Remarks on the quantum numbers of $X(3872)$ from the invariant mass distributions of the $\rho J/\psi$ and $\omega J/\psi$ final states

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We reanalyze the two- and three-pion mass distributions in the decays $X(3872) \rightarrow \rho J/\psi$ and $X(3872) \rightarrow \omega J/\psi$ and argue that the present data favor the 1^{++} assignment for the quantum numbers of the X.

I. INTRODUCTION

Since its discovery in 2003 [1], the $X(3872)$ charmonium is subject of many experimental and theoretical efforts aimed at disclosing its nature—for a recent review see [2]. The most recent data on the mass of the X is [3]

$$M_X = (3871.85 \pm 0.27(\text{stat}) \pm 0.19(\text{syst})) \text{ MeV}, \quad (1)$$

with a width of $\Gamma_X < 1.2$ MeV. However, the problem of the quantum numbers for the X is not fully resolved yet: while the analysis of the $\pi^+\pi^- J/\psi$ decay mode of the $X(3872)$ yields either 1^{++} or 2^{-+} quantum numbers [3,4], the recent analysis of the $\pi^+\pi^-\pi^0 J/\psi$ mode seems to favor the 2^{-+} assignment [5], though the 1^{++} option is not excluded. This question is clearly very central, for the most promising explanations for the X in the S-wave $D\bar{D}^*$ molecule model [6] as well as in the coupled-channel model [7] require the quantum numbers 1^{++}. In addition, the X cannot be a naive $c\bar{c}$ 2^{-+} state, for its large branching fraction for the $D^0\bar{D}^0\pi^0$ mode [8] is not compatible with the quark-model estimates for the 2^{-+} charmonium [9]. So, for the 2^{-+} quantum numbers, very exotic explanations for the X would have to be invoked.

The aim of the present paper is to perform a combined analysis of the data on the $\pi^+\pi^-$ and $\pi^+\pi^-\pi^0$ mass distribution in the $\pi^+\pi^- J/\psi$ and $\pi^+\pi^-\pi^0 J/\psi$ mode, respectively. We find that the S-wave amplitudes from the decay of a 1^{++} state provide a better overall description of the data than the P-wave ones from the 2^{-+}, especially when the parameter range is restricted to realistic values. We conclude then that the existing data favor 1^{++} quantum numbers of the X, however, improved data in the $\pi^+\pi^-\pi^0 J/\psi$ mode are necessary to allow for definite conclusions regarding the X quantum numbers.

II. EXPERIMENTAL SITUATION

Recently Belle announced [3] the updated results of the measurements for the reaction $X(3872) \rightarrow \pi^+\pi^- J/\psi$:

$$\mathcal{B}_{2\pi}^+ = [8.61 \pm 0.82(\text{stat}) \pm 0.52(\text{syst})] \times 10^{-6},$$
$$\mathcal{B}_{2\pi}^0 = [4.3 \pm 1.2(\text{stat}) \pm 0.4(\text{syst})] \times 10^{-6},$$

for the charged $B^+ \rightarrow J/\psi\rho K^+$ and neutral $B^0 \rightarrow J/\psi\rho K^0$ mode, respectively, with $\mathcal{B}_{2\pi}$ being the product branching fraction $\text{Br}(B \rightarrow KX(3872)) \times \text{Br}(X(3872) \rightarrow \pi^+\pi^- J/\psi)$ in the corresponding mode. The number of events in the background-subtracted combined distribution is

$$N_{2\pi} = 196.0^{+18.9}_{-15.2}.$$

For the decay $X(3872) \rightarrow \pi^+\pi^-\pi^0 J/\psi$ Babar reports [5]

$$\mathcal{B}_{3\pi}^+ = [0.6 \pm 0.2(\text{stat}) \pm 0.1(\text{syst})] \times 10^{-5},$$
$$\mathcal{B}_{3\pi}^0 = [0.6 \pm 0.3(\text{stat}) \pm 0.1(\text{syst})] \times 10^{-5},$$

for the charged mode $B^+ \rightarrow J/\psi\omega K^+$ and for the neutral mode $B^0 \rightarrow J/\psi\omega K^0$, respectively. Similarly to the two-pion case above, $\mathcal{B}_{3\pi}$ stands for the product branching fraction $\text{Br}(B \rightarrow KX(3872)) \times \text{Br}(X(3872) \rightarrow \pi^+\pi^-\pi^0 J/\psi)$ in the corresponding mode. The number of events in the combined distribution is

$$N_{3\pi}^{\text{sig+bg}} = 34.0 \pm 6.6, \qquad N_{3\pi}^{\text{bg}} = 8.9 \pm 1.0, \quad (2)$$

and we assume a flat background. Note, the spectrum reported in [5] and used below appears not to be efficiency corrected. However, since only the shape of this spectrum plays a role for the analysis (see number-of-event distributions (11) below) and we can reproduce the theoretical spectra of [5], which have the efficiency of the detector convoluted in via a Monte Carlo simulation, the invariant mass dependence of the efficiency corrections is expected to be mild and therefore should not affect our analysis significantly.

Thus, the updated ratio of branchings reads [3]

$$\frac{\mathcal{B}_{3\pi}}{\mathcal{B}_{2\pi}} = \frac{\text{Br}(X(3872) \rightarrow \pi^+\pi^-\pi^0 J/\psi)}{\text{Br}(X(3872) \rightarrow \pi^+\pi^- J/\psi)} = 0.8 \pm 0.3. \quad (3)$$

In our analysis we use the ratio (3) as well as

$$N_{2\pi} = 196, \qquad N_{3\pi} = 25.1, \qquad (4)$$

and the corresponding bin sizes are $\Delta E_{2\pi} = 20$ MeV and $\Delta E_{3\pi} = 7.4$ MeV.

III. THEORETICAL $\pi^+\pi^-$ AND $\pi^+\pi^-\pi^0$ INVARIANT MASS DISTRIBUTIONS

As in previous analyses, we assume that the two-pion final state is mediated by the ρ in the intermediate state, while the three-pion final state is mediated by the ω. It was shown in [4] that the description of the $\pi^+\pi^- J/\psi$ spectrum with the 2^{-+} assumption is improved drastically if the isospin-violating ρ-ω mixing is taken into account. Theoretical issues of the ρ-ω mixing are discussed, for example, in [10]. Here we include this effect with the help of the prescription used in [11], where the transition amplitude is described by the real parameter $A_{\omega\to\rho} = A^\dagger_{\rho\to\omega} = \epsilon$. Thus, the amplitudes for the decays $X \to \pi^+\pi^- J/\psi$ and $X \to \pi^+\pi^-\pi^0 J/\psi$ take the form

$$A_{2\pi} = A_{X\to J/\psi\rho} G_\rho A_{\rho\to 2\pi} + A_{X\to J/\psi\omega} G_\omega \epsilon G_\rho A_{\rho\to 2\pi},$$

$$A_{3\pi} = A_{X\to J/\psi\omega} G_\omega A_{\omega\to 3\pi} + A_{X\to J/\psi\rho} G_\rho \epsilon G_\omega A_{\omega\to 3\pi},$$

where the vector meson propagators are

$$G_V^{-1} = m_V^2 - m^2 - i m_V \Gamma_V(m), \qquad V = \rho, \omega, \quad (5)$$

with $m = m_{\pi\pi}$ ($m = m_{\pi\pi\pi}$) being the $\pi^+\pi^-$ ($\pi^+\pi^-\pi^0$) invariant mass in the $\pi^+\pi^- J/\psi$ ($\pi^+\pi^-\pi^0 J/\psi$) final state. Masses of the vector mesons used below are [12]

$$m_\rho = 775.5 \text{ MeV}, \qquad m_\omega = 782.65 \text{ MeV}.$$

The complex mixing amplitude multiplying the ω propagator used, for example, in [4] to analyze the two-pion spectrum, in our notation reads as $\epsilon G_\rho(m_\omega)$; in particular, we reproduce naturally the phase of $95°$ quoted in [4]. Note, as we shall only study the invariant mass distributions of the two final states, we do not need to keep explicitly the vector nature of the intermediate states. For the "running" ρ meson width we use

$$\Gamma_\rho(m) \approx \Gamma_{\rho\to 2\pi}(m) = \Gamma_\rho^{(0)} \frac{m_\rho q(m)}{m q(m_\rho)} \left[\frac{f_{1\rho}(q(m))}{f_{1\rho}(q(m_\rho))} \right]^2,$$

where $q(m) = \sqrt{m^2 - 4m_\pi^2}/2$, $f_{1\rho}(q) = (1 + r_\rho^2 q^2)^{-1/2}$, with $r_\rho = 1.5$ GeV^{-1} and with the nominal ρ meson width $\Gamma_\rho^{(0)} = 146.2$ MeV.

For the ω meson running width (the nominal width being $\Gamma_\omega^{(0)} = 8.49$ MeV), the 3π and $\pi\gamma$ decay modes are summed, with the branchings

$$\text{Br}(\omega \to 3\pi) = 89.1\%, \quad \text{Br}(\omega \to \pi\gamma) = 8.28\%. \quad (6)$$

In particular,

$$\Gamma_{\omega\to\pi\gamma}(m) = \Gamma_{\omega\to\pi\gamma}^{(0)} \left[\frac{m_\omega(m^2 - m_\pi^2)}{m(m_\omega^2 - m_\pi^2)} \right]^3, \qquad (7)$$

while, for the $\Gamma_{\omega\to 3\pi}(m)$, we resort to the expressions derived in [13], with a reduced contact term which provides the correct nominal value of the $\omega \to 3\pi$ decay width [14].

The transition amplitudes for the decays $X \to J/\psi V$ are parameterized in the standard way, namely,

$$A_{X\to J/\psi V} = g_{XV} f_{lX}(p), \qquad (8)$$

with the Blatt-Weisskopf "barrier factor"

$$f_{0X}(p) = 1, \quad \text{and} \quad f_{1X}(p) = (1 + r^2 p^2)^{-1/2}, \qquad (9)$$

for the 1^{++} and 2^{-+} assignment, respectively. Here, p denotes the J/ψ momentum in the X rest frame. The "radius" r is not well understood. If one associates it with the size of the $X \to \rho(\omega)J/\psi$ vertex, it might be related to the range of force. In the quark model this radius is $r \sim 0.2$ fm $= 1$ GeV^{-1}. This is also in line with the inverse mass of the lightest exchange particle allowed between J/ψ and ρ/ω, namely, $f_0(980)$. On the other hand, a larger value $r = 5$ GeV^{-1} is used in the experimental analysis of [3]. Therefore, in the analysis presented below we use both values $r = 1$ GeV^{-1} as well as $r = 5$ GeV^{-1}, keeping in mind that smaller values of r are preferred by phenomenology.

The theoretical invariant mass distributions for the $\pi^+\pi^-$ and $\pi^+\pi^-\pi^0$ final state take the form:

$$\frac{d\text{Br}_{2\pi}}{dm} = \mathcal{B} m_\rho \Gamma_{\rho\to 2\pi} p^{2l+1} f_{lX}^2(p) |R_X G_\rho + \epsilon G_\rho G_\omega|^2,$$

$$\frac{d\text{Br}_{3\pi}}{dm} = \mathcal{B} m_\omega \Gamma_{\omega\to 3\pi} p^{2l+1} f_{lX}^2(p) |G_\omega + \epsilon R_X G_\omega G_\rho|^2, \qquad (10)$$

where $R_X = g_{X\rho}/g_{X\omega}$ and the parameter $\mathcal{B}$ absorbs the details of the short-ranged dynamics of the X production.

The theoretical number-of-event distributions read

$$N_{2\pi}(m) = \frac{N_{2\pi}\Delta E_{2\pi}}{B_{2\pi}} \times \frac{d\text{Br}_{2\pi}}{dm},$$

$$N_{3\pi}(m) = \frac{N_{3\pi}\Delta E_{3\pi}}{B_{3\pi}} \times \frac{d\text{Br}_{3\pi}}{dm}. \qquad (11)$$

The ρ-ω mixing parameter ϵ is extracted from the $\omega \to 2\pi$ decay width (Br($\omega \to 2\pi$) = 1.53%). The corresponding amplitude reads

$$A_{\omega\to 2\pi} = \varepsilon G_\rho A_{\rho\to 2\pi}, \qquad (12)$$

and we find that

$$\epsilon \approx \sqrt{m_\omega m_\rho \Gamma_\rho \Gamma_{\omega\to 2\pi}} \approx 3.4 \times 10^{-3} \text{ GeV}^2. \qquad (13)$$

TABLE I. Sets of parameters and the quality of the combined fits to the data. The last column refers to the presentation of the different fits in Fig. 1.

Fit	Wave	J^{PC}	r, GeV^{-1}	$R_X = g_{X\rho}/g_{X\omega}$	χ^2/N_{dof}	CL	Curve/Markers in Fig. 1
S	S	1^{++}	-	$0.26^{+0.08}_{-0.05}$	1.07	37%	Solid (red)/Triangles
P_5	P	2^{-+}	5	$0.15^{+0.05}_{-0.03}$	1.33	14%	Dash-dotted (blue)/Squares
P_1	P	2^{-+}	1	$0.09^{+0.03}_{-0.02}$	2.77	10^{-5}	Dashed (green)/Diamonds

IV. FITTING STRATEGY AND RESULTS

The number-of-event distributions (11) possess 3 free parameters: the "barrier" factor r, the ratio of couplings R_X, and the overall normalization parameter $\mathcal{B}$. As outlined above, we perform the analysis for two values of r, namely, the preferred value of 1 GeV^{-1} and a significantly larger value of 5 GeV^{-1} used in earlier analyses. Since the normalization factor $\mathcal{B}$ drops out from the ratio of the two branchings, we extract the ratio R_X directly from the integrated data, that is from the relation

$$\left(\int \frac{d\text{Br}_{3\pi}}{dm} dm\right) \Big/ \left(\int \frac{d\text{Br}_{2\pi}}{dm} dm\right) = \mathcal{B}_{3\pi}/\mathcal{B}_{2\pi}, \quad (14)$$

where the value of the ratio on the right-hand side is fixed by Eq. (3), and in the integration above we have cut off the $\pi^+\pi^-$ invariant mass at 400 MeV, as in [3], and the $\pi^+\pi^-\pi^0$ invariant mass at 740 MeV, as in [5]. Therefore, the norm $\mathcal{B}$ is our only fitting parameter which governs the overall strength of the signal in both channels simultaneously, while the shape of the curves is fully determined from other sources.

In Table I we list the parameters of the 3 combined fits to the data, found for the 2 values of the Blatt-Weisskopf parameter r. The corresponding line shapes and the result of the integration in bins are shown in Fig. 1.

One can see from Table I and Fig. 1 that the best overall description of the data for the two channels under consideration is provided by the S-wave fit. The P-wave fit is capable to provide the description of the data of a comparable (however somewhat lower) quality, only for large values of the Blatt-Weisskopf parameter r, $r = 5$ GeV^{-1}. The P-wave fit becomes poorer when the Blatt-Weisskopf parameter is decreased, and for values of r of order 1 GeV^{-1}, the quality of the P-wave fit is unsatisfactory, which is the result of a very poor description of the two-pion spectrum—see the dashed (green) curve in Fig. 1. Varying the ratio of branchings $\mathcal{B}_{3\pi}/\mathcal{B}_{2\pi}$ around its central value within the experimental uncertainty interval [see Eq. (3)] leads only to minor changes in the fits and does not affect the conclusions.

V. DISCUSSION

Since no charged partners of the $X(3872)$ are observed experimentally, it is supposed to be (predominantly) an isoscalar. Then, the ratio $R_X = g_{X\rho}/g_{X\omega}$ measures the strength of the isospin violation in the $X \to VJ/\psi$ decay vertex. As discussed above, this ratio is extracted directly from the data on the ratio of the branchings (3).

An isospin-violating observable for a compact charmonium is the ratio of the branching fractions for the $\psi(2S)$ decays into $\eta J/\psi$ and $\pi^0 J/\psi$ final states as
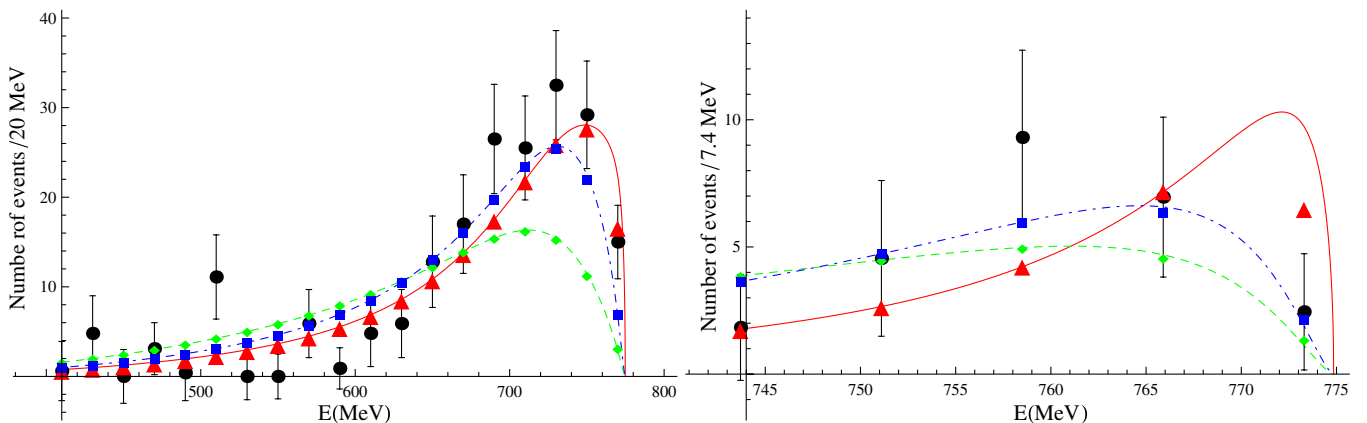

FIG. 1 (color online).   Comparison of the theoretical predictions with the data. Experimental data, taken from [3,5] for the two-pion (left plot) and three-pion (right plot) spectrum, respectively, are given by black circles with error bars, while the theoretical results are shown both as the continuous number-of-event distributions (11) as well as by markers for the corresponding integrals over the bins. Fit S is shown as the (red) solid line and triangles, fit P_1 is shown as the (green) dashed line and diamonds, while fit P_5 is shown as the (blue) dash-dotted line and squares. See Table I for the details.

$$R_{\psi(2S)} = \frac{g_{\pi^0 J/\psi}}{g_{\eta J/\psi}} = \sqrt{\frac{\mathrm{Br}(\psi(2S) \to \pi^0 J/\psi)}{\mathrm{Br}(\psi(2S) \to \eta J/\psi)} \frac{k_\eta^3}{k_{\pi^0}^3}} \approx 0.03, \quad (15)$$

where k_{π^0} and k_η are the center-of-mass momenta of π^0 and η, respectively, and the $\psi(2S)$ branching fractions are taken from [12]. However, since here also the denominator violates a symmetry, namely, SU(3), and there might be significant meson-loop contributions [15], the estimate (15) is to be regarded as a conservative upper bound for the isospin violation strength for compact charmonia.

In contrast to this, in the S-wave molecular picture for the X, isospin violation is enhanced significantly compared to the strength (15) for it proceeds via intermediate $D\bar{D}^*$ states and is therefore driven by the mass difference $\Delta \approx 8$ MeV of the neutral and charged $D\bar{D}^*$ threshold—see, for example, [16,17]. An order-of-magnitude estimate is provided by the expression

$$R_X^{\mathrm{mol}} \sim \left| \frac{I_0(M_X) - I_c(M_X)}{I_0(M_X) + I_c(M_X)} \right| \sim \frac{\sqrt{m_D \Delta}}{\beta} \sim 0.13, \quad (16)$$

where m_D is the D meson mass, while $I_0(M_X)$ and $I_c(M_X)$ denote the amplitudes corresponding to loop diagrams with neutral and charged $D\bar{D}^*$ intermediate states, respectively, evaluated at the X mass. They are composed of two terms, the strongly channel-dependent analytic continuation of the unitarity cut, proportional to the typical momentum of the meson pair, and the weakly channel-dependent principle value term, whose size is identified with the inverse range of forces of order of 1 GeV (see above). This estimate is within a factor of 2 consistent with the value $R_X \sim 0.26$ found from our S-wave fit—see Table I.

On the other hand, if the X has the quantum numbers 2^{-+}, one should expect the isospin violation in the X wave function to be of the natural charmonium size, and thus of the order of 10^{-2}—see discussion below Eq. (15), since the $D\bar{D}^*$ loop effects are suppressed by the additional centrifugal barrier: the estimate analogous to Eq. (16) now reads $R_X^{\mathrm{mol}} \sim (\sqrt{M_D \Delta}/\beta)^3 \sim 2 \times 10^{-3}$. Thus, for the state with the quantum numbers 2^{-+}, one expects values of at most $R_X \sim 10^{-2}$, that are significantly smaller than those following from the data (see Table I). One is led to conclude therefore that for $R_X \gtrsim 0.1$, needed for the quantum numbers 2^{-+} to be consistent with the data on the X decays, a new, yet unknown, isospin violation mechanism would have to be invoked.

VI. CONCLUSIONS

We conclude therefore that, although the present quality of the data in the $X \to \pi^+ \pi^- \pi^0 J/\psi$ channel is not sufficient to draw a definite conclusion concerning the quantum numbers of the $X(3872)$, the combined analysis of the existing two- and three-pion spectra favors the S-wave fit, related to the 1^{++} assignment for the $X(3872)$, over the P-wave fit, related to the 2^{-+} assignment. We notice that an acceptable P-wave fit calls for a large range parameter in the Blatt-Weisskopf form factor which meets certain difficulties with its phenomenological interpretation. In addition, while the value $R_X = g_{X\rho}/g_{X\omega}$ can be understood theoretically for the 1^{++} assignment, the value extracted for the 2^{-+} assignment is too large to be explained from known mechanisms of the isospin violation.

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