Entropy, contact interaction with horizon, and dark energy

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We present some arguments suggesting that the mismatch between Bekenstein-Hawking entropy and the entropy of entanglement for vector fields is due to the same gauge configurations that saturate the contact term in topological susceptibility in QCD. In both cases, the extra term with the "wrong sign" is due to distinct topological sectors in gauge theories. This extra term has a nondispersive nature, cannot be restored from the conventional spectral function through dispersion relations, and cannot be associated with any physical propagating degrees of freedom. We make a few comments on some of the profound consequences of our findings. In particular, we speculate that the source of the observed dark energy may also be related to the same type of gauge configurations that are responsible for the mismatch between black hole entropy and the entropy of entanglement in the presence of a causal horizon.

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I. INTRODUCTION

The relation between black hole entropy and the entropy of entanglement for matter fields has been a subject of intense discussion for the last couple of years, see the reviews in [1–4] and references therein. There are many subtleties in relating these two things. The present work is concentrated just on one specific subtlety first discussed in [5]. Namely, it has been claimed [5] that for spin zero and one-half fields, the one loop correction to the black hole entropy is equal to the entropy of entanglement, while for the spin one field black hole entropy has an extra term describing the contact interaction with the horizon. Precisely, this contact interaction with the horizon is the main topic of the present work. Before we elaborate on this subject we want to make one preliminary remark regarding the term "black hole entropy." As it has been argued in a number of papers, see e.g.[6–8] and also review papers [1–4], the notion of black hole entropy should apply not just to black holes but to any causal horizon ("black hole entropy without black holes"). We adopt this viewpoint, and in fact we shall not discuss black hole physics in this paper at all. Rather, the main application of our studies will be the cosmology of the expanding universe and its causal horizon. For short, we shall use term "entropy" throughout the paper.

The unique features of the contact term related to the vector gauge field in the entropy computations can be summarized as follows [5] (see also the follow-up paper in [9]):

(a) The contact term being a total derivative can be represented as a surface term determined by the behavior at the horizon;
(b) This term makes a *negative* contribution to black hole entropy.
(c) Therefore, it cannot be identified with the entropy of entanglement, which is intrinsically positive quantity.
(d) This contribution does not vanish even in two dimensions when the entropy of entanglement is identically zero as no physical propagating degrees of freedom are present in the system.
(e) This contribution is gauge invariant in two dimensions and gauge dependent in the four-dimensional (4D) case [9].
(f) The technical reason for this phenomenon is as follows: One cannot use the physical Coulomb gauge (when only physical degrees of freedom are present in the system) as it breaks down at the origin, where A_θ is ill-defined. Therefore, an alternative description in terms of a covariant gauge (when unphysical degrees of freedom inevitably appear in the system) should be used instead.
(g) In this covariant description the entropy is obtained by varying the path integral with respect to the deficit angle of the cone as explained in [5]. Such a procedure can (in principle) lead to a negative value for the entropy. In fact, it does come out negative [5].

The main goal of the present work is to argue that the presence of this "weird" term is intimately related to the well-known property of gauge theories where the summation over all topological sectors must be performed for the path integral to be properly defined. We explain how all the features denoted as (a–g) in the list above can be naturally understood within our framework when the sum over topological sectors is properly taken into account. In Minkowski space the corresponding procedure is known to produce a nondispersive contact term with the "wrong sign," which plays a crucial role in the resolution of the so-called $U(1)_A$ problem in QCD. Precisely, this nondispersive contact term eventually becomes the weird term with the properties (a–g) when we go from conventional Minkowski space into a curved/time-dependent background with a causal horizon.

Our consideration in this paper will be based on an analysis of the local characteristics (such as topological susceptibility, free energy density, etc.) computed deep inside the horizon region. It is very different from the computation of a global characteristic such as the total entropy of a black hole when the closest vicinity of the horizon (just outside of it) plays the crucial role in the computations. Nevertheless, we remain sensitive to the existence of the horizon because our analysis is based on consideration of some specific topologically protected quantities. Essentially, by analyzing the very unusual features listed above, i.e. (a–g), we learn some important lessons regarding the behavior of the ground state resulting from the mere existence of a causal horizon in the presence of the gauge degrees of freedom in the system.

Thpaper is organized as follows. In Sec. II we present our arguments for the two-dimensional (2D) case when all computations can be explicitly performed. We generalize our arguments for the four-dimensional case in Sec. III. We argue that this term is indeed gauge dependent in four dimensions in the Abelian case as the explicit computations of Ref. [9] suggest. However, we shall argue that this term becomes gauge independent in the non-Abelian case. We make a few comments on some of the profound consequences of our findings in Sec. IV, where we speculate that the source of the observed dark energy might be related to the same gauge configurations that are responsible for the mismatch between black hole entropy and the entropy of entanglement.

II. TOPOLOGICAL SECTORS, CONTACT TERM WITH THE WRONG SIGN, AND ALL THAT FOR 2D QED IN RINDLER SPACE

First of all, we shall demonstrate below the presence of a nonconventional contribution into the energy with a wrong sign in Minkowski space. This contribution is gauge invariant, and it exists even in pure photodynamics when no propagating degrees of freedom are present in the system. It cannot be removed by any means (such as redefinition of the energy) as it is a real physical contribution. In particular, the anomalous Ward identities (which emerge when the massless fermions are added into the system) cannot be satisfied without this term. We shall argue that this term can be treated as a contact term, and in fact is related to the existence of different topological sectors in this (naively trivial) two-dimensional photodynamics. In other words, the presence of different topological sectors in the system, which we call the ''degeneracy'' for short,[1] is the source of this contact term, which is not related to any physical propagating degrees of freedom.

[1]Not to be confused with the conventional term ''degeneracy'' when two or more physically distinct states are present in the system. In the context of this paper the ''degeneracy'' implies the existence of winding states $|n\rangle$ constructed as follows: $\mathcal{T}|n\rangle = |n+1\rangle$. In this formula the operator $\mathcal{T}$ is the large gauge transformation operator which commutes with the Hamiltonian $[\mathcal{T}, H] = 0$. The physical vacuum state is *unique* and constructed as a superposition of $|n\rangle$ states. In the path integral approach, the presence of n different sectors in the system is reflected by the sum over $k \in \mathbb{Z}$ in Eqs. (10)–(12).

In the next step, we discuss the same system in the presence of the horizon in the Rindler space. We shall argue that the contact term (which emerges as a result of topological features of gauge theory) demonstrates the weird and strange properties listed above in the presence of the horizon.

In what follows it is convenient to study the topological susceptibility χ (rather than free energy itself) which is related to the θ dependent portion of the free energy density[2] as follows

$$\chi(\beta, \theta = 0) = -\frac{\partial^2 F_{\rm vac}(\beta, \theta)}{\partial \theta^2}\bigg|_{\theta=0}, \tag{1}$$

where θ is the conventional θ parameter which enters the Lagrangian along with topological density operator, see a precise definition below. We always assume that $\theta = 0$, however $\chi(\theta = 0) \neq 0$ does not vanish, and in fact is the main ingredient of the resolution of the $U(1)_A$ problem in QCD [10–12], see also [13–15]. Free energy itself $F_{\rm vac}(\theta)$ can be always restored from χ as dependence on θ is known to be $F_{\rm vac} \sim \cos\theta$. As we show below, the topological susceptibility χ (and therefore $F_{\rm vac}$), being the local characteristics of the system, nevertheless are quite sensitive to the mere existence of the horizon, even when computed far away from it. As we shall see this sensitivity is related to the degeneracy of the system and topological nature of χ.

A. Topological susceptibility and contact term

The simplest (and physically attractive) choice is the Coulomb gauge when no physical propagating degrees of freedom are present in the system, and therefore the dynamics must be trivial. It is well known why this naive argument fails: the vacuum in this system is degenerate, and one should consider an infinite superposition of the winding states $|n\rangle$ as originally discussed in [16]. Such a construction in the Coulomb gauge restores the cluster and other important properties of quantum field theory. The vacuum in this gauge is characterized by long-range forces (if charged physical fermions are introduced into the system). This long-range force prevents distant regions from acting independently. We believe that precisely this feature leads to the difficulties mentioned in [5] in the computations of the entropy in the physical Coulomb gauge in two dimensions, where a covariant gauge has been used instead.

As our goal is to make a connection with the computations of Ref. [5], we shall not elaborate on the Coulomb gauge in the present paper any further, but rather consider a covariant gauge to study this system. In the covariant Lorentz gauge, there are no long-range forces. Instead, new (unphysical) degrees of freedom emerge in the system, see precise definition below.

[2]in case of infinite manifold (rather than finite size $\beta = T^{-1}$) the free energy from relation (1) becomes the conventional vacuum energy as employed in study of the $U(1)_A$ problem in QCD in [10–12].

We want to study the topological susceptibility χ in the Lorentz gauge defined as follows,[3]

$$\chi \equiv \frac{e^2}{4\pi^2} \lim_{k\to 0} \int d^2x e^{ikx} \langle TE(x)E(0)\rangle, \tag{2}$$

where $Q = \frac{e}{2\pi} E$ is the topological charge density and

$$\int d^2x Q(x) = \frac{e}{2\pi} \int d^2x E(x) = k \tag{3}$$

is the integer valued topological charge in the 2D $U(1)$ gauge theory, $E(x) = \partial_1 A_2 - \partial_2 A_1$ is the field strength. The expression for the topological susceptibility in the 2D Schwinger QED model is known exactly as [17]

$$\chi_{\rm QED} = \frac{e^2}{4\pi^2} \int d^2x \left[\delta^2(x) - \frac{e^2}{2\pi^2} K_0(\mu|x|) \right], \tag{4}$$

where $\mu^2 = e^2/\pi$ is the mass of the single physical state in this model, and $K_0(\mu|x|)$ is the modified Bessel function of order 0, which is the Green's function of this massive particle. The expression for χ for pure photodynamics is given by (4) with coupling $e = 0$ in the brackets,[4] which corresponds to the decoupling from the matter field ψ, i.e.

$$\chi_{E\&M} = \frac{e^2}{4\pi^2} \int d^2x [\delta^2(x)]. \tag{5}$$

The crucial observation here is as follows: any physical state contributes to χ with negative sign

$$\chi_{\rm dispersive} \sim \lim_{k\to 0} \sum_n \frac{\langle 0| \frac{e}{2\pi} E|n\rangle\langle n| \frac{e}{2\pi} E|0\rangle}{-k^2 - m_n^2} < 0. \tag{6}$$

In particular, the term proportional $-K_0(\mu|x|)$ with the negative sign in Eq. (4) is resulted from the only physical field of mass μ. However, there is also a contact term $\int d^2x[\delta^2(x)]$ in Eqs. (4) and (5) which contributes to the topological susceptibility χ with the *opposite sign*, and which cannot be identified according to (6) with any contribution from any physical asymptotic state.

This term has a fundamentally different, nondispersive nature. In fact, it is ultimately related to different topological sectors of the theory and the degeneracy of the ground state [18] as we will shortly review below. Without this contribution it would be impossible to satisfy the Ward identity (WI) because the physical propagating degrees of freedom can only contribute with sign $(-)$ to the correlation function as Eq. (6) suggests, while WI requires $\chi = 0$ in the chiral limit $m = 0$. One can explicitly check that WI is indeed automatically satisfied[5] only as a result of exact cancellation between the conventional dispersive term with sign $(-)$ and the nondispersive term (5) with sign $(+)$,

$$\chi = \frac{e^2}{4\pi^2} \int d^2x \left[\delta^2(x) - \frac{e^2}{2\pi^2} K_0(\mu|x|) \right] = \frac{e^2}{4\pi^2} \left[1 - \frac{e^2}{\pi} \frac{1}{\mu^2} \right] = \frac{e^2}{4\pi^2} [1 - 1] = 0. \tag{7}$$

B. The origin of the contact term—the sum over topological sectors

The goal here is to demonstrate that the contact term in exact formulae (4) and (5) is a result of the sum over different topological sectors in the 2D pure $U(1)$ gauge theory as we now show. We follow [17] and introduce the classical "instanton potential" in order to describe the different topological sectors of the theory, which are classified by integer number k defined in Eq. (3). The corresponding configurations in the Lorentz gauge on the two-dimensional Euclidean torus with total area V can be described as follows [17]:

$$A_\mu^{(k)} = -\frac{\pi k}{eV} \epsilon_{\mu\nu} x^\nu, \qquad eE^{(k)} = \frac{2\pi k}{V}, \tag{8}$$

such that the action of this classical configuration is

$$\frac{1}{2} \int d^2x E^2 = \frac{2\pi^2 k^2}{e^2 V}. \tag{9}$$

This configuration corresponds to the topological charge k as defined by (3). The next step is to compute the topological susceptibility for the theory defined by the following partition function:

$$\mathcal{Z} = \sum_{k\in\mathbb{Z}} \int \mathcal{D}A e^{-(1/2)\int d^2x E^2}. \tag{10}$$

All integrals in this partition function are Gaussian and can be easily evaluated using the technique developed in [17]. The result is determined essentially by the classical configurations (8) and (9) as real propagating degrees of freedom are not present in the system of pure $U(1)$ gauge field theory in two dimensions. We are interested in computing χ defined by Eq. (2). In the path integral approach, it can be represented as follows:

$$\chi = \frac{e^2}{4\pi^2 \mathcal{Z}} \sum_{k\in\mathbb{Z}} \int \mathcal{D}A \int d^2x E(x)E(0) e^{-(1/2)\int d^2x E^2}. \tag{11}$$

This Gaussian integral can be easily evaluated[6] and the result can be represented as follows [18]:

[3]Here we use Euclidean metric where path integral computations (4) have been performed.

[4]Factor $\frac{e^2}{4\pi^2}$ in front of (4) does not vanish in this limit as it is due to our definition (2) rather than the result of dynamics.

[5]When $m \neq 0$, the WI takes the form $\chi \sim m|\langle\bar\psi\psi\rangle|$. It is also automatically satisfied because $\mu^2 = \frac{e^2}{\pi} + \mathcal{O}(m)$, and cancellation in Eq. (7) is not exact resulting in behavior $\chi \sim m$ in complete accord with WI [19].

[6]One can check that the contribution resulting from the quantum fluctuations about the classical background (8) does not change the result (13). Indeed, the corresponding extra "quantum" contribution $\frac{e^2}{4\pi^2} \cdot \int d^2x[\delta^2(x) - \frac{1}{V}] = 0$ vanishes as expected.

$$\chi = \frac{e^2}{4\pi^2} \cdot V \cdot \frac{\sum_{k\in\mathbb{Z}} \frac{4\pi^2 k^2}{e^2 V^2} \exp(-\frac{2\pi^2 k^2}{e^2 V})}{\sum_{k\in\mathbb{Z}} \exp(-\frac{2\pi^2 k^2}{e^2 V})}. \tag{12}$$

In the large volume limit $V \to \infty$, one can evaluate the sums entering (12) by replacing $\sum_{k\in\mathbb{Z}} \to \int dk$ such that the leading term in Eq. (12) takes the form

$$\chi = \frac{e^2}{4\pi^2} \cdot V \cdot \frac{4\pi^2}{e^2 V^2} \cdot \frac{e^2 V}{4\pi^2} = \frac{e^2}{4\pi^2}. \tag{13}$$

A few comments are in order. First, the obtained expression for the topological susceptibility (13) is finite in the limit $V \to \infty$, coincides with the contact term from exact computations (4) and (5) performed for 2D Schwinger model in Ref. [17], and has the wrong sign in comparison with any physical contributions (6). Second, the topological sectors with very large $k \sim \sqrt{e^2 V}$ saturate the series (12). As one can see from the computations presented above, the final result (13) is sensitive to the boundaries, infrared regularization, and many other aspects that are normally ignored when a theory from the very beginning is formulated in infinite space with the conventional assumption about trivial behavior at infinity. Last, but not least, the contribution (13) does not vanish in a trivial model when no any propagating degrees of freedom are present in the system! This term is entirely determined by the behavior at the boundary, which is conveniently represented by the classical topological configurations (8) describing different topological sectors (3), and accounts for the degeneracy of the ground state.[7] We know that this term must be present in the theory when the dynamical quarks are introduced into the system. Indeed, it plays a crucial role in this case as it saturates the WI as formula (7) shows.

C. The ghost as a tool to describe the contact term

The goal here is to precisely describe the same contact term (5) and (13) without the explicit sum over different topological sectors, but rather, using the auxiliary ghost fields as it was originally done in Ref. [16] (using the so-called the Kogut-Susskind dipole). This auxiliary ghost field effectively accounts for the degeneracy of the ground state as discussed above. The computations in both Refs. [5,16] are performed precisely in terms of the same auxiliary scalar field defined as follows:

$$A_\mu = \epsilon_{\mu\nu}\partial^\nu \Phi. \tag{14}$$

This formal connection allows us to make a link between expressions (5) and (13) for the contact term with the wrong sign computed in our framework in terms of the auxiliary scalar field as described below and the entropy computations performed in Ref. [5] featuring the "weird properties" (e–g) as listed in the Introduction.

[7] See footnote [1] for clarification of the term "degeneracy." In the given context the degeneracy implies the sum over $k \in \mathbb{Z}$ in Eqs. (10)–(12).

Our starting point is the effective Lagrangian describing the same two-dimensional gauge system. However, now the theory is formulated in covariant Lorentz gauge in terms of the scalar fields [16]. The crucial element accounting for different topological sectors of the underlying theory, and corresponding degeneracy of the ground state, does not go away in this description. Rather, this information is now coded in terms of unphysical ghost scalar field which provides the required wrong sign for the contact term (5) and (13).

A precise construction is as follows. The effective Lagrangian describing the low-energy physics (in Minkowski metric) is given by [16]

$$\mathcal{L} = \tfrac{1}{2}\partial_\mu\hat{\phi}\partial^\mu\hat{\phi} + \tfrac{1}{2}\partial_\mu\phi_2\partial^\mu\phi_2 - \tfrac{1}{2}\partial_\mu\phi_1\partial^\mu\phi_1 - \tfrac{1}{2}\mu^2\hat{\phi}^2 + m|\langle\bar{q}q\rangle|\cos 2\sqrt{\pi}[\hat{\phi} + \phi_2 - \phi_1]. \tag{15}$$

The fields appearing in this Lagrangian are

$$\phi_1 = \text{ the ghost,} \qquad \phi_2 = \text{ its partner,} \tag{16}$$

while $\hat{\phi}$ is the only physical massive degree of freedom. It is important to realize that the ghost field ϕ_1 is always paired up with ϕ_2 in each and every gauge invariant matrix element, as explained in [16]. The condition that enforces this statement is the Gupta-Bleuler-like condition on the physical Hilbert space $\mathcal{H}_{\rm phys}$, which reads like

$$(\phi_2 - \phi_1)^{(+)}|\mathcal{H}_{\rm phys}\rangle = 0, \tag{17}$$

where the $(+)$ stands for the positive frequency Fourier components of the quantized fields. One can easily understand the origin of a wrong sign for the kinetic term for ϕ_1 field. It occurs as a result of the $\Box^2$ operator when the Maxwell term $E^2 \sim \Box^2$ is expressed in terms of the scalar field (14). As usual, the presence of fourth-order operator is a signal that the ghost is present in the system. Indeed, the relevant operator $[\Box\Box + \mu^2\Box]$ which emerges for this system can be represented as the combination of the ghost ϕ_1 and a massive physical $\hat{\phi}$ using the standard trick by writing the inverse operator as follows:

$$\frac{1}{\Box\Box + \mu^2\Box} = \frac{1}{\mu^2}\left(\frac{1}{-\Box - \mu^2} - \frac{1}{-\Box}\right). \tag{18}$$

This is a simplified explanation of how the sign $(-)$ emerges in the Lagrangian (15) describing auxiliary ϕ_1 field, see Ref. [16] for details.

The contact term in this framework is precisely represented by the ghost contribution[18,19] replacing the standard procedure of summation over different topological sectors as discussed above II B. Indeed, the topological density $Q = \frac{e}{2\pi}E$ in 2D QED is given by $\frac{e}{2\pi}E = (\frac{e}{2\pi})\frac{\sqrt{\pi}}{e} \times (\Box\hat{\phi} - \Box\phi_1)$ [16]. The relevant correlation function in coordinate space which enters the expression for the topological susceptibility (2) can be explicitly computed using the ghost as follows:

$$\chi(x) \equiv \left\langle T \frac{e}{2\pi} E(x), \frac{e}{2\pi} E(0) \right\rangle = \left(\frac{e}{2\pi}\right)^2 \frac{\pi}{e^2} \int \frac{d^2 p}{(2\pi)^2} p^4 e^{-ipx} \left[-\frac{1}{p^2+\mu^2} + \frac{1}{p^2} \right] = \left(\frac{e}{2\pi}\right)^2 \left[\delta^2(x) - \frac{e^2}{2\pi^2} K_0(\mu|x|) \right] \quad (19)$$

where we used the known expressions for the Green's functions (the physical massive field $\hat{\phi}$ as well as the ghost ϕ_1 field) determined by Lagrangian (15) and switched back to Euclidean metric for comparison with previous results from Secs. II A and II B.

The obtained expression precisely reproduces the exact result (4) as claimed. In the limit $e \to 0$ when matter fields decouple from gauge degrees of freedom we reproduce the contact term (5) and (13) which was previously derived as a result of the summation over different topological sectors of the theory. The nondispersive contribution manifests itself in this description in terms of an unphysical ghost scalar field which provides the required wrong sign for the contact term.

At the same time, this unphysical ghost scalar field does not violate unitarity or any other important properties of the theory as consequence of Gupta-Bleuler-like condition on the physical Hilbert space (17). Indeed, while the ghost's number density operator, N may look dangerous due to the sign $(-)$ in the commutation relations

$$\mathrm{N} = \sum_k (b_k^\dagger b_k - a_k^\dagger a_k) \quad [b_k, b_{k'}^\dagger] = \delta_{kk'}, \quad [a_k, a_{k'}^\dagger] = -\delta_{kk'} \quad (20)$$

one can in fact check that the expectation value for any physical state vanishes as a result of the subsidiary condition [16,18]:

$$\langle \mathcal{H}_{\mathrm{phys}} | \mathrm{N} | \mathcal{H}_{\mathrm{phys}} \rangle = 0, (a_k - b_k) | \mathcal{H}_{\mathrm{phys}} \rangle = 0. \quad (21)$$

This vanishing result (21) obviously implies that no entropy may be produced in Minkowski space. In other words, the fluctuations of unphysical fields described by operator (20) do not lead to any physical consequences (except for merely existence of the contact term (5) as already discussed).

We shall see in next subsection how this simple picture drastically changes when we consider the very same system but in the presence of the horizon. We shall argue that the number density N of "fictitious particles" with wrong commutation relations starts to fluctuate in the presence of the horizon, in contrast with Eq. (21). Therefore, we formulate a conjecture that precisely these fluctuations are responsible for a term with a wrong sign in entropy computations [5,9]. The corresponding contribution, as we already mentioned, is not related to any physical propagating degrees of freedom but rather, is due to the presence of topological sectors in gauge theories (and the degeneracy of the ground state as its consequence, see footnote [1] for clarification of the terminology) which eventually lead to a nondispersive contribution in topological susceptibility. To simplify things in what follows we consider a simple Rindler space when the Bogolubov's coefficients are exactly known. However, we argue that a generic case (when horizon is present in the system) leads to very similar conclusion.

D. Rindler space

The total entropy with weird properties listed in the Introduction was computed a while ago [5], and there is no reason to review these results in the present paper. These original results have been reproduced in [9] by using another technique. Furthermore, in the same paper [9] it has been demonstrated that in the two-dimensional case the final result is gauge invariant, and therefore, it obviously represents a physically observable quantity. As we mentioned earlier, we are not interested in computing global characteristics such as total entropy. Rather, we are interested in computing some local properties, such as topological susceptibility, or the $\theta-$ dependent portion of the energy density (1) in the presence of the horizon. However, we shall argue below, the source of weird features in both cases is the same, and, in fact, related to the fundamental properties of gauge theories as discussed in Sec. II B.

As we explained above, the presence of different topological sectors in gauge theory (and the degeneracy of the ground state as its consequence) leads to the contact term (5) even when no physical propagating degrees of freedom are present in the system. In the physical Coulomb gauge this term manifests itself as the presence of a long-range force which prevents distant regions from acting independently. The same feature but in covariant Lorentz gauge is expressed in terms of new (unphysical) degrees of freedom (16) which emerge in the system and effectively reproduce the contact term as explicit computations show (19). While these unphysical degrees of freedom do fluctuate, these fluctuations do not lead to any physical observable expectation values in Minkowski space (21) as a result of the cancellation between two unphysical fields similar to the conventional Gupta-Bleuler condition in QED when two unphysical photon's polarizations cancel each other. We want to see how this conclusion changes when a horizon is present in the system.

One can repeat the construction in Sec. II C to describe (unphysical) degrees of freedom but in Rindler space [18]. A Rindler observer in a (R,L) wedge will measure the number density of unphysical states using density operator $\mathrm{N}^{(R,L)}$ which is given by

$$\mathrm{N}^{(R,L)} = \sum_k (b_k^{(R,L)\dagger} b_k^{(R,L)} - a_k^{(R,L)\dagger} a_k^{(R,L)}). \quad (22)$$

The subsidiary condition (17) defines the physical subspace for accelerating Rindler observer

$$(a_k^{(R,L)} - b_k^{(R,L)})|\mathcal{H}_{\rm phys}^{(R,L)}\rangle = 0, \tag{23}$$

such that the exact cancellation between ϕ_1 and ϕ_2 fields holds for any physical state defined by Eq. (23), i.e.

$$\langle\mathcal{H}_{\rm phys}^{(R,L)}|{\rm N}^{(R,L)}|\mathcal{H}_{\rm phys}^{(R,L)}\rangle = 0, \tag{24}$$

as it should. However, if the system is prepared as the Minkowski vacuum state $|0\rangle$, then a Rindler observer using the same operator for ${\rm N}^{(R,L)}$ (22) will observe the following number density in mode k,

$$\langle 0|{\rm N}^{(R,L)}|0\rangle = \langle 0|(b_k^{(R,L)\dagger} b_k^{(R,L)} - a_k^{(R,L)\dagger} a_k^{(R,L)})|0\rangle = \frac{2e^{-\pi\omega/a}}{(e^{\pi\omega/a} - e^{-\pi\omega/a})} = \frac{2}{(e^{2\pi\omega/a} - 1)}, \tag{25}$$

where we used known Bogolubov's coefficients mixing the positive and negative frequency modes for operators $b_k^{(R,L)}$, $a_k^{(R,L)}$ describing unphysical fluctuations [18].

One can explicitly see why the cancellation (21) of unphysical degrees of freedom in Minkowski space fail to hold for the accelerating Rindler observer (25). The technical reason for this effect to occur is the property of Bogolubov's coefficients which mix the positive and negative frequencies modes. The corresponding mixture cannot be avoided because the projections to positive-frequency modes with respect to Minkowski time t and positive-frequency modes with respect to the Rindler observer's proper time η are not equivalent. The exact cancellation of unphysical degrees of freedom which is maintained in Minkowski space cannot hold in the Rindler space because it would be not possible to separate positive frequency modes from negative frequency ones in the entire space-time, in contrast with what happens in Minkowski space where the vector $\partial/\partial t$ is a constant Killing vector, orthogonal to the $t =$ const hypersurface. The Minkowski separation is maintained throughout the whole space as a consequence of Poincaré invariance. It is in drastic contrast to the accelerating Rindler space [18].

The nature of the effect is the same as the conventional Unruh effect[20] when the Minkowski vacuum $|0\rangle$ is restricted to the Rindler wedge with no access to the entire space-time. An appropriate description in this case, as is known, should be formulated (for the R observer) in terms of the density matrix by "tracing out" over the degrees of freedom associated with the L-region. In this case the Minkowski vacuum $|0\rangle$ is obviously not a pure state but a mixed state with a horizon separating two wedges, which is the source of the entropy. In contrast with Unruh effect[20], however, one cannot speak about real radiation of real particles as the ghost ϕ_1 and its partner ϕ_2 are not the asymptotic states and the corresponding positive frequency Wightman Green function describing the dynamics of these fields vanishes [18]. In different words, these auxiliary fields contribute to the nondispersive portion of the correlation function in Eqs. (4), (5), and (7) but not to conventional dispersive part which is unambiguously determined by the absorptive function as conventional dispersion relation dictates.

Few more comments on (25) are in order. The effect is obviously sensitive to the presence of the horizon, and, therefore is infrared (IR) in nature. The IR nature of the effect was anticipated from the very beginning as formulation of the problem in terms of auxiliary fields (16) is simply a convenient way to deal with different topological sectors of the gauge theory in covariant gauge (and the degeneracy of the ground state as their consequence) instead of dealing with the long-range forces in the unitary Coulomb gauge as discussed in Secs. II B and II C. Also, the contribution of higher frequency modes are exponentially suppressed $\sim\exp(-\omega/a)$ as expected. The interpretation of Eq. (25) in terms of particles is very problematic (as usual for such kinds of problems) as typical frequencies when the effect (25) is not exponentially small, are of order $\omega \sim a$, and the notion of "particle" for such ω is not well defined.

We do not attempt to reproduce the known results on entropy from Ref. [5] based on the nonvanishing expectation value for the number density operator (25). First of all, it is not obvious what the physical meaning of such a computation would be based on expectation value (25) for the operator which satisfies the wrong commutation relation (20). Furthermore, it is not obvious how to interpret N particles from Eq. (25) when the entire notion of particles is not even defined for relevant parameters. Indeed, as we argued above the effect is large $\langle 0|{\rm N}|0\rangle \sim 1$ only for a very large wavelength $\lambda \geq a^{-1}$ which is the size of the horizon scale.

Our goal here is in fact quite different. We want to argue that the source of the "wrong sign" in entropy computations [5] (featuring the "weird properties" as listed in the Introduction) and the source of the "wrong sign" for the contact nondispersive term (discussed in present paper) are in fact of the same origin. In addition to the arguments presented above, we note that the technical computations of the entropy performed in [5] are actually based precisely on the same representation for the A_μ field (14) describing fluctuations of unphysical auxiliary degrees of freedom. This representation of the A_μ field in our formalism eventually leads to the expression for the contact nondispersive contribution $\sim\delta^2(x)$ with the wrong sign (19) and nonvanishing number density (25), while in Ref. [5] the very same representation for the A_μ field (14) leads to the wrong sign for the entropy. Furthermore, the contact term (5) can be represented as a surface term,

$$\chi_{E\&M} \sim \int {\rm d}^2x[\delta^2(x)] = \int {\rm d}^2x\partial_\mu\left(\frac{x^\mu}{2\pi x^2}\right), \tag{26}$$

analogous to the weird contribution in the entropy computations [5,9]. It is important to realize that the contact term (5), (13), and (26) is a result of the sum over all topological sectors with the inclusion of all quantum fluctuations which account for the degeneracy of the ground state as discussed in Sec. II B. At the same time, quite miraculously, the final result (5), (13), and (26) can be interpreted as a surface integral of a single classical configuration of a pure gauge field $A_\mu^{cl} \sim \partial_\mu \phi^{cl}$ defined on a distance surface S_1 and characterized by unit winding number

$$\chi_{E\&M} \sim \oint_{S_1} \frac{A_\mu^{cl} dl^\mu}{2\pi} = \oint_{S_1} \frac{r A_\theta^{cl} d\theta}{2\pi} = \int \frac{d\theta}{2\pi} \frac{\partial \phi^{cl}}{\partial \theta}. \quad (27)$$

These observations strongly suggest that the term with a wrong sign in the expression for the entropy derived in [5] has exactly the same *origin* as the wrong sign for the contact term (5) as in both cases the relevant physics is determined by the surface integrals, not related to any physical propagating dynamical degrees of freedom. Furthermore, in both cases the sign of the effect is opposite to what one should expect from physical degrees of freedom, and, finally, in both cases the starting point [formal representation for A_μ field (14)] is the same.

• Therefore, we *conjecture* that the surface term with the "wrong sign" in entropy computations [5,9] and the "wrong sign" in topological susceptibility (2), (5), and (13) both originated from the same physics, and are both related to the same (topologically nontrivial) gauge configurations, and must be present (or absent) in both computations simultaneously. In both cases the "wrong sign" emerges due to unphysical degrees of freedom fluctuating in the far infrared (IR) region. The technical treatments of these terms in our framework and in Ref. [5] of course is very different: we use the conventional Hamiltonian approach supplemented by the condition (17), while in Ref. [5] the Rindler Hamiltonian is ill-defined on the cone, and computations are performed using some alternative methods. Nevertheless, in our framework, we interpret the fluctuations (25) of "fictitious particles" with wrong commutation relations (20) as a different manifestation of the same physics which led to the wrong sign in entropy computations [5,9]. An additional argument supporting our *conjecture* will be presented in the next section where we show that these very different quantities nevertheless behave very similarly when the system is generalized from two to four dimensions, and therefore, they must be originated from the same physics.

Our final comment here is this: the IR physics penetrates into the physical gauge invariant correlation function (2) not due to the massless degrees of freedom in the physical spectrum (there are in fact none), but rather, as a result of the degeneracy of the ground state and the summation over all topological sectors in gauge theory as discussed in Secs. II A and II B. The ghost (16) in this framework is simply a convenient tool to account for this far IR physics as it effectively accounts for the nondispersive contact term with the wrong sign (19). It fluctuates in the presence of the horizon (25), and is responsible for the wrong sign in entropy computations, according to our conjecture. However, it remains as an unphysical auxiliary field, as it does not belong to the physical Hilbert space (and it never becomes an asymptotic state capable to propagate to infinity) [18]. It is interesting to note that there are other known examples where the degeneracy of the ground state in the presence of the horizon leads to a mismatch between black hole entropy and the entropy of entanglement, see Appendix for references and details.

III. GENERALIZATION TO THE 4D CASE

The goal of this section is twofold. First, in the Sec. III A we make a few comments on the generalization of the two-dimensional results to four-dimensional QED as discussed above. In this case the corresponding calculations of the entropy are known [5,9]. Analysis of these results further support our conjecture on the common nature of the surface term with the wrong sign in entropy computations [5,9] and the wrong sign in topological susceptibility as the behavior of the system follows precisely the pattern dictated by the *conjecture*. Second, in Sec. III B we discuss four-dimensional non-Abelian gauge theories where corresponding computations of the entropy are not yet known. Nevertheless, based on our conjecture on a common origin of these two different phenomena, we predict a possible outcome if the corresponding computations are performed.

A. Four-dimensional Abelian QED

We start by reviewing the basic results of Refs. [5,9] on entropy computations in the four-dimensional case. In the Ref. [5] the gauge invariance of the "surface term" has not been tested. This question has been specifically discussed in the follow-up paper in [9] where it has been demonstrated that in two dimensions the result is indeed gauge invariant and coincides with the original expression found in Ref. [5]. However, a similar analysis in four dimensions turned out to be much more subtle, see details in [9]. In particular, it has been found that this term is gauge dependent in the four-dimensional Abelian case, and therefore, it was discarded [9].

How can one understand such puzzling behavior of the system when one jumps from two to four dimensions? If one accepts our *conjecture* formulated above, then this puzzle has a very natural explanation. Indeed, the photon field in two dimensions has nontrivial topological properties formally expressed by the first homotopy group $\pi_1[U(1)] \sim \mathcal{Z}$. It implies the degeneracy of the ground state when each topological sector $|n\rangle$ is classified by an integer. Precisely, this feature leads to nonvanishing topological susceptibility with the wrong sign in two dimensions (5). The same degeneracy leads to nontrivial instanton solutions (8) interpolating between different topological sectors which saturate the topological susceptibility (13) with the wrong sign.

In contrast to the two-dimensional case, in four dimensions one should not expect any contact term with a wrong sign similar to (5) as the third homotopy group is trivial, $\pi_3(U(1)) \sim 1$, there is no degeneracy of the ground state as there is only a single trivial vacuum state. Therefore, one should not expect any nontrivial surface terms in entropy computations in four dimensions. This expectation based on our conjecture is supported by explicit computations [9] where it was shown that in four-dimensional QED the surface term is gauge dependent and must be consistently discarded. In fact, we consider these arguments as further support for our *conjecture* formulated above as the behavior of the system follows precisely the pattern dictated by this conjecture.[8]

Our final comment is on the interpretation of the surface term with the wrong sign given in the conclusion of Ref. [9], where it has been suggested that, quote, "effective low-energy string theory which does not coincide with the ordinary QFT" in principle may produce some surface terms with the wrong sign. We want to comment here, that in fact, very ordinary QFT may produce such kind of terms, which however, are nondispersive in nature, and not related to any physical propagating degrees of freedom as explained in previous Sec. II B for the two-dimensional case. As we argue in next section, such behavior is not a specific feature of two-dimensional physics, but in fact a very generic property in four dimensions as well. However, these nontrivial properties emerge in four dimensions only for non-Abelian gauge fields when the third homotopy group is nontrivial, $\pi_3[SU(N)] \sim \mathcal{Z}$, the ground state is degenerate and each topological sector $|n\rangle$ is classified by an integer similar to the two-dimensional case considered in Sec. II. The contact term with the wrong sign, similar to Eq. (5), is expected to emerge in this case as a result of the nontrivial topological features of four-dimensional non-Abelian gauge theories.

B. Four-dimensional non-Abelian QCD

The goal of this section is to argue that all key elements from the previous section (Sec. II) are also present in four-dimensional QCD. In fact, the presence of the contact term with the "wrong sign" in topological susceptibility in QCD is a crucial element of resolution of the so-called $U(1)_A$ problem [10,11]. The difference with the two-dimensional case is that in strongly coupled QCD we cannot perform exact analytical computations similar to (5) and (13). However, one can use an effective description in terms of the auxiliary ghost field [11] to compute the nondispersive contribution to topological susceptibility with the "wrong sign." This computation, which employs the Veneziano ghost,[9] is a direct analog of the derivation of Eq. (19) where the Kogut-Susskind ghost was used. Essentially, our goal here is to point out that the relevant features in 2D QED (discussed in Sec. II where all computations can be explicitly performed) and in 4D QCD (where the final word is expected to come from the lattice numerical computations) are almost identical. To further support these similarities we present some QCD lattice numerical results explicitly measuring the term with the wrong sign in topological susceptibility similar to Eqs. (4) and (19). Based on these observations, our *conjecture* essentially implies that the entropy computations in four-dimensional non-Abelian gauge theories must reveal a contribution with the wrong sign as the crucial element, the degeneracy of the ground state, is present in the system. Moreover, it must be gauge invariant (and therefore, physical) in contrast with 4D QED computations where it has been shown to be gauge variant [9], and therefore, was discarded.

Our starting remark is that the expression for topological density operator

$$q = \partial_\mu K^\mu = \frac{g^2}{64\pi^2}\epsilon_{\mu\nu\rho\sigma}G^{a\mu\nu}G^{a\rho\sigma} = \Box\Phi \qquad (28)$$

being represented in terms of the auxiliary scalar field Φ has exactly the same form as in the 2D Schwinger model, see Sec. II C. The Φ field in Eq. (28) is defined as $K_\mu \equiv \partial_\mu\Phi$ and is the direct analog of representation (14) for the 2D model. Our next remark is that the four-derivative operator $\int d^4xq^2 \sim \int d^4x(\Box\Phi)^2$ is expected to be induced in the effective low-energy Lagrangian as argued by Veneziano [11,12] in his resolution of the $U(1)_A$ problem. As a result of generating the q^2 operator, the relevant structure that emerges in the effective Lagrangian, and describing this system is identical to the 2D QED case, i.e. it has precisely the same structure $\sim\Phi[\Box\Box + m_{\eta'}^2\Box]\Phi$. The corresponding path integral $\int \mathcal{D}\Phi$ can be treated exactly in the same way as it was treated in 2D QED, i.e. it can be represented as the combination of the ghost ϕ_1 and a massive physical $\hat{\phi}$ field using the same trick by writing the inverse operator as follows:

$$\frac{1}{\Box\Box + m_{\eta'}^2\Box} = \frac{1}{m_{\eta'}^2}\left(\frac{1}{-\Box - m_{\eta'}^2} - \frac{1}{-\Box}\right), \qquad (29)$$

in complete analogy with the 2D case, see Eq. (18). In fact, one can show that the relevant part of the low-energy QCD Lagrangian in the large N_c limit in the form suggested by Veneziano [11,12] is *identical* to that proposed by Kogut

[8]The argument presented above is based on observation that 4D space-time (where computations [9] have been performed) has trivial topological properties. One can consider, instead, the less trivial case where 4D space-time is represented, for example, by a torus, in which case the relevant homotopy group could be nontrivial, $\pi_1(U(1)) \sim \mathcal{Z}$, and the contact term with the wrong sign in entropy computations may occur. In principle, this is a testable proposal. Technically, though, it could be quite a challenging problem.

[9]Not to be confused with conventional Fadeev Popov ghosts.

and Susskind for the 2D Schwinger model (15), where one should replace $2\sqrt{\pi} \to f_{\eta'}^{-1}$ and $\mu \to m_{\eta'}$ such that the scalar fields $\phi_1, \phi_2, \hat{\phi}$ have an appropriate (for the 4D case) canonical dimension one, see [21] for details. This formal similarity leads to almost identical computations (in terms of the ghost) of the topological susceptibility in 4D QCD and in 2D QED. Indeed, by repeating all our previous steps leading to Eq. (19) with known Green's functions which follow from (15) and with known expression for topological density operator $q(x) \sim (\Box\hat{\phi} - \Box\phi_1)$ one arrives at the following expression for topological susceptibility[10] in 4D QCD in the chiral limit $m = 0$,

$$\chi_{\rm QCD} \equiv \int d^4x \langle T\{q(x), q(0)\}\rangle = \frac{f_{\eta'}^2 m_{\eta'}^2}{4} \cdot \int d^4x[\delta^4(x) - m_{\eta'}^2 D^c(m_{\eta'}x)], \quad (30)$$

where $D^c(m_{\eta'}x)$ is the Green's function of a free massive particle with standard normalization $\int d^4x m_{\eta'}^2 D^c(m_{\eta'}x) = 1$. In this expression, $\delta^4(x)$ represents the ghost contribution, while the term proportional to $D^c(m_{\eta'}x)$ represents the physical η' contribution, see [21,22] for details. The ghost's contribution can be also thought of as the Witten's contact term [10] with the wrong sign, which is not related to any propagating degrees of freedom. The topological susceptibility $\chi_{\rm QCD}(m = 0) = 0$ vanishes in the chiral limit as a result of exact cancellation of two terms entering (30) in complete accordance with WI. When $m \neq 0$, the cancellation is not complete and $\chi_{\rm QCD} \simeq m\langle\bar{q}q\rangle$ as it should.

Similar to Eq. (26) for the two-dimensional system, the nondispersive term with the wrong sign in the topological susceptibility (30) in four-dimensional QCD can be also represented as a surface integral

$$\chi \sim \int d^4x[\delta^4(x)] = \int d^4x \partial_\mu \left(\frac{x^\mu}{2\pi^2 x^4}\right). \quad (31)$$

In the case of 2D QED we could compare our ghost-based computations (19) with exact results (4) and (5) and with an explicit sum over different topological sectors in pure *E&M* when no propagating degrees of freedom are present in the system (13). We do not have such a luxury in the case of 4D QCD. Nevertheless, we can compare the ghost-based computations in 4D QCD given by Eq. (30) with the lattice results, see e.g. [23]. We reproduce Fig. 1 from Ref. [23] to illustrate few elements that are crucial for this work and that are explicitly present on the plot. First of all, there is a narrow peak around $r \simeq 0$ with the wrong sign. Second, one can observe a smooth behavior in the extended region of $r \sim$ fm with the opposite sign. Both these elements are present in the lattice computations as one can see from Fig. 1. The same important elements are also present in our ghost-based computations given by Eq. (30). In other words, the QCD ghost does model the crucial property of the topological susceptibility related to the sum over topological classes in gauge theories. This feature cannot be accommodated by any physical asymptotic states as it is related to the nondispersive contribution in the topological susceptibility as explained above in Sec. II B, and elaborated further in Appendix where this feature is explained as a result of differences in the definitions of the Dyson's T-product and Wick's T- product.

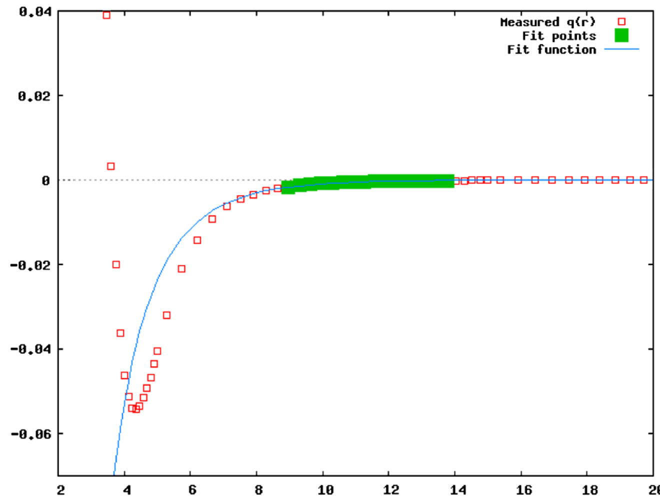

FIG. 1 (color online). The density of the topological susceptibility $\chi(r) \sim \langle q(r), q(0)\rangle$ as a function of separation r such that $\chi \equiv \int dr\chi(r)$, adapted from [23]. The plot explicitly shows the presence of the contact term with the wrong sign (narrow peak around $r \simeq 0$) represented by the Veneziano ghost in our framework.

Our next step is to describe the behavior of the same system (more precisely, the behavior of the nondispersive term in Eqs. (30) and (31) proportional to the $\delta^4(x)$ function) in Rindler space in the presence of the horizon. We consider a simple case when the acceleration is sufficiently large $\Lambda_{\rm QCD}^4 \gg a^4 \gg m|\langle\bar{q}q\rangle|$ such that the interaction term in (15) can be neglected and the Bogolubov's coefficients are exactly known. In this limit one can repeat all previous steps to arrive to the same Planck spectrum (25) for number density fluctuations of "fictitious particles" with the wrong commutation relations [18,24]. This formula (up to some irrelevant numerical coefficient) has been reproduced in Ref. [25] using a different technique. We interpret these fluctuations precisely in the same way as we did in Sec. II D in the 2D case where we interpreted these fluctuations of "fictitious particles" with wrong commutation relations as a different manifestation of the same physics which led to

[10]Of course $\chi = 0$ to any order in perturbation theory because $q(x)$ is a total divergence $q = \partial_\mu K^\mu$. However, as we learned from [10,11], $\chi \neq 0$ due to the nonperturbative infrared physics. One can interpret field K^μ as a unique collective mode of the original gluon fields. It describes the dynamics of the degenerate states $|n\rangle$ representing the topologically nontrivial sectors of the ground state, it leads to a pole in unphysical subspace in the infrared, and finally, it saturates the contact term with the wrong sign in topological susceptibility (30).

the wrong sign in the entropy computations [5,9]. As we emphasized before, the corresponding contribution is not related to any physical propagating degrees of freedom but rather, is due to topological sectors in gauge theories, which eventually lead to a nondispersive contribution in topological susceptibility.

Analysis of 2D case led us to the conjecture formulated at the end of Sec. II that both phenomena (the wrong sign in entropy computations and the wrong sign in topological susceptibility) are originated from the same physics determined by the surface dynamics of the ''fictitious particles.'' In this section we demonstrated that all relevant features are also present in 4D non-Abelian QCD. Therefore, based on the series of arguments presented above, it is natural to assume that there will be a mismatch between black hole entropy and the entropy of entanglement in 4D QCD with the same weird features as those listed in the Introduction. However, in contrast with 4D QED we expect that the surface term with the wrong sign in entropy will be a gauge invariant quantity similar to the 2D QED case discussed in Sec. II. This is essentially a prediction that follows from the *conjecture*. As we already mentioned there are other known examples where the degeneracy of the ground state in the presence of the horizon leads to a mismatch between black hole entropy and the entropy of entanglement, see Appendix for the details and references. One could argue that the dynamics of ''fictitious particles'' on a surface should be governed by the corresponding Chern-Simons action. We leave this subject for a future study[26].

IV. CONTACT INTERACTION AND PROFOUND CONSEQUENCES FOR AN EXPANDING UNIVERSE

This portion of the paper is much more speculative in nature than the previous sections. However, these speculations may have some profound consequences on our understanding of the expanding Universe we live in where the horizon is an inherent part of the system. Therefore, we opt to present these speculation in the present work.

Nondispersive contribution with the wrong sign in topological susceptibility (30) obviously implies, as Eq. (1) states, that there is also some energy related to this contact term determined by the surface dynamics of ''fictitious particles.'' This $\theta-$ dependent portion of the energy, unrelated to any physical propagating degrees of freedom, is well-established phenomenon and tested on the lattice; it is not part of the debate. What is the part of the debate and speculation is the question as to how this energy changes when background varies. In other words, the question we address in this section can be formulated as follows. How does the nondispersive contribution to the $\theta-$ dependent portion of the energy vary when the conventional Minkowski background is replaced by an expanding universe with the horizon size $L \sim H^{-1}$ determined by the Hubble constant H [11]?

The motivation for this question is as follows. We adopt the paradigm that the relevant definition of the energy which enters the Einstein equations is the difference $\Delta E \equiv (E - E_{\rm Mink})$, similar to the well-known Casimir effect where the observed energy is in fact the difference between the energy computed for a system with conducting boundaries (positioned at finite distance d) and infinite Minkowski space. In this framework it is quite natural to define the ''renormalized vacuum energy'' to be zero in Minkowski vacuum, wherein the Einstein equations are automatically satisfied as the Ricci tensor identically vanishes. From this definition it is quite obvious that the ''renormalized energy density'' must be proportional to the deviation from Minkowski space-time geometry. This is in fact the standard subtraction procedure which is normally used for description the horizon's thermodynamics [27,28] as well as in the course of computations of different Green's function in a curved background by subtracting infinities originated from the flat space [29]. In the present context such a definition $\Delta E \equiv (E - E_{\rm Mink})$ for the vacuum energy for the first time was advocated in 1967 by Zeldovich [30] who argued that $\rho_{\rm vac} \sim G m_p^6$ with m_p being the proton's mass. Later on such a definition for the relevant energy $\Delta E \equiv (E - E_{\rm Mink})$ which enters the Einstein equations has been advocated from different perspectives in a number of papers, see e.g. relatively recent works [31–38] and references therein.

This is exactly the motivation for the question formulated in the previous paragraph: how does ΔE scale with H? The difference ΔE must obviously vanish when $H \to 0$ as it corresponds to the transition to flat Minkowski space. How does it vanish? A naive expectation based on common sense suggests that $\Delta E \sim \exp(-\Lambda_{\rm QCD}/H) \sim \exp(-10^{41})$ as QCD has a mass gap $\sim\Lambda_{\rm QCD}$, and therefore, ΔE must not be sensitive to the size of our Universe $L \sim H^{-1}$. Such a naive expectation formally follows from the dispersion relations similar to (6) which dictate that a sensitivity to very large distances must be exponentially suppressed when the mass gap is present in the system.

However, as we discussed at length in this paper, along with the conventional dispersive contribution we also have the nondispersive contribution (30) and (31) which emerges as a result of topologically nontrivial sectors in

[11]Here and in what follows we use the parameter $H \sim L^{-1}$ as a typical dimensional factor characterizing the visible size of our Universe. We do not assume at this point that it is described byFriedmann-Robertson-Walker metric with a single parameter H. In fact, it could be a much more generic construction where the spatial hypersurfaces are embedded in a compact three-dimensional manifold such as, for example, the Bianchi I geometry with a few additional parameters. We refer to Appendix B of Ref. [21] for a short review on this subject in a given context.

four-dimensional QCD. This contact term may lead to a power-like scaling $\Delta E \sim H + \mathcal{O}(H^2) + \ldots$ rather than exponential like $\Delta E \sim \exp(-\Lambda_{\rm QCD}/H)$ because this term (in our framework) is described by a massless ghost field (29) as discussed in Sec. III B. The position of this unphysical massless pole is topologically protected as Eq. (28) states, which eventually may result in power-like scaling $\Delta E \sim H + \mathcal{O}(H^2) + \ldots$ rather than exponential like.[12]

In fact it was precisely the assumption postulated in [21,22] that the observable dark energy (DE) being identified with ΔE could be small but not exponentially small. A similar assumption based on very different arguments was also advocated in [31–34]. This postulate on Casimir-like scaling $\Delta E \sim H + \mathcal{O}(H)^2$ has recently received solid theoretical support as reviewed below. It is important to emphasize that this term with power-like behavior emerges as a result of the nondispersive nature of topological susceptibility (30), such that no violation of unitarity, gauge invariance, or causality occur when the theory is formulated in terms of the unphysical ghosts [18]. If true, the difference between two metrics (expanding universe and Minkowski space-time) would lead to an estimate

$$\Delta E \sim H\Lambda_{\rm QCD}^3 \sim (10^{-3}{\rm eV})^4, \qquad (32)$$

which is amazingly close to the observed DE value today. It is interesting to note that expression (32) reduces to Zeldovich's formula $\rho_{\rm vac} \sim Gm_p^6$ if one replaces $\Lambda_{\rm QCD} \rightarrow m_p$ and $H \rightarrow G\Lambda_{\rm QCD}^3$. The last step follows from the solution of the Friedmann equation

$$H^2 = \frac{8\pi G}{3}(\rho_{\rm DE} + \rho_M), \; \rho_{\rm DE} \sim H\Lambda_{\rm QCD}^3 \qquad (33)$$

where the DE component dominates the matter component, $\rho_{\rm DE} \gg \rho_M$. In this case the evolution of the universe approaches a de Sitter state with constant expansion rate $H \sim G\Lambda_{\rm QCD}^3$ as follows from (33).

There are a number of arguments supporting the power-like behavior $\Delta E \sim H + \mathcal{O}(H)^2$ in gauge theories. First of all, it is an explicit computation in exactly solvable two-dimensional QED discussed in Sec. II and defined in a box size L. The model has all of the elements crucial for the present work: a nondispersive contact term (5) which emerges due to the topological sectors of the theory (13), and which can be described using auxiliary fictitious ghost fields (19). This model is known to be a theory of a single physical massive field. Still, one can explicitly compute $\Delta E \sim L^{-1}$ which is in drastic contrast with naively expected exponential suppression, $\Delta E \sim e^{-L}$ [19]. It is important to emphasize that this correction $\Delta E \sim L^{-1}$ while computed in terms of the ghost's (unphysical) degrees of freedom in our framework, nevertheless represents a gauge invariant physical result. In other words, the final result $\Delta E \sim L^{-1}$ is not related to any violation of gauge invariance though it is computed using auxiliary fictitious ghost fields similar to computation of the contact term (19).

One more support in power-like behavior is an explicit computation in the simple case of Rindler space-time in four-dimensional QCD in the limit where a Rindler observer is moving with acceleration $\Lambda_{\rm QCD}^4 \gg a^4 \gg m|\langle\bar{q}q\rangle|$ where the interaction term in Eq. (15) can be neglected [18,24,25]. These computations explicitly show that the power like behavior emerges in four-dimensional gauge systems in spite of the fact that the physical spectrum is gapped. In other words, a power like behavior is not a specific feature of two-dimensional physics as some people (wrongly) interpret the results of Ref. [19].

Another argument supporting the power-like corrections is the computation of the contact term in four-dimensional QCD defined in a box size L. The computations are performed using the so-called instanton liquid model [39]. While the motivation for analysis [39] was quite different from our motivation, these model-based computations nevertheless explicitly show the emergence of power-like corrections to the nondispersive portion of the topological susceptibility.

Power-like behavior $\Delta E \sim L^{-1}$ is also supported by recent lattice results [40], see also the earlier paper in [41] with some hints on power-like scaling in drastic contrast with naive expectations $\Delta E \sim \exp-(\Lambda_{\rm QCD}L)$. The approach advocated in Ref. [40] is based on the physical Coulomb gauge where the nontrivial topological structure of the gauge fields is represented by the so-called Gribov copies. It is very different from our approach where we advocate that the auxiliary ghost's description accounts for this physics. Eventually, the physical results must not depend on the different technical tools which are used in different frameworks. However, it is not a simple task to demonstrate an independence of the results from an employed technique in a strongly coupled gauge theory!

Finally, Casimir-like scaling $\Delta E \sim L^{-1}$ can be tested in the so-called "deformed QCD" in a weakly coupled regime where all computations are under complete theoretical control [42]. One can explicitly demonstrate that for the system defined on a manifold size L the θ-dependent portion of the energy shows the Casimir-like scaling $E = -A \cdot [1 + \frac{B}{L} + \mathcal{O}(\frac{1}{L^2})]$ despite the presence of a mass gap in the system, in contrast with naive expectation $E = -A \cdot [1 + B\exp(-L)]$ which would normally originate from any physical massive propagating degrees of freedom consequent to conventional dispersion relations.

Another remark worth mentioning is that the sign of $\Delta E \equiv (E - E_{\rm Mink})$ is always expected to be *negative* in conventional quantum field theory computations. This is

[12]If a system is characterized by a single parameter, the curvature, then one should expect, on a dimensional ground, that the first nonvanishing term in this expansion should be quadratic $\Delta E \sim H^2$ rather than linear. However, in a generic case one expects a linear nonvanishing term $\Delta E \sim H + \mathcal{O}(H^2) + \ldots$, see Appendix B of Ref. [21] for the details.

due to the fact that some modes cannot be accommodated in a system with nontrivial geometry/boundaries, and therefore the absolute value of $E_{\rm Mink} > E$ which corresponds to $\Delta E < 0$. The Casimir effect is the well-known example where the sign $(-)$ emerges as a result of this subtraction procedure. The nondispersive contribution into the energy, on the other hand, being represented by the ghost in our framework will lead to an opposite sign $\Delta E > 0$. The positive sign for $\Delta E > 0$ is supported by explicit computations in a simplified setting [18,25,42], and is consistent with observations corresponding to the accelerating universe with $\Delta E > 0$.

To conclude this section, the contact term with the wrong sign in topological susceptibility which is present in gauge theories as a result of a nontrivial topological structure of the theory and which is ultimately related to the wrong sign contribution in the entropy computations [5,9], as argued in this paper, has another profound consequence. Namely, the very same physics, and the very same gauge configurations may lead to a power-like sensitivity from the distant regions $\Delta E \sim L^{-1}$ in drastic contrast with naive expectations $\Delta E \sim \exp-(\Lambda_{\rm QCD} L)$ which should occur from any conventional physical massive propagating degrees of freedom. If true, one can interpret the extra contribution to the energy (32), which we identify with DE, as a result of contact interaction with the horizon. This interpretation is consistent with interpretation of the term with the wrong sign in the entropy computation in two dimensions [5,9], and it is also consistent with our interpretation presented in Sec. II D, see comments after Eq. (25). This interpretation is also consistent with arguments [5] suggesting that this term corresponds to a contact interaction with horizon in the description of black hole entropy within a string theory formulation [43].

V. CONCLUSION. FUTURE DIRECTIONS

The main result of this paper is presented in the form of a *conjecture* formulated at the end of Sec. II and elaborated in Sec. III. Essentially, the basic idea is that the surface term with the wrong sign in the entropy computations [5,9] and the contact term with the wrong sign in topological susceptibility both originated from the same physics, and are both related to the same gauge configurations related to the nontrivial topological structure of the theory. If this conjecture turns out to be correct, it would unambiguously identify the nature of the well-known mismatch between computations of black hole entropy and the entropy of entanglement for vector gauge fields. A similar mismatch (but in a quite different context) was also discussed in [44–46]. In both cases, the mismatch is a result of the degeneracy of the ground state in the presence of the horizon, see Appendix for the details.

Another, much more profound consequence is that the same physics which is responsible for the wrong sign contact term in topological susceptibility, and the wrong sign contribution in the entropy computations [5,9], may in fact lead to extra vacuum energy (32) [which is identified with observed DE] in an expanding universe in comparison with Minkowski space. This extra energy emerges in gauge theories with multiple topological sectors as a result of the mere existence of a causal horizon at distance $L \sim H^{-1}$. A similar phenomenon as we already mentioned occurs in different systems [45,46] where extra energy emerges as a result of dynamics of the "soft modes" at the horizon. The degeneracy of the vacuum state in the system discussed in [45,46] is achieved by nonminimal coupling with a scalar field, see Appendix for the details, while in our case the presence of topologically distinct sectors in the system is an inherent feature of the QCD dynamics.

Here are some features of these unique gauge configurations which are responsible for the wrong sign in the entropy computations and the wrong sign in topological susceptibility and which are characterized by very exotic properties which are drastically different from everything previously known:

(a) A typical wavelength of fluctuations of the auxiliary "fictitious particles" is determined by the horizon scale, $\lambda_k \sim 1/H \sim 10$ Gyr, while smaller $\lambda_k \ll 1/H$ are exponentially suppressed (25). Therefore, these modes do not gravitationally clump on distances smaller than the Hubble length, in contrast with all other types of matter, and can be identified with the observed properties of DE. Such very large wavelengths prevent us from adopting a meaningful scattering-based description, as the notion of particle is not even defined;

(b) The corresponding fluctuations are observer dependent, similar to the Unruh radiation, in contrast with any other types of radiation, see Sec. II D and also [18] for detailed discussions on the problem of measurements in these circumstances;

(c) The coexistence of the two drastically different scales ($\Lambda_{\rm QCD} \sim 100$ MeV and $H \sim 10^{-33}$ eV) is a direct consequence of the auxiliary conditions (17) and (23) on the physical Hilbert space rather than an *ad hoc* built-in feature such as small coupling or/and extra symmetries in a Lagrangian.

So, essentially our proposal for the DE can be formulated as follows. The source for both DE and the *mismatch* between black hole entropy and the entropy of entanglement is the same and related to the dynamics of topological sectors of a gauge theory in the presence of the horizon. In other words, the relevant gauge configurations responsible for DE are exactly the same as those responsible for the wrong sign contribution in the computations of Refs [5,9]. Precisely this contribution represents the mismatch between black hole entropy and the entropy of entanglement. In both cases the source of the extra term is the degeneracy of the vacuum state which is represented by different topological sectors, and in our framework is described by the Veneziano ghost. One should also add that a phenomenological analysis of the DE model based on this idea and represented by Eqs. (32) and (33) has been recently performed in [47] with the conclusion that this model is consistent with all presently available observational data.

Another important result of this work can be formulated as follows. When a system is promoted from Minkowski space to a an expanding universe, we expect power-like corrections $\Delta E \sim H + \mathcal{O}(H)^2$ rather than exponentially suppressed corrections, $\Delta E \sim \exp-(\Lambda_{\mathrm{QCD}}/H)$. This happens in spite of the fact that the physical Hilbert subspace contains only massive propagating degree of freedom, and naively the sensitivity to very distant regions should be exponentially suppressed. However, the presence of the nondispersive contributions (originated from degenerate topological sectors of the theory), which cannot be associated with any physical asymptotic states *falsifies this naive argument*. Explicit computations in 2D QED [19], in 4D weakly coupled "deformed QCD" [42] and numerical studies in real four-dimensional QCD [40] where power-like behavior $\sim L^{-1}$ is indeed observed, supports our claim.

What is more remarkable is the fact that some of fundamental properties of gauge theories discussed in this paper can be, in principle, experimentally tested in the Relativistic Heavy Ion Colliderat Brookhaven and Heavy Ion program at LHC. In the "little bang" at the Relativistic Heavy Ion Collider the horizon appears as a result of induced acceleration $a \sim \Lambda_{\mathrm{QCD}}$ which itself emerges as a consequence of high energy collision. The acceleration $a \sim \Lambda_{\mathrm{QCD}}$ is a universal number which is determined by strong QCD dynamics, does not depend on energy or other properties of the colliding particles, and plays the role of Hubble constant $H \sim 10^{-33}$ eV of an expanding universe, see [24] for the details.

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SUBTLETIES IN THE DEFINITION OF ENERGY

It is quite natural to expect that there should be some energy associated with fluctuations of fictitious particles (25). As we argued above these fluctuations, which reflect the nontrivial topological structure of the gauge theory, are the source of the mismatch between black hole entropy and the entropy of entanglement in the presence of the horizon. It is clear that there is some ambiguity in the definition of this type of energy due to a number of reasons. First of all, as we discussed in the text the relevant physics is determined by the dynamics on the surface rather than in the bulk of the space-time. Therefore, it is not obvious if a simple insertion ω_k into definition (22) would properly reflect this feature. Another comment is that the physics of fluctuations (25) is the observer dependent property similar to the Unruh effect as discussed in Sec. II D. Therefore, all subtleties related to the Unruh effect are also present here. Also, very large wavelengths of these fluctuations prevent us from adopting a conventional description in terms of particles, as the notion of particle is not even defined.

Finally, and most importantly, the contact interaction which is the main subject of this paper, cannot be expressed in terms of any physical states, but rather is formulated in terms of fluctuations of fictitious particles with wrong commutation relations. These unphysical states contribute to the nondispersive portion of the correlation function, not to the dispersive part which is unambiguously determined by the physical spectral function through conventional dispersion relations. Our description in terms of the ghost is simply a convenient way to study this IR physics in a covariant gauge. The same physics in the Coulomb (physical) gauge where the ghost degrees of freedom are not present in the system leads to the long-range forces as discussed in the simple two-dimensional model long ago [16]. These long-range forces prevent distant regions from acting independently. The vacuum in this system is degenerate, and one should consider an infinite superposition of the winding states $|n\rangle$ as originally discussed in [16]. We think that precisely this feature prevented the author of Ref. [5] from using the physical Coulomb gauge in two dimensions in the computations of entropy, where a covariant gauge has been used instead. The same physics in the Coulomb gauge in 4D QCD is reflected by existence of the so-called Gribov copies, and one should use some numerical lattice methods to study the relevant physics [40].

The reason why we pay so much attention to the topological susceptibility χ and the corresponding contact term $\sim\delta(x)$ which enters the expressions for topological susceptibility (4) and (30) is due to its relation to the θ dependent portion of the vacuum energy as Eq. (1) states. Therefore, the presence of nondispersive contributions in χ automatically implies the presence of the corresponding nondispersive contribution in the vacuum energy E_{vac}. At the same time, as we discussed in the text, the nondispersive contribution in χ (and in the vacuum energy) can be interpreted as a result of nontrivial topological structure of the gauge theories. These discussions explicitly demonstrate some subtleties on the possible definition of the vacuum energy which should accommodate the physics related to the contact term discussed in this paper.

It is quite fortunate that in the specific case with computations of χ we can easily separate nondispersive and physical contributions. This is due to the fact that these two very different terms contribute to χ with opposite signs as discussed in the text. Indeed, the topological susceptibility in pure Yang-Mills gauge theory (no quarks, and no η' contribution) according to [10,11] is given by[13]

$$\chi_{YM} \equiv i \lim_{k\to 0} \int d^4x e^{ikx} \langle T\{q(x), q(0)\}\rangle_W = -\lambda_{YM}^2 < 0,$$
$$\lambda_{YM}^2 = \frac{f_{\eta'}^2 m_{\eta'}^2}{4}, \quad \text{(A1)}$$

where $\langle \ldots \rangle_W$ stands for the Wick T- product, see below. The expression on right-hand side of Eq. (A1) corresponds to the subtraction constant $\sim\delta^4(x)$ in Eq. (30). This term has the wrong sign (in comparison with contribution from any real physical states), the property which motivated the term "Veneziano ghost." Indeed, a physical state of mass m_G, momentum $k \to 0$ and coupling $\langle 0|q|G\rangle = c_G$, contributes to the topological susceptibility with the sign which is opposite to (A1),

$$i \lim_{k\to 0} \int d^4x e^{ikx} \langle T\{q(x), q(0)\}\rangle_D$$
$$\sim i \lim_{k\to 0} \langle 0|q|G\rangle \frac{i}{k^2 - m_G^2} \langle G|q|0\rangle \simeq \frac{|c_G|^2}{m_G^2} \geq 0, \quad \text{(A2)}$$

where $\langle \ldots \rangle_D$ stands for Dyson T-product, see below. However, the negative sign for the topological susceptibility (A1) is what is required to extract the physical mass for the η' meson, see Ref. [12] for a thorough discussion. The difference between the behavior (A1) and (A2) is related to inequivalent definitions of these correlation functions. The behavior (A2) corresponds to the usual Dyson T-product where only physical states contribute, while Eq. (A1) corresponds to the Wick T-product obtained by variation of the partition function over the θ parameter. The difference in the definitions constitutes precisely the subtraction constant $\sim\delta^4(x)$ in Eq. (30). The WI expressed as $\chi_{\rm QCD}(m_q = 0) = 0$ is satisfied for the Wick T-product, but not for the Dyson T-product.

It is interesting to note that an analogous phenomenon (but in quite different context) was discussed in Refs. [45,46], where it was observed that there are two definitions of energy when the difference is saturated by the so-called "soft modes" fluctuating in far infrared at the horizon. The first definition is the canonical energy determined by the Hamiltonian which is generator of translations of the system along the timelike Killing vector field ξ^μ. Since the Killing vector ξ^μ vanishes at the bifurcation surface of the Killing horizons, the corresponding Hamiltonian is degenerate. Therefore, one can add to the system an arbitrary number of "soft modes" without changing the canonical energy. These "soft modes" contribute to the surface integral which was interpreted as a Noether charge of some nonminimally coupled scalar field ϕ_s. Precisely this contribution of the "soft modes" distinguishes two different definitions of the energy. As argued in Refs. [45,46] precisely the dynamics of "soft modes" constitutes the difference between Bekenstein-Hawking entropy and the entropy of entanglement.

It is very similar to our case where we argued that the wrong sign in the entropy computations of Refs. [5,9] is a result of degeneracy in the gauge theory represented in our framework by the fluctuations of the fictitious particles with a typical wavelength of order of the horizon scale, $\lambda_k \sim H^{-1}$. In other words, the "soft modes" from [45,46] play the same role as pure gauge fields which describe different topological sectors of the theory in our case. In both cases the effect emerges as a result of the degeneracy of the ground state in the presence of the horizon, and in both cases the difference is determined by some surface integrals. Further to this analogy, two different types of energies which can be reconstructed from two different definitions of topological susceptibility (A1) and (A2) precisely correspond to two different types of energies discussed in Refs. [45,46].

A final comment on the similarities between the two very different systems is as follows. The relevant gauge configurations which are responsible for mismatch between black hole entropy and the entropy of entanglement from computations [5,9] are exactly the same which are responsible for the contact term in topological susceptibility (30). This mismatch is very similar to the extra term discussed in [45,46] which resulted from the dynamics of the "soft modes" at the horizon. In both cases the mismatch is a result of the degeneracy present in both systems. In Refs. [45,46] this degeneracy is a result of nonminimally coupling with scalar field ϕ_s. In our cases this degeneracy is a reflection of the topological structure of gauge theories, and an inherent feature of QCD. However, the outcome of the degeneracy is very similar in both cases as we argued above, and can be interpreted as a contact interaction with horizon, see the discussion at the end of Sec. IV.

[13]All formulae in this Appendix are written in Minkowski space in contrast with our discussions in the text, where a comparison with path integral computations and lattice numerical computations (which are always performed in Euclidean space) was made. In particular, there is factor "i" in the definition of the correlation function (A1).

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