1/4 BPS string junctions and N^3 problem in 6-dimensional (2,0) superconformal theories

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We explore 1/4-BPS objects in the Coulomb phase of the ADE-type 6-dim (2,0) superconformal theories. By using the previous work on the junctions of strings in 5-dim gauge theories and 6-dim superconformal theories, we count the number of 1/4-BPS objects, which are made of waves on self-dual strings and junctions of self-dual strings, and show that for all cases the number matches exactly one-third of the anomaly constant $c_G = d_G h_G$ which is the product of dimension d_G and dual Coxeter number h_G. This suggests the long sought after N^3 degrees of freedom are these 1/4-BPS objects at least in the Coulomb phase.

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I. INTRODUCTION

The 6-dim (2,0) superconformal field theories form an important cornerstone for the M-theory structure and for the whole hierarchy of supersymmetric field theories. They come as the ADE type and are realized as the low energy dynamics of type IIB string theory on ADE-type singularities [1]. The AD-type theories are also realized as the low energy dynamics on parallel M5 branes, maybe with OM5 orientifold. These theories are purely quantum and the exact nature of its nonabelian description is not well-understood. It is known from gravity dual that the entropy scales like N^3 for $A_{N-1} = SU(N)$ theory [2,3]. In the generic Coulomb phase, there exist obviously 1/2-BPS self-dual strings whose multiplicity is of order N^2. Sometime ago it has been suggested that the additional degrees of freedom could manifest in the Coulomb phase as 1/4-BPS three self-dual string junctions [4]. We want here to make further developments and elaborations on this idea.

A finer counting of the degrees of freedom was proposed by the anomaly calculation under the global $SO(5)$ R-symmetry of the (2,0) theories [5–7]. This has been further supported by recent works on M5 branes via the conformal field theory [8,9]. The anomaly coefficient for the ADE type is the product $c_G = d_G * h_G$ of dimension d_G and dual Coxeter number h_G. The number c_G for the ADE-type group is divisible by 6. In this work, we explore the 1/4-BPS objects in the generic Coulomb phase of the (2,0) theories of ADE type. Irreducible 1/4-BPS objects consist of BPS waves on self-dual strings and junctions made of three self-dual strings. By irreducible we mean that they are the simplest and indivisible 1/4-BPS objects. While a self-dual string can turn to its anti-self-dual string by a spatial rotation, these 1/4-BPS objects and its anti-objects are distinguishable. In this work we count the number of 1/4-BPS objects and its anti-objects by considering their charges only, and found the number is exactly $c_G/3$ for all ADE-type theories. Our counting suggests that there may be no further degrees of freedom to account.

In Ref. [4], it has been argued that there are further less supersymmetric nonplanar BPS webs of self-dual strings. Their basic elements are 1/4-BPS junctions and so it may be not necessary to count them as the fundamental degrees of freedom. The key for these BPS webs of strings is the locking of the internal $SO(5)$ R-symmetry with the spatial $SO(5)$ rotational symmetry. In Ref. [7], the $SO(5)_R$ anomaly is studied in the Coulomb phase by the Wess-Zumino-Witten term for the five scalar fields, where self-dual strings appear as skyrmions with the topology $\pi_4(S^4) = \mathbb{Z}$. Our counting suggests that there may be a way to include 1/4-BPS junctions to this argument. (See [10] for the anomaly analysis for the monopole strings in 5-dim gauge theory.) One can take a somewhat different approach in the Coulomb phase. One can ask for the Wilsonian effective Lagrangian for the abelian modes, which would be expressed in terms of the derivative expansion. The contributions to higher order derivative terms would contain various bubbles of BPS and anti-BPS objects. It would be great if one can obtain these terms and identify the contributions. The quantum bubbles of junctions and anti-junctions may have contributed to these effective Lagrangian.

Another approach to the 6-dim (2,0) theory is to compactify them on a circle of radius R_6. The resulting theory in the low energy limit becomes the 5-dim maximally supersymmetric Yang-Mills theory with the dimensionful coupling of order R_6. Surprisingly, instanton solitons in this 5-dim theory play the role of the Kaluza-Klein modes of the underlying (2,0) theory [11]. While the 5-dim theory is weakly coupled in the low energy, it has

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the dimensionful coupling constant and so is nonrenormalizable theory. It is presumably divergent in ultraviolet region, and so UV incomplete in perturbative sense. As the 6-dim (2,0) theory, even after the circle compactification, is expected to be complete. If the instantons represent all KK modes of the 6-dim theory, the 5-dim theory would be complete by its own if one keeps all nonperturbative effects [12,13].

Since the 5-dim gauge theory entropy scales like N^2, at least at small energy or perturbative regime, the N^3 states must be hidden in some way. Our paper is in part stimulated by these recent conjectures although we will not rely directly on them in our arguments. So what could be the possible solutions to the N^3 problem? We can divide the possible solutions in two categories: (I) The fundamental degrees of freedom are not present in the 5d description at all. They must thus be in the extra KK modes and not accessible when the temperature is small. (II) The fundamental degrees of freedom are already there but hidden in some way.

There are some reasons to be skeptical about option (I). In this case we would be forced to consider two different kind of fundamental degrees of freedom, those of the 5d theory and the N^3 ones in higher KK modes. But instantons of the 5d theory are KK modes. There may be more KK modes besides those captured by instantons, but there is no concrete evidence for that yet. The naive counting of BPS instantons does not generate N^3. Classically they have many zero modes, but quantum mechanically one expects that there is only single threshold bound state for each instanton number besides the spin counting. There is a proposal that instantons are made of N instanton partons of mass $1/(NR_6)$ and instanton partons fall to the adjoint representation [14,15]. The number of instanton partons would then lead to the N^3 counting. Recently we have proposed more concrete realization of this instanton partons [16], but there is no evidence yet that instanton partons are in adjoint representation. Especially the instanton partons are intrinsically related to the compactification as they are presumably arising from a single M5 brane wrapping the compactification circle N times. It remains to see whether instanton partons play any role in counting N^3 degrees of freedom. (For a somewhat completely different approach to introduce N^3 d.o.f. in 3,4 dim, see Ref. [17]. Also for somewhat interesting approach to nonabelian tensor field, see Ref. [18].)

Option (II) sounds better but still requires some further specifications. Additional states could be formed as a bound or confinement of BPS objects, for example, dyonic instantons. When temperature goes up, the bound states would be broken to their fundamental components. However the N^3 entropy is what we see even at arbitrary high temperature. Furthermore we should also have a good reason to select N^3 instead of N^4 or N^5. Bounded states can be created with any number of legs and so there seems to be no criterion to prefer N^3.

We thus conclude that N^3 states if they emerge from somewhere must be strange, at least at first glance. First they must have three legs, and being absolutely stable, no matter what the energy is. Higher leg object could exists, but they must be just composite of the three leg ones. Second they cannot be states with finite energy. This could sound in contrast with the fact that the entropy counts the number of states *up* to a certain energy, and infinite energy objects are not counted.

One way out is that these states could be confined objects like quarks or gluons in QCD. The transition from N^2 to N^3 of the entropy, as the temperature increases in 5-dim gauge theories, is analogous to the deconfinement transition in $U(N)$ QCD where the entropy jumps from order N^0 to order N^2. The energy of a single quark is divergent in the confined phase. The free energy of a single quark becomes instead finite when the temperature reaches the deconfinement transition and so they can count in the entropy. Actually the gluons dominate in the count because they are of order N^2.

Infinite energy objects with three legs have been found in Ref. [4] as $1/4$-BPS junctions of monopole strings in the Coulomb phase of the 5-dim theories. The junctions have all the requirements we are looking for to be a good candidate for the N^3 degrees of freedom. They are already present in the 5d theory. Each junction has three legs and absolutely stable, and then BPS objects with more legs are just composite objects of the junctions. They have infinite energy due to the string legs. As one needs half of each monopole string to make the junction, it is absolutely stable once the boundary configurations are fixed. Our approach in this work can be regarded as a refined point of view of the pant diagram made by M2 branes connecting three M5 branes [19]. The pant diagram can be regarded as a bound state of junction and anti-junction.

Still many questions remain to be answered though. What is the exact nature of this deconfinement phase transition? The flat direction disappears in any finite temperature, leading to the gauge symmetry restoration. At zero temperature, a junction and its anti-junction are bounded by a linear potential and so confined. An interesting problem which we do not attempt here is to count the number of independent unstable massive mesons in the Coulomb phase of 5d theories or 6d theories. They would contribute to the scattering amplitude and the high derivative low energy effective Lagrangian for the abelian degrees of freedom.

The anomaly polynomial under the general background of the $SO(5)_R$ gauge field strength F and graviational curvature R for a single M5 brane [20] is

$$I_8(1) = \frac{1}{48}\left[p_2(F) - p_2(R) + \frac{1}{4}(p_1(F) - p_1(R))^2\right], \quad (1)$$

where p_k is the k-th Pontryagin class. The similar anomaly polynormial for the 6-dim (2,0) theories of the group G is

TABLE I. r_G, d_G, h_G and $c_G/3$ for simple-laced groups ADE.

Group	r_G	d_G	h_G	$c_G/3$
$A_{N-1} = SU(N)$	$N-1$	N^2-1	N	$\frac{1}{3}N(N^2-1)$
$D_N = SO(2N)$	N	$N(2N-1)$	$2(N-1)$	$\frac{2}{3}N(2N-1)(N-1)$
E_6	6	78	12	312
E_7	7	133	18	798
E_8	8	248	30	2480

calculated for AD-type and also conjectured for the E-type to be [5–7]

$$I_8[G] = r_G I_8(1) + c_G \times \frac{p_2(F)}{24}, \tag{2}$$

where r_G is the rank of the group and c_G is the anomaly coefficient. The anomaly coefficient c_G is the product of the dimension d_G of the group and the dual Coxeter number h_G so that

$$c_G = d_G * h_G. \tag{3}$$

Table I enlists all r_G, d_G, h_G and $c_G/3$ for the ADE-type groups.

More recently there were several works exploring the M5 branes wrapped on a certain kind of 4-dim manfolds, resulting in 2d (2,0) superconformal field theory with $U(1)$ R-symmetry. The central charge of the Toda theory of similar gauge group takes a similar structure [8,9] as it is given as

$$c_{\rm Toda}[G] = r_G + c_G Q^2, \tag{4}$$

where $Q = (\epsilon_1 + \epsilon_2)^2/(\epsilon_1\epsilon_2)$ if the 6-dim theory is compactified on R^4 with equivariant parameters $\epsilon_{1,2}$.

The plan of the paper is as follows. In Sec. II, we first review the BPS junctions in 5-dim gauge theories. In Sec. III, we count 1/4-BPS objects and anti-objects in the (2,0) theories of ADE types. Finally in Sec. IV, we close with some concluding remarks.

II. 5-DIM GAUGE THEORIES AND JUNCTIONS

Let us start with the (2, 0) $A_{N-1} = SU(N)$ superconformal theory in 5 + 1 which describes the low energy physics of N M5 branes. Upon compactification on a circle with $x^5 \sim x^5 + 2\pi R_5$, the physics below KK scale is described by the 5-dim maximally supersymmetric $SU(N)$ Yang-Mills theory. This in turn is the description of the low energy dynamics of parallel N D4 branes. This 5d theory has the coupling constant

$$8\pi/g_5^2 = 1/R_6. \tag{5}$$

The theory has 16 supercharges and consist of a gauge multiplet, which is composed of the gauge field, A_M, $M = 0, 1, 2, 3, 4$, the scalar field ϕ_I, $I = 1, 2, 3, 4, 5$ and the gaugino spinor field Ψ. The bosonic part of Lagrangian is

$$L_B = \frac{1}{2g_5^2}{\rm tr}(-F_{MN}F^{MN} - 2D_M\phi_I D^M\phi_I + [\phi_I, \phi_J]^2). \tag{6}$$

We have three main topological charges in the Coulomb phase: the electric charge Q_E, the magnetic charge Q_M of monopole strings and the instanton charge Q_I. The supersymmetry transformation for the gaugino field in 10-dim notation is

$$\delta\lambda = \frac{1}{2}\Gamma^{MN}F_{MN} + \Gamma^{M(I+4)}D_M\phi_I - \frac{i}{2}\Gamma^{(I+4)(J+4)}[\phi_I, \phi_J], \tag{7}$$

where Γ^M, $M = 0, 1, \ldots, 9$ is the 10-dim gamma matrices. Under the $SO(5)$ R-symmetry, the spinor transforms as **4** and $\bar{\mathbf{4}}$ and the scalars transform as **5**. In the Coulomb phase, the scalar field takes a nonzero expectation value and the R and gauge symmetries are spontaneously broken. The diagonalized scalar vacuum expectation values denote the position of M5 or D4 branes in normal R^5 space modulo string scale. Massive W-bosons, monopole strings, and instantons are 1/2-BPS objects.

Let us consider first the simplest case: the $SU(2)$ group with the scalar field expectation $\langle\phi_5\rangle = (v, -v)/2$. There exist nonlocal 1/2-BPS monopole strings, corresponding to D2 branes connecting two D4 branes. Monopole strings lying along x^4 axis are described by the self-dual equation

$$F_{ij} = \epsilon_{ijk}D_k\phi_5, \qquad i, j, k \in \{1, 2, 3\}, \tag{8}$$

and the unbroken susy parameter satisfies

$$\Gamma^{1239}\epsilon = \epsilon. \tag{9}$$

The tension of a single monopole string is

$$T_s = \frac{4\pi v}{g_5^2}. \tag{10}$$

Interestingly anti-monopole strings are equivalent to monopole strings. A rotation x^3, x^4 axis by 180 degrees flips the sign of the supersymmetric condition (9). This is related to the fact that a single monopole string can make a closed loop of trivial quantum number. In the strong coupling limit, the monopole strings would become 1/2-BPS self-dual strings once we keep v/g_5^2 constant, which characterizes the relative position of two M5 branes.

There exists one 1/2-BPS massless vector multiplet for the unbroken abelian subgroup $U(1)$. For example, we can choose the momentum direction along x^4 axis, and the conserved supersymmetry would be fixed by the condition

$$\Gamma^{04}\epsilon = \epsilon. \tag{11}$$

There are also one 1/2-BPS massive charged vector multiplet and instanton multiplet whose supersymmetric conditions are, respectively,

$$\Gamma^{09}\epsilon = \epsilon, \qquad \Gamma^{1234}\epsilon = \epsilon. \tag{12}$$

The 5-dim massive W-bosons are interpreted as self-dual strings wrapping the compactified circle, and the 5-dim instanton solitons are just KK modes along x^5 direction. There are 1/2-BPS dyonic strings which are monopole strings with uniform electric charge density. These dyonic strings describe the self-dual strings which are tilted in, say, x^4, x^5 plane and so are wrapping x^5 circle but are extended along x^4 axis.

Let us now consider the $1/4$-BPS objects in the 5-dim $SU(2)$ gauge theory. Dyonic instantons [21,22] are $1/4$-BPS objects with their supersymmetric parameters satisfying both conditions in Eq. (12). Similarly, one could have $1/4$-BPS objects made of massless waves on magnetic monopole strings lying along, say, x^4 axis. Their supersymmetric parameter satisfies

$$\Gamma^{1239}\epsilon = \epsilon, \qquad \Gamma^{04}\epsilon = \pm\epsilon. \tag{13}$$

Depending on the sign of the second condition above, the massless wave on the string would left or right moving. A way to approach the wave on monopole strings is to consider the zero modes of BPS monopoles, and lift them to the zero modes of monopole strings. Instead of the position and phase of BPS monopoles, we would get the massless modes along monopole strings.

Both dyonic instantons and waves on monopole strings get lifted to the same kind of $1/4$-BPS objects in 6-dim: massless waves on self-dual strings. The self-dual strings with left and right moving waves form $1/4$-BPS objects and anti-objects. For the $SU(2)$ case, there exists no more $1/4$-BPS objects and so the number of $1/4$-BPS object in the 6-dim theory is $2 = c_{A_1}/3$.

Let us now consider the 5-dim theory with $A_{N-1} = SU(N)$ gauge group. There exist $1/2$-BPS monopole strings for any root, say, $\alpha = e_i - e_j$, in the generic Coulomb phase which correspond to the D2 branes connecting i-th D4 brane to j-th D4 branes. As none of three D4 branes are lined up in the generic Coulomb phase, there would be only four zero modes for each $1/2$-BPS monopole strings. The tension of the $1/2$-BPS monopole strings would be proportional to the distance between two D4 branes so that

$$T_\alpha \sim \frac{1}{g_5^2}|\alpha \cdot \phi_I|. \tag{14}$$

The string for the opposite $-\alpha$ root is again obtained just by a spatial rotation. Lifting to the 6d (2,0) theories in the generic Coulomb phase, there exist $1/2$-BPS self-dual strings with four zero modes and the same tension.

From our understanding of the left and right moving waves on monopole strings and also dyonic instantons, one sees that the $1/4$-BPS moving waves on self-dual strings could have a finite transverse energy profile. As the wave is moving with speed of light, the profile of the wave is stationary and has no dissipation. Figure 1 shows two kinds of representations for both left and right moving waves on the self-dual string corresponding to the root $\alpha = e_i - e_j$.

To see less supersymmetric BPS objects in 5-dim theory, let us recall the minimally supersymmetric, or $1/16$, BPS webs of monopole strings arise from the locking of the

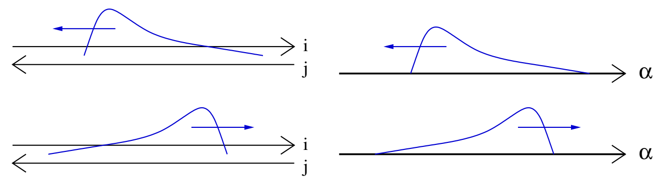

FIG. 1 (color online). Two representations of $1/4$ left and right moving BPS waves on a self-dual string.

spatial $SO(4)$ rotation to $SO(4)$ of $SO(5)$ R-symmetry. In 5d Yang-Mills theory, the BPS equation for the BPS dyonic webs of monopole strings [4,23] is

$$\begin{aligned} F_{ab} - \epsilon_{abcd}D_c\phi_d + i[\phi_a, \phi_b] = 0, \\ D_a\phi_a = 0, \\ F_{a0} = D_a\phi_5, D_a^2\phi_5 - [\phi_a, [\phi_a, \phi_5]] = 0, \end{aligned} \tag{15}$$

where $a, b, c, d = 1, 2, 3, 4$ and the Gauss law is used with the gauge $A_0 = \phi_5$.

Besides the $1/2$-BPS monopole strings, the simplest ones are $1/4$-BPS planar junctions of monopole strings [4], which can exist when $N \geq 3$. One needs generic scalar expectation value so that any given three D4 branes characterized by indices i, j, k are not aligned. Three corresponding roots $\alpha = e_i - e_j$, $\beta = e_j - e_k$, $\gamma = e_k - e_i$ have the vanishing sum. There are $1/2$-BPS self-dual strings for each root defined by a pair of D4 branes. Once three strings are on the plane and form a junction such that the junction form a dual lattice to the triangle defined by $\alpha \cdot \phi_I$, $\beta \cdot \phi_I$, $\gamma \cdot \phi_I$, the tension of self-dual string gets balanced and the junction becomes $1/4$-BPS [4]. If the string junction lies on x^3, x^4 plane with ϕ_3, ϕ_4 involved, the preserved supersymmetric parameter satisfies

$$\Gamma^{1238}\epsilon = \Gamma^{1247}\epsilon = \epsilon \tag{16}$$

The BPS equation for the $1/4$-BPS junction becomes

$$\begin{aligned} &F_{12} = D_3\phi_4 - D_4\phi_3 - i[\phi_3, \phi_4], \quad F_{23} = D_1\phi_4, \\ &F_{31} = D_2\phi_4, \quad F_{14} = D_2\phi_3, \quad F_{42} = D_1\phi_3, \\ &D_3\phi_3 + D_4\phi_4 = 0. \end{aligned} \tag{17}$$

Anti-junction has the opposite charge orientation. Figure 2 shows both such $1/4$-BPS junctions and anti-junctions. Dyonic string webs would be also $1/4$-BPS as they can be obtained from the self-dual string webs tilted along x^5 direction and stacked periodically along x^5 direction.

Before we count the $1/4$-BPS objects in the (2,0) theories, let us consider an important issue of the energy scales of the theory in the Coulomb branch of the 5-dim theory. What follows is summarized in Fig. 3. The Coulomb branch becomes quantitative tractable when the brane separation is big enough so that

$$v \gg 1/g_5^2. \tag{18}$$

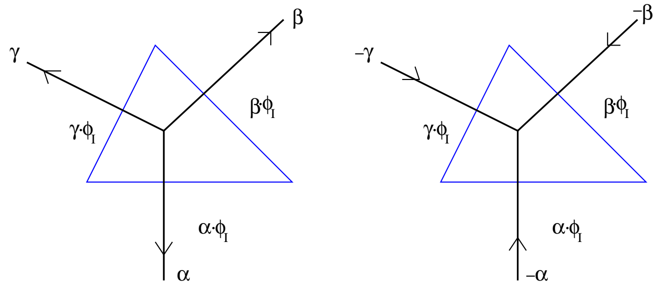

FIG. 2 (color online). Triangle and 1/4-BPS junction and anti-junction.

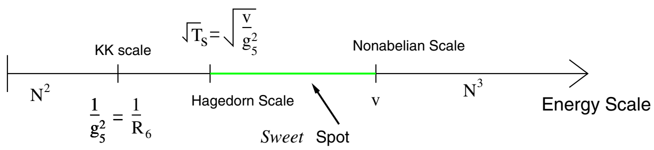

FIG. 3 (color online). Energy scale in the Coulomb phase.

At low energies the theory is 5-dim abelian gauge theory. At the KK scale $1/R_6 \sim 1/g_5^2$, the theory becomes 6-dim but still abelian. Strings junctions are confined in the Coulomb phase with linearly divergent energy due to the three infinite strings attached. Only junction and anti-junction bound states can have finite energy and thus contribute to the entropy counting. With the monopole string tension $T_s \sim v/g_5^2$, the region from $\sqrt{T_s}$ to v is our *sweet* spot. In this region, the theory is still weakly coupled and since there is a parametric separation between the string scale $\sqrt{T_s}$ and the UV completion scale v we can treat the string as if it were fundamental and quantize it. For a quantum free string the exponential growth of the number of state with the energy level sets a superior limit for the possible temperature, the so called Hagedorn temperature given by scale set by the tension $\sqrt{T_s}$. At this temperature the free energy of the string $F = E - TS$ becomes zero. The reason is that both entropy S and energy E are linearly growing with the length, and so at the right temperature the entropy term dominates over the energy term. For a interacting string, with parametrically suppressed thickness, the Hagedorn temperature clearly can be crossed as the energy density becomes bigger than the UV completion scale v. Above this limit the underling UV completion should dictate the phase of the theory. So it is now important to stress that from the string perspective we will never be able to tell what is above the Hagedorn temperature. But we can infer indirectly that the strings is not in the fundamental degrees of freedom, because its partition function would diverge, and thus we can think of it as deconfining. (In QCD something similar happens, although without parametric suppression we do not have a range of parameters in which we can quantitatively treat the string as fundamental. But the qualitative relation between Hagedorn transition and deconfinement should remain true.)

Another way to approach the nonabelian degrees of freedom is to find the 5d low energy effective Lagrangian for the abelian degrees of freedom. While there is the usual perturbative contributions by W-boson loops, one expects also the nonperturbative contributions by instanton loops, self-dual string virtual bubbles and also by junction bubbles. The ultraviolet completeness of the 5d theories, if it is true, should include all these nonperturbative effects.

III. COUNTING 1/4-BPS OBJECTS IN ADE-TYPE (2,0) THEORIES

We now start to count the number of 1/4-BPS objects characterized by what kinds of charge they carry. Let us recall how to count the degrees of freedom for the 4-dim $SU(N)$ gauge theory on N D3 branes in the Coulomb phase in the weak coupling limit. First of all the $SU(N)$ gauge symmetry is spontaneously broken in the Coulomb phase to $U(1)^{N-1}$. For each pair of distinct branes we have a charged W-boson and anti W-boson, which count as $N(N-1)$. For each brane we have a photon which count as $N-1$, subtracting the global $U(1)$. Total light 1/2-BPS object is $N-1+N(N-1) = N^2-1$, corresponding to the adjoint representation.

Let us now consider the (2,0) theory in the generic Coulomb phase with gauge group $A_{N-1} = SU(N)$. The $N(N-1)$ root vectors of the Lie algebra A_{N-1} can be represented by $e_i - e_j$, where e_i with $i = 1, \cdots N$ are N-dim orthonormal vectors. There are 1/2-BPS $N-1$ massless particles or waves corresponding to the Cartan elements of the Lie algebra and there are 1/2-BPS $N(N-1)/2$ self-dual strings. 1/2-BPS massless particles of $N-1$ kinds have finite energy, but 1/2-BPS strings of $N(N-1)/2$ kinds have infinite energy. It is difficult to say they together form an adjoint representation.

Let us turn our attention to the 1/4-BPS objects in the generic Coulomb phase. There are 1/4-BPS objects made of left and right moving waves on 1/2-BPS self-dual strings. As there are left and right moving waves for a given self-dual string, there exist $2\times N(N-1)/2 = N(N-1)$ such objects. The 1/4-BPS junctions can exist for any three choice of M5 branes, or any three roots α, β, γ such that their sum vanishes. One way to represent such roots is $\alpha = e_i - e_j$, $\beta = e_j - e_k$, $\gamma = e_k - e_i$. The junction and its anti-junction for such three roots are shown in Fig. 4. The total number of junctions and anti-junctions would be $2\times N(N-1)(N-2)/6 = N(N-1)(N-2)/3$. The total number of 1/4-BPS objects in the 6d (2,0) theory of the $A_{N-1} = SU(N)$ gauge is then

$$N(N-1)+\frac{1}{3}N(N-1)(N-2)=\frac{1}{3}N(N^2-1)=\frac{1}{3}c_{A_{N-1}}. \tag{19}$$

This number matches exactly one-third of the anomaly coefficient $c_{A_{N-1}}$. We would readily admit that this counting is naive at best as we have ignored the spin and other

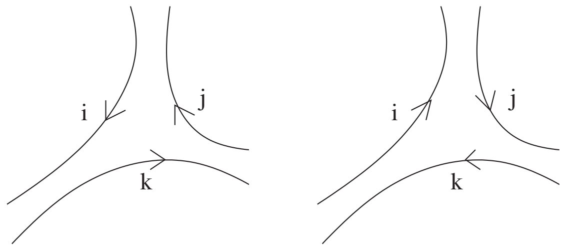

FIG. 4. A-type junction and anti-junction.

structures of these object. Also we have ignored additional degeneracy for the wave structure. But it is not implausible to think that this additional factors should be the same for all 1/4-BPS objects and thus contribute as an overall multiplicative factor.

The counting of 1/4-BPS objects in the (2,0) theory of $D_N = SO(2N)$ gauge group goes quite similarly. The root vectors are $\pm(e_i - e_j)$ or $\pm(e_i + e_j)$ where $i \neq j$ and $i, j = 1, 2, \cdots N$. There are N 1/2-BPS massless particles and thus $N(N-1)$ 1/2-BPS self-dual strings. Now the number of the 1/4-BPS waves on self-dual strings would be $2N(N-1)$. The counting of junctions and anti-junctions is a bit more complicated. At the junction, the sum of the root should vanish. The Fig. 5 shows the types of junctions and anti-junctions in this theory. As shown in this figure, there are eight types of junctions for a given $i \neq j \neq k \neq i$, leading to the total number of 1/4-BPS junctions to be $8 \times N(N-1)(N-2)/6 = 4N(N-1) \times (N-2)/3$. The total number of the 1/4-BPS objects in the 6d (2,0) theory of $D_N = SO(2N)$ gauge algebra in the generic Coulomb phase is then

$$\begin{aligned} 2N(N-1) &+ \frac{4}{3}N(N-1)(N-2) \\ &= \frac{2}{3}N(N-1)(2N-1) \\ &= \frac{1}{3}c_{D_N}. \end{aligned} \tag{20}$$

Again it matches one-third of the anomaly coefficient.

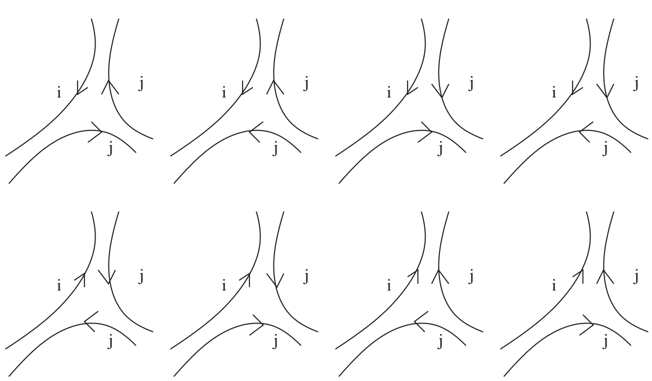

FIG. 5. D-type junctions and anti-junctions.

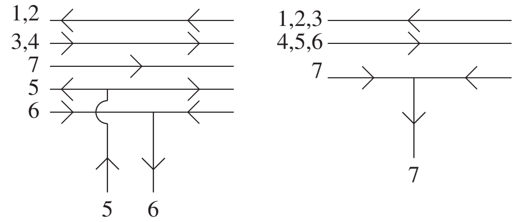

FIG. 6. Two examples of E_6 junctions.

The root diagram of E_6 is made of the roots $e_i - e_j, (i, j = 1, 2, \ldots 6)$ of $A_5 = SU(6)$, $\pm\sqrt{2}e_7$ and $\frac{1}{2}(\pm e_1 \pm e_2 \pm e_3 \pm e_4 \pm e_5 \pm e_6) \pm e_7/\sqrt{2}$ with the number of plus sign for $e_1 \cdots e_6$ being three. Note that $d_{E_6} = 78$ and $h_{E_6} = 12$ and so $c_{E_6}/3 = 312$. The number of the 1/4-BPS objects of $SU(6)$ is 70. The number of the additional 1/4-BPS objects for waves on self-dual strings is 42. The number of additional 1/4-BPS junctions with one end being e_7 is 20. Finally the number of additional 1/4-BPS junctions with one end of type $e_i - e_j$ is 180. The total 1/4-BPS object is then

$$70 + 42 + 20 + 180 = 312 = \frac{1}{3}c_{E_6}. \tag{21}$$

Figure 6 shows two types of additional junctions besides those from A_5. Note that the sum of the roots for these junctions vanishes.

The root diagram of E_7 is made of the roots $e_i - e_j, (i, j = 1, 2 \ldots 8)$ of $A_7 = SU(8)$, and the roots $(\pm e_1 \pm e_2 \cdots \pm e_8)/2$ with four plus and four minus signs. The number of 1/4-BPS objects from $A_7 = SU(8)$ is 168. The number of additional 1/4-BPS objects for waves on self-dual strings is 70. The number of additional 1/4-BPS junction is $(8 * 7/2) * 2 * 1/2 * (6 * 5 * 4/6) = 560$. The total number of 1/4-BPS objects is

$$168 + 70 + 560 = 798 = \frac{1}{3}c_{E_7}. \tag{22}$$

Figure 7 shows an example of additional junctions of E_7 case besides those from A_7.

The root system of E_8 is made of the roots $\pm e_i \pm e_j$ $(i \neq j, 1, 2 \cdots 8)$ of $D_8 = SO(16)$, and $\frac{1}{2}(\pm e_1 \pm e_2 \cdots \pm e_8)$ with the product of the signs being plus one. The number of 1/4-BPS objects from $D_8 = SO(16)$ theory is 560. T he number of 1/4-BPS objects for waves on additional

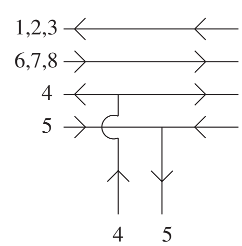

FIG. 7. An example of E_7 junctions.

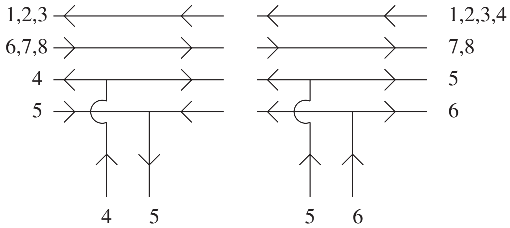

FIG. 8. Two examples of E_8 junctions.

self-dual strings is 128. The number of additional junction is $8 * 7/2 * 2^8/2/2 = 1792$. Thus, the total number of 14 BPS objects for E_8 case is

$$560 + 128 + 1792 = 2480 = \frac{1}{3} c_{E_8}, \tag{23}$$

which is exactly the number we expect. Figure 8 shows two examples of additional junctions besides those from A_7.

So far our counting of 1/4-BPS objects has been somewhat naive as we did not include the additional degrees of freedom living on the junctions. For example, magnetic monopole strings would have the $\mathcal{N} = (4, 4)$ world sheet supersymmetries on the string. The 1/2-BPS self-dual strings would have the $\mathcal{N} = (4, 4)$ world sheet supersymmetries with torsion so that the left and right complex structure being different for two parallel identical strings. 1/4-BPS junctions would have more complicated spin structure, which is beyond the scope of the current work. There is a classification for the representation of this superalgebra in terms of superfields [24]. This could be a starting point and we hope to return to this issue in future.

We would like to add some further comments on the mathematical structure behind our observation. By definition, the Coxeter number h_G is the number of roots $d_G - r_G$ divided by the rank r_G, and so $d_G = (h_G + 1) r_G$. Note that Coxeter number and dual Coxeter number coincide for simple-laced group. Our relation for ADE type theories is then translated into

$$\frac{1}{3} c_G = h_G r_G + \frac{1}{3} r_G h_G (h_G - 2). \tag{24}$$

The first term of RHS represents the 1/4-BPS objects for waves on strings, and the second term of RHS represents the 1/4-BPS junctions and anti-junctions. The number of independent BPS junctions and anti-junctions is twice the number of embeddings $SU(3)$ roots to the simple-laced group and is given by the last number [25]. We make a further observation in relation to the central charge (4) of the Toda models for the simple-laced group [26] where it appears via the Freudenthal and de Vries' strange formula,

$$\frac{1}{3} c_A = \frac{1}{3} h_G^V d_G = 4\boldsymbol{\rho}^2, \tag{25}$$

with the Weyl vector

$$\boldsymbol{\rho} = \frac{1}{2} \sum_{\alpha > 0} \alpha \tag{26}$$

being the sum over the positive root with the convention that the length square of a long root is two. Thus we get

$$\frac{1}{3} c_A = \sum_{\alpha > 0} \alpha^2 + \sum_{\alpha, \beta > 0, \alpha \neq \beta} \alpha \cdot \beta. \tag{27}$$

Note that the first term of RHS counts the number of roots $h_G r_G$ as $\alpha^2 = 2$ and so the second term of RHS should count twice the number of $SU(3)$ root embedding, or the number of junctions and anti-junctions.

IV. CONCLUDING REMARKS

Let us conclude with some remarks. We have identified all 1/4-BPS objects and anti-objects in the 6d (2,0) superconformal theories in a generic point on the Coulomb phase. These 1/4-BPS objects consist of waves on self-dual strings and (anti)-junctions of self-dual strings. The total number of 1/4-BPS objects and anti-objects is exactly one-third of the anomaly coefficient c_G for all (2,0) theories. For A_N, D_N type theories, $c_G \sim N^3$ for large N, which suggest that these 1/4-BPS objects may be fundamental ones in the (2,0) theories, even if they appear as infinite energy states in the Coulomb phase. For example, after local heating of M5 branes in Coulomb phase there could be "generalized Hagedorn phase transition" which not only release self-dual string loops but also junctions and anti-junction nets or more complicated webs of strings. While the numbers $c_G/3$ for AD-type theories coincide with the dimensions of some representations, it is not the case for E type theories. This seems to imply that the number $c_G/3$ cannot be represented as some objects of irreducible representation of the group G in general. Further studies are needed to relate precisely these 1/4-BPS objects to the anomaly calculation.

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