Improving the tunings of the MSSM by adding triplets and singlet

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We study an extension of the minimal supersymmetric standard model (MSSM) which includes both new $SU(2)$ triplets with hypercharge ± 1 and a standard model gauge singlet (*à la* the next-to-minimal supersymmetric standard model [NMSSM]) which are coupled to each other. We are motivated by the little hierarchy problem, as well as by the μ problem of the MSSM. We show that the NMSSM and the triplet-extended MSSM can successfully solve problems of one another: while triplets are responsible for large correction to the lightest physical Higgs mass, the singlet's vacuum expectation value (VEV) explains why the μ terms (for the Higgs doublets and the new triplets) are naturally of order the electroweak (EW) scale. We also show that singlet-triplet coupling significantly changes the renormalization group evolution of the singlet mass squared, helping to render this mass squared negative, as required for the singlet to acquire a VEV. We analyze constraints on this scenario from EW precision measurements and find that a relatively large region of the parameter space of this model is viable, especially with the triplet fermions (including doubly charged) being light.

I. INTRODUCTION

Weak-scale supersymmetry (SUSY) is a very well-motivated extension of the standard model (SM): it naturally explains why the electroweak (EW) symmetry-breaking scale is much smaller than the Planck scale, it can incorporate a dark matter candidate, and the minimal supersymmetric SM (MSSM) predicts the unification of the three SM gauge coupling constants. However, the MSSM comes with its own shortcomings: it cannot *fully* solve the hierarchy problem since, even with the most optimistic assumptions, it is fine-tuned to the level of 1% (see e.g. [1]). This residual fine-tuning originates in a tree-level prediction that the SM-like Higgs boson in the MSSM is always lighter than the Z boson. Although this result is slightly modified by radiative corrections, it is hard to comply with the LEP2 bounds [2] on the Higgs mass, without rendering the stops unnaturally heavy.

Another puzzle of the MSSM has to do with the μ term, which is required to be of order EW scale. Since this term is completely supersymmetric and *a priori* has nothing to do with SUSY breaking, it is not easy to explain this coincidence of scales. While the Giudice-Masiero mechanism [3] can provide a solution for high-scale SUSY breaking, finding such a solution, for example, for low-scale SUSY breaking is much harder.

Of course, extensions of the MSSM have been proposed to solve these problems. For example the extension of the MSSM by addition of a (SM) gauge singlet which is coupled to Higgs doublets (the next-to-minimal supersymmetric SM [NMSSM]) (see [4,5] for review) has been proposed to solve the μ problem. The idea is that a bare μ term is forbidden, while an effective μ term is *dynami-*

cally generated by a vacuum expectation value (VEV) of the singlet. Thus the effective μ term can naturally coincide with the scale of soft SUSY breaking. However, in practice, it is typically difficult to realize a tachyonic singlet as is required for it to get a VEV. It is true that the above-mentioned coupling of singlet to Higgs doublet tends to drive the singlet mass squared negative in renormalization group evolution (RGE), but the up-type Higgs doublet mass squared is being driven negative in its own RGE from UV to IR, precisely as it happens in a regular MSSM. The tachyonic H_u tends to make the singlet mass squared more *positive* in the latter's RGE mentioned above.[1]

Unfortunately the NMSSM with all the couplings being perturbative[2] up to the GUT scale also does not really ameliorate the little hierarchy problem [7–12]. This contradicts a naive expectation that the little hierarchy problem can be addressed in the NMSSM due to the extra Higgs quartic, which arises from the interaction with the singlet. A reason for this "disappointment" is that the extra quartic coupling for the Higgs doublets, which directly contributes to the physical Higgs mass, is suppressed in the large $\tan\beta$ limit. The problem is that these are precisely the values of $\tan\beta$ where the tree-level MSSM quartic coupling for Higgs doublets (and thus the Higgs mass) tends to be maximized. Moreover, in the NMSSM there is an

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[1]Of course, the resulting tachyonic H_u is otherwise a feature (rather than a bug) since it results in *radiative* electroweak symmetry breaking.

[2]For the discussion of the NMSSM with large coupling between singlet and doublets, see [6].

additional *negative* contribution to the physical Higgs mass squared which tends to cancel the positive effect of the extra Higgs quartic. This effect arises due to singlet-doublet mass mixing term, which is proportional to the singlet-doublet coupling and singlet VEV. The point is that the singlet mass is also proportional to the singlet VEV so that this negative contribution does not decouple with the singlet VEV.[3] Therefore if one takes the little hierarchy problem seriously, then we should consider another source for the Higgs quartic coupling, which would not decouple in the large $\tan\beta$ limit.

One of the models which naturally possesses such a Higgs quartic coupling is the extension of the MSSM by the addition of $SU(2)$ triplets (dubbed TMSSM) [14–16]. This model is especially attractive when the triplets with non-zero hypercharge are included, such that they can couple to H_u only (unlike a NMSSM singlet or triplet with a zero hypercharge [14,17–20]). In this case, the extra quartic coupling for the Higgs boson is unsuppressed in the large $\tan\beta$ limit such that the stops significantly lighter than 1 TeV can be consistent with the LEP bounds on the Higgs mass. The second bug of the NMSSM in this regard is also avoided by the TMSSM. The triplet VEV is required to be small, of order a few GeV (see e.g. [21]) in order to be consistent with the ρ parameter, and so is the mass mixing term between doublets and triplets (arising in analogy to that in the NMSSM). The point is that the triplet mass term is not proportional to its VEV, and thus can still be large so that the resulting negative contribution to the physical Higgs mass squared can be negligible.

In this paper we propose combining these two extensions of the MSSM (namely NMSSM and TMSSM) showing that they can solve one another's problems if we couple the triplets to the singlet. Evidently the TMSSM introduces an additional μ problem (for the triplet), but this can be solved by the singlet VEV, along the lines of the solution to the usual μ problem. On the other hand, we show that the couplings of the singlet to the triplet help drive the soft mass^2 of the singlet negative: what is crucial here is that the triplet is not driven tachyonic in the IR, unlike H_u in the case of NMSSM mentioned above. Thus we can solve the problem of getting sufficiently large singlet VEV of the NMSSM. We also do not run into the usual problems of the NMSSM as far as the little hierarchy is concerned since the singlet interactions do not play any important role in raising the physical Higgs mass: we rely on triplets instead in achieving this goal.

Our paper is organized as follows. In Sec. II we present the basics of the model in terms of parameters at the weak scale and discuss the minimization conditions of the extended Higgs potential. We show that this model indeed

addresses the little hierarchy problem such that even the tree-level mass of the Higgs boson can easily evade the LEP bounds. In Sec. III we perform further analysis of the model. We start by discussing the constraints on the model from EW precision tests, mainly the ρ parameter. It is well-known that models with EW triplets in general are subject to stringent constraints from EW precision tests since they *a priori* have a new contribution to the ρ parameter from the triplet VEV. A neutral component of the triplets always acquires a VEV in these models. A trivial solution to this problem is of course to render the entire triplet (superfield) heavy, which has been discussed in detail in the literature. We concentrate instead on another part of the parameter space, where the soft masses of the triplet scalars are relatively big, but the associated μ term is small. We show that the physical Higgs mass is larger in this region due to a suppression of the negative contribution to it from triplet-doublet mixing driven by the triplet μ term. In Sec. IV we discuss the renormalization group evolution of this model to higher energy scales and discuss its implications. Finally in Sec. V we conclude. Important RGE equations are summarized in the Appendix.

II. THE MODEL

As in any supersymmetric theory which is broken softly, the Lagrangian of the next-to-minimal supersymmetric standard model with triplets (TNMSSM) (which is being proposed here) is characterized by the superpotential, the supersymmetric gauge interactions, and the various soft-breaking couplings (soft masses and trilinear terms).

To begin with, we consider the terms in the superpotential of the TNMSSM. As we have already mentioned in the introduction, the terms in the Higgs sector depend exclusively on the gauge singlet superfield S, the $SU(2)_L$ triplet superfields T and $\bar{T}$, and the MSSM Higgs doublets, $H_{u,d}$. In addition, the model contains only dimensionless Yukawa couplings which will be given below, i.e., there exist no dimensionful supersymmetric parameters such as μ and μ_T (μ terms for the doublets and triplets, respectively) in the superpotential. The superpotential of the Higgs sector is given as follows:

$$W_{\text{Higgs}} = S(\lambda H_u \cdot H_d + \lambda_T \,\text{tr}(\bar{T}T))$$
$$+ \frac{\kappa}{3} S^3 + \chi_u H_u \cdot \bar{T} H_u + \chi_d H_d \cdot T H_d, \quad (1)$$

where λ, λ_T, κ, χ_u, and χ_d are dimensionless Yukawa couplings. Note that compared to the MSSM, there is an additional physical *CP*-violating phase coming from the superpotential, $\text{Arg}(\chi_u\chi_d\kappa\lambda_T^*(\lambda^*)^2)$. However, we defer from studying constraints from electric dipole moments (EDMs) (for example) on this new phase; instead, in this paper, we simply assume *CP* conservation.

As usual, one should add the Yukawa couplings of the quark and the lepton superfields,

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[3]However, the MSSM with singlet can solve the little hierarchy problem if one *does not insist* that the singlet gets a sufficient VEV to solve the μ problem [13].

$$W_{\text{Yukawa}} = h_u H_u \cdot Q\bar{u} + h_d H_d \cdot Q\bar{d} + h_e H_d \cdot L\bar{e}. \quad (2)$$

Here the triplet superfields with hypercharge $Y = \pm 1$ are defined as follows:

$$T \equiv T^a \sigma^a = \begin{pmatrix} T^+/\sqrt{2} & -T^{++} \\ T^0 & -T^+/\sqrt{2} \end{pmatrix}, \quad (3)$$

$$\bar{T} \equiv \bar{T}^a \sigma^a = \begin{pmatrix} \bar{T}^-/\sqrt{2} & -\bar{T}^0 \\ \bar{T}^{--} & -\bar{T}^-/\sqrt{2} \end{pmatrix}, \quad (4)$$

where σ^a are the usual 2×2 Pauli matrices, and the respective definitions of the products between two $SU(2)_L$ doublets and between a $SU(2)_L$ doublet and a $SU(2)_L$ triplet are given as follows:

$$H_u \cdot H_d = H_u^+ H_d^- - H_u^0 H_d^0, \quad (5)$$

$$H_u \cdot \bar{T} H_u = \sqrt{2} H_u^+ H_u^0 \bar{T}^- - (H_u^0)^2 \bar{T}^0 - (H_u^+)^2 \bar{T}^{--}, \quad (6)$$

$$H_d \cdot T H_d = \sqrt{2} H_d^- H_d^0 T^+ - (H_d^0)^2 T^0 - (H_d^-)^2 T^{++}. \quad (7)$$

The soft terms in the Lagrangian include

$$\begin{aligned}
-\mathcal{L}_{\text{soft}} = {} & m_{H_u}^2 |H_u|^2 + m_{H_d}^2 |H_d|^2 + m_S^2 |S|^2 + m_T^2 \, \text{tr}(|T|^2) \\
& + m_{\bar{T}}^2 \, \text{tr}(|\bar{T}|^2) m_Q^2 |Q|^2 + m_{\bar{u}}^2 |\bar{u}|^2 + m_{\bar{d}}^2 |\bar{d}|^2 \\
& + m_L^2 |L|^2 + m_{\bar{e}}^2 |\bar{e}|^2 (A_{h_u} Q \cdot H_u \bar{u} - A_{h_d} Q \cdot H_d \bar{d} \\
& - A_{h_e} L. H_d \bar{e} A S H_u \cdot H_d + A_T S \, \text{tr}(T\bar{T}) + \frac{A_k}{3} S^3 \\
& + A_u H_u \cdot \bar{T} H_u + A_d H_d \cdot T H_d + \text{H.c.}). \quad (8)
\end{aligned}$$

Note that, in addition to R parity, we explicitly imposed a $\mathbb{Z}_3$ symmetry which forbids bare μ and $B\mu$ terms both for Higgs doublets and triplets.[4] The effective μ, $B\mu$ terms form when S gets a VEV. We also assume that all A terms are small for simplicity, and thus one can expect a light pseudoscalar, the R axion, which will be discussed in detail in Sec. III B. This assumption is also well motivated in the context of flavor-safe mediation mechanisms for SUSY breaking, for example, gauge mediation.

As advertised in the introduction, one can briefly see that this model, i.e., combining the singlet and triplet extensions of the MSSM, can solve both μ problem(s) and little hierarchy problem. For the μ problem(s), like in the NMSSM, a vacuum expectation value v_s of singlet of the

order of the weak or SUSY-breaking scale will generate an effective μ term for the Higgs doublet and the triplet with

$$\mu^{\text{eff}} = \lambda v_s \qquad \mu_T^{\text{eff}} = \lambda_T v_s. \quad (9)$$

For the little hierarchy problem, clearly one can see that the coupling of triplet to *up*-type Higgs H_u introduces the extra quartic couplings for Higgs without any mixture with *down*-type Higgs H_d in the Higgs (tree-level) potential,

$$V_{\text{Higgs}} \ni \sim \chi_u^2 (H_u)^4. \quad (10)$$

In the next section, we will see that this leads to an enhancement of SM-like Higgs mass even in the large $\tan\beta$ limit.

A. SM-like vacuum of TNMSSM

Plugging the VEVs into the full Higgs potential one gets

$$\begin{aligned}
V_{\text{Higgs}} = {} & (2\chi_u v_u v_{\bar{T}} + \lambda v_s v_d)^2 + (2\chi_d v_d v_T + \lambda v_s v_u)^2 \\
& + (\kappa v_s^2 - \lambda v_u v_d - \lambda_T v_T v_{\bar{T}})^2 + (\chi_u v_u^2 + \lambda_T v_s v_T)^2 \\
& + (\chi_d v_d^2 + \lambda_T v_s v_{\bar{T}})^2 + \frac{g^2 + g'^2}{8}(v_u^2 - v_d^2 + 2v_T^2 \\
& - 2v_{\bar{T}}^2)^2 + m_{H_u}^2 v_u^2 + m_{H_d}^2 v_d^2 + m_S^2 v_s^2 + m_T^2 v_T^2 \\
& + m_{\bar{T}}^2 v_{\bar{T}}^2 - 2A v_s v_u v_d - 2A_T v_s v_T v_{\bar{T}} + \frac{2}{3} A_\kappa v_s^3 \\
& - 2A_u v_u^2 v_{\bar{T}} - 2A_d v_d^2 v_T, \quad (11)
\end{aligned}$$

where $g'(\approx 0.35)$ and g denote $U(1)_Y$ and $SU(2)_L$ gauge couplings, respectively. In the TNMSSM, the mass of the Z boson has the same form as in the MSSM, but the electroweak symmetry breaking (EWSB) VEV for the doublets is modified due to the presence of triplet VEVs,

$$M_Z^2 = \frac{g'^2 + g^2}{2} v^2 \equiv \hat{g}^2 v^2, \quad (12)$$

$$v^2 = v_u^2 + v_d^2 + 4v_T^2 + 4v_{\bar{T}}^2 \approx (174 \text{ GeV})^2, \quad (13)$$

and $\tan\beta$ is defined by the ratio of v_u to v_d as usual: $\tan\beta \equiv v_u/v_d$.[5]

Since we introduced a singlet and two triplets, we have five minimization equations including the ones for usual up- and down-Higgses. In general, the vacuum expectation values for the triplets must be small to avoid large ρ parameter correction, which will be fully investigated in Sec. III A. Assuming small VEVs for the triplets, i.e., v_T, $v_{\bar{T}} \approx 0$, one can easily derive the following relation for the ratio of the v_u, v_d using minimization equations for H_u and H_d:

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[4]For the study of NMSSM without $\mathbb{Z}_3$ symmetry, see [22].

[5]Given that in our case $(v_u^2 + v_d^2)$ does not sum to the measured value of the SM Higgs VEV[2] (unlike in the MSSM or even the NMSSM), this definition might be somewhat misleading. Nonetheless, since we know that the corrections from the triplet VEVs are small (as required by the T parameter), we will still loosely use this definition.

$$\frac{v_u v_d}{v_u^2 + v_d^2} = \frac{1}{2} \sin 2\beta = \frac{v_s(\lambda \kappa v_s + A)}{2(\chi_u^2 v_u^2 + \chi_d^2 v_d^2) + \lambda^2(2v_s^2 + v_u^2 + v_d^2) + m_{H_u}^2 + m_{H_d}^2}. \tag{14}$$

Note that this relation reduces to the usual NMSSM relation in the limit χ_u, $\chi_d = 0$ as expected. In order to have nonzero v_u and v_d the numerator should not vanish, i.e., $\lambda \kappa v_s + A \neq 0$. Indeed, the TNMSSM accommodates such a nonzero numerator. To see this, one notices that a large v_s is required in order to generate a sufficiently large effective μ term [see Eq. (9)] like the case of the NMSSM [4]. In addition, since small A terms are assumed as mentioned before, this condition for nonvanishing v_u and v_d is readily attained in most of the parameter space.

One can easily find that the minimization condition for v_s reduces to the corresponding NMSSM-like form under the assumption of small VEVs for the triplets [4],

$$(2\kappa^2 v_s^2 + \lambda^2(v_u^2 + v_d^2) - \kappa \lambda v_u v_d + A_\kappa v_s + m_S^2)v_s = 0. \tag{15}$$

One can see that for small A_κ and $v_s \gtrsim v_d$, one should demand $m_S^2 < 0$. Here we simply assume this condition, but we show in Sec. IV that our model can naturally have this feature.

B. Higgs mass in the TNMSSM

To begin the discussion of the SM-like Higgs mass in the TNMSSM, one may consider a few interesting limits, depending on the hierarchy between the soft mass term and the μ term for triplets. As we will discuss in more detail in Sec. III A, one is required to have large masses for the triplets to avoid a significant correction to the ρ parameter. To obtain large triplet scalar masses, either a soft mass for triplets, or a μ term for triplets, or both of them should be large enough (again see Sec. III A for details) so that we will discuss three distinct cases. One of them has large μ_T and small triplet soft masses, which implies that the triplets can be integrated out in the supersymmetric limit. The opposite regime is the highly nonsupersymmetric one, where $m_T \gg \mu_T$, and we will find it to be the most interesting phenomenologically. One can think about the third regime, $m_T \sim \mu_T$ as a kind of an intermediate case.

It is easy to estimate the Higgs mass in the limiting cases, and the intermediate case can be understood as the admixture of the two extreme limits. Let us begin with the limit where the soft mass is dominant and the μ_T is small. The tree-level Higgs mass spectra, in general, can be obtained by diagonalizing the associated mass mixing matrices. To find the mass of the SM-like Higgs boson we have to (typically) find the lightest eigenvalue of the (5×5) mass matrix of the CP-even neutral scalars. The associated calculations will be made numerically in our parameter scans, but in order to develop some intuition we

can look at the following analytical upper bound on the mass of the lightest Higgs boson [14]:

$$m_{h^0}^2 \leq M_Z^2 \Big(\cos^2 2\beta + \frac{\lambda^2}{\hat{g}^2} \sin^2 2\beta + \frac{\chi_d^2}{\hat{g}^2} \cos^4 \beta$$
$$+ \frac{\chi_u^2}{\hat{g}^2} \sin^4 \beta \Big). \tag{16}$$

Note that the last term in Eq. (16) is proportional to $\sin^4 \beta$ (originating from the coupling of only the up-type Higgs to triplet) which is good for solving the little hierarchy problem since it is maximized precisely where the SM-like Higgs mass is in the MSSM, i.e., $\beta \to \frac{\pi}{2}$. On the contrary, as mentioned in the introduction, in the NMSSM, the enhancement of the Higgs mass at the tree level is suppressed in this large $\tan\beta$ limit [see the second term in Eq. (16)]. Also note that the bound Eq. (16) is saturated in the limit of small mixing between doublets and singlet or triplets. Such mixing arises from various F terms and reduces the mass of the Higgs boson below the bound in Eq. (16). In the case of NMSSM [4], the doublet-singlet mixing from the F term of H_d then results in the following estimate for the Higgs mass (in the large $\tan\beta$ limit):

$$m_{h^0}^2 \sim M_Z^2 - \frac{\lambda^4 v^2}{\kappa^2}. \tag{17}$$

Now let us look at the effects coming from the doublet-triplet mixing in the TNMSSM. The effects of doublet-triplet mixing from F terms of singlet and H_u are proportional to the VEV of the triplet. Since bounds on the T parameter require the VEV of the triplet to be very small (see discussion in Sec. III A), this mixing is negligible. Similarly, the doublet-triplet mixing from the F terms of $H_{u,d}$ (which is proportional to μ term for doublets) is suppressed in the large $\tan\beta$ limit (which is our interest here). Thus, the most important contribution to the doublet-triplet mixing arises from the F term of the triplet

$$|F_T|^2 \sim \chi_u \mu_T \bar{T}^\dagger H_u H_u + \text{H.c.}, \tag{18}$$

which results in a shift in the Higgs mass,

$$\delta m_{h^0}^2 \sim -\frac{(\chi_u \mu_T v_u)^2}{\mu_T^2 + m_T^2}. \tag{19}$$

However in the limit $m_T \gg \mu_T$ this effect (which does not depend on the triplet VEV) is suppressed and we can estimate the overall correction to the Higgs mass to be

$$m_{h^0}^2 \sim M_Z^2 - \frac{\lambda^4 v^2}{\kappa^2} + \chi_u^2 v^2. \tag{20}$$
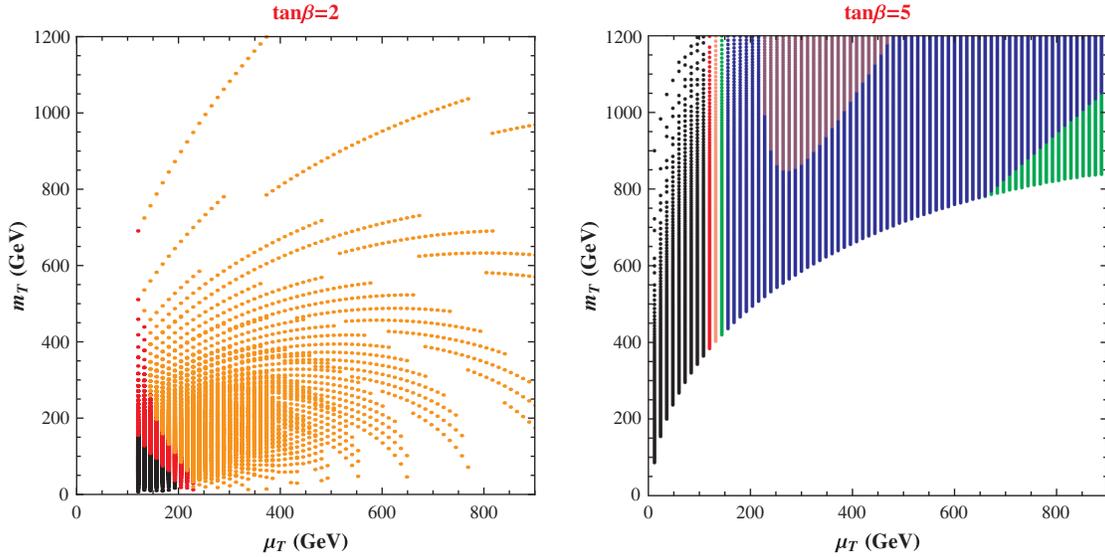
FIG. 1 (color online).   Contour plots of the SM-like Higgs mass (including one-loop radiative correction from stop) in the plane of $m_T \approx m_{\tilde{T}}$ and μ_T. The relevant parameters (see text for an explanation) are chosen as follows: $\lambda = 0.30$, $\kappa = 0.30$, $\lambda_T = -0.60$, $\chi_u = -0.30$, $\chi_d = -0.55$, $m_{\tilde{t}} = 300$ GeV and all A terms are assumed to vanish. The black, the red, and the orange points correspond to the Higgs mass in the ranges $[0, 110]$ GeV, $[110, 114.4]$ GeV, and $[114.4, 120]$ GeV, respectively. For the remaining colors (in the order pink, green, blue, and purple), the upper end points of this range are increased successively by 5 GeV, i.e., pink is $[120, 125]$ GeV and so on. On the gray scale, black corresponds to the darkest part of the grid, while green corresponds to the brightest part. Note that the large blank region on the plot with $\tan\beta = 5$ is due to the constraint from the ρ parameter. This constraint is weakened for the case of $\tan\beta = 2$ because of an accidental cancellation between F terms of the $\tilde{T}$ and H_d which results in a smaller VEV for the triplet. The vacant regions appearing for large soft masses are an artifact of the numerical calculation.

We see that by choosing appropriate values of the couplings λ, χ_u, and κ (for example, $\chi_u \sim 0.5 > \lambda \sim \kappa$), we can easily be above the LEP2 bound on the Higgs mass (even at tree level).

On the other hand, the estimate of the Higgs mass in the opposite limit, where μ_T is dominant, is different. In this case, the shift in Higgs mass given in Eq. (19) actually cancels the last term in Eq. (16).[6] An equivalent way to see this cancellation is that, in this limit, the triplets must be supersymmetrically integrated out, and thus their remnant effects appear via nonrenormalizable effective superpotential [23–26]. After the supersymmetric integrating out is performed, one finds no triplet contribution in the Higgs mass proportional to $\sin^4\beta$, but rather

$$\delta m_{h^0}^2(\text{triplet only}) \simeq \frac{\mu}{\mu_T}\chi_u\chi_d v^2 \sin2\beta$$
$$= \frac{\lambda}{\lambda_T}\chi_u\chi_d v^2 \sin2\beta + \cdots, \qquad (21)$$

where the dots stand for corrections, coming from hard SUSY-breaking terms, proportional to powers of m_T/μ_T. Clearly, this contribution to the SM-like Higgs mass favors moderate $\tan\beta$ (i.e., $\beta \sim \pi/4$), where the MSSM contribution is not saturated while the NMSSM contribution is saturated.

It is interesting to compare the size of the correction to the Higgs mass due to the triplets only (i.e., ignoring the singlet) in SUSY to the non-SUSY limit,

$$R \equiv \frac{(\delta m_{h^0}^2)_{\text{SUSY}}}{(\delta m_{h^0}^2)_{\text{non-SUSY}}} \sim \frac{\frac{\lambda}{\lambda_T}\chi_u\chi_d v^2 \sin2\beta}{M_Z^2(\frac{\chi_d'^2}{\tilde{g}^2}\cos^4\beta' + \frac{\chi_u'^2}{\tilde{g}^2}\sin^4\beta')}$$
$$= \frac{r_\lambda r_\chi \sin2\beta}{r_\chi^2\cos^4\beta' + \sin^4\beta'}, \qquad (22)$$

where $r_\lambda = \lambda/\lambda_T$ and $r_\chi = \chi_d\chi_u/\chi_u'^2$. To compare the maximum contributions in the respective limits, one should take $\beta \to \pi/4$ and $\beta' \to \pi/2$ so that R is simplified to $R = r_\lambda r_\chi$. Since in the SUSY limit μ_T is larger than the weak scale (whereas μ is at the weak scale for naturalness), we have $r_\lambda \ll 1$. So, unless we choose the χ-type couplings in the two limits so that $r_\chi \gg 1$, we see that it is more difficult to get a larger size of Higgs mass correction by introducing the triplets in the SUSY limit than that in the nonsupersymmetric limit.

Figure 1 is a contour plot of SM-like Higgs mass in the TNMSSM in the plane of triplet soft mass term versus μ term for the triplet. For the scan we take the one-loop correction from stop into consideration [27],

$$\delta m_h^2 \sim \frac{3}{4\pi^2}\sin^2\beta y_t^2 m_t^2 \ln\left(\frac{m_{\tilde{t}_1}m_{\tilde{t}_2}}{m_t^2}\right), \qquad (23)$$

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[6]See Ref. [19] for a similar discussion in the model with a zero-hypercharge triplet.

neglecting any stop mixing in order to be conservative. Namely, the mixing between $\tilde{t}_R$ and $\tilde{t}_L$ pushes the Higgs mass even higher so that we are generically underestimating the Higgs mass. We show only the points which respect the ρ-parameter constraint which will be discussed in detail in Sec. III A 1. Note that we completely neglect the A terms for all of the Higgs fields in this scan. From this figure, we see that (as expected from the estimates above) it is easier to get large enhancement of the physical Higgs mass (namely $m_h > 120$ GeV) when $\tan\beta$ is large *and* when there is a large hierarchy between the soft triplet mass term and the μ term for the triplet. It does not necessarily mean that moderate $\tan\beta$ is excluded by the Higgs mass limit, but getting $m_h > 120$ GeV in this case is difficult.

C. Remarks about mass spectrum

Analysis of a full mass spectrum (i.e., including squarks and sleptons) is highly model dependent and heavily relies on underlying assumptions about the mediation scheme. Here we just make several comments regarding the spectrum of EW Higgses, charginos, and neutralinos, which directly follows from our previous considerations of the Higgs sector.

We begin with the spectrum of the neutral scalars and pseudoscalars. One can easily derive the relevant mass matrix elements $(M_0^2)_{ij}$ in the basis of $(H_u^0, H_d^0, T^0, \bar{T}^0, S)$. We notice that in the limit of the vanishing A terms and gaugino masses our Lagrangian is invariant under extra $U(1)_R$ symmetry under which all the chiral fields carry a charge 2/3. This leads to the appearance of the light

pseudoscalar, i.e., the pseudo-Nambu-Goldstone boson associated with the spontaneous breaking of this symmetry. More detailed discussion of this R axion is left to Sec. III B.

Moving onto charged scalar particles, in the TNMSSM these are of two types: singly charged and doubly charged. In the singly charged Higgs sector, i.e., $(M_\pm^2)_{ij}$, the basis consists of H_u^+, H_d^{-*}, T^+, $\bar{T}^{-*}$. For the doubly charged Higgs sector, there are contributions only from triplets (i.e., T^{++} and $\bar{T}^{--*} = \bar{T}^{++}$), and thus the associated mass matrix $(M_{2\pm}^2)_{ij}$ is simply a 2×2 matrix.

Similarly, the fermionic mass matrices can be constructed. The EW gauginos contribute in the neutral and singly charged sectors, but not in the doubly charged sector (where only the triplet contributes, just like for scalars above).

As an illustration we show in Fig. 2 representative spectra of Higgses, charginos, and neutralinos in all the three different cases that we mentioned above. In all these cases we take 200 GeV and 220 GeV for the $U(1)_Y$ and $SU(2)_L$ gaugino masses, respectively, and the other input parameters are tabulated in Table I. Regarding the Higgs mass correction, we again include a one-loop correction from the stop for the SM-like Higgs mass. Conforming to the usual convention, we denote scalars and fermions by dashed lines and solid lines, respectively. States labeled H correspond to CP-even or charged Higgses depending on their electric charge. In the same manner, the labels A, N, and C correspond to CP-odd Higgses, neutralinos, and charginos, respectively.

Of course, we can choose the gaugino masses to be larger than what we assumed here, i.e., closer to the TeV
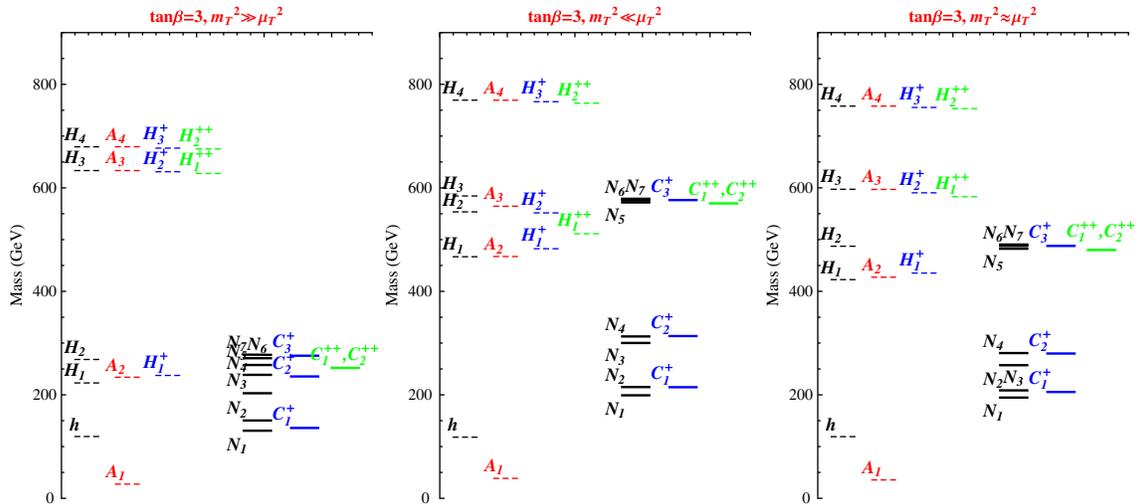

FIG. 2 (color online). Sample particle spectra for the three cases in the TNMSSM. The dashed and the solid lines denote scalars and fermions. H, A, N, and C stand for CP-even/charged Higgses, CP-odd Higgses, neutralinos, and charginos, respectively (with superscripts indicating electric charges). The mass spectrum for the lightest CP-even Higgs boson includes the one-loop radiative correction from stop (we assumed $m_{\tilde{t}_R} = m_{\tilde{Q}_3} = 300$ GeV), whereas the other masses are tree level. The light state A_1 corresponds to the R axion which is further discussed in Sec. III B.

TABLE I.   Parameter sets chosen for the three cases.

Items in GeV²	$m_{H_u}^2$	$m_{H_d}^2$	m_S^2	m_T^2	$m_{\bar{T}}^2$	μ_T^2
Non-SUSY limit	-132^2	177^2	-181^2	601^2	601^2	252^2
SUSY limit	-245^2	401^2	-404^2	312^2	312^2	570^2
Intermediate case	-212^2	338^2	-340^2	472^2	472^2	480^2
SUSY parameters	$\chi_u = -0.25,\ \chi_d = -0.55,\ \lambda = 0.3,\ \kappa = 0.3,\ \lambda_T = -0.6$					
A terms (GeV)	$A_u = -0.1,\ A_d = 0.1,\ A = 0.9,\ A_\kappa = 0.5,\ A_T = -0.1$					

scale, since they do not (at least directly) enter the consideration of the little hierarchy problem that we had so far. Gauginos contribute to neutralino and singly charged fermion spectra so that some of these particles can be heavier than shown in Fig. 2. However, in the non-SUSY limit, μ_T is always at the weak scale and the triplet has an admixture in all three (i.e., neutral, singly charged, and doubly charged) fermionic sectors, so that (at least) one eigenvalue in all three sectors is always at the weak scale. In the SUSY limit, μ_T is larger, but the μ term for doublets (which contribute to the neutral and singly charged fermionic sectors) is still at the weak scale (based on the usual consideration of naturalness of the weak scale). Thus, (at least) one eigenvalue in the neutralino and singly charged chargino sector is always at the weak scale in this case, but the doubly charged fermions are always heavier.

III. ANALYSIS OF THE MODEL

A. Electroweak precision tests in the models with triplets

One of the most stringent tests on new physics models comes from the EW precision observables. The VEV of the electroweak triplet will modify the relation between the masses of the W and Z bosons from the one in the SM and is thus constrained to be very small. On the other hand, once the MSSM doublet H_u and singlet S get VEVs, the F term of the triplet inevitably leads to the tadpole term for the triplet,

$$|F_{\bar{T}}|^2 \sim \chi_u \lambda_T H_u H_u S^\dagger T^\dagger. \qquad (24)$$

So even in the case when the triplet soft mass is not tachyonic, this tadpole will result in the triplet VEV of the order

$$\langle T \rangle \sim \frac{\chi_u \mu_T v_u^2}{(m_T^2 + \mu_T^2)}, \qquad (25)$$

where μ_T is the triplet effective μ term $\mu_T = \lambda_T v_S$.[7] So we can see that the only way to accommodate the experi-

mental data is to make the total mass $\sqrt{\mu_T^2 + m_T^2}$ of the scalar triplet large.[8] The limit where only μ_T is large corresponds to the supersymmetric integrating out of the triplet considered in [24,25] and discussed in Sec. II. This limit was shown to be viable and it can ameliorate the little hierarchy problem.

As mentioned in Sec. II, here we will be interested in a different region, when μ_T is of order soft masses or even smaller. This can give large corrections to the Higgs quartic for *any* $\tan\beta$ (as discussed in Sec. II), but we should of course check the constraint from the T parameter which we do in detail in this section. A similar model with triplets but without singlet (TMSSM), where the μ terms for Higgses and triplets are just bare terms, has very similar properties in the EW symmetry-breaking sector. So, in order to understand better the qualitative features of the parameter space of the TNMSSM, we will first analyze the EW precision physics in the TMSSM.

1. EW precision in TMSSM

We first review the bounds on the VEV of the triplet coming from ρ parameter. For example, the triplet with hypercharge -1 will have a VEV of the following form:

$$\langle T \rangle = \begin{pmatrix} 0 & 0 \\ v_T & 0 \end{pmatrix}. \qquad (26)$$

Then the contribution to the Peskin-Takeuchi T parameter will be

$$\delta T = \frac{1}{\alpha} \frac{m_{W_1}^2 - m_{W_3}^2}{m_W^2} = -\frac{1}{\alpha} \frac{2v_T^2}{v^2}, \qquad (27)$$

so that the constraint $T \gtrsim -0.1$ requires

$$v_T^2 \lesssim (4 \text{ GeV})^2. \qquad (28)$$

On the other hand, we estimated in (25) a triplet VEV in large $\tan\beta$ limit,

$$v_T \approx 4 \times \frac{(\frac{\mu_T}{130})}{(\frac{m_T}{600})^2 + (\frac{\mu_T}{130})^2} \left(\frac{\chi_u}{0.4}\right)\left(\frac{v_u}{174}\right)^2. \qquad (29)$$

---

[7]There is also a tadpole for a triplet from the F term of $H_{u,d}$, which is proportional to the μ term for doublets, but it is suppressed in the large $\tan\beta$ limit (which is our interest here). See also Ref. [19] for a similar discussion, including a detailed analysis of constraint from the T parameter, in the model with *zero*-hypercharge triplet.

[8]Note that demanding a small triplet VEV does not necessarily mean tuning because this VEV is triggered by a dynamical tadpole (and not by a tachyonic mass). We will further justify this point using Eq. (29).
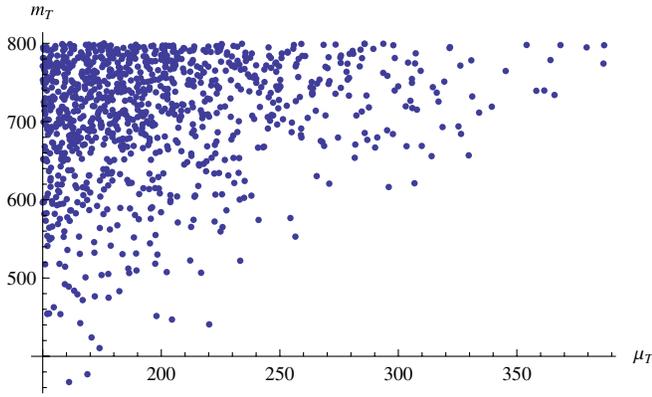
FIG. 3 (color online).   Sample viable points of the TMSSM in the plane of soft SUSY breaking ($m_T \approx m_{\bar{T}}$) and supersymmetric mass terms for triplets (μ_T). Every point on the plot has a tree-level T parameter consistent with data and *tree-level* Higgs mass above 110 GeV. The other parameters (see text for an explanation) are varied in the following ranges: $\chi_{u,d} \in [-0.5, 0.5]$, μ terms—$\in [150, 400]$ GeV, and $B\mu$ terms for doublets and triplets: $\in [-500^2, 500^2]$ GeV^2, $\tan\beta = 10$.

So we can see that with the soft mass of the order ~ 600 GeV[9] and smaller μ_T (i.e., with only a mild hierarchy between the two triplet mass terms), we can accommodate the bound from the T parameter. In this case, the estimate for Higgs mass is given by Eq. (16), except that we drop the singlet contribution (second term) here. Thus, we obtain a large enhancement of the SM-like Higgs mass. Another possibility for obtaining small v_T is μ_T being

larger than weak scale (and smaller m_T). Here, the enhancement in Higgs mass is smaller; see Eq. (21). We have checked these estimates by numerical calculations shown in Fig. 3 for ($\tan\beta = 10$), where one can indeed see that points with small μ_T for triplets and large triplet soft masses are preferred. In fact, this plot is similar to that for the TNMSSM, i.e., adding the singlet, shown in Fig. 1 so that the lesson here is that the singlet is not so relevant for consideration of the little hierarchy problem and the T parameter.

2. One-loop analysis

So far we have been focusing only on the tree-level effects in EW precision tests arising from the VEV of the scalar triplet. Let us see now what will happen at one-loop level. We know that the new fermionic and bosonic states will also contribute at the loop level to the S and T parameters. This contribution arises from the diagrams, where components of the triplet and doublet fields mix due to the Higgs VEV. However in the non-SUSY limit that is our focus, the one-loop contributions with doublet-triplet scalar mixing are suppressed by the large soft mass of the triplets, but the same is not true for the fermions as follows. In order to maximize the increase in Higgs mass, we need to be in the region of the parameter space with small μ_T so that generically contributions of the fermion loops with triplet fermion-Higgsino mixing are important. The mass matrix for the neutralino fields (treated as two-component/Weyl spinors) will be given by

$$\hat{H}^T M_{mf} \hat{H} = (\tilde{H}_u^0, \tilde{H}_d^0, \tilde{T}^0, \tilde{\bar{T}}^0, \lambda_1, \lambda_2^3) \begin{pmatrix} -2\chi_u v_{\bar{T}} & -\mu & 0 & -2\chi_u v_u^{\sim} & \frac{g' v_u}{\sqrt{2}} & -\frac{g v_u}{\sqrt{2}} \\ -\mu & -2\chi_d v_T & -2\chi_d v_d & 0 & -\frac{g' v_d}{\sqrt{2}} & \frac{g v_d}{\sqrt{2}} \\ 0 & -2\chi_d v_d & 0 & -\mu_T & \sqrt{2}g' v_T & -\sqrt{2}g v_T \\ -2\chi_u v_u & 0 & -\mu_T & 0 & -\sqrt{2}g' v_{\bar{T}} & \sqrt{2}g v_{\bar{T}} \\ \frac{g' v_u}{\sqrt{2}} & -\frac{g' v_d}{\sqrt{2}} & \sqrt{2}g' v_T & -\sqrt{2}g' v_T & M_1 & 0 \\ -\frac{g v_u}{\sqrt{2}} & \frac{g v_d}{\sqrt{2}} & -\sqrt{2}g v_T & \sqrt{2}g v_T & 0 & M_2 \end{pmatrix} \begin{pmatrix} \tilde{H}_u^0 \\ \tilde{H}_d^0 \\ \tilde{T}^0 \\ \tilde{\bar{T}}^0 \\ \lambda_1 \\ \lambda_2^3 \end{pmatrix}.$$

$$(30)$$

We need to know the couplings of these spinors in the mass eigenstate basis and in our analysis we will follow the discussion of Majorana spinors presented in [28,29]. The mass eigenstates will be related to the weak interaction eigenstates by orthogonal transformation O,

$$\hat{H}^i = O^{i\alpha} N^\alpha, \qquad (31)$$

where N^α are Majorana mass eigenstates, such that

$$O^T.M_{mf}.O \qquad (32)$$

is a diagonal matrix. Then using properties of the Majorana fields one can show that the couplings of the mass eigenstates to the gauge bosons will be given by

$$A^\mu \bar{H}_L^i \gamma_\mu G_A^i \hat{H}_L^i = -\frac{1}{2} A^\mu \bar{N}^\alpha \gamma_\mu \gamma_5 N^\beta (O^T G O)_{\alpha\beta},$$

$$G_{W_3}: \frac{g}{2} \text{Diag}(-1, 1, -2, 2, 0, 0), \qquad (33)$$

$$G_B: \frac{g'}{2} \text{Diag}(1, -1, 2, -2, 0, 0).$$

where $G_{W,B}$ is gauge coupling matrix in the EW basis (for the left-handed fermion fields).
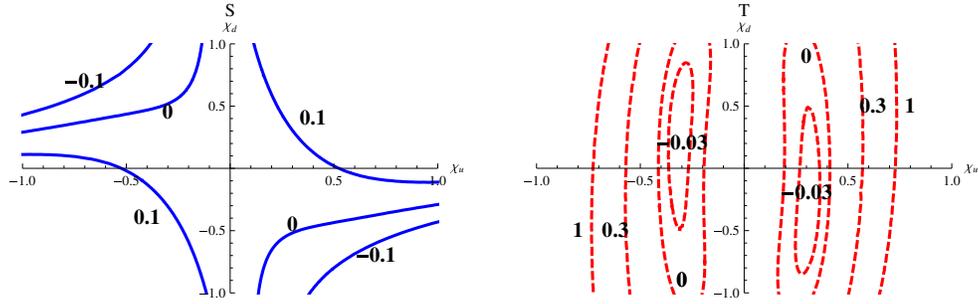
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[9]Further discussion of such a size of a soft mass term for triplet relative to those for doublets is in Sec. IV.

FIG. 4 (color online). Contour plot of S (blue, solid line) and T (red, dashed line) parameters in the (χ_u, χ_d) plane with $\mu_T = \mu = 150$ GeV, $M_1 = M_2 = 200$ GeV contributed by the fermions at one loop (see text for an explanation of the parameters).

The charge-one fields have the following mass matrix:

$$(H_d^-, T^-, \lambda^-)_R \begin{pmatrix} \mu & \sqrt{2}\chi_d v_d & g v_d \\ \sqrt{2}\chi_u v_u & \mu_T & \sqrt{2}g v_{\bar{T}} \\ g v_u & \sqrt{2}g v_T & M_2 \end{pmatrix} \begin{pmatrix} H_u^+ \\ T^+ \\ \lambda^+ \end{pmatrix}_L. \tag{34}$$

These charge-one fermions have the following vectorlike couplings to neutral gauge bosons:

$$B: \frac{g'}{2} \, \text{Diag}(1, 2, 0) \qquad W_3: \frac{g}{2} \, \text{Diag}(1, 0, 2). \tag{35}$$

Similarly we can calculate the couplings of the fermions to the charged gauge bosons. Now we calculate the contribution of the Higgsino-triplet fermion sector to the S, T parameters. Results are presented on Figs. 4 and 5. In these plots we have included only the contribution of the fermion sector. One can see that contribution to the T parameter is almost always positive, compared to the tree-level contribution which was always negative, so we will have some relaxation of the tree-level bounds. Also it is interesting to note that a significant part of the parameter space predicts negative S parameter, which relaxes EW precision bounds even more.

Finally, we would like to mention the one-loop contributions of the triplet might raise the Higgs mass even beyond the tree-level effect, as calculated by Ref. [19] for the model with *zero*-hypercharge triplet. Since in our

model it is rather easy for the tree-level Higgs mass to be beyond the LEP2 limit, we defer such study of loop effects of the triplet on Higgs mass for future work. We also notice that the contribution to the Higgs mass from this effect is expected to be subdominant to that of the tree-level effects already discussed above.

B. Light pseudoscalar

So far in our analysis we always assumed that soft SUSY-breaking A terms are small. This assumption was motivated by low-scale gauge mediation which we considered as a possible UV completion of our model. As briefly discussed in Sec. II B, in the limit when all A terms are zero the potential is invariant under $U(1)_R$ symmetry,

$$H_u, H_d, T, \bar{T}, S \rightarrow e^{i\phi_R} H_u, H_d, T, \bar{T}, S. \tag{36}$$

This symmetry is broken spontaneously by the VEVs of the H_u, H_d, T, $\bar{T}$, S so that our spectrum contains a massless pseudoscalar, the pseudo-Nambu-Goldstone boson of spontaneous breaking of $U(1)_R$ symmetry of the model (called the R axion). This $U(1)_R$ is explicitly broken by the nonvanishing A terms so that the mass of the R axion will be suppressed by the value of the A terms. In the limit where the soft masses of the triplets are large such that the VEVs of the triplet are small ($v_T, v_{\bar{T}} \rightarrow 0$), i.e., the triplet plays a negligible role in $U(1)_R$ breaking, the R axion is an
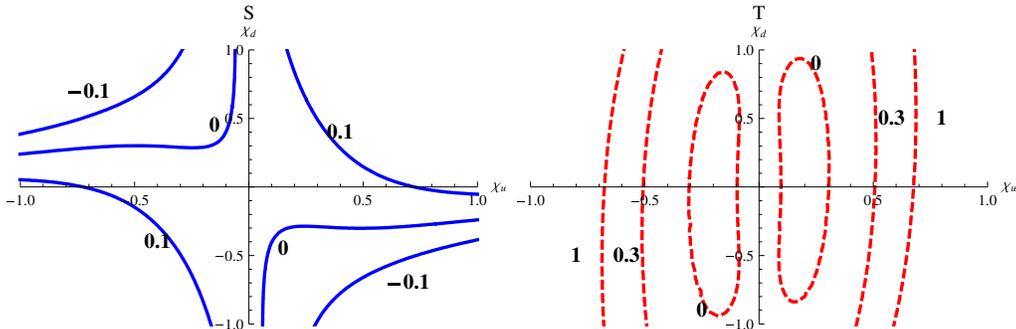

FIG. 5 (color online). Same as the previous figure, except $M_1 = M_2 = 500$ GeV.

admixture of the singlet and SM doublet, just like in the usual NMSSM,

$$R_{\text{axion}} \approx \frac{v_s S_I + 2 v c_\beta s_\beta (c_\beta H_{uI} + s_\beta H_{dI})}{\sqrt{v_s^2 + v^2 \sin^2 2\beta}}. \qquad (37)$$

In the limit when A terms are small and $\tan\beta$ is large we will get the following expression for the axion mass:

$$m_{\text{axion}}^2 \approx -3 A_S v_s - \frac{14 A v_u v_d}{v_s}$$
$$- \frac{2 A_\kappa \kappa \lambda v_d (-8 v_s^2 \lambda^2 + 2 v_u^2 (g^2 + g'^2 + 8\chi_u^2))}{v_s v_u (-8\lambda^4 + 4\kappa^2 (g^2 + g'^2 + 8\chi_u^2))}. \qquad (38)$$

As expected, tree-level mass of the axion vanishes for the zero values of A terms.

The couplings of R axion to the SM fermions in the limit of large v_S, large $\tan\beta$, and small values of triplet VEVs v_T, $v_{\bar{T}}$ are given by the same formula as in the NMSSM [30],

$$\sim \frac{\sqrt{2}}{v_s} \left(\frac{m_u}{\tan^2\beta} \bar{u}\gamma_5 u + m_d \bar{d}\gamma_5 d \right) i R_{\text{axion}}. \qquad (39)$$

In the case when $m_{\text{axion}} < m_\Upsilon$, the decays of $\Upsilon \to \gamma R_{\text{axion}}$ at B factories can provide an important test of the model, but it is possible to evade this bound by simply making the R axion a bit heavier than m_Υ. Bounds from $Z \to R_{\text{axion}}\gamma$ are very weak (see for example [30] and references therein) due to the effective coupling involved in this decay arising from a loop of SM fermions, combined with the suppressed nature of the R-axion couplings to the SM fermions (as above). The LEP searches for the $e^+ e^- \to Z^* \to h A^0$ in MSSM [31] can be reinterpreted as searches for the light axion. However, these bounds are expected to be rather weak because the axion is always mostly singlet [see Eq. (37)]. In the large $\tan\beta$ limit, there is a further suppression due to the dependence on $\tan\beta$ in the admixtures of $H_{u,d}$ in the R axion relative to those in the lightest CP-even Higgs boson. Thus, in this limit, the $Z(h \overleftrightarrow{\partial} R_{\text{axion}})$ coupling is estimated to be

$$\sim \sqrt{g^2 + g'^2} \left(\frac{2v}{v_s} \frac{1}{\tan\beta^2} \right). \qquad (40)$$

Numerical calculations show that this coupling is indeed is very small. For a more detailed analysis of the constraints on the light axion interactions in NMSSM, which applies to TNMSSM as well, we refer interested readers to [4,30].

C. Neutrino mass

In the TNMSSM, gauge invariance and renormalizability actually allow one more superpotential term other than the terms shown in Eqs. (1) and (2),

$$W = \xi L \cdot T L. \qquad (41)$$

This is potentially dangerous because it could lead to too large Majorana mass for neutrinos once the triplet with $Y = +1$ acquires its vacuum expectation value. Indeed, the Lagrangian contains the following mass term:

$$\mathcal{L} \ni \frac{1}{2} (2\xi v_T \psi_\nu \psi_\nu + \text{H.c.}), \qquad (42)$$

i.e., the neutrino mass, which must be sufficiently small, is given by $2\xi v_T$.

Even if we are required to have the triplet VEVs not more than ~ 4 GeV by the constraint from the ρ parameter (see Sec. III A 1), it would need an enormous tuning in order to suppress v_T much below $\mathcal{O}(1$ GeV), as required to approach the scale of neutrino masses: see the issue of tadpole for triplets in Sec. III A. We assumed that the coupling ξ is $O(1)$ in the above argument. In order to avoid large neutrino masses, it is of course technically natural to choose this coupling ξ to be very small. Indeed, we can forbid the superpotential given in Eq. (41) by imposing a symmetry. One possibility is a $\mathbb{Z}_6$ under which the supermultiplets in the TNMSSM are charged as follows:

$$+1 \quad \text{for} \quad L, \qquad +3 \quad \text{for} \quad \bar{e}, \qquad +2 \text{ for other fields.} \qquad (43)$$

Even though this discrete symmetry is not anomaly free, one can think about it as an effective symmetry which holds up to very high energy scales.

Under this symmetry, all terms in the TNMSSM that we had earlier survive except for the (unwanted) large Majorana mass term for neutrinos in Eq. (41). Nevertheless, the (different) Majorana mass term, arising from the usual seesaw mechanism and thus naturally highly suppressed

$$W \sim \frac{1}{M_s} (H_u L)^2, \qquad (44)$$

is allowed. Note that this symmetry forbids the usual renormalizable lepton-number violation operators ($W \ni LL\bar{e}$, $LQ\bar{d}$, and LH_u), allowing only the operator $W \ni \bar{u}\bar{d}\bar{d}$. Thus, an intriguing possibility is that we do not impose R parity, allowing the above baryon-number violating term. Note that such baryon-number violating couplings (i.e., appearing without the lepton-number violating terms) are relatively poorly constrained since they do not induce proton decay. This would completely change the phenomenology of our model, but studying this possibility is beyond the scope of our paper.

IV. EVOLUTION OF PARAMETERS TO HIGH SCALES

In the previous section we discussed preferred values of TNMSSM parameters at the EW scale based on minimizing fine-tuning and EW precision tests. In this section, we

TABLE II.   Values of parameters at the weak scale for a sample point.

Gauge couplings	Yukawa couplings	VEVs ($\tan\beta = 5$) (GeV)	A terms (GeV)
$g_1 = 0.45$	$\lambda = 0.294$	$v_u = 170.6$	$A = -7.48$
$g_2 = 0.65$	$\kappa = 0.360$	$v_d = 34.1$	$A_\kappa = 1.46$
$g_3 = 1.18$	$\lambda_T = -0.615$	$v_s = -519.4$	$A_T = 5.22$
	$\chi_u = -0.242$	$v_T = 2.85$	$A_u = -2.25$
	$\chi_d = -0.430$	$v_{\bar{T}} = -0.96$	$A_d = 4.62$
			$A_{h_t} = -335$
			$A_{h_b} = -40$
			$A_{h_e} = -45$

Gaugino mass (GeV)	Soft mass (light scalars) (GeV^2)	Soft mass (heavy scalars) (GeV^2)	Soft mass (Higgses) (GeV^2)
$M_1 = 90$	$m_{Q_1}^2 = 525^2$	$m_{Q_3}^2 = 470^2$	$m_{H_u}^2 = -154^2$
$M_2 = 100$	$m_{u_1}^2 = 510^2$	$m_{u_3}^2 = 390^2$	$m_{H_d}^2 = 372^2$
$M_3 = 570$	$m_{d_1}^2 = 505^2$	$m_{d_3}^2 = 500^2$	$m_S^2 = -266^2$
	$m_{L_1}^2 = 180^2$	$m_{L_3}^2 = 180^2$	$m_T^2 = 657^2$
	$m_{e_1}^2 = 115^2$	$m_{e_3}^2 = 110^2$	$m_{\bar{T}}^2 = 656^2$

will renormalization group evolve these parameters to higher energy scales in order to determine if there are any additional UV considerations. It is well known that the MSSM has two remarkable features when extrapolated beyond the EW scale: all its parameters stay perturbative up to very high scale (practically, the Planck scale) and in fact the three gauge couplings meet rather precisely at $\sim 10^{16}$ GeV. In this sense TNMSSM (by itself) is less appealing since (as we show below) it lacks the nice feature of gauge coupling unification. Nonetheless, we will show that a simple modification of the TNMSSM has hints of unification. On top of that we show that all the couplings which are important for the solution to the little hierarchy and μ problems that we presented here can be kept perturbative up to the GUT scale.

Let us start with gauge couplings unification. When the triplets are added to the MSSM the one-loop β-function coefficients for the three gauge couplings are modified from the MSSM: $(b_1, b_2, b_3) = (\frac{51}{5}, 5, -3)$ [we are using $SU(5)$ normalization for the g_1 coupling]. With these coefficients, we find the $SU(2)_L \times U(1)_Y$ gauge couplings meet around the usual GUT scale, i.e., $\sim 10^{16}$ GeV. However the value of the gauge couplings at this scale ($\alpha \approx 1$) cannot quite be regarded as perturbative so that one-loop RGE equations might not suffice.[10] The $SU(3)_c$ gauge coupling does not unify with these two couplings, but it can attain a similar value (albeit large) at the GUT scale if eight color triplets—inert under $SU(2)_L \times U(1)_Y$ [16]—are added not far from the EW scale. Thus, even though perturbative/one-loop unification is lost in the TNMSSM, with a suitable modification, there is the pos-

sibility of a "strong" unification right below the Planck scale.

Next, we consider the RGE of the new couplings (relative to the MSSM) which we introduced in order to address the little hierarchy problem and the μ problem of the MSSM, i.e., the couplings involving the singlet, triplet, and the usual Higgs doublet fields. A full set of relevant equations is given in the Appendix. As seen there, all these couplings tend to grow in the UV due to contributions from these couplings themselves. In addition, for the couplings involving Higgs doublets, the contribution of the (large) third-generation Yukawa couplings make matters worse here. On the other hand, as is well known, the (EW) gauge contributions have the opposite effect on the RGE of these couplings. The point is that the Casimir invariant involved in these asymptotically free effects is larger for the triplet couplings than for the others.

In the light of the above properties of the RGE, we expect λ and κ (the couplings of the NMSSM part of our model) to hit Landau poles before the other couplings ($\chi_{u,d}$ and λ_T) if they all have similar values at the weak scale. However, note that we only need χ's to be relatively large at the weak scale in order to solve the little hierarchy problem, i.e., we are not using the λ coupling to enhance the Higgs mass (unlike in the NMSSM). In fact, we would like λ to be relatively small since the μ term for the doublets ($\sim \lambda v_S$) should be at the weak scale for naturalness. Thus, Landau poles should not be a problem for our model.

For illustration purposes we consider a sample point in parameter space, described in Table II. This point is fairly representative and one gets very similar results considering other values in parameter space consistent with naturalness and EW precision measurements. Running of the Yukawa couplings is depicted in the left panel of Fig. 6 which

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[10]Adding extra matter charged under these gauge groups only makes the couplings more strong at the GUT scale.
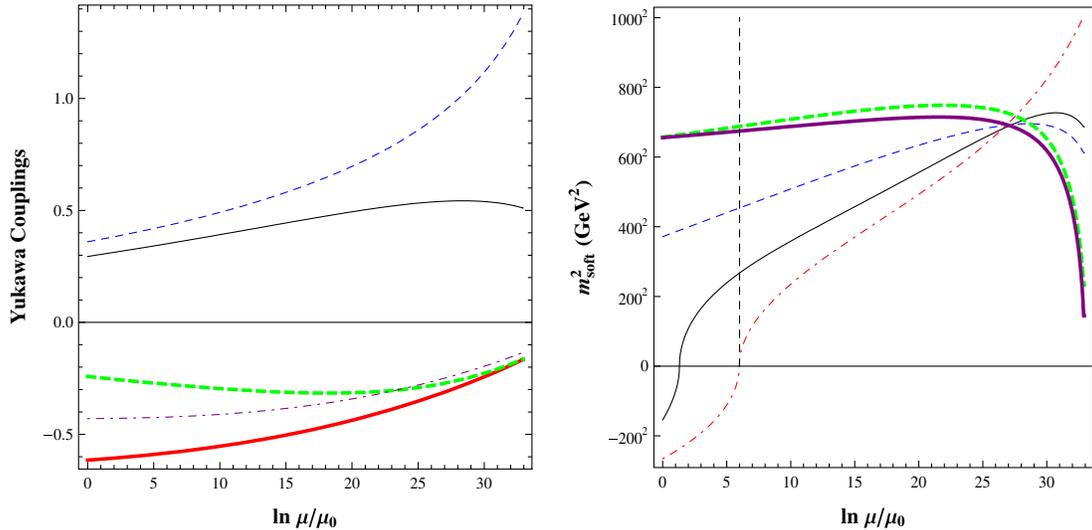
FIG. 6 (color online).   On the left, evolution of Yukawa/superpotential couplings between the Higgses up to the GUT scale. [black/ thin solid line, blue/dashed line, red/thick solid line, green/thick dashed line, and purple/dot-dashed line] correspond (respectively) to $[\lambda, \kappa, \lambda_T, \chi_u,$ and $\chi_d]$. On the right, evolution of soft masses in the Higgs sector. [black/thin solid line, blue/dashed line, red/dot-dashed line, green/thick dashed line, and purple/thick solid line] correspond (respectively) to $[m_{H_u}^2, m_{H_d}^2, m_S^2, m_T^2, m_{\bar{T}}^2]$ and $\mu_0 = 100$ GeV. No new particles between the weak and the GUT scale are assumed in either case.

confirms the above expectations. In particular, we can see that the values of χ required in order to enhance the Higgs mass remain rather easily perturbative up to GUT scale.

Let us now briefly discuss the running of the soft masses in the Higgs sector. The running for the same reference point is illustrated in the right panel of Fig. 6 (renormalization group equations are again given in the Appendix). Recall that we assumed that the soft masses squared for H_u and S are negative at the weak scale (as required for these scalars to acquire VEVs), but those for triplet are positive (and of course similarly for squarks and sleptons). A very attractive scenario would be that all of these soft masses squared renormalization group evolve to positive (and roughly similar) values in the UV, i.e., radiative symmetry breaking (as happens for EW symmetry, i.e., H_u in the MSSM).

From the right panel of Fig. 6, we see that (not surprisingly) this indeed happens for H_u just like in the MSSM. What is more remarkable (compared to the usual NMSSM) is that singlet behaves similarly.[11] We would like to stress that this difference between the TNMSSM and the NMSSM is due to the interaction of the singlet with triplets giving an additional negative contribution to the running (from UV to IR) of m_S^2. Thus, by (radiatively) generating a sufficiently large VEV for the S, the singlet-triplet coupling significantly enlarges the viable parameter space. Note that driving the singlet mass squared negative this way does require the singlet-triplet coupling to be larger

than ~ 0.1. Combining this condition with μ_T being weak scale (or ~ 100 GeV) implies that singlet VEV should be less than ~ 1 TeV. On the other hand, the soft masses squared of the triplets undergo very moderate change in renormalization group evolving to the UV, mainly because positive Yukawa contributions are largely compensated by negative terms proportional to the gauge couplings (which again come with a large Casimir invariant). This feature is crucial in allowing for the possibility that the soft masses squared for the triplet and doublet are (roughly) similar in the UV.[12]

V. CONCLUSIONS/DISCUSSIONS

In this paper, we have presented an extension of the MSSM, based on adding $SU(2)_L$ triplet fields and a SM gauge singlet (as in the NMSSM) coupled to each other, which solves the little hierarchy and μ problems of the MSSM. Our focus was on presenting a complete model, performing a thorough analysis of electroweak precision tests, and providing an origin for all mass scales in the model. We have started from a completely scaleless superpotential, such that the only scale in the problem is a soft mass scale, and showed that one can get a completely viable model which dynamically generates all the necessary scales, including effective μ and $B\mu$ terms for the doublet and triplet fields.

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[11]Note that, in the context of gauge mediation of SUSY breaking, the renormalization group scale where singlet mass squared vanishes can be taken to the messenger scale.

[12]In the context of gauge mediation of SUSY breaking, the mild hierarchy between the soft masses in the UV, i.e., at the messenger scale, for triplet and doublets (see the right panel of Fig. 6) can arise due to the larger Casimir invariant for triplet versus doublet.

As the first step we reanalyzed the TMSSM (triplet-extended MSSM) as a good candidate for solving the little hierarchy problem. We explicitly showed that in this model the triplet inevitably gets a VEV (even if it is not tachyonic), which is of course severely constrained by the ρ parameter. This problem has usually been circumvented in the literature by assuming a large μ term for the triplet and thus analyzing a model where these fields can be safely integrated out supersymmetrically, and all the interesting effects can be incorporated in the MSSM Lagrangian augmented by higher-dimensional operators. This approach has the important drawbacks that the correction to the Higgs quartic is small in the large $\tan\beta$ limit and decouples with the μ term for the triplet.

In this paper we took another approach, showing that the part of parameter space with relatively heavy triplet scalar and light triplet fermions (i.e., big soft masses, but small μ term) can be even more attractive. It solves the little hierarchy problem for any $\tan\beta$ and even for very large soft mass for the triplet. At the same time the S and T parameters are well under control since the largest (potentially) dangerous contribution (to the T parameter) comes from the triplet scalar VEV at the tree level, which however is suppressed by the large soft mass. We also note that adding a zero-hypercharge triplet, whose VEV gives an opposite contribution to T parameter, can also potentially ameliorate a tension with EW precision tests [32–35].

We then analyzed a full singlet plus triplet extension of the MSSM (TNMSSM). We showed that coupling the singlet to the triplet has two major advantages. First, it naturally generates a weak-scale μ term for the triplet, just like the singlet-doublet coupling of the NMSSM solves the doublet μ problem. On the other hand, the triplet-singlet coupling helps to render the soft mass squared of the singlet negative along the RGE trajectories thus enabling the singlet to acquire a VEV. In summary, we discover that the "sum" of NMSSM and TMSSM is significantly more appealing than each of its components, taken separately.

Finally let us comment on the issue of how the current LHC searches for SUSY might apply to this model. It is well known that these searches put very stringent bounds on squarks and gluinos below the TeV scale. However, these bounds heavily rely on several highly model-dependent assumptions. First, in order to put strong bounds on squark mass one needs the squarks of different generations to be (roughly) degenerate in order to have big production cross sections. Superpartners are much harder to find if the third generation is somewhat special such that the stops and sbottoms are (much) lighter than rest of the squarks. It is also well known that one can "hide" SUSY by squeezing the superpartner spectrum so that the energy available to SM particles in superpartner decays is small. Usually this possibility is considered to be marginal. However in the TNMSSM one finds lots of new EW scale particles (including scalars and fermions) which are ex-pected to be at the EW scale, thus making it easier to hide superpartners. To the best of our knowledge the bounds on these kinds of spectra are not well understood. Moreover, as we have already mentioned this entire scenario can be easily accompanied by R parity (in particular baryon-number, but not simultaneously lepton-number) violation, which would significantly complicate the study. This would result in collider signatures without large missing transverse energy since the lightest SUSY particle would just decay into jets. Needless to say that such searches are much more difficult than standard SUSY searches and the current bounds on such a scenario are expected to be rather mild.

It would be very interesting to understand better these bounds, as well as the phenomenology of our model in general. The latter can be of special interest (however, also possibly experimentally challenging) due to the enlarged neutralino, chargino, and Higgs sectors. In particular, there is a light doubly charged fermion (coming from the triplet) in the non-SUSY limit that was our focus, unlike in the SUSY limit of this model or in the MSSM, NMSSM and extension of the MSSM with *zero*-hypercharge triplet.

APPENDIX A: RENORMALIZATION GROUP EQUATIONS

In this appendix we provide the renormalization group equations in the $\overline{DR}$ scheme for the parameters of the TNMSSM. The notations are $t = \ln(\mu/\mu_0)$ with μ the renormalization group scale, and $g_2 = g$, $g_1^2 = \frac{5}{3} g'^2$ (with $e = g \sin\theta_W = g' \cos\theta_W$).

Running of the new (relative to the MSSM) couplings is given by the following equations:

$$16\pi^2 \frac{d\lambda}{dt} = \lambda\left[3h_t^2 + 3h_b^2 + h_\tau^2 + 4\lambda^2 + 6\chi_d^2 + 6\chi_u^2 \right.$$
$$\left. + 3\lambda_T^2 + 2\kappa^2 - 3g_2^2 - \frac{3}{5}g_1^2\right] \tag{A1}$$

$$16\pi^2 \frac{d\kappa}{dt} = \kappa[6\lambda^2 + 9\lambda_T^2 + 6\kappa^2] \tag{A2}$$

$$16\pi^2 \frac{d\lambda_T}{dt} = \lambda_T\left[2(\chi_d^2 + \chi_u^2 + \lambda^2 + \kappa^2) + 5\lambda_T^2 \right.$$
$$\left. - 8g_2^2 - \frac{12}{5}g_1^2\right] \tag{A3}$$

$$16\pi^2 \frac{d\chi_u}{dt} = \chi_u\left[6h_t^2 + 2\lambda^2 + 14\chi_u^2 + \lambda_T^2 - 7g_2^2 - \frac{9}{5}g_1^2\right]$$

(A4)

$$16\pi^2 \frac{d\chi_d}{dt} = \chi_d\left[6h_b^2 + 2h_\tau^2 + 2\lambda^2 + 14\chi_d^2 + \lambda_T^2\right.$$
$$\left. - 7g_2^2 - \frac{9}{5}g_1^2\right].$$

(A5)

Running of the *A* terms associated with the above new couplings is given by

$$16\pi^2 \frac{dA_\lambda}{dt} = A_\lambda\left[3h_t^2 + 3h_b^2 + h_\tau^2 + 12\lambda^2 + 6\chi_d^2 + 6\chi_u^2 + 3\lambda_T^2\right.$$
$$\left. + 2\kappa^2 - 3g_2^2 - \frac{3}{5}g_1^2\right] + \lambda\left[6h_tA_{h_t} + 6h_bA_{h_b}\right.$$
$$+ 2h_\tau A_{h_\tau} + 12\chi_dA_d + 12\chi_uA_u$$
$$\left. + 6\lambda_TA_T + 4\kappa A_\kappa + 6g_2^2M_2 + \frac{6}{5}g_1^2M_1\right]$$

(A6)

$$16\pi^2 \frac{dA_\kappa}{dt} = 3A_\kappa[2\lambda^2 + 3\lambda_T^2 + 6\kappa^2]$$
$$+ 3\kappa[4\lambda A + 6\lambda_TA_T]$$

(A7)

$$16\pi^2 \frac{dA_T}{dt} = A_T\left[2\lambda^2 + 2\chi_d^2 + 2\chi_u^2 + 15\lambda_T^2 + 2\kappa^2\right.$$
$$\left. - 8g_2^2 - \frac{12}{5}g_1^2\right] + \lambda_T\left[4\lambda A + 4\chi_dA_d + 4\chi_uA_u\right.$$
$$\left. + 4\kappa A_\kappa + 16g_2^2M_2 + \frac{24}{5}g_1^2M_1\right]$$

(A8)

$$16\pi^2 \frac{dA_u}{dt} = A_u\left[6h_t^2 + 2\lambda^2 + 42\chi_u^2 + \lambda_T^2 - 7g_2^2 - \frac{9}{5}g_1^2\right]$$
$$+ \chi_u\left[12h_tA_{h_t} + 4\lambda A + 2\lambda_TA_T + 14g_2^2M_2\right.$$
$$\left. + \frac{18}{5}g_1^2M_1\right]$$

(A9)

$$16\pi^2 \frac{dA_d}{dt} = A_d\left[6h_b^2 + 2h_\tau^2 + 2\lambda^2 + 42\chi_d^2 + \lambda_T^2 - 7g_2^2\right.$$
$$\left. - \frac{9}{5}g_1^2\right] + \lambda_t\left[12h_bAh_b + 4h_\tau A_{h_\tau} + 4\lambda A\right.$$
$$\left. + 2\lambda_TA_T + 14g_2^2M_2 + \frac{18}{5}g_1^2M_1\right].$$

(A10)

In order to describe the running of the soft masses, it is convenient to define following quantities:

$$X_t \equiv h_t^2(m_{H_u}^2 + m_{Q_3}^2 + m_{\bar{u}_3}^2) + A_{h_t}^2$$

(A11)

$$X_b \equiv h_b^2(m_{H_d}^2 + m_{Q_3}^2 + m_{\bar{d}_3}^2) + A_{h_b}^2$$

(A12)

$$X_\tau \equiv h_\tau^2(m_{H_d}^2 + m_{L_3}^2 + m_{\bar{e}_3}^2) + A_{h_\tau}^2$$

(A13)

$$X \equiv \lambda^2(m_{H_u}^2 + m_{H_d}^2 + m_S^2) + A^2$$

(A14)

$$X_T \equiv \lambda_T^2(m_T^2 + m_{\bar{T}}^2 + m_S^2) + A_T^2$$

(A15)

$$X_u \equiv \chi_u^2(2m_{H_u}^2 + m_{\bar{T}}^2) + A_u^2$$

(A16)

$$X_d \equiv \chi_d^2(2m_{H_d}^2 + m_T^2) + A_d^2$$

(A17)

$$X_\kappa \equiv 3\kappa^2m_S^2 + A_\kappa^2$$

(A18)

$$S \equiv m_{H_u}^2 - m_{H_d}^2 + 3m_T^2 - 3m_{\bar{T}}^2$$
$$+ \text{tr}[\mathbf{m}_Q^2 - \mathbf{m}_L^2 - 2\mathbf{m}_{\bar{u}}^2 + \mathbf{m}_{\bar{d}}^2 + \mathbf{m}_{\bar{e}}^2],$$

(A19)

where **m**'s denote squark and slepton soft mass matrices in the generation space.

The renormalization group flows for the soft masses in the Higgs sector are then given by

$$16\pi^2 \frac{dm_{H_u}^2}{dt} = 6X_t + 2X + 12X_u - 6g_2^2M_2^2$$
$$- \frac{6}{5}g_1^2M_1^2 + \frac{3}{5}g_1^2S$$

(A20)

$$16\pi^2 \frac{dm_{H_d}^2}{dt} = 6X_b + 2X_\tau + 2X + 12X_d$$
$$- 6g_2^2M_2^2 - \frac{6}{5}g_1^2M_1^2 - \frac{3}{5}g_1^2S$$

(A21)

$$16\pi^2 \frac{dm_T^2}{dt} = 4X_d + 2X_T - 16g_2^2M_2^2 - \frac{24}{5}g_1^2M_1^2 + \frac{6}{5}g_1^2S$$

(A22)

$$16\pi^2 \frac{dm_{\bar{T}}^2}{dt} = 4X_u + 2X_T - 16g_2^2M_2^2 - \frac{24}{5}g_1^2M_1^2 - \frac{6}{5}g_1^2S$$

(A23)

$$16\pi^2 \frac{dm_S^2}{dt} = 4X + 6X_T + 4X_\kappa.$$

(A24)

The running of the other Yukawa couplings (for example, that of top quark) and soft masses (for example, that of stop) changes accordingly, i.e., taking into account the effect of the new couplings and soft masses. We find that these changes are typically not important for our purposes so that we do not provide a complete list here. One can easily obtain these equations using the generic formula given in Refs. [27,36].

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