Towards a holographic marginal Fermi liquid

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(Received 27 June 2011; published 2 December 2011)

We present an infinite class of $2 + 1$-dimensional field theories which, after coupling to semiholographic fermions, exhibit strange metallic behavior in a suitable large N limit. These theories describe lattices of hypermultiplet defects interacting with parity-preserving supersymmetric Chern-Simons theories with $U(N) \times U(N)$ gauge groups at levels $\pm k$. They have dual anti-de Sitter (AdS) gravity descriptions in terms of lattices of probe M2-branes in $AdS_4 \times S^7/Z_k$ (for $N \gg 1$, $N \gg k^5$) or probe D2-branes in $AdS_4 \times CP^3$ (for $N \gg k \gg 1$, $N \ll k^5$). We discuss several challenges one faces in maintaining the success of these models at finite N, including backreaction of the probes in the gravity solutions and radiative corrections in the weakly coupled field theory limit.

DOI: 10.1103/PhysRevD.84.126002 PACS numbers: 11.25.Tq, 04.50.Gh, 71.10.Hf

I. INTRODUCTION

Local quantum criticality, an invariance under rescaling of energies that leaves the spatial momenta fixed, has been invoked as a potential explanation of interesting phases seen in a variety of condensed matter systems [1]. One leading approach for explaining the anomalous transport properties of the strange metallic phase, the marginal Fermi liquid [2], involves a locally critical sector of spin and charge fluctuations, coupled to a Fermi sea.

In general, the theory of non-Fermi liquids is still in its infancy. One recently developed method of obtaining controlled models of non-Fermi liquids uses holography. The study of fermion probes in black brane backgrounds with $AdS_2 \times R^2$ near-horizon geometries [3–6], or equivalently the semiholographic prescription of [7], readily gives rise to non-Fermi liquid behavior. In the latter approach, free fermions are mixed with fermionic operators from a large-N locally critical sector, dual to fermions living in AdS_2. A distinct holographic mechanism realizing non-Fermi liquid transport arises on probe branes in Lifshitz backgrounds [8].

Much of the work on the holographic approach to non-Fermi liquids has so far been at the level of four-dimensional effective AdS gravity theories, with the scaling dimensions of operators in the dual field theory appearing as free parameters (masses of bulk fields). It would be useful to have microscopic dual pairs where the field theory dynamics giving rise to local criticality is visible in a conventional field-theoretic Lagrangian, and the scaling properties of the non-Fermi liquid can be predicted by the concrete dual field theory instead of being parameterized as unknowns [9]. One goal of our work is to provide an infinite class of such theories where it is natural to obtain precisely the scaling dimensions required for marginal Fermi liquid behavior.

A second goal has been to remedy one of the residual defects in the models of [4]; there, the precise nature of the non-Fermi liquid depends sensitively on the Fermi momentum $\mathbf{k_F}$ (since the dimensions of the relevant fermionic operators depend on $\mathbf{k_F}$). In the models we describe here, the relevant scaling dimension Δ, which (with the right value) gives rise to marginal Fermi liquid behavior, is independent of $\mathbf{k_F}$. This allows an arbitrary shape of the Fermi surface, a useful feature since this is not protected from renormalization group flow.

A third goal has been to clarify when and how locally critical behavior can occur in a higher-dimensional ($D \geq$ 2-dimensional) quantum field theory. Local criticality is a rather exotic property, which needs to be better understood. By definition, it entails quantum mechanical degrees of freedom propagating independently at every point in space, not suppressed by gradient terms. On the other hand, in higher-dimensional quantum field theories, the ultraviolet physics contains itinerant fields which propagate in all directions, with gradient terms in their Lagrangian. Even if one begins with a sector of localized degrees of freedom (like the defects we study), which in itself exhibits local criticality, this sector generically mixes with the itinerant fields through interaction terms. These can, and generally would be expected to, induce gradients. Yet surprisingly, among holographic gravity systems dual to very strongly coupled field theories, one often finds solutions with AdS_2 symmetry (using either the AdS-Reissner-Nordström black brane, or the world volumes of appropriate probe branes [12] as we shall do here). These solutions are common because they are not terribly hard to obtain, whether by the relatively prosaic matter of stabilizing the extra dimensions of string theory or by stably embedding a probe brane along an AdS_2 slice. However, even in the large-N approximation of a gauge theory with N colors, strong effects of the itinerant fields are included, so this is a nontrivial result of gauge/gravity duality.

Therefore, we wish to begin an analysis of whether this emergence of local criticality is only an artifact of the extreme strongly coupled limit where the gravity description is appropriate, or whether instead a similar mechanism exists also at weaker coupling and finite N. In the second part of this paper, we discuss the interaction between impurities, which is a finite N effect but becomes important at low energies. In some cases, this spoils the local criticality, but in others this may survive to the IR.

II. THE BRANE SYSTEM

Instead of obtaining AdS_2 in the near-horizon limit of an AdS-Reissner-Nordström black brane, a setup which incurs various instabilities, we choose to obtain the AdS_2 regions on the world volumes of lattice defects, as in [12,13]. A variety of field-theoretic toy-models suggest that lattices of defects interacting with itinerant electrons could be a reasonable starting point for strange metal phenomenology (see e.g. [14–16]).

Such lattices can be implemented in various ways, differing in their symmetries and in the quantum numbers of the operators in the theory. The model of [12] involves a lattice of defect fermions interacting with the four-dimensional $\mathcal{N} = 4$ supersymmetric Yang-Mills theory, and is engineered by intersecting D3- and D5-branes (with the D5-branes wrapping $AdS_2 \times S^4$ regions in the near-horizon $AdS_5 \times S^5$ geometry of the D3-branes). The supersymmetry preserved by that lattice model is somewhat unconventional (allowing e.g. purely fermionic defect representations); therefore, we will mostly focus on a different lattice system which is $2 + 1$ dimensional and enjoys a more powerful supersymmetry algebra for some values of our discrete parameters. This, however, entails extraneous bosonic degrees of freedom at the lattice sites, and the examples containing only fermions on the defects can be analyzed similarly.

In the most symmetric case, the brane configuration we study is given, in M-theory, by M2- and M2′-branes:

	0	1	2	3	4	5	6	7	8	9	10	
M2	x	x	x								.	(1)
M2′	x	::	::	x	x							

Here, an x denotes a dimension wrapped by the given brane stack, blanks denote dimensions where the given branes are localized at a common point, and :: denotes dimensions in which the given branes are individually localized but form a lattice. In this configuration, the two stacks intersect along a lattice in the 1–2 plane.

Our family of theories will depend on two parameters: N and k. N denotes the number of M2-branes in the stack above; the M2′branes are equally spaced in a square lattice, and the lattice spacing is the only scale in the problem (so it does not constitute a new parameter). The second parameter k arises as follows. We consider a Z_k orbifold which acts as follows on the four complex coordinates transverse to the M2s:

$$g_k: z_i = x_{2i+1} + i x_{2i+2}, \quad z_i \rightarrow e^{(i2\pi/k)} z_i, \quad i = 1 \ldots 4. \tag{2}$$

The set of M2′-branes wraps the locus [17]

$$z_1 = z_2 = 0, \qquad z_3 = \bar{z}_4, \tag{3}$$

and their orbifold images under (2). For $k = 1$ this embedding is equivalent to the one in (1). We treat even and odd k symmetrically, defining the orbifold action to identify points on different, mirror branes (rather than taking the $g_k^{k/2}$ element to identify points on the same brane in the case k even).

The global symmetry of the M2-brane theory is partially broken by the orbifolding and the presence of the M2′ probes; from $SO(8) \times SO(2)$ to $SO(6) \times U(1) \times Z_4$ for $k = 1$, and down to $SU(2) \times U(1)^2 \times Z_4$ for $k > 1$. The Z_4 factor here represents the symmetry of the lattice. At large k (such that $k^5 \gg N \gg 1$), it follows from the analysis in [18] that the near-horizon region of the system of M2- and M2′-branes is described more accurately using different variables in terms of type-IIA string theory with D2- and D2′-branes on a nontrivial geometry with background 2-form gauge flux.

III. THE FIELD THEORY

The field theory on the M2-branes in these geometries has been studied in great detail [18]. A general three-dimensional supersymmetric Chern-Simons theory with at least $\mathcal{N} = 2$ supersymmetry has an action including the terms [19]:

$$\begin{aligned} S = \int d^3x \frac{k}{4\pi} \mathrm{Tr}\left(A \wedge dA + \frac{2}{3}A^3\right) + D_\mu \bar{\phi}_i D^\mu \phi_i \\ + i\bar{\psi}_i \gamma^\mu D_\mu \psi_i - \frac{16\pi^2}{k^2} (\bar{\phi}_i T^a_{R_i} \phi_i)(\bar{\phi}_j T^b_{R_j} \phi_j) \\ \times (\bar{\phi}_k T^a_{R_k} T^b_{R_k} \phi_k) - \frac{4\pi}{k} (\bar{\phi}_i T^a_{R_i} \phi_i)(\bar{\psi}_j T^a_{R_j} \psi_j) \\ - \frac{8\pi}{k} (\bar{\psi}_i T^a_{R_i} \phi_i)(\bar{\phi}_j T^a_{R_j} \psi_j). \end{aligned} \tag{4}$$

Here, T^a_R are the generators of the gauge group in representation R, and the scalars ϕ_i and fermions ψ_i are superpartners in a chiral multiplet. These terms arise from integrating out the scalars and fermions of the massive vector multiplet and flowing to the deep infrared limit of the theory.

The field theory on our M2-branes is a special case of this theory, with gauge groups $U(N) \times U(N)$ appearing at levels $\pm k$. The ‘t Hooft coupling of this theory is N/k and so is large in the holographic limits. The matter fields ϕ_i are four bifundamental fields $A_{1,2}$ and $B_{1,2}$, in the $(\bar{N}, N)$ and $(\bar{N}, N)$ representations, respectively. In addition to the basic supersymmetric action written above for these fields, we add an $\mathcal{N} = 3$ superpotential

$$W = \frac{2\pi}{k}\epsilon^{ab}\epsilon^{\dot{a}\dot{b}}\,\mathrm{Tr}(A_a B_{\dot{a}} A_b B_{\dot{b}}). \tag{5}$$

Here, $a, b = 1, 2$ and the superpotential has been written in a manifestly $SU(2) \times SU(2)$ symmetric manner. The full symmetry of the field theory is in fact enhanced to an $SO(6) \times U(1)_b$ [with the baryonic $U(1)_b$ acting with charge ± 1 on the A and B fields], and the theory with these choices enjoys an enhanced $\mathcal{N} = 6$ supersymmetry [18,20].

The probe M2′-branes give rise to localized degrees of freedom; in the type-IIA string theory limit of the brane construction, these arise from strings stretching between the D2-branes and a lattice of probe D2′-branes. In the simplest case of $k = 1$, these are hypermultiplets, with the fermions transforming as spinors in the dimensions transverse to both branes (and the bosons transforming as spinors along 1234). The infrared Chern-Simons theory is more difficult to analyze directly, since the appropriate type-IIB brane construction involves nonperturbative ingredients. However, by generalizing the methods of [18] one can obtain a plausible hypothesis for the spectrum [17], in which defect hypermultiplets are added to both gauge groups. One reason that this is plausible is that the dual probe branes respect parity, which in the field theory exchanges the gauge group factors. The bosonic quantum mechanical degrees of freedom $Q_{1,2}$ and $\tilde{Q}_{1,2}$ at each site transform as follows. Q_i transforms in the N of the ith $U(N)$ gauge group (and is a singlet under the other), while $\tilde{Q}_i$ transforms in the conjugate manner; these also transform as spinors under the Lorentz group in the 1234 directions. Each boson is accompanied by a fermion partner so there are also defect fermions $\chi_{1,2}$, $\tilde{\chi}_{1,2}$; these do not transform as spinors in the 1234 directions, but do in the remaining directions. Starting from the ABJM theory [18], the defect probe branes preserve 8 supercharges in the special case of $k = 1$, and more generally they preserve 4 supercharges [17]. We expect a similar spectrum of localized degrees of freedom on the defects for all k.

While the overall system preserves at least 4 supercharges in all cases, the superspace structure is unconventional and we have not been able to find a packaging in the standard superspace arising in four-dimensional $\mathcal{N} = 1$ supersymmetry. (For instance, from the IIB-brane configuration used to obtain the $\mathcal{N} = 6$ theories in [18], supplemented by our defects as in [17], it is clear that there are no spatial directions along which one could T dualize to obtain a higher-dimensional theory with a conventional superspace; either the probe branes or the ABJM configuration itself breaks the needed higher-dimensional translation symmetries.) However, the couplings of the A_i, B_j fields to the Qs and $\tilde{Q}$s can be inferred by the following logic. Under translations of the M2-branes along the 34 directions, the Q, $\tilde{Q}$ degrees of freedom should remain massless, while other motions should separate the M2s and M2′s and give $Q2$, $\tilde{Q}$ a mass. In a standard way, one can identify motion in the transverse space to the M2-branes with (eigenvalues of) appropriate gauge-invariant composites of the A, B fields. First, we identify motion in the 34 directions with $A_1B_1 + A_2B_2$. Then, we expect component couplings localized at the defects depending on the other bilinears in A_i, B_i; these are of the form

$$\Delta S = \int dt \sum_i |(A_1B_1 - A_2B_2)Q_i|^2 + |(A_1B_2 - A_2B_1)Q_i|^2 + |(A_1B_2 + A_2B_1)Q_i|^2 \tag{6}$$

with similar couplings to $\tilde{Q}_i$. For the fermions, there are related couplings

$$\Delta S = \int dt \tilde{\chi}^\alpha \Gamma^M_{\alpha\beta} X^M \chi^\beta \tag{7}$$

with X^M corresponding to the real and imaginary parts of $A_1B_1 - A_2B_2$, $A_1B_2 \pm A_2B_1$ and α, β spinor indices running over the directions transverse to both the M2s and the M2′s.

The dimensions of the fields determined from their kinetic terms at weak coupling are $\Delta(Q) = \Delta(\tilde{Q}) = -\frac{1}{2}$, $\Delta(\chi) = \Delta(\tilde{\chi}) = 0$, and $\Delta(A) = \Delta(B) = \frac{1}{2}$. Gauge-invariant composite operators can be formed from these fields. We will shortly compute the dimensions of low-lying defect operators at strong 't Hooft coupling and large N using the gravity side of the correspondence, and then comment on the field theory description of these operators.

IV. COMPUTATION OF OPERATOR DIMENSIONS USING HOLOGRAPHY

A standard extension of the holographic dictionary relates the dimensions Δ of scalar operators localized at the lattice sites in our construction, to the masses of scalar KK modes arising in the M2′-brane world-volume action, via the formula

$$m^2_{\mathrm{localized}} = \Delta(\Delta - 1). \tag{8}$$

The fermionic spectrum may be inferred by supersymmetry.

We briefly discuss the calculation in the simplest case, $k = 1$. The fluctuations of the transverse scalars to a given M2′-brane (the $x^I = x^5, x^6, \ldots, x^{10}$ directions in space) are all related by an $SO(6)$ symmetry, so we may focus on a single scalar. The M2′-brane wraps an $\mathrm{AdS}_2 \times S^1$ geometry. The fluctuations can be expanded in Fourier modes on the S^1. If we let r denote the radial coordinate in AdS_2 and focus on static fluctuations, then

$$\delta x^I(r, \phi) = \sum_l \delta x^{I,l}(r) e^{il\phi} \tag{9}$$

with ϕ the angular coordinate on the wrapped S^1. The resulting Laplace equation for $\delta x^{I,l}(r)$ reveals that

$$m_l^2 = -\frac{1}{4} + \frac{l^2}{4}, \tag{10}$$

which corresponds to scalar operators of dimension

$$\Delta_l = \frac{1}{2} + \frac{l}{2}. \tag{11}$$

The lowest operator in the tower, with $l = 0$, gives a sextet of scalar primaries with $\Delta = 1/2$; its Fermi partner is a quartet of $\Delta = 1$ fermionic defect operators. We will see in the next subsection that this $\Delta = 1$ multiplet of fermionic operators plays an important role in obtaining semiholographic descriptions of marginal Fermi liquids.

There is also a second tower of operators, arising from fluctuations of the M2′-branes along the two transverse spatial directions to their world volume in AdS_4, i.e. the $x^{1,2}$ directions in (1). The tower arising from these fluctuations is distinguished from the tower above by global quantum numbers. For example, the fluctuations in the AdS directions transform under the $SO(2)$ rotation symmetry of the $x^{1,2}$ plane (which is broken to Z_4 by the lattice), and are singlets under the $SO(6)$ global symmetry discussed above, while the fluctuations in the $x^{5,\cdots 10}$ directions transform nontrivially under $SO(6)$ but are Z_4 invariant. While this second tower contains some fermionic operators of $\Delta = 1/2$ which would be dangerous if they coupled to the semiholographic fermions, such couplings can be forbidden by the $SO(6) \times Z_4$ symmetry in a ‘‘natural’’ way (in the sense of the renormalization group).

The spectrum for higher k may be most easily inferred from the $k = 1$ case by the following logic. We can obtain the higher k-brane configurations by Z_k orbifolds of appropriate lattice configurations on $AdS_4 \times S^7$. The orbifold action is free on the S^7 (the fixed point at $z_i = 0$ in C^4 is removed in the near-horizon limit), and therefore, all of the low-lying modes in the orbifold theory are Z_k-invariant modes in the original $k = 1$ theory. Correlation functions of the dual operators will enjoy large N inheritance from the parent $k = 1$ theory, similarly to the theories discussed in [21]. (New degrees of freedom that might be introduced by the orbifolding, analogous to twisted states in string theory, are very massive in the supergravity regime, due to the free orbifold action.) A simple analysis following this logic implies that the spectrum is the same for all $k > 1$; so, in particular, $\Delta = 1$ fermionic operators arise in these theories (and any lower Δ fermionic operators from the second tower can be rendered safe as above, by using global quantum numbers). A careful discussion of the KK spectra of these theories, and the matching with operators in the dual defect field theories, will appear in [22].

V. COUPLING TO SEMIHOLOGRAPHIC FERMIONS

The theory we have constructed above is locally critical in the large N limit. That is, because the probe M2′-branes wrap AdS_2 slices of the AdS_4 geometry, the excitations of the bulk fields localized on the probe branes can be classified by the quantum numbers of a locally critical quantum theory, and the correlation functions of the operators dual to localized bulk excitations (computed using the standard AdS/CFT dictionary) obey the constraints following from local criticality. These are precisely correlation functions of operators involving defect fields in the dual field theory.

Now, we couple the defect field theory we have constructed to semiholographic fermions, following [7]. Namely, if we call the full action of the lattice system above (including both the bulk gauge theory and the defect fields) S_{LC}, we consider the theory with

$$\begin{aligned} S_{\text{total}} = {}& S_{LC}(A, B, Q, \tilde{Q}) \\ &+ \sum_{J,J'} \int dt c_J^\dagger (i\delta_{J,J'}\partial_t + \mu\delta_{J,J'} + t_{J,J'}) c_{J'} \\ &+ g\sum_J \int dt (c_J^\dagger \mathcal{O}_J^F + \text{Hermitian conjugate}). \end{aligned} \tag{12}$$

In (12), we are coupling a normal theory of a weakly coupled Fermi surface (governing the excitations of the c fermion) to the strongly coupled locally critical sector, through the coupling constant g mixing c with (in any natural theory) the lowest-dimension fermionic operator $\mathcal{O}_F$ that has the right quantum numbers to couple to c.

Using large N factorization, it is then easy to show that the $g = 0$ Green's function of the c fermion

$$G_0(\mathbf{k}, \omega) \sim \frac{1}{\omega - v|\mathbf{k} - \mathbf{k_F}(\mathbf{k})|} \tag{13}$$

is modified to

$$G_g(\mathbf{k}, \omega) \sim \frac{1}{\omega - v|\mathbf{k} - \mathbf{k_F}(\mathbf{k})| - g^2\mathcal{G}(\mathbf{k}, \omega)}, \tag{14}$$

where

$$\mathcal{G}(\omega) = \int dt e^{i\omega t}\langle \mathcal{O}_J^F(t)\mathcal{O}_J^{F\dagger}(0)\rangle. \tag{15}$$

This two-point function is fixed by the scaling symmetry of the LC theory to be $\mathcal{G}(\omega) = c_\Delta \omega^{2\Delta-1}$ where Δ is the dimension of $\mathcal{O}^F$ [and, importantly, $\mathcal{G}(\omega) \sim c\omega \log(\omega)$ in the degenerate case $\Delta = 1$].

The correction term in the denominator of G_g will dominate the low-frequency behavior if $\Delta \leq 1$. Unitarity allows any $\Delta \geq \frac{1}{2}$ and this scaling dimension is a free parameter in the general approaches of [4,7]. The marginal Fermi liquid behavior of [2] appears in the case that the dimension of $\mathcal{O}^F$ is precisely 1. Therefore, the question is, are there natural circumstances in which the theory $S_{\text{LC}}(A, B, Q, \tilde{Q})$ has a leading fermionic operator of $\Delta = 1$ which can couple to c?

The theories we have constructed above naturally come with defect operators of $\Delta = 1$, as indicated by our calculation of the KK spectrum on the probe M2′-branes. It is interesting to consider where these come from in field theory language. The field theory has gauge-invariant operators of the form

$$\partial_t \tilde{Q}_1 A \chi_2,\ \partial_t \tilde{Q}_2 B \chi_1,\ \partial_t Q_1 B \tilde{\chi}_2,\ \partial_t Q_2 A \tilde{\chi}_1, \tag{16}$$

(as well as related quartets of operators of the schematic form $\tilde{\chi}_1 \psi_A \chi_2, \cdots$ and $\tilde{\chi}_1 A \partial_t Q_2, \cdots$). These have $\Delta = 1$ at weak coupling, and are good candidates for the duals of the probe defect operators we computed on the gravity side (arising in the tower of fluctuations of the M2′-branes along $x^{5,\cdots,10}$). Suppose that upon extrapolating to strong coupling (at large N), the weak-coupling dimensions of these operators are indeed protected, i.e. that the weak-coupling engineering dimensions of the fields correspond to their scaling dimensions under the locally critical scaling governing the defect sector in the probe limit. Then, assigning appropriate global quantum numbers to c, one can choose one of these as the lowest-dimension fermionic operator that c can couple to in the localized sector.

Returning to the dual gravitational description, we can see that the idea above does work at least in the probe approximation. By appropriate choice of global quantum numbers [under the Z_4 lattice symmetry and the (subgroup of) $SO(6)$ preserved by the brane configuration], one can guarantee that no lower Δ operators from the second tower of fluctuations in the previous subsection infect the leading-order c-fermion correlators (14) after coupling to the large N sector. We conclude that we can work directly in the probe limit and obtain a marginal Fermi liquid by identifying $\mathcal{O}^F$ with the lowest fermionic operator in the first tower of defect fields computed above. This has $\Delta = 1$, and as emphasized in the Introduction, this dimension is independent of momentum.

VI. BACKREACTION

Up until now, we have ignored the backreaction of the impurities on the itinerant fields, and therefore on each other. Thus, we have been studying the dynamics of a single impurity interacting strongly with itinerant fields. The gravity side exhibits the successes it does because the probe branes each wrap an AdS_2 region, and the symmetries of local quantum criticality are manifest, even including the highly nontrivial field theory interactions that are resummed by the tree-level gravity solution.

At scales of order the lattice spacing, the backreaction is a $1/N$ effect, but at lower energies it must become important. The scale symmetry of the itinerant fields, which the impurity system inherits, acts on the spatial coordinates. At energies of order $N^{-1/2}$ times the fundamental scale, the number of impurities in a scaling volume is of order N, and the effect of the impurities on the itinerant fields and on each other can no longer be neglected. Do these effects inevitably generate corrections to the action which destroy the locally critical behavior—is the behavior seen in the gravity regime a peculiarity of very strongly coupled large N theories, which would not extrapolate to any more realistic systems—or can it be robust in some circumstances? And, if locally critical behavior survives to the far IR, how do the operator dimensions there relate to those we have found at higher energy?

Staying in the limit of strong ’t Hooft coupling, gauge/gravity duality transforms this field theory question into the problem of finding the supergravity solution with backreaction. This can still be a challenging problem, but one can get insight from a simple energetics argument. We start with the M-theory brane configuration (1). We are looking for an IR geometry $\mathrm{AdS}_2 \times R^2 \times X$, which we will for convenience compactify to $\mathrm{AdS}_2 \times T^2 \times X$. We study this with the ansatz $X = S^7$, averaging the energy density of the impurity 2′-branes over the compact dimensions. Let A, T, and S be the respective radii of the three factors $\mathrm{AdS}_2 \times T^2 \times S^7$. The effective action dimensionally reduced to $1+1$ dimensions is of the form

$$\mathcal{S} = \int d^2x \left(-T^2 S^7 + A^2 T^2 S^5 - N_2' A^2 S - \frac{N_2^2 A^2 T^2}{S^7} \right). \tag{17}$$

We work in units where the M-theory scale is one, and ignore order one coefficients. The respective terms come from the curvatures of AdS_2 and S^7, the 2′-brane tensions, and the 7-form flux from the 2-branes. In other situations, it would be natural to Weyl transform to an effective potential, but this is not possible for AdS_2; instead, we directly extremize with respect to A in addition to T and S.

One finds that there is an extremum (with physically acceptable positive values for the moduli) such that

$$A \sim S \sim N_2^{1/6}, \qquad T \sim N_2'^{1/2}/N_2^{1/3}. \tag{18}$$

The radius S is parametrically the same as for the pure M2 system. The density of defects is $N_2'/T^2 = N_2^{2/3}$.

What is happening is that the lattice defects provide a force acting against the contraction of the two spatial dimensions, hence helping to drive the system towards a fixed point where the bulk modes are locally critical. In the probe approximation, the itinerant fields retained their relativistic scaling, and each independent impurity was invariant under a scale transformation leaving its position fixed. Here, there is a common locally critical scaling of the whole geometry.

This result is encouraging, but we should improve the ansatz. We have averaged the action of the 2′-branes over the S^7, but in fact they are wrapped on a circle and we should consider moduli corresponding to the contraction of this circle. Thus, we represent S^7 as a circle over CP^3, with radius F for the fiber circle and B for the base. The action becomes

$$\mathcal{S} = \int d^2x \bigg(-T^2 F B^6 + A^2 T^2 F B^4 - A^2 T^2 F^3 B^2 - N_2' A^2 F - \frac{N_2^2 A^2 T^2}{F B^6} \bigg). \tag{19}$$

One now finds that there is no physical extremum; the contraction of the fiber is not stabilized.

Nevertheless, there are brane systems that realize the solution (18). Consider a system with several kinds of impurity brane, with different orientations in the transverse spacetime. These will be dual to field theories with an action similar to that in Eqs. (6) and (7), but with couplings of the defect fields to the bulk fields given by suitable $SO(8)$ rotations of those appearing in (6) and (7). If the configuration of M2$'$-branes is sufficiently uniform and isotropic, the spherical ansatz will be a good approximation [23]. Such a configuration will necessarily break supersymmetry (for supersymmetric configurations, at least with $\mathcal{N} \geq 2$, there will always be an unstable fiber circle). It is also necessary to stabilize the angular configuration, for example, by taking a sufficiently symmetric configuration, and by keeping relatively nonsupersymmetric branes far enough apart to avoid tachyons. With the scaling (18), the typical transverse distance between the branes is larger than the M-theory scale, so one expects that the latter difficulty may be avoided. Although with a symmetric distribution there should be a solution of the equations of motion, it may be an unstable saddle point; with the lack of supersymmetry, there is no *a priori* guarantee against disallowed tachyons. Without having addressed all the possible instabilities, something that might benefit from further model building, we simply take from this construction the lesson already noted that lattice flavors contribute to producing local criticality on the gravity side.

As an aside, the absence of supersymmetric solutions could also be anticipated from another point of view. We are looking for solutions where the color branes remain localized in the 3–4 directions in which the impurity branes are extended. In Ref. [25], it is shown that these do not exist for brane intersections of spatial dimension 0 (as here) or 1. The interpretation was that the scalar fields Q on the intersection are spread out on their moduli space due to low-dimensional quantum effects, which implies that the brane intersection delocalizes and the AdS IR region disappears. In nonsupersymmetric systems, masses will generically be generated for these scalars. In the Appendix, we analyze an impurity system that has no such impurity scalars.

Orbifolding by Z_k does not affect the energetics, and so the discussion above can be applied with $N_2 \rightarrow Nk$, giving in M-theory units

$$A \sim S \sim N^{1/6}k^{1/6}, \quad R_{11} \sim N^{1/6}/k^{5/6}, \quad T \sim N_2'^{1/2}/N^{1/3}k^{1/3} \tag{20}$$

and in string units

$$A \sim S \sim N^{1/4}k^{1/4}, \quad g_s \sim N^{1/4}/k^{5/4}, \quad T \sim N_2'^{1/2}/N^{1/4}k^{1/4}. \tag{21}$$

The same applies if the orbifold action (2) is replaced by one acting only on two complex coordinates $z_{3,4}$, generating the brane configuration

	0	1	2	3	4	5	6	7	8	9	10
D2	x	x	x								
D6	x	x	x	x	x	x	x				
D2$'$	x	::	::	x	x						

(22)

with N color D2-branes and k D6-branes. This is a nice example, having a weakly coupled conformal point for $N_2 \ll N_6$ (as in Ref. [26]) and an AdS_4 dual description for $N_2 \gg N_6$ [27]. The radius S and coupling g_s are parametrically the same as for the pure D2-D6 system. In particular, one sees that the condition that the radius be large (in string units) is $N_2 \gg N_6$, and that there then is a weakly coupled IIA dual for $N_2 \ll N_6^5$ and an M-theory dual for $N_2 \gg N_6^5$. The density of defects is $N_2'/T^2 = N_2^{1/2}N_6^{1/2}$.

Even if we find a supergravity solution, there is a general argument that suggests that the local critical scaling cannot persist indefinitely into the IR. The scaling would imply a density of states

$$\rho(E) = A\delta(E) + B/E \tag{23}$$

per energy (and exponential in the volume). The first term is the widely noted zero-temperature entropy. If only this term is present, the Hamiltonian in the critical sector is zero: there is no dynamics (e.g. a dimension-1 operator would have a correlator $\delta'(t)$ rather than $1/t^2$). So, the B term is necessary, but its integral diverges, so local criticality must always break down at sufficiently low energy. In the gravity description, the density B comes from bulk states, and so is of order $1/N^2$. Thus, the breakdown takes place at exponentially small scales, which seems more promising than the $N^{-1/2}$ breakdown scale of the probe approximation.

Ref. [8] identified a specific breakdown mechanism, whereby the scaling exponents of the spatial directions were shifted (at all scales) from 0 to $O(1/N)$, thus rendering the density of states convergent. This is a rather special property of the system studied there. More generally, local criticality might persist until the finite density of states per volume forces it to break down.

VII. BACKREACTION AT WEAK COUPLING

It is encouraging that we have found possible stable systems with the desired IR properties, but the gravity methods are still only controlled in a peculiar limit, from the field theory perspective. Here, we discuss some related issues in direct analysis of the dual field theory. We start with the field theory corresponding to the brane system (22). This is an $\mathcal{N} = 8$ supersymmetric three-dimensional Yang-Mills theory, with defect hypermultiplets. In such theories, with a Maxwell action, the conformal symmetry that will emerge in the IR is far from manifest. A second approach, via the Chern-Simons theories of [18], has been the one we have followed in the bulk of the paper. The IR conformal behavior of the bulk theory is much clearer here, as the gauge fields do not appear with a dimensionful coupling, and the starting (bulk) Lagrangian has no dimensionful parameters. It is interesting to contrast our expectations for radiative corrections arising from the two approaches.

Starting from the three-dimensional $\mathcal{N} = 8$ Yang-Mills theory with hypermultiplet defects, and following the techniques of [28], it is easy to write a superspace Lagrangian. The problems with finding a four-dimensional $\mathcal{N} = 1$ superspace do not arise in this perspective; the additional complications of the ABJM brane construction [18] are not present, and one can straightforwardly T dualize to find an $\mathcal{N} = 1$ presentation. In terms of the brane construction with D2-branes wrapping $x^{1,2}$ and D2′-branes wrapping $x^{3,4}$, it is convenient to perform the T duality along the 7, 8, 9 directions and to treat those as the spatial directions of the $\mathcal{N} = 1$ field theory, with $x^{1,2}$ being internal dimensions. The bulk action, without explicitly writing out the Wess-Zumino-Witten (WZW) term, is

$$\begin{aligned} S = \frac{1}{g_3^2} \int dt d^2 x \,\mathrm{Tr}\bigg[\int d^2\theta \frac{1}{2} W^\alpha W_\alpha \\ + \epsilon^{ijk} \phi_i (\partial_j \phi_k - [\phi_j, \phi_k]/3\sqrt{2}) + \text{H.c.} \\ + 2 \int d^4\theta (\sqrt{2}\bar{\partial}^i + \bar{\phi}^i) e^{-V} (-\sqrt{2}\partial_i + \phi_i) e^V \\ + \bar{\partial}^i e^{-V} \partial_i e^V \bigg] + \text{WZW term}. \end{aligned} \tag{24}$$

Here, $\partial_1 = \partial_{x^1} + i\partial_{x^2}$, while $\partial_{2,3} \to 0$, and $(\phi^i)^\dagger = \bar{\phi}^i$. W_α is an $SU(N)$ gauge field strength superfield, while V is the vector superfield. In three-dimensional $\mathcal{N} = 4$ language, one should think of $\phi_{1,2}$ as the scalars in a hypermultiplet and ϕ_3 as the complex adjoint scalar in the vector multiplet. In Wess-Zumino gauge, the WZW term vanishes. The fields in the above action can be interpreted as follows: D2 gauge field Wilson lines along $x^{1,2}$ and D2 motions along $x^{3,4}$ are packaged in $\phi_{1,2}$; D2 motions along $x^{5,6}$ are contained in ϕ_3; and the vector multiplet V has $\theta\bar{\theta}$ components consisting of A_0 and $x^{7,8,9}$.

The hypermultiplets H, which transform in the fundamental of $SU(N)$, have localized actions

$$\begin{aligned} \sum_n \int dt \int d^4\theta (H_n^c e^{V_n} \bar{H}_n^c + \bar{H}_n e^{-V_n} H_n) \\ - \int d^2\theta H_n^c \phi_{3,n} H_n - \text{H.c.} \end{aligned} \tag{25}$$

The index n runs over the lattice sites, and n subscripts on a bulk field simply indicate that the field is to be evaluated at position of the nth site. This has the intuitively expected features; for instance, motions of the D2-branes along $x^{5,6,7,8,9}$, given the correspondence with fields above, can be seen to mass up the defect hypermultiplets.

Integrating out the auxiliary D field in the gauge multiplet generates interdefect interactions. For simplicity, we focus on the Abelian ($N = 1$) case; defect hypermultiplet scalars are denoted by η. Then, the couplings of the auxiliary field are

$$\begin{aligned} S_D = \frac{1}{g_3^2} \int dt d^2 x \bigg(\frac{1}{2} D^2 - 2\sqrt{2}(\phi 1 \bar{\partial}^1 D + \bar{\phi}^1 \partial_1 D) + \dot{\bar{\phi}}_1 \dot{\phi}_1 \bigg) \\ + \frac{1}{2} \sum_n D_n (|\eta_n^c|^2 - |\eta_n|^2). \end{aligned} \tag{26}$$

Integrating out D, the action becomes

$$S_D = \frac{1}{g_3^2} \int dt d^2 x (-2[\bar{\partial}^1 Z_1 + \partial_1 \bar{Z}^1]^2 + |\dot{Z}_1 - \dot{\zeta}|^2), \tag{27}$$

where we have defined

$$\zeta(z_1) = \frac{1}{8\pi\sqrt{2}} \sum_n \frac{(|\eta_n^c|^2 - |\eta_n|^2)}{z_1 - z_{1n}} \tag{28}$$

and

$$\phi_1 = Z_1 - \zeta. \tag{29}$$

The $|\dot{\zeta}|^2$ term in (27) exhibits cross couplings between the η hypermultiplet fields that would naively ruin local criticality. One would also get similar terms by integrating out A_0 and ϕ_3. The generation of interdefect interactions is not tied to supersymmetry, but these terms sum to a cross-coupling term in the Kähler potential for the defect hypermultiplets [29]. This makes it seem unlikely that the local criticality of the gravity regime can survive to finite N and coupling, where a field theory analysis should be reliable. However, it is important to remember that our starting point here has been the three-dimensional $\mathcal{N} = 8$ Yang-Mills theory, and this UV Lagrangian is valid only far from the IR fixed point which we know governs the physics on the N M2-branes (even at finite N).

To get an alternate perspective, we can also try to compute the interdefect corrections arising from coupling the defect hypermultiplets to the doubled Chern-Simons theory which captures the fixed-point physics. In fact, a simple toy model already illustrates the important difference between the Chern-Simons defect theories and the Yang-Mills defect theories. An Abelian Chern-Simons gauge field coupled to defect fermions χ_n would be governed by an action

$$\begin{aligned} S = \int dt d^2 z [A_0(\partial_z A_{\bar{z}} - \partial_{\bar{z}} A_z) - A_z(\partial_0 A_{\bar{z}} - \partial_{\bar{z}} A_0) \\ + A_{\bar{z}}(\partial_0 A_z - \partial_z A_0) + \sum_n \delta^{(2)}(z - z_n) \chi_n^\dagger A_0 \chi_n]. \end{aligned} \tag{30}$$

One can see directly that integrating out A_0 will *not* generate a dangerous interdefect coupling here, as it is a nonpropagating field. The A and B fields do propagate, but these couple to the defect fields only quadratically as in Eqs. (6) and (7) and so do not generate tree-level corrections.

A full field-theoretic analysis of the radiative corrections to the ABJM theory coupled to hypermultiplet defects is beyond the scope of our work. It will be interesting to see to what extent the absence of induced interdefect couplings applies in the full model; the simple computation above suggests that at least the most obvious dangerous cross couplings visible from the Yang-Mills perspective do not characterize the physics of the IR fixed-point theory coupled to hypermultiplet defects. Especially in the cases $k = 1, 2$, where the full model enjoys enhanced supersymmetry, nonrenormalization theorems strongly constrain the possible generation of four-fermion cross-coupling terms (see, for instance, [30]); constraints on higher multifermion terms are less obvious. It would be most interesting to push this analysis further, and construct systems of defect fermions interacting with itinerant fields where local criticality can be seen robustly directly from field-theoretic arguments.

ACKNOWLEDGMENTS

We would like to thank S. Kivelson, M. Mulligan, S. Sachdev, and C. Varma for interesting discussions. We thank S. Hartnoll and J. McGreevy for a correction to the volume dependence of (20). K. J., S. K., A. K. and E. S. acknowledge the hospitality of the Aspen Center for Physics while this work was in progress. S. K. also acknowledges the hospitality of the organizers of the 5th Asian Winter School at Jeju Island, and thanks the participants for asking many interesting questions about related subjects. This research was supported in part by the National Science Foundation under Grants No. PHY-02-44728, No. PHY05-51164, and No. PHY07-57035, and by the DOE under Contracts No. DE-AC03-76SF00515 and No. DE-FG02-96ER40956. K. J. was supported by NSERC, Canada.

APPENDIX: THE 3.5 SYSTEM

To begin, let us consider a variant of the construction of [12], who studied the brane configuration

	0	1	2	3	4	5	6	7	8	9	10
D3	x	x	x	x							
D5($\bar{5}$)	x	::	::	::	x	x	x	x	x	x	

. (A1)

As before, an x indicates a direction in which the given branes are extended, and a :: indicates a direction in which they are in a lattice configuration. The 3–5 intersections are $0 + 1$ dimensional, representing defects in the dual gauge theory. For this system, with 8 ND directions, only fermions live on the intersections, which is very natural for the intended applications.

In the limit that the 5-branes are probes, the D3-branes generate an $\mathrm{AdS}_5 \times S^5$ spacetime, with each 5-brane wrapped on an $\mathrm{AdS}_2 \times S^4$ subspace. However, the spatial directions contract in the IR of the AdS_5 geometry, so the 5-brane density diverges there and their backreaction cannot be neglected. At large N, the backreaction becomes a large effect at energies which are parametrically small compared to the lattice scale (as noted in [12]) [31].

We are looking for an IR geometry $\mathrm{AdS}_2 \times R^3 \times X$, which we will for convenience compactify to $\mathrm{AdS}_2 \times T^3 \times X$. We study this with the ansatz $X = S^5$, averaging the energy density of the 5-branes over the compact dimensions. Let A, T, and S be the respective radii of the three factors $\mathrm{AdS}_2 \times T^3 \times S^5$. The effective action dimensionally reduced to $1 + 1$ dimensions is of the form

$$\mathcal{S} = \int d^2x\left(-\frac{T^3S^5}{g_s^2} + \frac{A^2T^3S^3}{g_s^2} - \frac{N_5A^2S^4}{g_s} - \frac{N_3^2A^2T^3}{S^5}\right). \tag{A2}$$

We work in units where the string length is one, and ignore order one coefficients. The respective terms come from the curvatures of AdS_2 and S^5, the 5-brane tensions, and the RR 5-form flux, and do not distinguish between pure D5-branes and a mix of D5s and $\overline{\mathrm{D}}$5s. In other situations, it would be natural to Weyl transform to an effective potential, but this is not possible for AdS_2; instead, we directly extremize with respect to A. One readily verifies that the action has no stationary points for finite values of the moduli A,T,S,g_s. This analysis precludes an $\mathrm{AdS}_2 \times T^3 \times S^5$ solution in the case that the 5-branes are oriented in many directions on the S^5, averaging to a symmetric source.

One way of understanding the absence of an AdS_2 solution in the infrared in this case is that the $\mathcal{N} = 4$ super Yang-Mills sector has a line of fixed points, parameterized by the string coupling g_s. The additional lattice branes source this mode and altogether there are not enough independent forces to fix g_s, T, S, and A. If we include electric and magnetic flavors, these can fix g_s. Having done this, an AdS_2 solution fixing the other moduli does arise.

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