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Lorentz violation from torsion trace and non-minimal coupling in radio galactic dynamos

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Earlier Kostelecky *et al.* [A. Kostelecky, N. Russell, and J. Tasson, Phys Rev Lett **100**, 111102 (2008).] have obtained torsion bounds from Lorentz violation, where torsion components are taken from the axial part of torsion. In this brief report it is shown that more stringent bounds may be obtained by using nearly minimal magnetogenesis torsion trace instead of the minimal coupling between photons and axial torsion used by Kostelecky and his group. Just for comparison, in Kostelecky *et al.*, the most stringent limit is estimated to be 10^{-31} GeV while here one obtains 10^{-33} GeV. This estimate is obtained by constraining the torsion to galactic astronomy data. From the point of view of magnetogenesis, an interesting physical consequence is that dynamo action is obtained when the torsion trace background is negative, while the magnetic field energy decays when torsion is positive. Polarization of radio-galaxies can be used to obtain an even more stringent limit of $T \sim 1.7 \times 10^{-46}$ GeV to Lorentz violation. Using WMAP, Kostelecky and Mewes [A. Kostelecky and M. Mewes, Astrophys. J. **689**, L1 (2008)] have found limits of the order of 10^{-43} GeV. These results are obtained by making use of flat torsion modes [L. Garcia de Andrade, Phys Lett B **696**, 1 (2011)], but may easily be extended to Riemann-Cartan spacetime.

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I. INTRODUCTION

Previously, Turner and Widrow [1] have investigated the electrodynamic coupling with gravity as a nonminimal coupling of the Lagrangian type $\mathcal{L} \sim R^* F^2$, where $F^2 = F_{\mu\nu}F^{\mu\nu}$ ($\mu = 0, 1, 2, 3$), is the electromagnetic invariant and $F_{\mu\nu} = 2\partial_{[\mu}A_{\nu]}$ is the electromagnetic field tensor. Here, R^* is the Riemannian-Ricci scalar. They considered just the Riemannian geometry of general relativity. One of the drawbacks of this sort of Riemannian coupling was that in vacuum it is trivial or its contribution to the Lagrangian is nil. Recently [2], one used a QED-coupled Lagrangian, where photons A^μ and torsion are nonminimally coupled or, as Prokopec *et al.* [3] prefer to say, nearly minimal, since they only couple through the presence of torsion in the curvature. One of the big differences between that approach and the one we adopt in this brief report is that here we extend the Turner-Widrow Lagrangian by minimally replacing the Riemannian-Ricci scalar by the Riemann-Cartan-Ricci one, while previously we considered a Lagrangian of the type $R_{\mu\nu\rho\sigma}F^{\mu\nu}F^{\rho\sigma}$, where the electromagnetic field couples to the Riemann-Cartan [4] curvature $R_{\mu\nu\rho\sigma}$. In this brief report, we consider nearly minimal coupling in the language of Prokopec *et al.* to show that the magnetic field amplification or dynamo effect is possible when the torsion trace is negative, while the magnetic field energy density decays when the torsion is positive. The paper is organized as follows: In Sec. II we review the basic relation between dynamos and Lorentz violation as given by Campanelli *et al.* [5], where general electric and magnetic fields are considered and where, contrary to their work, torsion is already introduced. Some few new results are discussed concerning torsion effects on Lorentz violation (LV). In Sec. III we consider the derivation of the magnetic energy in terms of torsion trace from the Ricci-Cartan scalar Lagrangian, and derive the estimates of the LV from astronomical data, while in Sec. IV, making use of negative torsion, we compute the LV from polarization of radio-galaxies, computed by Ruzmaikin *et al.* [6] to be $\frac{\delta B}{B} \sim 1.7$. In this section, stronger assumptions are made about the homogeneity of the magnetic field as well as the absence of electric fields. The basic difference between our result and Kostelecky and Mewes [7] is that they used a Chern-Simmons Lagrangian while our Lagrangian is simpler. Sec. V contains conclusions and discussions.

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II. COSMOLOGICAL MAGNETIC HELICITY, DYNAMOS AND BIREFRIGENCE FROM SEMIMINIMAL TORSION-PHOTON COUPLING

Since torsion effects are highly suppressed in comparison with the curvature effects of the Einstein gravity sector, here we only consider Minkowski space. The Turner and Widrow Lagrangian [1] is

$$S = \frac{1}{m^2}\int d^4x(-g)^{(1/2)}\left(-\frac{1}{4}[1 - 4R(\Gamma)]F^2]\right), \quad (1)$$

where Γ is the Riemann-Cartan connection. Euler-Lagrange equations yields the following field equations: The Maxwell equations

$$\partial_\mu F^{\mu\nu} = F^{\mu\nu}\frac{\partial_\mu R}{(1 - R)}, \quad (2)$$

the Bianchi identities

$$\partial_{[\mu}F_{\alpha\nu]} = 0, \tag{3}$$

and the torsion trace $T_\nu = T^\mu{}_{\nu\mu}$. Here we shall assume that only the time component of the torsion trace survives. This time component is represented simply by T. Thus the torsion trace equation is

$$\Box T - T = \partial_\eta (E^2 - B^2), \tag{4}$$

where η is the conformal coordinate of Minkowski space given by the line element

$$ds^2 = a^2(\eta)(d\eta^2 - d\mathbf{x}^2). \tag{5}$$

By assuming the following ansatz

$$\mathbf{B}_k = \mathbf{B}_0 e^{\omega_B \eta}, \tag{6}$$

$$\mathbf{E}_k = \mathbf{E}_0 e^{\omega_E \eta}, \tag{7}$$

and

$$\mathbf{T}_k = \mathbf{T}_0 e^{\omega_T \eta}, \tag{8}$$

Fourier-transforming the torsion equation and substitution of the last three equations yields

$$T_k = \frac{2\omega(E_0^2 - B_0^2)}{(1 + k^2 + 4\omega^2)} e^{2\omega\eta}, \tag{9}$$

where $\omega = \omega_B = \omega_E = \frac{1}{2}\omega_T$ is a kind of degeneracy in the frequency. Let us now consider the Maxwell equations in the form

$$\partial_\eta \mathbf{B} + \nabla \times \mathbf{E} = 0 \tag{10}$$

$$\partial_\eta \mathbf{E} + \nabla \times \mathbf{B} = \frac{\dot{R}}{(1 - R)} \mathbf{E} \tag{11}$$

$$\nabla . \mathbf{B} = 0 \tag{12}$$

$$\nabla . \mathbf{E} = 0. \tag{13}$$

Taking the curl of Eq. (10) and substitution of Eq. (9) yields

$$-\partial_\eta^2 \mathbf{B} + \nabla^2 \mathbf{B} = -\frac{\dot{R}}{(1 - R)} \partial_\eta \mathbf{B}. \tag{14}$$

Computing the Fourier spectrum one obtains

$$\partial_\eta{}^2 \mathbf{B}_k + k^2 \mathbf{B}_k = \frac{\dot{R}}{(1 - R)} \partial_\eta \mathbf{B}_k. \tag{15}$$

By considering the plasma effects, one has for the electrical conductivity

$$\sigma = \frac{\dot{R}}{(1 - R)}. \tag{16}$$

Note that in the early Universe, where torsion or curvature effects are strong, the electrical conductivity is negative and the effective conductivity decreases, which means more electric resistivity in the cosmic plasma due to torsion effects. By making use of the above ansatz for the magnetic field and substituting into expression (10), one obtains

$$\omega^2 + k^2 - \frac{\dot{R}}{(1 - R)} \omega = 0. \tag{17}$$

Since the phase velocity is given by $v_{ph} = \frac{\omega}{k}$, the dispersion relation (16) shows the well-known result that the photons propagated with two distinct polarization states. Dobado and Maroto [8] have proven this in the axial torsion context, and birefrigence is also present here. Nevertheless, they worked out in the fermionic sector of QED and our approach is in the photonic sector of the QED with torsion. They also consider a tiny torsion to explain birefrigence. This fact allow us a simple interpretation of Lorentz violation associated with the torsion theory here as done previously by Kostelecky *et al.* [9]. The dispersion relation yields

$$\omega_\pm = \frac{\partial_\eta R}{[1 - R]} \pm ik. \tag{18}$$

Note that this result reduces to the vacuum Maxwell equation one when the Ricci-Cartan scalar vanishes. The helicity expression is

$$\mathcal{H}_k = \frac{k^2}{2\pi^2}(|B_k^+|^2 - |B_k^-|^2). \tag{19}$$

Recently, Semikoz and Sokoloff [10] have discussed and contested the paradigm that the weak field that seeds the magnetic galactic dynamo being weak necessarily implies a weak helicity. Let us now consider the computation of the last expression of helicity in terms of torsion, to check if the flat torsion modes induce a weak or strong helicity. Before computing the helicity, let us consider the comoving dissipation length

$$\epsilon_{\rm diss}^2 = \int \frac{d\eta}{\sigma}. \tag{20}$$

For modes well within the horizon, $k|\eta| >>> 1$ then the expression

$$\partial^2{}_\eta \mathbf{B}_k + k^2 \mathbf{B}_k = \sigma \partial_\eta \mathbf{B}_k \tag{21}$$

reduces to

$$k^2 \mathbf{B}_k \approx \sigma \partial_\eta \mathbf{B}_k. \tag{22}$$

A simple solution of this equation is given by

$$B^\pm{}_k \approx B_k^{\pm 0} \exp\left[\int d\eta \frac{k^2}{\sigma}\right]. \tag{23}$$

From the definition of dissipation length above

$$B_k^{\pm} \approx B_k^{\pm 0} \exp[-\epsilon^2 k^2]. \tag{24}$$

Since the dissipation length $\epsilon_{\rm diss}^2 \sim \frac{1}{\sigma H}$, where H is the Hubble constant, $\sigma \sim \frac{1}{H}$. The magnetic helicity is then given by

$$\mathcal{H}_k \sim \exp[-\epsilon^2 k^2]. \tag{25}$$

Therefore, unless $R \gg > k\eta$ or the torsion is extremely strong, the magnetic field helicity is washed out and the magnetic field presents a fast decay and the cosmological magnetic field is not strong enough to seed galactic dynamos. Note that the dynamo equation can be obtained from the expression

$$\nabla^2 \mathbf{B}_k - \sigma \partial_\eta \mathbf{B}_k = 0. \tag{26}$$

The power spectra of the magnetic field

$$\mathcal{P}_k = \frac{k^3}{2\pi^2}(|B_k^+|^2 + |B_k^-|^2), \tag{27}$$

which shows that power spectra decay fast unless the torsion is extremely strong such as in black holes or in the very early Universe.

III. FLAT TORSION MAGNETOGENESIS AND LV VIOLATION

Only local spacetime is endowed with torsion trace $T_\mu = T_\mu{}^\nu{}_\nu$ and the Minkowski flat metric is considered. Here $T_{\mu\nu\sigma}$ is the torsion tensor of 24 components. Photon-torsion semiminimal coupling is given by Lagrangian

$$\mathcal{L} \sim \int d^4x(-g)^{(1/2)}\left(-\frac{1}{4}F^2[1-4R(\Gamma)] + J_\mu A^\mu\right), \tag{28}$$

where now we have introduced the current photon interaction last term. In the Minkowski metric $a = 1$ and the Ricci-Cartan scalar

$$R = g_{\mu\nu}R^{\mu\nu} = R^* + 2\nabla_\mu T^\mu - T^2 \tag{29}$$

may be substituted into the Lagrangian, where the field equations are obtained from the Euler-Lagrange equations

$$\frac{d}{dt}\frac{\partial\sqrt{g}\mathcal{L}}{\partial\dot{A}_\mu} - \frac{\partial\sqrt{g}\mathcal{L}}{\partial A_\mu} = 0 \tag{30}$$

$$\frac{d}{dt}\frac{\partial\sqrt{g}\mathcal{L}}{\partial\dot{T}} - \frac{\partial\sqrt{g}\mathcal{L}}{\partial T} = 0 \tag{31}$$

here $T = T^0$. Since our Riemannian space is flat, the Ricci scalar vanishes which greatly simplifies our computations. Let us start from the last equation to determine the time component T of torsion trace in terms of the magnetic field. To achieve this goal let us express the electromagnetic invariant as $F^2 = \mathbf{E}^2 - \mathbf{B}^2$ where $\mathbf{E}$ and $\mathbf{B}$ represent, respectively, the electric and magnetic fields. In principle, one adopts here the condition $\mathbf{E}.\mathbf{B} = 0$. To simplify matters one considers that the electric field vanishes. This yields

$$d_t B^2 + 2TB^2 = 0 \tag{32}$$

$$\partial_\mu F^{\mu\nu} = J^\nu. \tag{33}$$

The first equation describes the constraint between the magnetic field and torsion [4], while the second set of field equations describes the usual Maxwell's equation in cosmic plasma spacetime. It is easily shown that in the absence of cosmic plasma, free flat space homogeneous magnetic fields imply that torsion vanishes in the absence of electric fields. This can be seen by expanding the expression (32) as

$$\partial_t B^2 + \mathbf{v}.\nabla B^2 + 2TB^2 = 0. \tag{34}$$

From the Maxwell's equations in vector form

$$\nabla \times \mathbf{E} = -\partial_t \mathbf{B}. \tag{35}$$

Since the electric field vanishes, the right-hand side of the last equation also vanishes and Eq. (34) reduces to

$$\mathbf{v}.\nabla B^2 + 2TB^2 = 0, \tag{36}$$

which, in the absence of cosmic plasma, where $\mathbf{v}$ vanishes, the last expression reduces to

$$2TB^2 = 0, \tag{37}$$

which implies that either the magnetic field energy or torsion vanishes. Since magnetic fields exist by assumption, the only option is that the trace of the torsion vanishes. Therefore, this reasoning leads us to the need of a cosmic plasma background for electromagnetism and torsion. Also, this is useful when one considers MHD dynamos, which of course do not take place in empty space. Now let us solve the Eq. (32), which yields

$$B^2 = c_0 \exp[-2\int T dt], \tag{38}$$

where c_0 is an integration constant. By considering a constant torsion background and assuming that the product $T\Delta t$ is very weak, one may write down this solution as

$$B^2 = c_0[1 - 2T\Delta t]. \tag{39}$$

From expression (38), one immediatly sees that the magnetic energy density $\frac{B^2}{8\pi}$ grows (dynamo action) when torsion scalar T is negative $T = -T_0$, and this allows us to write

$${B^2}_{\rm torsion} \sim T_0 \Delta t. \tag{40}$$

This expression shall be used in this section to estimate the value of torsion trace and Lorentz violation in some specific situations. For example. imagine that one imposes the constraint that the magnetic field might satisfy the astronomical observation of the galactic magnetic field of micro-Gauss at just beyond where structure formation starts, at $\Delta t \sim 10^{13} s$. Substitution of these data into expression (40) yields

$$T \sim 10^{-33}\ \text{GeV}, \tag{41}$$

where one has already changed the units s^{-1} to cm^{-1}, which in turn changes to GeV, where $1\ \text{cm}^{-1} \sim 10^{-24}$ GeV. Expression (15) is 2 orders of magnitude more stringent than the estimate of 10^{-31} GeV obtained by Kostelecky *et al.* [7], making use of a fermionic sector of a LV with a many terms fermionic Lagrangian and axial torsion. Thus, to summarize this section, we showed that for torsion contribution to be able to amplify the magnetic field to observed values, it might be 2 orders of magnitude lower than the LV torsion trace value obtained in the fermionic sector [7].

IV. POLARIZATION IN RADIO-GALAXIES AND LV

In this section, we will estimate torsion trace bounds based on the equation of the last section, with a different coinstraint, namely, the one that comes from the polarization of radio-galactic sources. Ruzmaikin *et al.* [5] have computed the contrast of magnetic field as $\frac{\delta B}{B} \sim 1.7$. To be able to use this data, we will make use of expression (39) in the form

$$B \approx B_0[1 - T\Delta t]. \tag{42}$$

A simple manipulation of this formula yields

$$\frac{\delta B}{B} \sim \frac{B - B_0}{B_0} \sim T_0 \Delta t. \tag{43}$$

By choosing the $\Delta t \sim 10^{14} s$, one obtains 10^{-46} GeV, which is 3 orders of magnitude lower than the value obtained by Kostelecky and Mewes [7] using WMAP data.

V. DISCUSSION AND CONCLUSIONS

In Sec. II of this paper, a general framework of the introduction of torsion to dynamo and LV problems was addressed in detail and four important physical quantities were computed, namely, the cosmological magnetic helicity, birefrigence, and the power spectrum, as well as the setup used in general to obtain the dynamo equation in the cosmic plasma. In Sec. III, these ideas underwent stronger assumptions, such as the existence only of magnetic fields and the time component of torsion and the fact that only homogeneous component of the magnetic field were considered. From these strong assumptions, torsion bounds were obtained in two important physical cases by making use of a constant torsion which induces LV in the field equations, as happens in Dirac equation. The purpose of the paper was to make a comparison between our estimates, considered by using a photonic sector of the Lagrangian semiminimally coupled to the trace of torsion, and values previously obtained by Kostelecky *et al.* , using a fermionic sector coupled also with torsion. In their case, it is important to say that use was also made of the torsion trace. We have found out that there is a significantly more stringent limit of 2 or 3 orders of magnitude lower than the values obtained in laboratory and in the astrophysical case. The cosmic radio sources therefore prove to be an interesting source for testing LV in the realm of cosmic physics. As a by-product, we use the idea that negative torsion does amplify cosmological magnetic fields, while positive torsion trace imposes a decay on the magnetic field and therefore forbids an effective dynamo action for the amplification of these magnetic fields. CP-violating dynamos can also be used to place bounds on torsion [11]. The basic idea here is that lowering the orders of magnitude of torsion seems to increase the possibility of getting amplification of magnetic fields, which would motivate to develop more sensitive lab tests to improve torsion trace sensitivity for the time component T_T [12]. Another important conclusion of our study here is that torsion components used by Kostelecky *et al.* [9] are given by axial components which range from 10^{-27} GeV to 10^{-31} GeV lowest limit, while our limits here for LV are used from torsion trace and are definitely more stringent limits. Another important issue of higher-order Lorentz violation [13] terms in terms of torsion may be addressed elsewhere.

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