Holographic superconductor models with the Maxwell field strength corrections

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We study the effect of the quadratic field strength correction to the usual Maxwell field on the holographic dual models in the backgrounds of AdS black hole and AdS soliton. We find that in the black hole background, the higher correction to the Maxwell field makes the condensation harder to form and changes the expected relation in the gap frequency. This effect is similar to that caused by the curvature correction. However, in the soliton background we find that, unlike the curvature effect correction, the correction to the Maxwell field does not influence the holographic superconductor and insulator phase transition.

I. INTRODUCTION

As the most remarkable discovery in string theory, the anti-de Sitter/conformal field theory (AdS/CFT) correspondence states that a string theory on asymptotically AdS spacetimes can be related to a conformal field theory on the boundary [1–3]. Recently, this principle has been employed to study the strongly correlated condensed matter physics from the gravitational dual (for reviews, see [4–6]). It was shown that the instability of the bulk black hole corresponds to a second-order phase transition from normal state to superconducting state which brings the spontaneous U(1) symmetry breaking [7]. Because of the potential applications to the condensed matter physics, the gravity models with the property of the so-called holographic superconductor have been studied extensively; see, for example, [8–29] and references therein.

Recently, motivated by the application of the Mermin-Wagner theorem to the holographic superconductors, there has been a lot of interest in exploring the effect of the curvature correction on the (3 + 1)-dimensional superconductor [30] and higher-dimensional ones [31] by examining the charged scalar field together with a Maxwell field in the Gauss-Bonnet-AdS black hole background

$$S = \int d^d x \sqrt{-g}\left\{\frac{1}{16\pi G}\left[R + \frac{(d-1)(d-2)}{L^2} + \mathcal{L}_{R^2}\right] + \left(-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - |\nabla\psi - iA\psi|^2 - m^2|\psi|^2\right)\right\}, \quad (1)$$

with the curvature correction that reads

$$\mathcal{L}_{R^2} = \tilde{\alpha}(R_{\mu\nu\gamma\delta}R^{\mu\nu\gamma\delta} - 4R_{\mu\nu}R^{\mu\nu} + R^2), \quad (2)$$

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where $\tilde{\alpha}$ is the Gauss-Bonnet coupling constant with dimension (length)^2. It was observed that the higher curvature correction makes the condensation harder to form and causes the behavior of the claimed universal ratio $\omega/T_c \approx 8$ to become unstable [30–43].

As a matter of fact, in the low-energy limit of heterotic string theory, the higher-order correction term appears also in the Maxwell gauge field [44]. Thus, in order to understand the influences of the $1/N$ or $1/\lambda$ (λ is the 't Hooft coupling) corrections on the holographic superconductors [30], it is interesting to consider the high-order correction related to the gauge field besides the curvature correction to the gravity. In this work, in order to grasp the influence of the correction to the gauge field, we will turn off the curvature correction and study in a pure Einstein gravity background for simplicity. We will consider a gauge field and the scalar field coupled via a generalized Lagrangian

$$S = \int d^d x \sqrt{-g}\left\{\frac{1}{16\pi G}\left[R + \frac{(d-1)(d-2)}{L^2}\right] + \left(-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \mathcal{L}_{F^4} - |\nabla\psi - iA\psi|^2 - m^2|\psi|^2\right)\right\}, \quad (3)$$

where $\mathcal{L}_{F^4}$ is determined by a quadratic field strength correction to the usual Einstein-Maxwell field [45–49]

$$\mathcal{L}_{F^4} = c_1(F_{\mu\nu}F^{\mu\nu})^2 + c_2 F_{\mu\nu}F^{\nu\delta}F_{\delta\kappa}F^{\kappa\mu}, \quad (4)$$

with the real numbers c_1 and c_2. When c_1 and c_2 are zero, it reduces to the models considered in [8–10]. Interestingly, just as is shown in the following discussion, the constraint can be relaxed to the case $2c_1 + c_2 = 0$ where the model (3) reduces to the standard holographic superconductors studied in [8–10].

Recently the solutions of electrically charged black hole with the higher correction term in the Maxwell field have been discussed widely [45–48]; it is interesting to study the

coupling of the scalar field with the higher-order corrected Maxwell field, explore the effect of the higher correction in the Maxwell field on the scalar condensation, and compare it with the effect of the curvature correction.

Besides the black hole background, we will also extend our discussion to the AdS soliton background. There has been a lot of work discussing the holographic insulator and superconductor phase transitions in the five-dimensional AdS soliton background [50–57]. The discussion with the curvature correction in the Ricci flat AdS soliton in Gauss-Bonnet gravity was discussed in [31,57]. It would be of great interest to examine the influence of the correction to the Maxwell field on the holographic insulator and superconductor system. In this work, we will compare the correction to the gauge field with the correction to the curvature on the condensation in the AdS soliton background.

In order to extract the main physics, in this work we will concentrate on the probe limit to avoid the complex computation. The organization of the work is as follows. In Sec. II, we will study the holographic superconductor models with F^4 corrections in the Schwarzschild-AdS black hole background. In Sec. III we will extend our discussion to the Schwarzschild-AdS soliton background. We will conclude in the last section of our main results.

II. HOLOGRAPHIC SUPERCONDUCTING MODELS WITH F^4 CORRECTIONS

We consider the background of the d-dimensional planar Schwarzschild-AdS black hole

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 dx_i dx^i, \qquad (5)$$

where

$$f(r) = \frac{r^2}{L^2}\left(1 - \frac{r_+^{d-1}}{r^{d-1}}\right), \qquad (6)$$

L is the AdS radius and r_+ is the black hole horizon. The Hawking temperature can be expressed as

$$T = \frac{(d-1)r_+}{4\pi L^2}, \qquad (7)$$

which can be interpreted as the temperature of the CFT.

Taking the ansatz $\psi = |\psi|$, $A_t = \phi$ where ψ, ϕ are both real functions of r only, we can obtain the equations of motion from the action (3) in the probe limit

$$\psi'' + \left(\frac{d-2}{r} + \frac{f'}{f}\right)\psi' + \left(\frac{\phi^2}{f^2} - \frac{m^2}{f}\right)\psi = 0, \qquad (8)$$

$$(1 + 3\varepsilon\phi'^2)\phi'' + \frac{d-2}{r}(1 + \varepsilon\phi'^2)\phi' - \frac{2\psi^2}{f}\phi = 0, \qquad (9)$$

where we have set $\varepsilon = 8(2c_1 + c_2)$ which can be used to describe the F^4 correction to the usual Maxwell field. Obviously, Eqs. (8) and (9) reduce to the standard holographic superconductor models discussed in [8–10] when $\varepsilon = 0$.

The equations of motion (8) and (9) can be solved numerically by doing integration from the horizon out to the infinity. At the horizon $r = r_+$, the regularity gives the boundary conditions

$$\psi(r_+) = \frac{f'(r_+)}{m^2}\psi'(r_+), \qquad \phi(r_+) = 0. \qquad (10)$$

At the asymptotic AdS boundary $r \rightarrow \infty$, the solutions behave like

$$\psi = \frac{\psi_-}{r^{\lambda_-}} + \frac{\psi_+}{r^{\lambda_+}}, \qquad \phi = \mu - \frac{\rho}{r^{d-3}}, \qquad (11)$$

where μ and ρ are interpreted as the chemical potential and charge density in the dual field theory, respectively, and $\lambda_\pm = \frac{1}{2}[(d-1) \pm \sqrt{(d-1)^2 + 4m^2L^2}]$. It should be noted that the coefficients ψ_- and ψ_+ both multiply normalizable modes of the scalar field equations and they correspond to the vacuum expectation values $\langle \mathcal{O}_- \rangle = \psi_-$, $\langle \mathcal{O}_+ \rangle = \psi_+$ of an operator $\mathcal{O}$ dual to the scalar field according to the AdS/CFT correspondence. Just as in Refs. [8,9], we can impose a boundary condition so that either ψ_+ or ψ_- vanishes.

A. The condensation of the scalar operators

In order to discuss the effects of the F^4 correction terms ε on the condensation of the scalar operators, we will solve the equations of motion (8) and (9) numerically. Since we focus on the effects of the F^4 corrections, we will set $d = 4$ and $m^2L^2 = -2$ for concreteness. As a matter of fact, the other choices of the dimensionality of the spacetime and the mass of the scalar field will not qualitatively modify our results. It should be noted that, unlike the Gauss-Bonnet holographic superconductors which should be in $3 + 1$ dimensions at least, we can even construct $(2 + 1)$-dimensional holographic superconducting models with F^4 corrections.

In Fig. 1 we present the condensates of the scalar operators $\mathcal{O}_-$ and $\mathcal{O}_+$ as a function of temperature with various correction terms ε for the mass of the scalar field $m^2L^2 = -2$ in $d = 4$ dimension. Obviously, the curves in the right panel of Fig. 1 have similar behavior to the BCS theory for different ε, where the condensate goes to a constant at zero temperature. However, the curves for the operator $\mathcal{O}_-$ will diverge at low temperature, which are similar to those for the usual Maxwell electrodynamics in the probe limit, neglecting backreaction of the spacetime [8]. The behaviors of the condensates for the scalar operators $\mathcal{O}_-$ and $\mathcal{O}_+$ show that the holographic superconductors still exist even we consider F^4 correction terms to the usual Maxwell electrodynamics.
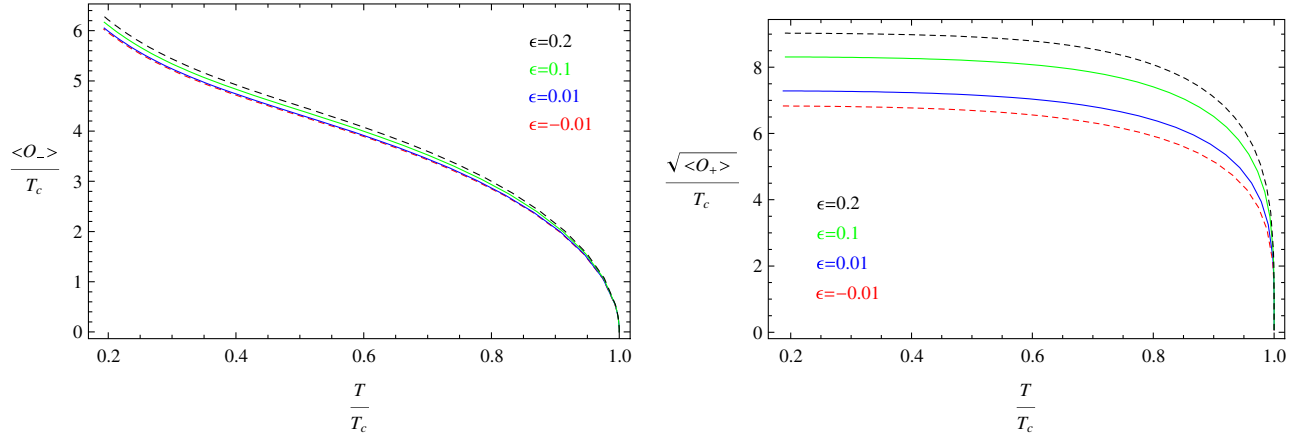
FIG. 1 (color online).   The condensates of the scalar operators $\mathcal{O}_-$ and $\mathcal{O}_+$ as a function of temperature for the mass of the scalar field $m^2 L^2 = -2$ in $d = 4$ dimension. The four lines from bottom to top correspond to increasing correction term, i.e., $\varepsilon = -0.01$ (red and dashed), 0.01 (blue), 0.1 (green) and 0.2 (black and dashed), respectively.

From Fig. 1, we see the higher correction term ε makes the condensation gap larger for both scalar operators $\mathcal{O}_-$ and $\mathcal{O}_+$, which means that the scalar hair is harder to be formed when adding F^4 corrections to the usual Maxwell field. In fact, the Table I shows that the critical temperature T_c for the operators $\mathcal{O}_-$ and $\mathcal{O}_+$ decreases as the correction term ε increases, which agrees well with the finding in Fig. 1. This behavior is reminiscent of that seen for the Gauss-Bonnet holographic superconductors, where the higher curvature corrections make condensation harder, so we conclude that the F^4 corrections to the usual Maxwell field and the curvature corrections share some similar features for the condensation of the scalar operators.

B. Conductivity

Now we are in a position to investigate the influence of the F^4 correction term on the conductivity. Since the condensation gap and the critical temperature depend on the correction term ε, which is similar to the Gauss-Bonnet correction term in the holographic superconductor, we want to know whether the correction term ε will change the expected universal relation $\omega_g/T_c \approx 8$ in the gap frequency [10] as the Gauss-Bonnet term did.

Considering the perturbed Maxwell field $\delta A_x = A_x(r)e^{-i\omega t}dx$, we obtain the equation of motion for δA_x, which can be used to calculate the conductivity

$$A_x'' + \left(\frac{d-4}{r} + \frac{f'}{f} + \frac{2\varepsilon\phi'\phi''}{1+\varepsilon\phi'^2}\right)A_x'$$
$$+ \left[\frac{\omega^2}{f^2} - \frac{2\psi^2}{f(1+\varepsilon\phi'^2)}\right]A_x = 0. \tag{12}$$

We still restrict our study to $d = 4$ for simplicity. Though the above equation is more complicated than that in usual Einstein-Maxwell electrodynamics, the ingoing wave boundary condition near the horizon is still given by

$$A_x(r) \sim f(r)^{-(i\omega/3r_+)}, \tag{13}$$

and in the asymptotic AdS region

$$A_x = A^{(0)} + \frac{A^{(1)}}{r}. \tag{14}$$

Thus, we can obtain the conductivity of the dual superconductor by using the AdS/CFT dictionary [8,9]

$$\sigma = -\frac{iA^{(1)}}{\omega A^{(0)}}. \tag{15}$$

For different values of F^4 correction term ε, one can obtain the conductivity by solving the Maxwell equation numerically. We will focus on the case for the fixed scalar mass $m^2 L^2 = -2$ in our discussion.

In Fig. 2 we plot the frequency dependent conductivity obtained by solving the Maxwell Eq. (12) numerically for $\varepsilon = -0.01,\ 0,\ 0.01,\ 0.05,\ 0.1$ and 0.2 at temperatures

TABLE I.   The critical temperature T_c for the operators $\mathcal{O}_-$ and $\mathcal{O}_+$ with different values of ε for $d = 4$ and $m^2 L^2 = -2$. We have set $\rho = 1$ in the table.

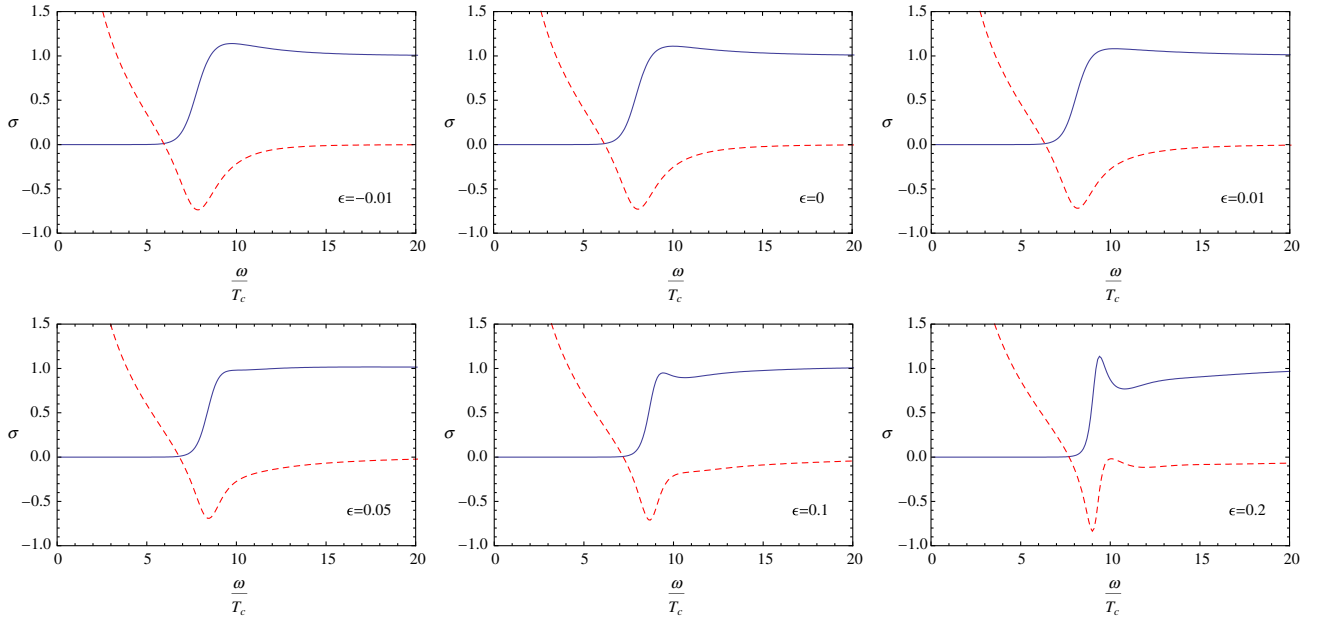| ε	-0.01	-0.001	0	0.001	0.01	0.1	0.2	0.3
$\mathcal{O}_-$	0.2260	0.2256	0.2255(4)	0.2255(0)	0.2251	0.2219	0.2189	0.2163
$\mathcal{O}_+$	0.1233	0.1188	0.1184	0.1181	0.1151	0.0993	0.0900	0.0836

FIG. 2 (color online). Conductivity of $(2+1)$-dimensional superconductors with the F^4 corrections for the fixed mass of the scalar field $m^2L^2 = -2$ and different correction terms, i.e., $\varepsilon = -0.01$, 0, 0.01, 0.05, 0.1 and 0.2.

$T/T_c \approx 0.2$. The blue (solid) line and red (dashed) line represent the real part and imaginary part of the conductivity $\sigma(\omega)$ respectively. We find a gap in the conductivity with the gap frequency ω_g. For the same mass of the scalar field, we observe that with the increase of the F^4 correction term ε, the gap frequency ω_g becomes larger. Also, for increasing F^4 correction term, we have larger deviations from the value $\omega_g/T_c \approx 8$. This shows that the high F^4 corrections really change the expected universal relation in the gap frequency, which is similar to the effect of the Gauss-Bonnet coupling.

III. HOLOGRAPHIC SUPERCONDUCTOR/ INSULATOR TRANSITIONS WITH F^4 CORRECTIONS

In Refs. [31,57], we discussed the holographic dual to Gauss-Bonnet-AdS soliton in the usual Maxwell electrodynamics. It shows that although the Gauss-Bonnet term has no effect on the Hawking-Page phase transition between AdS black hole and AdS soliton, it does have an effect on the scalar condensation and conductivity in Gauss-Bonnet-AdS soliton configuration. In this section we will examine the effect of F^4 correction in the Schwarzschild-AdS soliton background and explore its influence on the insulator and superconductor phase transition.

A. Superconductor/insulator phase in the AdS soliton

Making use of two wick rotations for the AdS Schwarzschild black hole given in (5), we can obtain the d-dimensional AdS soliton

$$ds^2 = -r^2 dt^2 + \frac{dr^2}{f(r)} + f(r)d\varphi^2 + r^2 dx_j dx^j, \quad (16)$$

with

$$f(r) = \frac{r^2}{L^2}\left(1 - \frac{r_s^{d-1}}{r^{d-1}}\right). \quad (17)$$

Note that there does not exist any horizon in this solution and $r = r_s$ is a conical singularity. Imposing a period $\beta = \frac{4\pi L^2}{(d-1)r_s}$ for the coordinate φ, we can remove the singularity.

Beginning with the generalized Lagrangian (3), we can get the equations of motion for the scalar field and gauge field in the probe limit

$$\psi'' + \left(\frac{d-2}{r} + \frac{f'}{f}\right)\psi' + \left(\frac{\phi^2}{r^2 f} - \frac{m^2}{f}\right)\psi = 0, \quad (18)$$

$$\left(1 + \frac{3\varepsilon f}{r^2}\phi'^2\right)\phi'' + \left[\frac{d-4}{r} + \frac{f'}{f} + \varepsilon\left(\frac{2f'}{r^2} - \frac{f}{r^3}\right)\phi'^2\right]\phi'$$
$$- \frac{2\psi^2}{f}\phi = 0. \quad (19)$$

Using the shooting method, we will solve these two equations numerically with appropriate boundary conditions at $r = r_s$ and at the boundary $r \to \infty$. At the tip $r = r_s$, the solutions behave like

$$\psi = \tilde{\psi}_0 + \tilde{\psi}_1(r - r_s) + \tilde{\psi}_2(r - r_s)^2 + \dots,$$
$$\phi = \tilde{\phi}_0 + \tilde{\phi}_1(r - r_s) + \tilde{\phi}_2(r - r_s)^2 + \dots, \quad (20)$$

where $\tilde{\psi}_i$ and $\tilde{\phi}_i$ $(i = 0, 1, 2, \dots)$ are integration constants, and we impose the Neumann-like boundary condition to
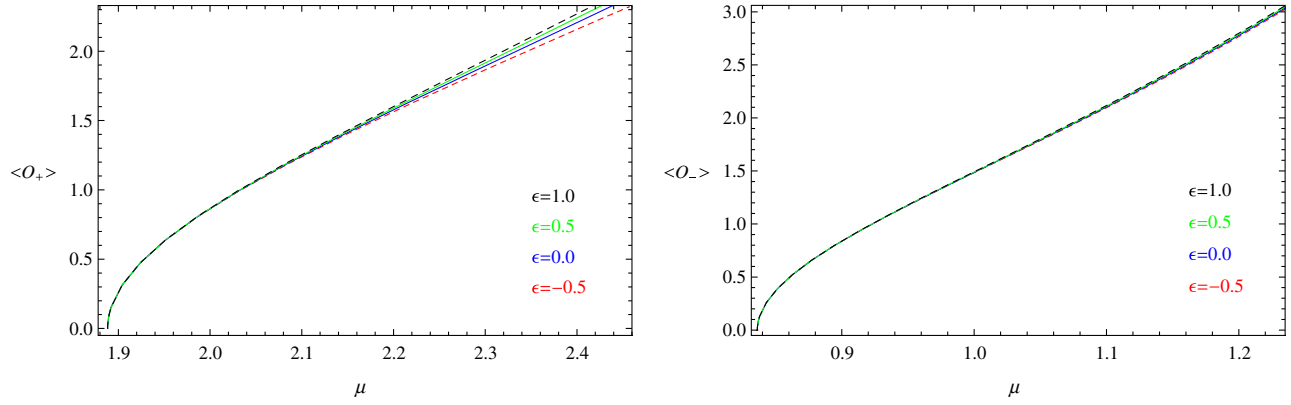
FIG. 3 (color online). The condensates of the scalar operators $\mathcal{O}_+$ and $\mathcal{O}_-$ with respect to the chemical potential μ for the mass of the scalar field $m^2 L^2 = -15/4$ in $d = 5$ dimension. In each panel, the four lines from bottom to top correspond to increasing correction term, i.e., $\varepsilon = -0.5$ (red and dashed), 0.0 (blue), 0.5 (green) and 1.0 (black and dashed), respectively.

keep every physical quantity finite [50]. Obviously, we can find a constant nonzero gauge field $\phi(r_s)$ at $r = r_s$. This is in strong contrast to the AdS black hole, where $\phi(r_+) = 0$ at the horizon. Near the AdS boundary $r \rightarrow \infty$, the solutions have the same form just as Eq. (11). For clarity, we will take $d = 5$ and still use the probe approximation in our calculation.

It is well-known that the solution is unstable and a hair can be developed when the chemical potential is bigger than a critical value, i.e., $\mu > \mu_c$. However, the gravitational dual is an AdS soliton with a nonvanishing profile for the scalar field ψ if $\mu < \mu_c$, which can be viewed as an insulator phase [50]. Thus, around the critical chemical potential μ_c there is a phase transition between the insulator and superconductor phases. We will examine the effect of the F^4 correction term on μ_c numerically.

In Fig. 3, we plot the condensations of scalar operators $\mathcal{O}_+$ and $\mathcal{O}_-$ with respect to the chemical potential μ in the 5-dimensional AdS Soliton for different F^4 correction terms with the fixed scalar mass $m^2 L^2 = -15/4$. From

this figure, we find that the critical chemical potential μ_c is independent of the correction term ε. As a matter of fact, selecting the mass of the scalar field in the range $-\frac{(d-1)^2}{4} < m^2 L^2 < -\frac{(d-1)^2}{4} + 1$ for $d = 5$ where both modes of the asymptotic values of the scalar fields are normalizable, we obtain μ_{c-} and μ_{c+} for scalar operators $\langle \mathcal{O}_- \rangle$ and $\langle \mathcal{O}_+ \rangle$ with different values of m and ε respectively

$$\mu_{c-} = 0.409 \quad \text{and} \quad \mu_{c+} = 2.261,$$
$$\text{for } m^2 L^2 = -13/4 \quad \text{and} \quad \forall\, \varepsilon,$$
$$\mu_{c-} = 0.598 \quad \text{and} \quad \mu_{c+} = 2.099,$$
$$\text{for } m^2 L^2 = -7/2 \quad \text{and} \quad \forall\, \varepsilon,$$
$$\mu_{c-} = 0.836 \quad \text{and} \quad \mu_{c+} = 1.888,$$
$$\text{for } m^2 L^2 = -15/4 \quad \text{and} \quad \forall\, \varepsilon. \tag{21}$$

It is shown the critical chemical potentials μ_{c-} and μ_{c+} are independent of the correction term ε for the fixed mass
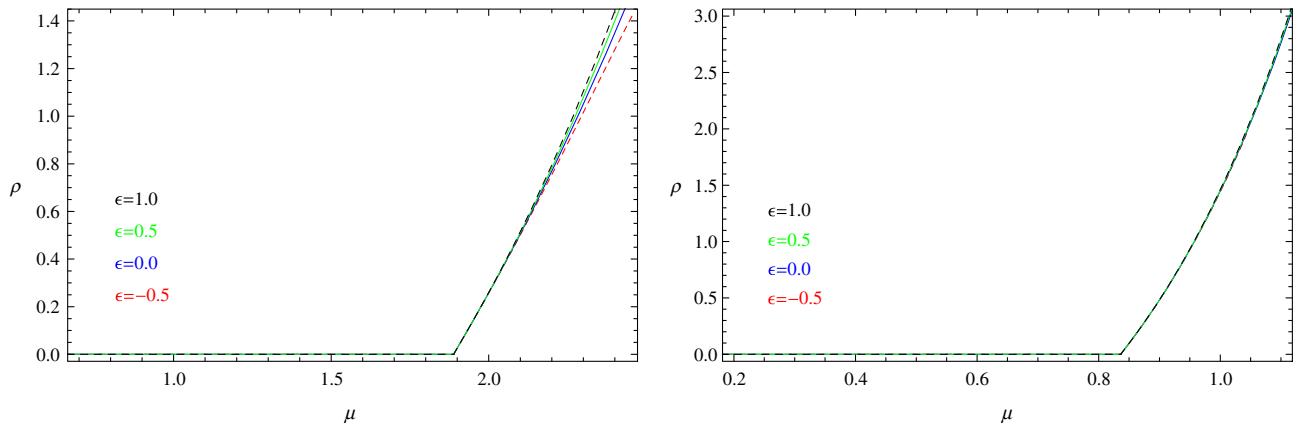

FIG. 4 (color online). The charge density ρ as a function of the chemical potential μ with fixed mass of the scalar field $m^2 L^2 = -15/4$ when $\langle \mathcal{O}_+ \rangle \neq 0$ (left) and $\langle \mathcal{O}_- \rangle \neq 0$ (right). Their derivatives jump at the phase transition points. In each panel, the four lines from bottom to top correspond to increasing correction term, i.e., $\varepsilon = -0.5$ (red and dashed), 0.0 (blue), 0.5 (green) and 1.0 (black and dashed), respectively.

of the scalar field, which is in contrast to the case of considering the Gauss-Bonnet correction term [31,57]. However, the critical chemical potentials μ_{c-} and μ_{c+} depend on the mass of the scalar field, i.e., μ_{c-} for the scalar operator $\mathcal{O}_-$ becomes smaller with the increase of the scalar field mass, but larger scalar field mass leads to higher μ_{c+} for the scalar operator $\mathcal{O}_+$, which is in agreement with the results in Refs. [31,50].

In Fig. 4, we plot the charge density ρ as a function of the chemical potential μ when $\langle\mathcal{O}_+\rangle \neq 0$ (left) and $\langle\mathcal{O}_-\rangle \neq 0$ (right) for $m^2L^2 = -15/4$. For each chosen ε, we see that when μ is small, the system is described by the AdS soliton solution itself, which can be interpreted as the insulator phase [50]. When μ reaches μ_{c-} or μ_{c+}, there is a phase transition and the AdS soliton reaches the superconductor (or superfluid) phase for larger μ. Still, we can find that the correction terms ε do not have any effect on the critical chemical potentials μ_{c-} and μ_{c+} for the fixed mass of the scalar field.

B. Analytical understanding of the superconductor/insulator phase transition

Here we will apply the Sturm-Liouville method [26] to analytically investigate the properties of holographic insulator/superconductor phase transition with F^4 corrections. We will analytically calculate the critical chemical potential which can accommodate the phase transition.

Introducing a new variable $z = 1/r$, we can rewrite Eqs. (18) and (19) for $d = 5$ into

$$\psi'' + \left(\frac{f'}{f} - \frac{1}{z}\right)\psi' + \left(\frac{\phi^2}{z^2f} - \frac{m^2}{z^4f}\right)\psi = 0, \quad (22)$$

$$(1 + 3\varepsilon fz^6\phi'^2)\phi'' + \left[\frac{1}{z} + \frac{f'}{f} + \varepsilon z^5(7f + 2zf')\phi'^2\right]\phi'$$
$$- \frac{2\psi^2}{z^4f}\phi = 0, \quad (23)$$

where the prime denotes the derivative with respective to z.

At the critical chemical potential μ_c, the scalar field $\psi = 0$. Thus, near the critical point Eq. (23) reduces to

$$(1 + 3\varepsilon fz^6\phi'^2)\phi'' + \left[\frac{1}{z} + \frac{f'}{f} + \varepsilon z^5(7f + 2zf')\phi'^2\right]\phi' = 0. \quad (24)$$

With the Neumann-like boundary condition (20) for the gauge field ϕ at the tip $r = r_s$, we can obtain the physical solution $\phi(z) = \mu$ to Eq. (24) when $\mu < \mu_c$. Considering the asymptotic behavior given in Eq. (11), close to the critical point μ_c, this solution indicates that $\rho = 0$ near the AdS boundary $z = 0$, which agrees with our previous numerical results.

As $\mu \to \mu_c$, the scalar field Eq. (22) reduces to

$$\psi'' + \left(\frac{f'}{f} - \frac{1}{z}\right)\psi' + \left(\frac{\mu^2}{z^2f} - \frac{m^2}{z^4f}\right)\psi = 0, \quad (25)$$

which is the master equation to give the critical chemical potential μ_c in the Sturm-Liouville method.

Before going further, we would like to comment Eq. (25). Although Eq. (24) for ϕ depends on ε, but the correction terms ε are absent in the master Eq. (25). Thus, we can immediately conclude that the F^4 correction term do not have any effect on the critical chemical potential μ_c for the fixed mass of the scalar field. However, for the black hole background, due to the difference of boundary conditions at the horizon, the physical solution $\phi(r)$ of Eq. (9) depends on ε, this results in the appearance of the correction term ε in the master equation derived from Eq. (8). Thus, in this case the F^4 correction terms do have effect on the critical temperature T_c for the AdS black hole, which agrees well to the numerical results.

Still, we will work on Eq. (25) to understand the dependence of the critical chemical potential on the mass of the scalar field analytically. As in [26], we introduce a trial function $F(z)$ near the boundary $z = 0$ which satisfies

$$\psi(z) \sim \langle\mathcal{O}_i\rangle z^{\lambda_i}F(z), \quad (26)$$

with $i = +$ or $i = -$. Note that the function $F(z)$ has the boundary condition $F(0) = 1$ and $F'(0) = 0$ [26]. So the equation of motion for $F(z)$ is

$$F'' + \left[\frac{2\lambda_i}{z} + \left(\frac{f'}{f} - \frac{1}{z}\right)\right]F' + \left[\frac{\lambda_i(\lambda_i - 1)}{z^2} + \frac{\lambda_i}{z}\left(\frac{f'}{f} - \frac{1}{z}\right)\right.$$
$$\left. + \frac{1}{z^4f}(\mu^2z^2 - m^2)\right]F = 0. \quad (27)$$

Defining a new function

$$T(z) = z^{2\lambda_i - 3}(z^4 - 1), \quad (28)$$

we can rewrite Eq. (27) as

$$(TF')' + T\left[\frac{\lambda_i(\lambda_i - 1)}{z^2} + \frac{\lambda_i}{z}\left(\frac{f'}{f} - \frac{1}{z}\right)\right.$$
$$\left. + \frac{1}{z^4f}(\mu^2z^2 - m^2)\right]F = 0. \quad (29)$$

According to the Sturm-Liouville eigenvalue problem [58], we obtain the expression which will be used to estimate the minimum eigenvalue of μ^2

$$\mu^2 = \frac{\int_0^1 T(F'^2 - UF^2)dz}{\int_0^1 VF^2dz}, \quad (30)$$

with

$$U = \frac{\lambda_i(\lambda_i - 1)}{z^2} + \frac{\lambda_i}{z}\left(\frac{f'}{f} - \frac{1}{z}\right) - \frac{m^2}{z^4f}, \qquad V = \frac{T}{z^2f}. \quad (31)$$

In the following calculation, we will assume the trial function to be $F(z) = 1 - az^2$, where a is a constant.

For clarity, we will take $i = +$ and one can easily extend the study to the case of $i = -$. From Eq. (30), we obtain the expression for $i = +$

$$\mu^2 = \frac{\Sigma(a, \alpha)}{\Xi(a, \alpha)}, \tag{32}$$

with

$$\Sigma(a, m) = -\frac{4a^2}{12 + m^2 + 6\sqrt{4 + m^2}}$$
$$-\frac{8 + m^2 + 4\sqrt{4 + m^2}}{2}$$
$$\times \left(\frac{1}{2 + \sqrt{4 + m^2}} - \frac{2a}{3 + \sqrt{4 + m^2}} + \frac{a^2}{4 + \sqrt{4 + m^2}} \right),$$

$$\Xi(a, m) = -\frac{1}{2(1 + \sqrt{4 + m^2})} + \frac{a}{2 + \sqrt{4 + m^2}}$$
$$-\frac{a^2}{2(3 + \sqrt{4 + m^2})}. \tag{33}$$

For different values of the mass of scalar field, we can get the minimum eigenvalue of μ^2 and the corresponding value of a, for example, $\mu^2_{\min} = 5.121$ and $a = 0.361$ for $m^2 L^2 = -13/4$, $\mu^2_{\min} = 4.416$ and $a = 0.348$ for $m^2 L^2 = -14/4$, and $\mu^2_{\min} = 3.574$ and $a = 0.330$ for $m^2 L^2 = -15/4$. Then, we have the critical chemical potential $\mu_c = \mu_{\min}$ [55], i.e.,

$$\begin{aligned} \mu_c &= 2.263, \quad \text{for } m^2 L^2 = -13/4, \\ \mu_c &= 2.101, \quad \text{for } m^2 L^2 = -7/2, \\ \mu_c &= 1.890, \quad \text{for } m^2 L^2 = -15/4. \end{aligned} \tag{34}$$

Comparing with numerical results in Eq. (21), we find that the analytic results derived from Sturm-Liouville method are in good agreement with the numerical calculation.

Thus, we conclude that, unlike the holographic superconductor models, the holographic superconductor/insulator transitions is not affected by the F^4 correction terms but only depends on the mass of the scalar field.

IV. CONCLUSIONS

We have investigated the behavior of the holographic superconductors in the presence of the a quadratic field strength correction F^4 to the usual Maxwell field. Different from the same order curvature correction in the Gauss-Bonnet holographic dual models which only appears in spacetime with dimension higher than $3 + 1$, F^4 correction can appear basically in all dimensions. We found that similar to the curvature correction, in the black hole background, the higher F^4 correction term can make the condensation harder to form and result in the larger deviations from the universal value $\omega_g / T_c \approx 8$ for the gap frequency. Thus, the F^4 corrections and the Gauss-Bonnet corrections share some similar features for the holographic superconductor system. However, the story is completely different if we study the holographic superconductor/insulator transitions with the F^4 correction. In contrast to the curvature correction effect, we observed in the AdS soliton background that the critical chemical potentials are independent of the F^4 correction term, which tells us that the correction to the Maxwell field will not affect the properties of the holographic superconductor/insulator phase transition. We confirmed our numerical result by using the Sturm-Liouville analytic method and concluded that different from the AdS black hole background, the corrections to the gravity and gauge field do play different roles in the holographic superconductor and insulator phase transition.

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