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Higgs inflation in minimal supersymmetric $SU(5)$ grand unified theory

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The standard model Higgs boson with large nonminimal coupling to the gravitational curvature can drive cosmological inflation. We study this type of inflationary scenario in the context of supersymmetric grand unification and point out that it is naturally implemented in the *minimal* supersymmetric $SU(5)$ model, and hence virtually in any GUT models. It is shown that with an appropriate Kähler potential the inflaton trajectory settles down to the standard model vacuum at the end of the slow roll. The predicted cosmological parameters are also consistent with the 7-year WMAP data.

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I. INTRODUCTION

Recently the idea that the standard model (SM) Higgs field may be identified with an inflaton field has attracted much attention [1–9] The major role is played by the nonminimal coupling to gravity, which renders the Higgs mass to be within the range of 126–194 GeV [1–4], while keeping the amplitude of the primordial curvature perturbation at the scale of $\sim 10^{-5}$. The idea of inflation by nonminimally coupled inflaton field itself is certainly not new [10]. Nevertheless, the striking agreement with the present-day cosmological data, combined with the minimalistic nature of the model, makes this type of scenario very attractive. The predicted mass range of the Higgs particle is also interesting for the physics of the Large Hadron Collider.

The Higgs potential in the SM is unstable against quantum corrections (the hierarchy problem) and it therefore is reasonable to reconsider Higgs inflation in supersymmetric theory [11,12]. It is shown in [11] that Higgs inflation cannot be implemented within the minimal supersymmetric standard model (MSSM), as the field content of the latter is too restrictive. Instead, with an extra field (i.e. in the next-to-minimal supersymmetric standard model, NMSSM) a sensible scenario of Higgs inflation is found to be possible. The NMSSM has tachyonic instability in the direction of the extra field, but this can be cured by considering a noncanonical Kähler potential [12].

In this paper we discuss the possibility of Higgs inflation in supersymmetric grand unified theory (GUT). There are several reasons to motivate this study. One obvious reason is that the energy scale of inflation is typically above the grand unification scale, and it is unnatural to suppose that the SM Lagrangian is valid all the way up to the scale of inflation; as the GUT scale destabilizes the electroweak scale without supersymmetry, it seems that supersymmetric GUT is an appropriate theory to start with. Another reason is the puzzling necessity of the extra field besides the MSSM fields for successful Higgs inflation, as alluded to above; going beyond the MSSM is somewhat against the minimalistic guiding principle of the original Higgs inflation, and as the NMSSM is structurally similar to the $SU(5)$ GUT model, it seems natural to conjecture that the $SU(5)$ GUT, rather than the NMSSM, may be a more appropriate minimal supersymmetric theory that accommodates Higgs inflation. Obvious questions are then whether it is possible to obtain enough inflation (e-folding) somewhere between the Planck scale and the GUT scale, and if so whether the prediction of the cosmological parameters is consistent with the present observation. We shall address these issues below, and find that a viable Higgs inflationary scenario nicely fits into the minimal $SU(5)$ model. We shall employ supergravity embedding of GUT [13], since the nonminimal coupling of the Higgs field to gravity naturally arises in that framework.

II. SUPERSYMMETRIC $SU(5)$ GUT

The minimal supersymmetric $SU(5)$ model consists of a vector supermultiplet transforming as an adjoint **24** of the $SU(5)$, as well as 5 types of chiral supermultiplets, namely $N_{\rm f}$ (the number of flavours) multiplets in $\bar{\mathbf{5}}$ (that include $\bar{d}$ and L of the MSSM), $N_{\rm f}$ multiplets in **10** (include Q, $\bar{u}$, and $\bar{e}$), one each in **24** (denoted Σ), **5** (H) and $\bar{\mathbf{5}}$ ($\bar{H}$). Σ is the Higgs multiplet responsible for breaking the GUT symmetry, while H and $\bar{H}$ respectively include the up- and down-type MSSM Higgs multiplets. Among these, only the three Higgs chiral multiplets Σ, H and $\bar{H}$ play rôles in the dynamics of inflation. We shall hence disregard the other fields. The superpotential of our model is,

$$W = \bar{H}(\mu + \rho\Sigma)H + \frac{m}{2}\,{\rm Tr}(\Sigma^2) + \frac{\lambda}{3}\,{\rm Tr}(\Sigma^3), \qquad (1)$$

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and the Kähler potential is $K = -3\Phi$, with

$$\Phi = 1 - \frac{1}{3}(\mathrm{Tr}\Sigma^\dagger\Sigma + |H|^2 + |\bar{H}|^2) - \frac{\gamma}{2}(\bar{H}H + H^\dagger\bar{H}^\dagger) + \frac{\tilde{\omega}}{3}(\mathrm{Tr}\Sigma^\dagger\Sigma^2 + \mathrm{Tr}\Sigma^{\dagger 2}\Sigma) + \frac{\zeta}{3}(\mathrm{Tr}\Sigma^\dagger\Sigma)^2, \quad (2)$$

where μ, ρ, m, λ, γ, ζ, $\tilde{\omega}$ are constant parameters (for simplicity we assume them to be real). The cubic and the quartic terms have been included in the Kähler potential, for reasons to be discussed shortly. We shall set the reduced Planck scale $M_P = 2.44 \times 10^{18}$ GeV to unity.

For the model to be phenomenologically consistent, the $SU(5)$ symmetry needs to be broken down to the SM gauge group $SU(3) \times SU(2) \times U(1)$. This is accomplished as usual by setting,

$$\Sigma = \sqrt{\frac{2}{15}} S\, \mathrm{diag}\left(1, 1, 1, -\frac{3}{2}, -\frac{3}{2}\right), \quad (3)$$

with S a chiral superfield. The MSSM Higgs doublets H_u, H_d and the Higgs color triplets H_c, $\bar{H}_c$ are embedded in H and $\bar{H}$ as

$$H = \begin{pmatrix} H_c \\ H_u \end{pmatrix}, \qquad \bar{H} = \begin{pmatrix} \bar{H}_c \\ H_d \end{pmatrix}. \quad (4)$$

The superpotential now reads

$$W = \left(\mu + \sqrt{\frac{2}{15}}\rho S\right)\bar{H}_c H_c + \left(\mu - \sqrt{\frac{3}{10}}\rho S\right) H_u H_d + \frac{m}{2}S^2 - \frac{\lambda}{3\sqrt{30}}S^3. \quad (5)$$

The masses of H_u and H_d are in the electroweak scale, which is negligibly smaller than the typical scale M_P of the inflationary dynamics. Thus the expectation value of the second term in (5) must vanish, $\mu = \sqrt{3/10}\rho\langle S\rangle$, where $\langle S\rangle = v \equiv 2 \times 10^{16}$ GeV is the GUT scale. The first term of (5) indicates that H_c and $\bar{H}_c$ have GUT scale masses. For the color symmetry to be unbroken we require that they are already stabilized at $\langle H_c\rangle = \langle\bar{H}_c\rangle = 0$, from the onset of the inflation. During inflation the dominant role is played by the MSSM Higgs fields H_u and H_d, which settle down to the present values after the inflation. When $H_u, H_d \ll 1$ (i.e. close to the end of inflation) the stationary condition $\delta W/\delta S = 0$ with $H_c = \bar{H}_c = 0$ yields $S(m - \lambda S/\sqrt{30}) = 0$. Since the GUT symmetry must be broken, $\langle S\rangle = v \neq 0$ and we must have $m = \frac{\lambda}{\sqrt{30}}v$. The charged Higgs can be consistently set to be zero,

$$H_u = \begin{pmatrix} 0 \\ H_u^0 \end{pmatrix}, \qquad H_d = \begin{pmatrix} H_d^0 \\ 0 \end{pmatrix}, \quad (6)$$

and parametrizing $S = se^{i\alpha}$, $H_u^0 = \frac{1}{\sqrt{2}}h_1 e^{i\alpha_1}$, $H_d^0 = \frac{1}{\sqrt{2}}h_2 e^{i\alpha_2}$, with $s, h_1, h_2, \alpha, \alpha_1, \alpha_2 \in \mathbb{R}$, and further setting $h_1 = h\sin\beta$ and $h_2 = h\cos\beta$, the model depends on five parameters ρ, λ, γ, $\tilde{\omega}$, ζ, and six real scalar fields s, h, α, β, α_1, α_2. Note that ρ and λ are parameters appearing in the GUT superpotential and are typically of order one, while there is no such restriction for γ, $\tilde{\omega}$, and ζ. Analyzing the scalar potential, we find stability at $\alpha = \alpha_1 = \alpha_2 = 0$. Furthermore, the D-flat condition sets the value of β to be $\pi/4$. Thus the model reduces to a system of two real scalars h and s, with the scalar-gravity part of the Jordan frame Lagrangian (cf. [12]),

$$\mathcal{L}_J = \sqrt{-g_J}\left[\frac{1}{2}\Phi R_J - \frac{1}{2}g_J^{\mu\nu}\partial_\mu h\partial_\nu h - \kappa g_J^{\mu\nu}\partial_\mu s\partial_\nu s - V_J\right]. \quad (7)$$

The subscript J denotes quantities in the Jordan frame, $\kappa \equiv K_{SS^\dagger} = 1 - 4\omega s - 4\zeta s^2$ is the nontrivial component of the Kähler metric, $\omega \equiv -\tilde{\omega}/\sqrt{30}$, and

$$\Phi = 1 - \frac{1}{3}s^2 + \frac{2\omega}{3}s^3 + \frac{\zeta}{3}s^4 + \left(\frac{\gamma}{4} - \frac{1}{6}\right)h^2. \quad (8)$$

V_J is the F-term scalar potential in the Jordan frame, computed in the standard way [14], as

$$V_J = \frac{3}{10}\left\{\frac{\rho^2}{2}(s-v)^2h^2 + \frac{1}{\kappa}\left[\frac{\rho}{4}h^2 - \frac{\lambda}{3}s(s-v)\right]^2\right\} - \frac{\left\{\frac{2\zeta s+\omega}{\kappa}\left[\frac{\rho h^2}{4} - \frac{\lambda s(s-v)}{3}\right]s^2 + \frac{\rho v h^2}{4} - \frac{\lambda v s^2}{6} - \frac{3\gamma\rho h^2(s-v)}{4}\right\}^2}{10\left[1 + \frac{\gamma}{4}\left(\frac{3}{2}\gamma - 1\right)h^2 + \frac{\zeta+\omega^2}{3\kappa}s^4\right]}. \quad (9)$$

III. THE INFLATION DYNAMICS

The dynamics of inflation is encoded in the scalar potential $V_E = \Phi^{-2}V_J$ in the Einstein frame. If we take the canonical form of the Kähler potential (i.e. $\omega = \zeta = 0$), the potential exhibits tachyonic instability in the direction of the s-field. Just as in the case of the NMSSM Higgs inflation [11,12] the instability is controlled by introducing a quartic term ($\zeta \neq 0$) in the Kähler potential. In the GUT model, however, this is not the whole story, as the quartic term has a serious side effect: the SM vacuum becomes disfavored and the $SU(5)$ symmetry tends to be restored at the end of inflation. This problem is resolved by allowing a cubic term,[1] $\omega \neq 0$. Note that these terms are perfectly consistent with the supergravity embedding. The bottom line is that for a wide range of the parameter space with up to quartic order terms in the Kähler potential, there exist reasonable trajectories of the inflaton field. In Fig. 1 we show the shape of the scalar potential V_E (the left panel), the inflaton trajectory (center), and the values of V_E at local minima (bottom of the valley) for given h (right). In this example we have taken $\rho = \lambda = 0.5$, $\omega = -100$, $\zeta = 10000$, and $\gamma = 1.86 \times 10^4$ (this value of γ is determined

[1]Higher (say sextic) terms in the Kähler potential can also solve this problem.

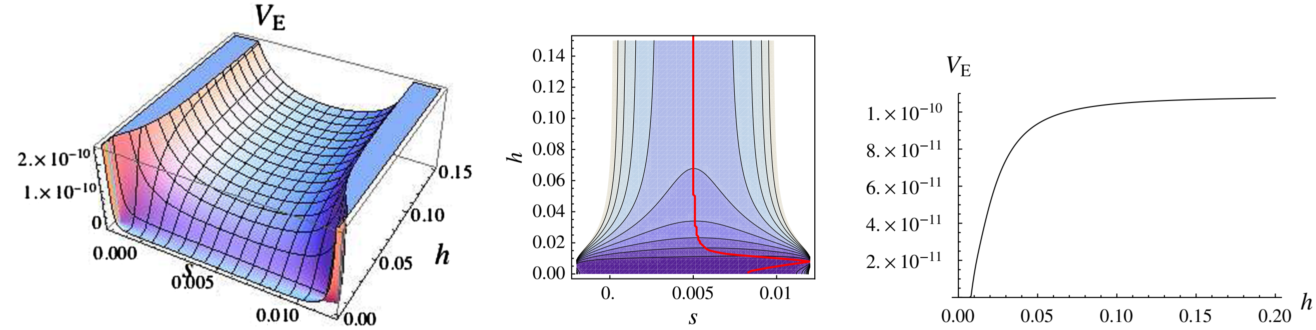

FIG. 1 (color online). The scalar potential V_E in the Einstein frame (left), the inflaton trajectory in the contour plot of the same potential (middle), and the minima of the scalar potential $V(s(h), h)$ plotted against h (right). In the middle panel the thick red curve is the inflaton trajectory. We have chosen $\rho = 0.5$, $\lambda = 0.5$, $\omega = -100$, $\zeta = 10000$. The nonminimal coupling $\gamma = 1.86 \times 10^4$ is fixed by the amplitude of the curvature perturbation, evaluated for e-folding $N_e = 60$.

for the e-folding number $N_e = 60$, as discussed below). The plateau of the potential at the large h values is a characteristic feature of Higgs inflation. As the field s controls breaking of the GUT symmetry, the trajectory shows that $SU(5)$ is broken from the onset, indicating that problematic topological defects are not produced during inflation. For this parameter set the dynamics of the slow roll inflation is dominated by the h field, as the displacement of s is negligibly small ($\Delta\tilde{s}/\Delta h \lesssim 2\%$, with suitable normalization $d\tilde{s} = \sqrt{2\kappa} ds$). Assuming that s is nearly constant,[2] the model simplifies to single field inflation. The Lagrangian (7) can then be written in a form similar to the SM Higgs inflation [1–8],

$$\mathcal{L}_J = \sqrt{-g_J}\left[\frac{M^2 + \xi h^2}{2} R_J - \frac{1}{2} g_J^{\mu\nu} \partial_\mu h \partial_\nu h - V_J\right], \tag{10}$$

with $M^2 = 1 - \frac{1}{3}s^2 + \frac{2}{3}\omega s^3 + \frac{1}{3}\zeta s^4$ and $\xi = \frac{1}{4}\gamma - \frac{1}{6}$.

IV. COSMOLOGICAL PARAMETERS

The slow roll parameters,

$$\epsilon = \frac{1}{2}\left(\frac{1}{V_E}\frac{dV_E}{d\hat{h}}\right)^2, \qquad \eta = \frac{1}{V_E}\frac{d^2V_E}{d\hat{h}^2}, \tag{11}$$

are defined for the scalar potential V_E and the canonically normalized inflaton field $\hat{h}$ in the Einstein frame. The latter is related to h by

$$d\hat{h} = \frac{\sqrt{M^2 + \xi h^2 + 6\xi^2 h^2}}{M^2 + \xi h^2} dh. \tag{12}$$

For given (λ, ρ, ω, ζ), the nonminimal coupling ξ is determined from the power spectrum of the curvature perturbation $\mathcal{P}_R = V_E/24\pi^2\epsilon$. The slow roll terminates when either of the slow roll parameters (ϵ in the present case) becomes $\mathcal{O}(1)$. The values of the inflaton $h = h_*$ at the end of the slow roll and h_k at the horizon exit of the comoving CMB scale k, are related by the e-folding number $N_e = \int_{h_*}^{h_k} dh V_E (d\hat{h}/dh)/(dV_E/d\hat{h})$. At $h = h_k$ the shape of V_E is constrained by the power spectrum $\mathcal{P}_R$. We have used the maximum likelihood value $\Delta_R^2(k_0) = 2.42 \times 10^{-9}$ from the 7-year WMAP data [15], where $\Delta_R^2(k) = \frac{k^3}{2\pi^2}\mathcal{P}_R(k)$ and the normalization is fixed at $k_0 = 0.002\ \text{Mpc}^{-1}$. With $\lambda = \rho = 0.5$, $\omega = -100$ and $\zeta = 10000$, we find $h_* = 0.0146$, $h_k = 0.128$ and $\xi = 4646$ for $N_e = 60$. For $N_e = 50$ we obtain $h_* = 0.0160$, $h_k = 0.130$ and $\xi = 3895$. With these parameters the prediction of the scalar spectral index $n_s \equiv d\ln\mathcal{P}_R/d\ln k = 1 - 6\epsilon + 2\eta$ and the tensor-to-scalar ratio $r \equiv \mathcal{P}_{gw}/\mathcal{P}_R = 16\epsilon$ can be evaluated. We find $n_s = 0.968$, $r = 0.00296$ for $N_e = 60$, and $n_s = 0.962$, $r = 0.00419$ for $N_e = 50$. These results are shown in Fig. 2 with observational constraints [15]. The prediction for n_s and r is insensitive to the change of λ and ρ, as long as they are $\mathcal{O}(1)$. With $(N_e, \lambda, \rho) = (60, 0.1, 0.5)$ and $(60, 0.5, 0.1)$, for

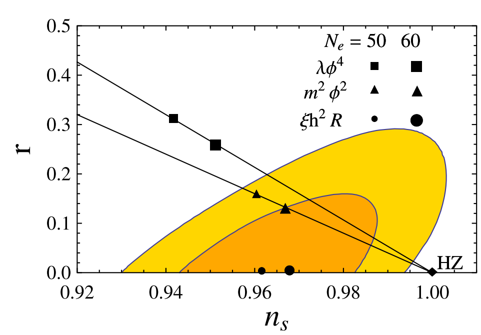

FIG. 2 (color online). The tensor-to-scalar ratio r and the scalar spectral index n_s, with the 68% and 95% confidence level contours from the WMAP7 + BAO + H_0 data [15]. The Harrison-Zel'dovich (HZ) values as well as the predictions of the ϕ^4 and ϕ^2 chaotic inflation models are also shown for comparison.

[2]The value of $s = s(h)$ is taken at the local minimum of V_E for a given h, and derivatives of s are set to be zero.

example, we obtain the same prediction $n_s = 0.968$ and $r = 0.00296$ as above. In contrast to the nonsupersymmetric case, the inflationary dynamics does not constrain the Higgs mass at the electroweak scale.

V. DISCUSSION

In this paper we have discussed Higgs inflation in supersymmetric GUT, taking the minimal $SU(5)$ model as a concrete example. In the early days the proposals of cosmological inflation were made for the Higgs field in the GUT models [16]. It is intriguing to see that the prediction based on the simplest GUT, with the help of nonminimal coupling to gravity, is in perfect fit with today's observational constraints.

The nonminimal coupling is consistent with the symmetries of general relativity and the SM, and it naturally arises in quantum field theory in curved spacetime [17]. The value of the coupling $\xi \sim 10^4$, however, is rather large. This is a generic feature of Higgs inflation, since successful slow roll requires $h^2 \lesssim M_{\rm P}^2 \lesssim \xi h^2$ [1]. It has been argued that such large nonminimal coupling could violate the unitarity bound, since the cut-off scale evaluated as M_P/ξ is considerably lower than the Planck scale [5–8]. Others contend that such a criticism is not valid, arguing that at large field values $\gtrsim M_{\rm P}/\xi$ the cut-off scale is actually field-dependent [4,9,12]. The large nonminimal coupling is, at any rate, a key feature of the Higgs inflation and it is certainly worthwhile understanding possible dangers arising from this. Another type of criticism concerns the quantum stability of the classical potential. This problem was studied using renormalization group (RG) analysis [2–4], and the effects of renormalization are found to be small except for some extreme values of parameters. We have also performed RG analysis in our GUT model and verified that the effects are small (less than 3% for r, less than 2% for ξ, and less than 0.1% for n_s). This is expected, since inflation takes place in a narrower energy range of 10^{16}–10^{18} GeV and the RG effects should be smaller than the SM case.

A closer look at the potential $V_{\rm E}$ shows that its minimum is at a small negative value, $\sim -2 \times 10^{-16} M_{\rm P}^4$, for our parameter choices. This is offset by a contribution from the supersymmetry breaking sector and the scenario does not suffer from the cosmological constant problem. In our scenario the energy scale of inflation is in the GUT scale and the Higgs fields are directly coupled to the SM particles. This indicates that the reheating temperature is high, typically from the intermediate to the GUT scale. It would be interesting to discuss further phenomenological implications, such as the gravitino problem and baryogenesis.

In this paper we considered a single-field Higgs inflation model appropriate for our parameter choice $\zeta = 10000$, $\omega = -100$ of the Kähler potential. These values are not too exotic, as $\langle \Phi \rangle$ is still very close to 1 and the Planck scale after inflation is nearly $M_{\rm P}$. For smaller values of ζ and $|\omega|$, the displacement of s during inflation becomes large. This leads to two-field inflation, which is also of interest, in particular, due to possible generation of detectable large non-Gaussianity. Supersymmetric models of Higgs inflation necessarily involve multiple fields [11]. The engendered isocurvature mode can, in principle, distinguish various models of Higgs inflation.

Finally, the scenario can also be extended to other GUT models whose gauge group contains $SU(5)$ as a subgroup. When the Higgs multiplets of the GUT model contain **5**, $\bar{\mathbf{5}}$ and **24** of the minimal $SU(5)$ GUT, a superpotential like (1) can be introduced. Then a viable model of Higgs inflation is implemented, as described in this paper. One such simple example is the $SO(10)$ GUT with Higgs multiplets in **10** and **54** representations.

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