the weak interaction scale M_W. Assuming X to be conserved (or nearly so) at low temperatures, the lightest particles carrying this quantum number would be stable and could play the role of dark matter. What would result from such a scenario is "asymmetric dark matter" [7]. Many other scenarios for generating asymmetric dark matter have been proposed [2–5]. In some of these scenarios, ordinary matter and dark matter are converted into each other by perturbative processes involving higher-dimension operators [2]; and in others by sphalerons (or by both sphalerons and higher-dimension operators) [3]. In some scenarios, the dark matter carries baryon number which compensates for the nonzero baryon asymmetry of ordinary matter [4]. Many papers propose still other mechanisms [5].

What we suggest here is the possibility that not only are dark matter and ordinary matter cogenerated but that they are "unified" in the sense of grand unification. The point is that grand unification naturally supplies several of the ingredients needed for the generation of asymmetric dark matter. First, grand unification based on groups larger than $SU(5)$ involves fermion multiplets that contain non-Standard-Model fermions that could play the role of dark matter. In particular, in $SU(N)$ with $N > 5$, the quark-lepton multiplets contain several fields that are singlets

under the standard model group $G_{SM} = SU(2)_L \times SU(3)_c \times U(1)_Y$. In $SU(6)$ or $SU(7)$ models with three families of quarks and leptons, for example, anomaly-free sets of fermion multiplets must contain *at least* six standard-model-singlet fermions. For larger groups, the number grows rapidly.

Second, it is not uncommon in simple unified models for there to be global quantum numbers that are "accidentally" (or "automatically") conserved, just as $B - L$ is accidentally conserved in the simplest $SU(5)$ model. Such quantum numbers could play the role of X that stabilizes the dark matter particles. We shall illustrate these ideas in a simple $SU(6) \times Z_N$ grand unified theory.

Another interesting feature of large unified groups is that they can contain additional non-Abelian factors at low energies (besides those of G_{SM}), whose sphalerons could convert baryons and leptons into dark matter particles; but we shall not explore that possibility in this paper. The illustrative model that we shall describe uses perturbative processes to convert matter and dark matter into each other [2].

II. AN $SU(6)$ MODEL

We shall now present the details of the model. Its symmetry group is $SU(6) \times Z_N$, where N may be any integer greater than 4. The fermions of each quark-lepton family consist of the anomaly-free set of $SU(6)$ multiplets $\mathbf{15} + \bar{\mathbf{6}} + \bar{\mathbf{6}}$ plus three $SU(6)$ singlets. These are shown in the left columns of Table I. The fundamental indices of $SU(6)$ are denoted by the capital latin letters, A, B, C, etc., which run from $1, \ldots, 6$. Here and throughout the paper, we suppress family indices. For those fermion multiplets that

TABLE I. The fermions of a family, and the Higgs fields.

Fermion field	$SU(6)$	Z_N	Higgs field	$SU(6)$	Z_N
$\Psi^{[AB]}$	**15**	1	$\phi^{[AB]}$	**15**	1
Ψ_A	$\bar{\mathbf{6}}$	1	ϕ_A	$\bar{\mathbf{6}}$	1
Ψ	**1**	1	H_A	$\bar{\mathbf{6}}$	ω
ψ'_A	$\bar{\mathbf{6}}$	ω^*			
η	**1**	ω	Ω^A_B	**35**	ω^*
ζ	**1**	ω^2			

contain the known quarks and leptons we use capital Ψ. The fermion multiplets denoted by the letters ψ and ζ and η contain only new fields.

The right columns of Table I show the Higgs fields and their $SU(6) \times Z_N$ transformation properties. The known fields all transform trivially under Z_N. The gauge symmetry breaking occurs in three stages:

(1) At the unification scale M_{GUT}, $SU(6)$ is broken to $[SU(3)_c \times SU(2)_L \times U(1)_Y] \times U(1)_6$ (which is contained in the $SU(5) \times U(1)_6$ subgroup of $SU(6)$). This breaking is done by an adjoint Higgs field Ω_B^A, whose vacuum expectation value (VEV) points in a direction that is a linear combination of the weak hypercharge generator $Y/2 = \text{diag}(\frac{1}{2}, \frac{1}{2}, -\frac{1}{3}, -\frac{1}{3}, -\frac{1}{3}, 0)$ and the $U(1)_6$ generator $T_6 = \text{diag}(-\frac{1}{5}, -\frac{1}{5}, -\frac{1}{5}, -\frac{1}{5}, -\frac{1}{5}, 1)$. We shall always denote an $SU(5)$ index (which takes values 1,2,3,4,5) by α, β, etc.; an $SU(2)_L$ index (which takes values 1,2) by i, j, etc.; and an $SU(3)_c$ index (which takes values 3,4,5) by a, b, etc. Thus, for example, $\psi_A = (\psi_\alpha, \psi_6) = (\psi_i, \psi_a, \psi_6)$.

(2) The $U(1)_6$ is broken at a scale M', which is somewhat larger than a TeV, by the fundamental Higgs multiplet H_A, whose VEV points in the 6 direction, i.e. $\langle H_6 \rangle \sim M'$. We assume that the mass of H_6 is of order M', but that its other components all have mass of order M_{GUT}. (This is the usual kind of split-multiplet problem of GUTs, analogous to the well-known "doublet-triplet splitting problem.") Below M', the gauge group is just the standard model group $G_{\text{SM}} = SU(3)_c \times SU(2)_L \times U(1)_Y$. There is, of course, a Z' gauge boson whose mass is of order M' that couples to T_6.

(3) The electroweak breaking is done by the two Higgs multiplets denoted by ϕ, which contain the $SU(2)_L$ doublets ϕ_i and ϕ_{i6}^*. We assume that one linear combination of these two doublets is tuned to be light (i.e. of order the weak scale in mass) and is the standard model Higgs doublet Φ_i that obtains a VEV, while all the other components of ϕ_A and $\phi^{[AB]}$ have superheavy mass. (In particular, the colored components, which can mediate proton decay, have mass of order M_{GUT}.)

Under the subgroup $SU(5) \subset SU(6)$, the nonsinglet fermions of Table I decompose as follows:

$$
\begin{aligned}
\Psi^{[AB]} &\rightarrow \Psi^{[\alpha\beta]} &+& \Psi^{\alpha6} \\
\mathbf{15} &\rightarrow \mathbf{10} &+& \mathbf{5}
\end{aligned}
$$

$$
\begin{aligned}
\Psi_A &\rightarrow \Psi_\alpha &+& \Psi_6 \\
\bar{\mathbf{6}} &\rightarrow \bar{\mathbf{5}} &+& \mathbf{1}
\end{aligned}
\qquad (1)
$$

$$
\begin{aligned}
\psi'_A &\rightarrow \psi'_\alpha &+& \psi'_6 \\
\bar{\mathbf{6}} &\rightarrow \bar{\mathbf{5}} &+& \mathbf{1}
\end{aligned}
$$

The standard model quarks and leptons are the $\Psi^{[\alpha\beta]} = \mathbf{10}$ and $\Psi_\alpha = \bar{\mathbf{5}}$. The extra $\bar{\mathbf{5}} + \mathbf{5}$ pair of $SU(5)$ will "mate" to get $O(M')$ masses. Specifically, the $\mathbf{5} = \Psi^{[\alpha6]}$ will obtain mass with $\bar{\mathbf{5}} = \psi'_\alpha$ through a term $(\Psi^{[\alpha6]}\psi'_\alpha)\langle H_6 \rangle$, as will be seen. There are also two $SU(5)$-singlet (and thus G_{SM}-singlet) fermions in the multiplets shown in Eq. (1). In order for these to get mass, we introduce gauge-singlets fermions denoted η and ζ that will mate with them to get Dirac masses.

The most general renormalizable Yukawa couplings allowed by $SU(6) \times Z_N$ are of the following forms (we suppress family indices):

$$
\begin{aligned}
\mathcal{L}_{\text{Yukawa}} &= \mathcal{L}_{\text{SM}} + \mathcal{L}_{F\bar{F}} + \mathcal{L}_{\text{singlet}} \\
\mathcal{L}_{\text{SM}} &= Y_u(\Psi^{[AB]}\Psi^{CD})\phi^{EF} + Y_d(\Psi^{[AB]}\Psi_A)\phi_B \\
&\quad + Y_\nu(\Psi_A\Psi)\phi^{*A} + M_R(\Psi\Psi) \\
\mathcal{L}_{F\bar{F}} &= f_1(\Psi^{[AB]}\psi'_A)H_B \\
\mathcal{L}_{\text{singlet}} &= f_2(\Psi_A\eta)H^{*A} + f_3(\psi'_A\zeta)H^{*A} + f_4(\psi'_A\eta)\phi^{*A}.
\end{aligned}
\qquad (2)
$$

The first three terms in $\mathcal{L}_{\text{SM}}$ have the effect of coupling the standard model Higgs doublet (which is a mixture of ϕ_i and ϕ_{i6}^*) to the known quarks and leptons (which, as noted above, are contained in the multiplets that are denoted by capital Ψ's). The term $\mathcal{L}_{F\bar{F}} = f_1(\Psi^{[AB]}\psi'_A)H_B$ gives $O(M')$ mass to the "extra" $\mathbf{5} + \bar{\mathbf{5}}$ pair of fermions, as already mentioned. The terms in $\mathcal{L}_{\text{singlet}}$ couple the standard-model-singlet fermions Ψ_6 and ψ'_6 (see Eq. (1)) to the gauge-singlet fermions denoted η and ζ so that all the singlet fermions can get mass. All these Yukawa terms and the masses that come from them will be examined in more detail shortly.

The most general Yukawa terms allowed by $SU(6) \times Z_N$ [shown in Eq. (2)] and the most general Higgs potential allowed by $SU(6) \times Z_N$ (which, incidentally, includes terms such as $\phi^{AB}\phi_A H_C \Omega_B^C$) happen accidentally to be invariant under a global $U(1)$ symmetry, whose generator we will call T. The T charges of the various multiplets in the model are as follows (given in parentheses): $\Psi^{[AB]}(1)$, $\Psi_A(-\frac{1}{2})$, $\Psi(0)$, $\psi'_A(-\frac{7}{2})$, $\eta(3)$, $\zeta(6)$, $\phi^{[AB]}(-2)$, $\phi_A(-\frac{1}{2})$, $H_A(\frac{5}{2})$, and $\Omega_B^A(0)$. This $U(1)_T$ symmetry is unbroken by grand unified theory-scale VEVs, but is spontaneously broken at the scale M' by $\langle H_6 \rangle$, which also breaks the gauge group $U(1)_6$, leaving an unbroken global symmetry $U(1)_X$, whose generator is given by

$$
X = \tfrac{1}{3}T + \tfrac{5}{6}T_6. \qquad (3)
$$

This generator X will play a crucial role in what follows as the quantum number that stabilizes dark matter. It is analogous to the conserved global symmetry $B - L$ in minimal $SU(5)$ models, where $B - L$ is a linear combination of a global charge that is accidentally conserved by the Yukawa couplings and a gauge generator (specifically, the weak

hypercharge Y). Here, both X and $B - L$ are conserved by low-energy couplings and VEVs.

All the Higgs fields in Table I have $O(M_{\mathrm{GUT}})$ masses except (a) the standard model doublet Φ_i, which is a linear combination of the doublets ϕ_i and $\phi^*_{[i6]}$, and has mass of order 100 GeV, and (b) the standard-model-singlet Higgs H_6, which has mass of order $M' > 1$ TeV. We will call Φ_i and H_6 "light Higgs fields" to distinguish them from "superheavy Higgs fields". It is easy to check explicitly that the light Higgs fields Φ_i and H_6 are neutral under X (as must be the case, of course, if X is left unbroken by their VEVs). Therefore, when fermions absorb or emit these light Higgs fields they do not change their X charge. The same is true of the emission and absorption of light gauge bosons (i.e. those of $SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)_6$).

In other words, except through extremely slow processes mediated by bosons with superheavy masses, fermions do not change their values of X. It is therefore very useful to classify the fermions of this model by their X charges. This is done in Table II.

Note that Table II also lists quantum numbers called B_0, L_0, B_1, L_1, L_2. These are defined as follows: $B_n \equiv B\delta_{|X|n}$, $L_n \equiv L\delta_{|X|n}$. In other words, we define separate baryon and lepton numbers for each $|X|$ sector. For example, a lepton with $X = -2$ has $L_2 = 1$, but $L_0 = L_1 = 0$. (It should be noted that $B = B_0 + B_1$, $L = L_0 + L_1 + L_2$, and $X = 3B_1 - L_1 - 2L_2$.) The reason for defining these

baryon and lepton numbers is that the emission and absorption of "light Higgs fields" and "light gauge bosons" do not change the values of B, L, and X of a fermion and therefore also separately conserve the quantum numbers B_n and L_n. This will be important in our later analysis. Later we shall introduce four-fermion operators, generated by the exchange of very heavy bosons, that conserve B, L and X but violate B_n and L_n. Such processes will be needed to redistribute particle asymmetries, i.e. convert a primordial asymmetry in one global charge into the other global charges, so that matter and dark matter asymmetries will end up being related to each other.

Observe that the fermions with $X = 0$ are just the known quarks and leptons of the standard model. Since the light Higgs fields have $X = 0$, their Yukawa couplings only couple these standard model fermions to each other and give them Dirac masses with each other. These couplings come from the terms $\mathcal{L}_{\mathrm{SM}}$ in Eq. (2). Specifically, the Y_u term gives mass to up-type quarks via $Y_u(\Psi^{[ab]}\Psi^{[c1]}) \times \langle\phi^{[26]}\rangle \propto Y_u(u^c u)v$. The Y_d term gives mass to down-type quarks and charged leptons via $Y_d(\Psi^{[a2]}\Psi_a + \Psi^{[21]}\Psi_1) \times \langle\phi_2\rangle \propto Y_d(dd^c + \ell^+\ell^-)v$. The Y_ν term gives the Dirac neutrino masses via $Y_\nu(\Psi_2\Psi)\langle\phi^{*2}\rangle \propto Y_\nu(\nu N^c)v$. And the M_R term gives the superlarge Majorana masses to the right-handed neutrinos: $M_R(\Psi\Psi) = M_R(N^c N^c)$.

As can be seen from Table II, the sector of fermions with $X = \pm 1$ contains (for each family) a **5** and $\bar{\mathbf{5}}$'s of $SU(5)$,

TABLE II. The fermions and Higgs fields, classified by their X values (0, ± 1, ± 2), where $X = \frac{1}{3}T + \frac{5}{6}T_6$. The baryon and lepton numbers are defined by $B_n \equiv B\delta_{|X|n}$, $L_n \equiv L\delta_{|X|n}$.

Field	X	T	T_6	B_0	L_0	B_1	L_1	L_2
$\Psi^{[\alpha\beta]} \rightarrow \Psi^{[ab]} = u^c$	0	1	$-\frac{2}{5}$	$-\frac{1}{3}$	0	0	0	0
$\rightarrow \Psi^{[ai]} = Q$	0	1	$-\frac{2}{5}$	$\frac{1}{3}$	0	0	0	0
$\rightarrow \Psi^{[12]} = l^c$	0	1	$-\frac{2}{5}$	0	-1	0	0	0
$\Psi_\alpha \rightarrow \Psi_a = d^c$	0	$-\frac{1}{2}$	$\frac{1}{5}$	$-\frac{1}{3}$	0	0	0	0
$\rightarrow \Psi_i = L$	0	$-\frac{1}{2}$	$\frac{1}{5}$	0	1	0	0	0
$\Psi \rightarrow \Psi = N^c$	0	0	0	0	-1	0	0	0
$\Psi^{[\alpha 6]} \rightarrow \Psi^{[a6]}$	1	1	$\frac{4}{5}$	0	0	$\frac{1}{3}$	0	0
$\rightarrow \Psi^{[i6]}$	1	1	$\frac{4}{5}$	0	0	0	-1	0
$\psi'_\alpha \rightarrow \psi'_i$	-1	$-\frac{7}{2}$	$\frac{1}{5}$	0	0	$-\frac{1}{3}$	0	0
$\rightarrow \psi'_i$	-1	$-\frac{7}{2}$	$\frac{1}{5}$	0	0	0	1	0
Ψ_6	-1	$-\frac{1}{2}$	-1	0	0	0	1	0
η	1	3	0	0	0	0	-1	0
ψ'_6	-2	$-\frac{7}{2}$	-1	0	0	0	0	1
ζ	2	6	0	0	0	0	0	-1
$\phi_\alpha \rightarrow \phi_i$	0	$-\frac{1}{2}$	$\frac{1}{5}$	0	0	0	0	0
$\phi^{[\alpha 6]} \rightarrow \phi^{[i6]}$	0	-2	$\frac{4}{5}$	0	0	0	0	0
H_6	0	$\frac{5}{2}$	-1	0	0	0	0	0

namely $\Psi^{[\alpha 6]}$ and ψ'_α. These mate to obtain masses of $O(M')$ via the Yukawa term in $\mathcal{L}_{F\bar{F}}$, which gives $f_1(\Psi^{[\alpha 6]}\psi'_\alpha)\langle H_6\rangle$.

The $X = \pm 1$ sector also contains (for each family) the singlet fermions Ψ_6 with $X = -1$, and η with $X = 1$. The term $f_2(\Psi_6\eta)H^{*6}$ in $\mathcal{L}_{\text{singlet}}$ couples them together into massive Dirac particles. (Note that the gauge-singlet fermions η have been introduced into the model just to give mass to Ψ_6, which otherwise would remain massless.) The term $f_4(\psi'_6\eta)\phi^{*6}$ in $\mathcal{L}_{\text{singlet}}$ has the effect of mixing these singlet fermions with neutrinos in the $\mathbf{5} + \bar{\mathbf{5}}$, so that the neutral fermions in the $X = \pm 1$ sector actually have a 2×2 mass matrix (actually 6×6 if one takes into account that there are three families) of the following form:

$$(\psi'_2, \Psi_6)\begin{pmatrix} f_1\langle H_6\rangle & f_4\langle\phi^{*2}\rangle \\ 0 & f_2\langle H^{*6}\rangle \end{pmatrix}\begin{pmatrix} \Psi^{[26]} \\ \eta \end{pmatrix}. \quad (4)$$

If all the Yukawa couplings in Eq. (4) were of order 1, then all the masses of the fermions in the $X = \pm 1$ sector would be of $O(M')$. There have to be, however, some particles to play the role of dark matter. Since in this scenario the present number densities of dark matter particles and baryons will turn out to be of the same order of magnitude, the mass m_{DM} of the dark matter particles should be roughly of order 1 GeV. There are various ways this can be the case. One simple way is that the Yukawa couplings that we have denoted f_2 in Eqs. (2) and (4) are of order $(1 \text{ GeV})/M' < 10^{-3}$. We shall assume this to be true and also assume that the Yukawa couplings denoted f_1 and f_3 are significantly larger than f_2. In that case, the lightest $X \neq 0$ fermions are the Dirac fermions made up of the gauge singlets Ψ_6 and η. These will be the dark matter particles

of the model. We will denote these dark matter particles sometimes as (Ψ_6, η). (As noted, and as can be seen from Eq. (4), these mix with angle $O(v/M')$ with weak-doublet neutrinos of mass $O(M')$. Thus the dark matter particles have $O((\frac{v}{M'})^2\frac{f_2}{f_1})$ couplings to the standard model Z boson.)

We come, finally, to the $X = \pm 2$ sector of fermions. This consists (for each family) of a standard-model-singlet fermions with $X = -2$ (namely, ψ'_6), and with $X = +2$ (namely ζ). The term $f_3(\psi'_A\zeta)H^{*A}$ in $\mathcal{L}_{\text{singlet}}$ couples these together to make massive Dirac particles. (Note that the gauge-singlet fermions denoted by the letter ζ have been introduced into the model just to give mass to the $X = -2$ fermions.)

III. THE PROCESSES THAT REDISTRIBUTE ASYMMETRIES

As noted before, there must be processes that conserve X and $B - L$ but violate B_n and L_n in order to redistribute asymmetries in quantum numbers and thus relate the matter and dark matter asymmetries. Such processes are needed for other reasons as well. For example, they are needed to allow the colored $X \neq 0$ particles (i.e. those with $B_1 \neq 0$) to decay. (Such particles if stable and light would have been seen at accelerators, and if stable and heavy would contribute too much to the dark matter density of the Universe.) These $\Delta B_1 \neq 0$ decays can be relatively slow, as long as they occur early enough not to interfere with primordial nucleosynthesis.

The processes that we postulate to violate B_n and L_n are very simple. They are given by four-fermion operators of the form $(\bar{\psi}'^A\psi'_B)(\bar{\Psi}\Psi_A)$, and more precisely by

$$\begin{aligned} \mathcal{O}_1 &= (M_1)^{-2}(\bar{\psi}'^6\psi'_i)(\bar{\Psi}^i\Psi_6), & \Delta(B_0, L_0, B_1, L_1, L_2) &= (0, -1, 0, 2, -1), \\ \mathcal{O}_2 &= (M_2)^{-2}(\bar{\psi}'^6\psi'_a)(\bar{\Psi}^a\Psi_6), & \Delta(B_0, L_0, B_1, L_1, L_2) &= \left(\tfrac{1}{3}, 0, -\tfrac{1}{3}, 1, -1\right), \\ \mathcal{O}_3 &= (M_3)^{-2}(\bar{\psi}'^a\psi'_i)(\bar{\Psi}^i\Psi_a), & \Delta(B_0, L_0, B_1, L_1, L_2) &= \left(-\tfrac{1}{3}, -1, \tfrac{1}{3}, 1, 0\right). \end{aligned} \quad (5)$$

It is easy to check that these conserve $B = B_0 + B_1$, $L = L_0 + L_1 + L_2$ and $X = 3B_1 - L_1 - 2L_2$. These operators can arise in a simple way from integrating out heavy fields as follows. Suppose there is a boson $\tilde{\phi}_A$ and gauge-singlet fermions $\tilde{\Psi}$ and $\tilde{\eta}$. Suppose $\tilde{\phi}_A$, $\tilde{\Psi}$, and $\tilde{\eta}$ have the same $SU(6) \times Z_N$, quantum numbers as the particles ϕ_A, ψ and η, respectively. We assume that $\tilde{\phi}_\alpha$ and $\tilde{\eta}$ have mass of a scale we will call M_Δ, where $M_\Delta \gg M'$, and that $\tilde{\phi}_6$ has mass of order 1 to 10 GeV, while $\tilde{\Psi}$ is massless. (It is also assumed that $\tilde{\phi}_A$ has vanishing VEV.) With these quantum numbers, these particles have the Yukawa couplings

$$\mathcal{L}_\Delta = f(\Psi_A\tilde{\Psi})\tilde{\phi}^{*A} + f'(\psi'_A\tilde{\eta})\tilde{\phi}^{*A}. \quad (6)$$

Note the similarity to the terms in Eq. (2) with coefficients Y_ν and f_4. We can ensure that these are the *only* Yukawa couplings that $\tilde{\phi}_A$, $\tilde{\Psi}$, and $\tilde{\eta}$ possess, either by assigning them suitable Z_N charges, or, even more simply, by positing another Z_M symmetry under which $\tilde{\phi}_A \to z\tilde{\phi}_A$, $\tilde{\Psi} \to z\tilde{\Psi}$, and $\tilde{\eta} \to z\tilde{\eta}$, while all other fields transform trivially. (We can assign the massless fermion $\tilde{\Psi}$ lepton numbers $L_0 = 0$, $L_1 = 1$, $L_2 = 0$, with the light boson $\tilde{\phi}_6$ having all baryon and lepton numbers zero.)

Then the box diagram shown in Fig. 1 gives rise to the operators in Eq. (5), with $M_1, M_2, M_3 \sim M_\Delta$. Note that the operator $\mathcal{O}_3$ directly gives the decay $\psi'_a \to \Psi_a + \bar{\Psi}^2 + \psi'_2$. The initial particle is an $X = -1$ antiquark with mass
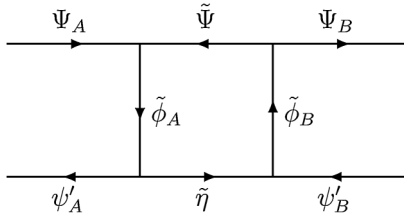
FIG. 1. The diagram that gives the operators $\mathcal{O}_1$ (if $A = 6$ and $B = i$), $\mathcal{O}_2$ (if $A = 6$ and $B = a$), and $\mathcal{O}_3$ (if $A = a$ and $B = i$).
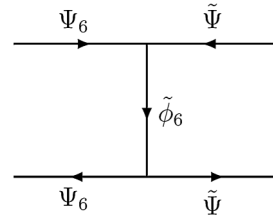

FIG. 2. The diagram by which dark matter particles and antiparticles can annihilate into massless fermion antifermion pairs: $\Psi_6 + \bar{\Psi}^6 \to \tilde{\Psi} + \bar{\tilde{\Psi}}$.

of $O(M')$. The first two final state particles are ordinary standard model particles, namely, an antiquark (d_L^c) and a lepton (ν_L). The third final state particle is ψ_2', which mixes with the dark matter particles (Ψ_6, η) as can be seen from Eq. (4). In fact, the operators in Eq. (5) allow all fermions of the model with masses of order M' to decay ultimately to ordinary quarks and leptons and the dark matter particles (Ψ_6, η).

IV. THE COSMOLOGICAL SCENARIO

Now let us outline the sequence of events in the early Universe that generate the current baryon and dark matter abundances in this model.

Stage 1, which happens when the Universe is above a superlarge temperature T_{LG}, is the genesis of an asymmetry in lepton number. Often this is assumed to happen through the decays of superheavy right-handed neutrinos (here called Ψ) [8]. However, as will be seen, this will not lead to the generation of any dark matter asymmetry in the present scenario, since the right-hand neutrinos have $X = 0$. We will therefore assume that the primordial asymmetry is of fermions that carry both L and X. In particular, we shall look at the case in which an asymmetry of the fermions ζ is generated. How this might happen will be discussed later.

Stage 2 is the period $T_\Delta < T < M_\Delta$, during which both $SU(2)_L$ sphaleron processes and the scattering processes involving the operators $\mathcal{O}_1$, $\mathcal{O}_2$ and $\mathcal{O}_3$ are in equilibrium. The freeze-out temperature T_Δ of those scattering processes is given roughly by $T_\Delta \sim M_\Delta (\frac{16\pi g^{1/2} M_\Delta}{M_{P\ell}})^{1/3}$, where g is the effective number of massless particle species at T_Δ. We shall assume that M_Δ is large enough that $T_\Delta > M'$, the scale at which $U(1)_6$ breaks and the non-Standard-Model quarks and leptons get their mass. During stage 2, the initial asymmetry in ζ (and thus in L and X) is reshuffled by sphalerons and scattering processes among the various particle types, leading to asymmetries in all of the quantum numbers B_0, L_0, B_1, L_1, and L_2 that are of the same order. These will be computed shortly.

Stage 3 is the period $T_{\text{dec}} < T < T_\Delta$, where T_{dec} is the temperature at which the particles with mass of $O(M')$ decay. When the temperature falls below T_Δ, the relative values of B_0, L_0, B_1, L_1, and L_2 freeze. Sphaleron processes continue until some temperature $T_{\text{sph}} \sim 200$ GeV,

but do not affect these ratios, which change again only when the fermions with mass of order $O(M')$ decay (out of equilibrium) via the operators $\mathcal{O}_1$, $\mathcal{O}_2$, and $\mathcal{O}_3$ at $T_{\text{dec}} \sim M'(\frac{M'^3}{96\pi^2 T_\Delta^3})^{1/2}$. It should be noted that when T falls below M' (which happens during stage 3, since $T_{\text{dec}} < M' < T_\Delta$), annihilations (mediated by $SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)_6$ gauge interactions wipe out almost all the particles of $O(M')$ except for the asymmetric components. (For example, if the asymmetry in B_1 is positive, the density of $B_1 < 0$ particles will be driven to a value much less than that of $B_1 > 0$ particles.)

Stage 4 is the period when $T < T_{\text{dec}}$. At this point, the particles with mass of $O(M')$ decay out of equilibrium via the operators $\mathcal{O}_1$, $\mathcal{O}_2$, and $\mathcal{O}_3$. These decays violate B_0, L_0, B_1, L_1, and L_2, and so again reshuffle the ratios of these quantum numbers. In particular, they set B_1 and L_2 to zero; but they leave a nonzero L_1 in the form of the dark matter particles (Ψ_6, η), which we will compute below. At this point, the only remaining fermions are the known standard model quarks and leptons, the dark matter particles (Ψ_6, η), and the massless $\tilde{\Psi}$ particles (whose contribution to the radiation density at the time of nucleosynthesis is equivalent to less than half of a neutrino species).

We assume that $T_{\text{dec}} > m_{\text{DM}} \sim 1$ GeV. (This means, for example, that if $M' \sim 1$ TeV, then T_Δ must be also be of order 1 TeV, while if $M' \sim 10$ TeV, then T_Δ must be less than about 100 TeV.) When T falls below m_{DM} the dark matter particles (Ψ_6, η) start to annihilate with their antiparticles through the diagrams shown in Fig. 2. Since the $\tilde{\phi}_6$ have been assumed to be light (of order 1 to 10 GeV) these annihilations efficiently reduce the number of dark matter antiparticles to much below the number of dark matter particles. That is, the dark matter is asymmetric.

V. COMPUTING THE RELATIVE ASYMMETRIES

We now calculate the matter and dark matter asymmetries that result in this model. During stage 2, all of the fermions of the model except Ψ and $\tilde{\eta}$ are relativistic, so we can neglect their masses. Call the fermion chemical potentials μ_{q_0}, μ_{ℓ_0}, μ_{q_1}, μ_{ℓ_1}, and μ_{ℓ_2}, for the quarks and leptons having various values of X. From the fact that the sphaleron processes and the scattering processes that

involve the operators $\mathcal{O}_1$ and $\mathcal{O}_3$ are in equilibrium, one has the following relations:

$$\begin{aligned}
\text{sphaleron:} \quad & 0 = 3\mu_{q_0} + \mu_{\ell_0}, \\
\mathcal{O}_1: \quad & 0 = \mu_{\ell_0} - 2\mu_{\ell_1} + \mu_{\ell_2}, \\
\mathcal{O}_3: \quad & 0 = \mu_{q_0} + \mu_{\ell_0} - \mu_{q_1} - \mu_{\ell_1}.
\end{aligned} \tag{7}$$

The equilibrium of the scattering processes involving $\mathcal{O}_2$ does not give an independent relation. From Table I, it can be seen that the number of species of fermions of each type (i.e. the statistical weight), counting the number of families, colors, and polarizations, is given by $g(q_0) = 3 \cdot 3 \cdot 4 = 36$, $g(\ell_0) = 3 \cdot 1 \cdot 3 = 9$, $g(q_1) = 3 \cdot 3 \cdot 2 = 18$, $g(\ell_1) = 3 \cdot 1 \cdot 6 + 1 = 19$, and $g(\ell_2) = 3 \cdot 1 \cdot 2 = 6$. (The extra 1 appearing in $g(\ell_1)$ is due to the fermion $\tilde{\Psi}$.) In a comoving volume of the Universe, the asymmetries in the quantum numbers are related to the chemical potentials by the relation $N_i \propto g_i \mu_i T^2 R(T)^3$, where $R(T)$ is the scale factor of the Universe when the temperature is T. Therefore, one has

$$\begin{aligned}
B_0 = \frac{1}{3}Q_0 = 12\mu_{q_0}K, \qquad & L_0 = 9\mu_{\ell_0}K, \\
B_1 = \frac{1}{3}Q_1 = 6\mu_{q_1}K, \qquad & L_1 = 19\mu_{\ell_1}K, \\
L_2 = 6\mu_{\ell_2}K, &
\end{aligned} \tag{8}$$

where K depends on the temperature and volume. Therefore, from Eqs. (7) and (8), one has that during stage 2

$$\begin{aligned}
0 &= \frac{1}{4}B_0 + \frac{1}{9}L_0, \\
0 &= \frac{1}{12}B_0 + \frac{1}{9}L_0 - \frac{1}{6}B_1 - \frac{1}{19}L_1, \\
0 &= \frac{1}{9}L_0 - \frac{2}{19}L_1 + \frac{1}{6}L_2.
\end{aligned} \tag{9}$$

We assume that during stage 1, primordial asymmetries were generated in the quantum numbers X and $B - L$. After stage 1, however, these quantum numbers are conserved. Thus we have two further relations,

$$\begin{aligned}
X &= 3B_1 - L_1 - 2L_2 = a, \\
B - L &= B_0 + B_1 - L_0 - L_1 - L_2 = b,
\end{aligned} \tag{10}$$

where a and b are constants. Equations (9) and (10) can be solved to obtain

$$\begin{aligned}
B_0 &= -\frac{4}{9 \cdot 119}(37a - 61b), \qquad L_0 = \frac{1}{119}(37a - 61b), \\
B_1 &= \frac{4}{9 \cdot 119}(83a - 50b), \qquad L_1 = -\frac{19}{3 \cdot 119}(a + 8b), \\
L_2 &= \frac{1}{3 \cdot 119}(-86a + 26b).
\end{aligned} \tag{11}$$

In stage 4, after T has fallen below T_{dec}, the particles with mass of order M' decay into the ordinary quarks and leptons of the standard model, plus the dark matter fields Ψ_6. We will assume for ease of discussion that the heaviest $O(M')$ particles are those with $B_1 \neq 0$ followed by those with $L_2 \neq 0$. Then the $B_1 \neq 0$ particles will decay via the operator $\mathcal{O}_3$. From Eq. (5) one sees that these decays will change the particle asymmetries in the proportions $\Delta B_0 = -\Delta B_1$, $\Delta L_0 = -3\Delta B_1$, and $\Delta L_1 = 3\Delta B_1$. Therefore, if $B_1 \to B_1' = B_1 + \Delta B_1 = 0$, one has

$$\begin{aligned}
B_0' &= B_0 + \Delta B_0 = \frac{2}{119}(a + 8b), \\
L_0' &= L_0 + \Delta L_0 = \frac{1}{3 \cdot 119}(277a - 283b), \\
L_1' &= L_1 + \Delta L_1 = \frac{1}{3 \cdot 119}(-185a - 52b), \\
L_2' &= L_2 + \Delta L_2 = \frac{1}{3 \cdot 119}(-86a + 26b).
\end{aligned} \tag{12}$$

The $L_2 \neq 0$ particles decay via the operator $\mathcal{O}_1$, changing the asymmetries in the proportions $\Delta L_0 = \Delta L_2$ and $\Delta L_1 = -2\Delta L_2$, as can be seen from Eq. (5). Therefore, if $L_2 \to 0$, one ends up with the final values of the quantum numbers being

$$B_{0f} = \frac{2}{119}(a + 8b), \qquad L_{0f} = \frac{1}{119}(121a - 103b), \tag{13}$$
$$L_{1f} = -a.$$

It should be noted that the final symmetry in L_1 is in the form of the massive dark matter particles (Ψ_6, η), and not in the form of the massless fermions $\tilde{\Psi}$. (It is easily shown in the following way that there is no asymmetry in $\tilde{\Psi}$. One can assign an exactly conserved quantum number Z to $\tilde{\Psi}$ and $\tilde{\phi}_6$, with both these particles having $Z = 1$ and all other particles having $Z = 0$. By conservation of Z, and the fact that no asymmetry of Z existed initially, one has that $N_{\tilde{\Psi}} - N_{\tilde{\phi}_6} = 0$. But eventually all the $\tilde{\phi}_6$ and their antiparticles decay by $\tilde{\phi}^6 \to \Psi_6 + \tilde{\Psi}$, which drives $N_{\tilde{\phi}_6}$ and thus $N_{\tilde{\Psi}}$ to zero.)

Let us suppose that the primordial asymmetry generated during stage 1 is in the number of ζ particles. Since these have $X = 2$ and $B - L = 1$, it follows that $a = 2b$. From this and Eq. (13), one has that

$$\frac{L_{1f}}{B_{0f}} = -\frac{119}{10}. \tag{14}$$

This is the present ratio of the number of dark matter particles to the number of baryons in the Universe. It is a rather remarkable feature of models of this type [1–3] that this ratio is predicted, and therefore that the mass of the dark matter particle is predicted. Of course, different grand unified models would give different predictions.

Finally, we come to the question of the primordial asymmetry generated during stage 1. One could imagine that this asymmetry was in ordinary leptons, from the decay of superheavy right-hand neutrinos Ψ, as has been

much studied [8]. However, that would give $a = 0$ and $b \neq 0$, which would yield no dark matter asymmetry, according to Eq. (13). But asymmetries in other species of particle can be generated in an analogous manner. For example, suppose that the discrete Z_N symmetry in Table I is Z_8 and there exist complex scalar fields S which are $SU(6)$ singlets and transform under Z_8 as $S \rightarrow \omega^4 S = -S$. Then, by Table I, one sees that the S can have the Yukawa coupling $\zeta\zeta S$ and also have explicit superheavy masses from terms of the form $M_S^2 S^* S$ and $\Delta_S^2 S S$ + H.c. The out-of-equilibrium decays of the superheavy S particles can generate a $\zeta - \bar{\zeta}$ asymmetry.

We conclude by observing that the model presented here is not unique, but is meant to illustrate the general point that grand unification, especially with large unitary groups, entails the existence of standard-model-singlet fermions that could be the dark matter. Such dark matter particles would be very difficult to detect. For example, in the model presented here, the dark matter particles have no standard model couplings except $O(v^2/M'^2)$ couplings to the Z boson through the mixing shown in Eq. (4). It is characteristic of these scenarios that there will be extra Z bosons at energies near the weak scale, and that the dark matter will couple to these extra Z bosons. Indeed, the primary means by which the dark matter particles would be produced in accelerators would be via the production of extra Z bosons and their subsequent decay into dark matter particle-antiparticle pairs.

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