# Geometrical spontaneous *CP* violation

Ivo de Medeiros Varzielas*

*Departamento de Física and Centro de Física Teórica de Partículas, Instituto Superior Técnico, Universidade Técnica de Lisboa, Avenue Rovisco Pais, 1049-001 Lisboa, Portugal and Fakultät für Physik, Technische Universität Dortmund, D-44221 Dortmund, Germany*

D. Emmanuel-Costa†

*Departamento de Física and Centro de Física Teórica de Partículas, Instituto Superior Técnico, Universidade Técnica de Lisboa, Avenida Rovisco Pais, 1049-001 Lisboa, Portugal*

(Received 1 July 2011; published 28 December 2011)

Spontaneous *CP*-violating phases that do not depend on the parameters of the Higgs sector—the so-called calculable phases—are investigated. The simplest realization is in models with 3 Higgs doublets, in which the scalar potential is invariant under non-Abelian symmetries. The non-Abelian discrete group $\Delta(54)$ is shown to lead to the known structure of calculable phases obtained with $\Delta(27)$. We investigate the possibility of accommodating the observed fermion masses and mixings.

DOI: 10.1103/PhysRevD.84.117901 PACS numbers: 11.30.Er, 12.15.Ff, 12.60.Fr

Since the discovery of *CP* violation in 1964, its origin remains a fundamental open question in particle physics. In the context of the standard model (SM), *CP* symmetry is explicitly broken at the Lagrangian level through complex Yukawa couplings which lead to *CP* violation in charged weak interactions via the Cabibbo-Kobayashi-Maskawa (CKM) matrix. Among many mechanisms that generate *CP* asymmetry beyond the SM, the possibility that *CP* is spontaneously broken together with the gauge symmetry group is a very attractive scenario [1,2]. One remarkable phenomenological implication of spontaneous *CP* violation (SCPV) is that it provides an appealing solution to the strong *CP* problem [3–10], since the only sources of *CP* violation are the vacuum phases. SCPV can also soften the well known supersymmetry (SUSY) *CP* problem [11,12]. Finally, it is relevant to point out that in perturbative string theory *CP* asymmetry can in principle only arise spontaneously through complex vacuum expectation values (VEVs) of moduli and matter fields [13–15].

In models of spontaneous *CP* violation one starts from a Lagrangian that conserves *CP*, which implies that all parameters of the scalar potential are real. Then, the *CP* asymmetry is achieved spontaneously when the gauge interactions are broken through complex VEVs of Higgs multiplets. In fact, just having complex VEVs is not sufficient to guarantee *CP* violation in the model. One has further to verify that it is not possible to find a unitary transformation, $U$, acting on the Higgs fields as

$$\phi_i \to \phi'_i = U_{ij}\phi_j, \tag{1}$$

such that the following condition holds

$$U_{ij}\langle\phi_j\rangle^* = \langle\phi_i\rangle, \tag{2}$$

while leaving the full Lagrangian invariant. If such a transformation is found, *CP* is a conserved symmetry even in the presence of complex Higgs VEVs.

The main purpose of this Letter is the search for a discrete symmetry that leads to a framework of SCPV where the VEVs of the Higgs multiplets have geometrical values, independently of any arbitrary coupling constants in the scalar potential—i.e. *calculable phases* [16]. If such a symmetry exists calculable phases are stable against radiative corrections [17,18]. It has been shown in Ref. [16] that calculable phases leading to geometrical SCPV require more than two Higgs doublets and non-Abelian symmetries, otherwise it is always possible to find an unitary transformation, $U$, which is a symmetry of the potential and fulfills Eq. (2). The authors found an interesting example of calculable phases with SCPV in the case of three Higgs doublets transforming under the discrete symmetry $\Delta(27)$. In order to find symmetries that generate calculable SCPV, we start by considering the most general $SU(2)\times U(1)$ potential $V(\phi)$ with three Higgs doublets $\phi_i$, having identical hypercharge,

$$\begin{aligned} V(\phi) = {} & \sum_i[-\lambda_i\phi_i^\dagger\phi_i + A_i(\phi_i^\dagger\phi_i)^2] \\ & + \sum_{i<j}\Big[\frac{\gamma_i}{2}(\phi_i^\dagger\phi_j + \text{H.c.},) + C_i(\phi_i^\dagger\phi_i)(\phi_j^\dagger\phi_j) \\ & + \bar{C}_i|\phi_i^\dagger\phi_j|^2 + \frac{D_i}{2}((\phi_i^\dagger\phi_j)^2 + \text{H.c.},)\Big] \\ & + \frac{1}{2}\sum_{i\neq j}[E_{1ij}(\phi_i^\dagger\phi_i)(\phi_i^\dagger\phi_j) + \text{H.c.},] \\ & + \frac{1}{2}\sum_{\substack{i\neq j\neq k \\ j<k}}[E_{2i}(\phi_i^\dagger\phi_j)(\phi_k^\dagger\phi_i) + E_{3i}(\phi_i^\dagger\phi_i)(\phi_k^\dagger\phi_j) \\ & + E_{4i}(\phi_i^\dagger\phi_j)(\phi_i^\dagger\phi_k) + \text{H.c.}], \end{aligned} \tag{3}$$

*ivo.de@udo.edu
†david.costa@ist.utl.pt

 

where the constants $\lambda_i$, $A_i$, $\gamma_i$, $C_i$, $\bar{C}_i$, $D_i$, $E_{2i}$, $E_{3i}$, $E_{4i}$, and $E_{1ij}$, $\forall_{i,j}$, $i, j = 1, 2, 3$ are taken real since $CP$ invariance is imposed at the Lagrangian level. In what follows it is convenient to parametrize the VEVs of the doublets with explicit phases:

$$\langle\phi_1\rangle = v_1 e^{i\varphi_1}, \qquad \langle\phi_2\rangle = v_2 e^{i\varphi_2}, \qquad \langle\phi_3\rangle = v_3 e^{i\varphi_3}, \tag{4}$$

with the requirement of satisfying the experimental constraint from the heavy $W^{\pm}$, $Z$ gauge boson masses:

$$v^2 \equiv v_1^2 + v_2^2 + v_3^2 = (\sqrt{2}G_F)^{-1}, \tag{5}$$

where $G_F$ is the Fermi constant.

Once the Higgs doublets are shifted according to their respective VEVs, only some terms in the potential depend on the phases $\varphi_i$: three $\gamma_i$ terms depend on $-\varphi_i + \varphi_j$; six terms of the type $E_1$, three $E_{2i}$ and three $E_{3i}$, share the same phase-dependence as the $\gamma_i$; three $D_i$ terms have phase-dependence $-2\varphi_i + 2\varphi_j$; and finally, three $E_{4i}$ terms with phase-dependence $\theta_i$,

$$\theta_i \equiv -2\varphi_i + \varphi_j + \varphi_k, \tag{6}$$

where we have assumed $i \neq j \neq k$.

Without any further assumption, there are many different coupling constants and the only calculable phase solution to the extremum conditions is the trivial one. We must therefore consider particular cases that can enable SCPV with calculable phases by reducing the number of parameters either by having coupling constants absent or related. If several coefficients are absent, then there are nontrivial calculable phase solutions that arise from terms which must vanish independently. For example, if only $D_i \neq 0$, there is a solution where

$$-2\varphi_i + 2\varphi_j = 0 \qquad (\mathrm{mod}\ \pi) \tag{7}$$

for $i \neq j$. In contrast, when the coefficients are related, nontrivial calculable phases may appear from cancellations among terms. If all quartic coupling constants share the same value, and the $\gamma$ bi-linears vanish, the extremum conditions admit a solution where the terms combine to make their appropriately weighted sum vanish; the common coupling constant factors out and calculable phases could appear—but the phases turn out to be trivial.

The most elegant way to justify the reduction of parameters of the potential in the search for SCPV with calculable phases is by requiring invariance under discrete non-Abelian symmetries. Therefore we consider $\mathsf{S}_3$, since it is the smallest non-Abelian group and affects the potential given in Eq. (3) by forcing coefficients of the same type to be equal. As the added symmetry commutes with $\mathsf{SU}(2)$ one just needs to compound the known $\mathsf{SU}(2)$ contractions to make them invariant under the added symmetry. The number of independent parameters is then reduced to ten as first proposed in Refs. [19,20]:

$$\lambda = \lambda_i, \quad A = A_i, \quad \gamma = \gamma_i, \quad C = C_i, \quad \bar{C} = \bar{C}_i,$$
$$D = D_i, \quad E_1 = E_{1ij}, \quad E_\alpha = E_{\alpha i}, \quad i, j = 1, 2, 3, \quad \alpha = 2, 3, 4. \tag{8}$$

Applying $\mathsf{S}_3$ invariance to the extremum conditions, solutions are enabled with cancellations in each type of term. These solutions do not give calculable SCPV unless there are further constraints. In the case $D = 0$, the presence of terms with $-\varphi_i + \varphi_j$ and $\theta_i$ dependence does not lead to interesting solutions even if the coupling constants are related. In contrast, if all terms with the $-\varphi_i + \varphi_j$ dependence are absent, there is a solution

$$\langle\phi\rangle^{\mathsf{T}} = \frac{v}{\sqrt{3}}(e^{i\varphi_1}, 1, 1), \tag{9}$$

with $\cos\varphi_1 = -E_4/6D$, which is a calculable phase only if the coupling constants $D$ and $E_4$ are related in some way by the underlying theory. Obtaining such a relationship between these coupling constants is beyond the scope of this Letter. Finally, if only $D \neq 0$ it is interesting to see that the same solutions are obtained that were already possible without $\mathsf{S}_3$.

In order to further reduce the parameters of the $\mathsf{S}_3$ invariant potential, a simple addition of a cyclic $\mathsf{C}_\mathsf{N}$ symmetry acts by eliminating terms. In particular, $\mathsf{C}_3$ can preserve the $E_4$-type terms while excluding every other phase-dependent term as long as each field $\phi_i$ transforms differently under $\mathsf{C}_3$. With this charge assignment the group $\mathsf{S}_3 \ltimes (\mathsf{C}_3 \times \mathsf{C}_3) \equiv \Delta(54)$ gives rise to the same potential as the group $\Delta(27)$ [16], since the only phase-dependent terms present are of the $E_4$ class—in what follows we consider only this kind of potential. We note that the $E_4$ terms are automatically preserved for other discrete subgroups of $\mathsf{SU}(3)$ within the $\Delta(3\mathsf{n}^2)$ [21] and $\Delta(6\mathsf{n}^2)$ [22] families, when $n$ is a multiple of 3; depending on the nature of the group chosen, one has in addition to select an appropriate representation for the Higgs multiplet, and some caution is required in order to avoid the presence of phase-dependent terms other than $E_4$. It is natural that different groups may lead to the same Higgs potential; however, this does not apply in general to the full Lagrangian, e.g. when the fermions are included.

The extremum conditions can be written in terms of the VEVs, $v_i$, and the phases $\theta_i$ defined in Eq. (6):

$$\frac{\partial V}{\partial v_i} = 0 = \lambda + 2Av_i^2 + (C + \bar{C})(v_j^2 + v_k^2) + E_4 v_1 v_2 v_3 \cos\theta_i, \tag{10a}$$

$$\frac{\partial V}{\partial \varphi_i} = 0 = -2v_i \sin\theta_i + v_j \sin\theta_j + v_k \sin\theta_k, \tag{10b}$$

with the restriction $i \neq j \neq k$. For $\lambda > 0$ spontaneous breaking occurs. Our potential is a particular case where the theorem stated in [19] applies and guarantees that the stable minimum is for equal magnitude components. From Eq. (10) we derive that the only VEVs with calculable phases are those presented in [16]:

$$\langle\phi\rangle^{\mathsf{T}} = \frac{v}{\sqrt{3}}(1, \omega, \omega^2), \tag{11a}$$

$$\langle\phi\rangle^{\mathsf{T}} = \frac{v}{\sqrt{3}}(\omega^2, 1, 1), \tag{11b}$$

with the phase $\omega \equiv e^{2\pi i/3}$ and up to cyclical permutations. The solution given in Eq. (11b) is a better candidate for SCPV since it is not removable by any symmetry of the potential, whereas the solution given in Eq. (11a) is.

The structure of the Yukawa couplings is determined by the fermion assignments and is restricted by the allowed contractions. In this work we restrict ourselves to renormalizable operators: $Q\tilde{\phi}u^c$ and $Q\phi d^c$ ($\tilde{\phi} \equiv i\sigma_2\phi^*$). Allowing higher order operators inevitably influences the scalar potential with the square of the $E_4$ term, even though it is present at a much higher order with four additional field insertions. We investigate what happens when placing the fields $\phi_i$ in an irreducible $\Delta(27)$ or $\Delta(54)$ triplet which acquires the VEV given in Eq. (11b) and without loss of generality we place $\phi$ in the $\mathbf{3^1}$ or $\mathbf{3_1^1}$ representation, respectively.

In $\Delta(27)$ we can assign the quark doublets $Q_i$ as a $\mathbf{3^1}$ triplet or as the conjugate triplet representation $\mathbf{3^2}$ or as a combination of the nine possible singlets [21]. We consider simultaneously the first two choices: one of the mass structures will always be given by the $\mathbf{3}^a \times \mathbf{3^a} \times \mathbf{3^a}$ invariant ($a = 1, 2$), with the respective right-handed (rh) quarks forced to be the same triplet $\mathbf{3^a}$ as $Q_i$, and is unable to accommodate the observed fermion hierarchy between the two heaviest generations—this choice had already been pointed out as nonviable in [16]. When the $Q_i$ are singlets, the rh quarks are then forced to be triplets. The structure for both sectors is then $\mathbf{3^1} \times \mathbf{3^2} \times \mathbf{1_{r,s}}$ ($r, s = 0, 1, 2$). The distinct rows of the mass matrix arise from their respective singlets, and both quark sectors share the same type of structure. Within this class we can assign all three $Q_i$ to a single representation (choice I), only one of the generations have a different singlet (choice II), or all three generations are different singlets (choice III)—this leads, respectively, to rank 1 mass matrices with only one nonzero eigenvalue, the decoupling of one generation, or diagonal matrices with three distinct eigenvalues. Notice that $M_u$ and $M_d$ must share the same structure (e.g. both I or II), since the choice of $\Delta(27)$ singlets is made on the $SU(2)$ doublets $Q$. The rank 1 solution of structure I accounts for the hierarchy of the third generation, and the decoupling case of structure II is also promising as a leading order structure. Both structures provide good first order approximations to the observed fermion hierarchy.

We turn now to $\Delta(54)$, where we assign $Q_i$ to its representations: two pairs of conjugate triplets $\mathbf{3_1^{1,2}}$, $\mathbf{3_2^{1,2}}$; four doublets $\mathbf{2_{1,\ldots,4}}$ and the singlets $\mathbf{1}$, $\mathbf{1'}$ [22]. If $Q_i$ is chosen to be a triplet, one quark sector has the mass matrix that arises from three-triplet invariants such as $\mathbf{3_1^a} \times \mathbf{3_1^a} \times \mathbf{3_1^a}$, $\mathbf{3_1^a} \times \mathbf{3_2^a} \times \mathbf{3_2^a}$, or $\mathbf{3_1^a} \times \mathbf{3_1^a} \times \mathbf{3_2^a}$. All these products lead to a structure with two degenerate eigenvalues, but the third product has the nondegenerate eigenvalue vanish. Even when $Q_i$ is a combination of doublet and singlet, the structures again have degenerate states. If the singlet is mismatched with the singlet of the two-triplet product, the nondegenerate value vanishes. Finally when $Q_i$ is a combination of singlets, the structures are rank 1 regardless of choice of singlets and these are the only promising assignments. We note that in an extension to the leptonic sectors, $\Delta(54)$ naturally enables charged leptons with an hierarchical structure and neutrinos with two generations degenerate.

To summarize, in the three Higgs doublet scenario $\Delta(27)$ and $\Delta(54)$ are the smallest groups that lead to complex VEVs with calculable phases stable against radiative corrections [16]. Geometrical SCPV requires the three Higgs to be assigned as a triplet of the respective groups. Within this framework, we have explicitly investigated the possible fermion mass matrices of all classes of renormalizable models. In some cases it is possible to simultaneously obtain promising first order approximations to the observed patterns of fermion masses and mixings of the up and down quark sectors. In conclusion, spontaneous *CP* violation with calculable phases may be viable, and a necessary condition is the correct interplay between the scalar content and an appropriate non-Abelian symmetry group. Since the number of physical scalar states is increased, one expects a richer phenomenology that could be accessible at high energy experiments, such as the Large Hadron Collider.

We thank Gustavo C. Branco for helpful discussion, reading the manuscript and Jean-Marc Gérard for encouragement. This work was partially supported by Fundação para a Ciência e a Tecnologia (FCT, Portugal) through the Projects Nos. CERN/FP/116328/2010 and CFTP-FCT UNIT 777 which are partially funded through POCTI (FEDER) and by PTDC/FIS/098188/2008. The work of I. dM. V. was supported by FCT under the Grant No. SFRH/BPD/35919/2007 and by DFG Grant No. PA 803/6-1.

[1] T. D. Lee, Phys. Rev. D **8**, 1226 (1973).
[2] G. C. Branco, Phys. Rev. Lett. **44**, 504 (1980).
[3] R. N. Mohapatra and G. Senjanovic, Phys. Lett. B **79**, 283 (1978).
[4] H. Georgi, Hadronic J. **1**, 155 (1978).
[5] S. M. Barr and P. Langacker, Phys. Rev. Lett. **42**, 1654 (1979).
[6] A. E. Nelson, Phys. Lett. B **136**, 387 (1984).
[7] S. M. Barr, Phys. Rev. Lett. **53**, 329 (1984).
[8] J. E. Kim, Phys. Rep. **150**, 1 (1987).
[9] R. D. Peccei, Adv. Ser. Dir. High Energy Phys. **3**, 503 (1989).
[10] H.-Y. Cheng, Phys. Rep. **158**, 1 (1988).
[11] S. Abel, S. Khalil, and O. Lebedev, Nucl. Phys. B **606**, 151 (2001).
[12] S. Khalil, Int. J. Mod. Phys. A **18**, 1697 (2003).
[13] E. Witten, Phys. Lett. B **149**, 351 (1984).
[14] A. Strominger and E. Witten, Commun. Math. Phys. **101**, 341 (1985).
[15] M. Dine, R. G. Leigh, and D. A. MacIntire, Phys. Rev. Lett. **69**, 2030 (1992).
[16] G. C. Branco, J. M. Gerard, and W. Grimus, Phys. Lett. B **36**, 383 (1984).
[17] S. Weinberg, Phys. Rev. D **7**, 2887 (1973).
[18] H. Georgi and A. Pais, Phys. Rev. D **10**, 1246 (1974).
[19] E. Derman, Phys. Rev. D **19**, 317 (1979).
[20] E. Derman and H.-S. Tsao, Phys. Rev. D **20**, 1207 (1979).
[21] C. Luhn, S. Nasri, and P. Ramond, J. Math. Phys. (N.Y.) **48**, 073501 (2007).
[22] H. Ishimori *et al.*, Prog. Theor. Phys. Suppl. **183**, 1 (2010).