$Z' \to ggg$ decay in left-right symmetric models with three and four fermion families

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We study the $Z' \to \bar{q}q, ggg$ decays in the context of a manifest left-right symmetric gauge theory with three and four generations. The Z' couplings to quarks are fixed essentially by the parameters of the standard model and we obtain $\Gamma(Z' \to q\bar{q}) \approx 14$ GeV for $M_{Z'} \approx 1$ TeV. For the $Z' \to ggg$ decay and three families we obtain a branching ratio $\mathrm{BR}(Z' \to ggg) = \frac{\Gamma(Z' \to ggg)}{\Gamma(Z' \to q\bar{q})} = 1.2 - 2.8 \times 10^{-5}$ for $m_{Z'} =$ 700–1500 GeV. The fourth generation produces an enhancement in the branching ratio for Z' masses close to the $\bar{b}'b'$ threshold and a dip for Z' masses close to the $\bar{t}'t'$ threshold. Using the values of the fourth-generation quark masses allowed by electroweak precision data, we obtain a branching ratio $\mathrm{BR}(Z' \to ggg) = (1 - 6) \times 10^{-5}$ for $m_{Z'} =$ (700–1500) GeV.

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I. INTRODUCTION

Additional Z' gauge bosons are ubiquitous in standard model (SM) extensions. Among them, models based on left-right symmetry groups have been extensively studied [1] and are particularly important from the point of view of LHC phenomenology. The basic assumption of manifest left-right symmetric models is that the fundamental weak interaction Lagrangian is invariant under parity symmetry, which is spontaneously broken at low energy due to a noninvariant vacuum. Models based on the smallest left-right symmetric gauge group $SU(3)_C \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$ have many additional appealing attributes (for a review see [2]). These include the same quark-lepton symmetry of the weak interaction; the possibility of writing electric charge in terms of purely physical quantum numbers such as weak-isospin, baryon, and lepton number; the natural accommodation of the seesaw mechanism for neutrino masses; understanding of the small CP violation in the quark sector [2]; and the solution to the strong CP problem[3].

In this work we study the $Z' \to \bar{q}q, ggg$ decays in the context of a left-right symmetric model based on the $SU(3)_C \otimes SU(2)_R \otimes SU(2)_L \otimes U(1)_{B-L}$ gauge group with three (LRSM) and four fermion families (LRSM4). The first decay is expected to give an important contribution to the Z' decay width and it is worthy to estimate the size of this channel for the search of Z' bosons in hadron colliders. The $Z' \to ggg$ decay is related to the production mechanism $gg \to gZ'$ and $gg \to \gamma Z'$ which could also be relevant to the searches of the Z' in hadron colliders. In general, the coupling $Vggg$ (with $V = Z, Z'$) is absent in the classical action of any renormalizable extension of the standard model. The process $V \to ggg$ is induced via quark loops and turns out to be a very interesting prediction which allows us to analyze the interplay between strong and weak sectors of a particular model. The $Vggg$ couplings are also important because they are much less suppressed than those coming from purely weak interactions, like $VVVV$.

A detailed analysis of the one-loop couplings $Vggg$ and $Vgg\gamma$, with $V = Z, Z'$, in the context of the minimal 3-3-1 model [4], was performed in [5,6]. It was explicitly shown there that the $Z \to ggg$ decay [7] does not receive sizable contribution from quarks in the loops with masses higher than $m_Z/2$ and therefore neither t nor an additional quark family is expected to contribute significantly to this process. These results remain valid in LRSM provided the mixing angle between neutral gauge bosons is small which is the case [8].

In LRSM the $Z' \to ggg$ decay is induced by quark loops and the necessary $Z'\bar{q}q$ couplings depend only on a mixing angle which is severely constrained by experimental data. As we shall show below, the most important contributions come from the third family of quarks which motivate us to study also the contributions of a fourth family to both $Z' \to \bar{q}q$ and $Z' \to ggg$ decays.

A fourth sequential fermion family is the simplest possible extension to the SM and can be easily adapted to left-right models. It is well-known that the number of families is not fixed by the theory and precision electroweak data do not exclude a fourth one [9–16]. Our results for LRSM suggest that the existence of a fourth generation of quarks could produce an enhancement of the $Z' \to ggg$ branching ratio and it is worthy to quantify this effect. An extensive review and an exhaustive list of references to the work on the possible existence of a fourth generation can be found in [17]. Recent highlights on the consequences of a fourth generation are contained in [18]. These include several appealing features:

(i) A fourth generation could relax the current SM low Higgs mass bounds from electroweak precision observables [11–13].

(ii) Mechanisms of dynamical electroweak symmetry breaking by a fourth generation of quarks and

TABLE I. Structure of the neutral currents for the quark sector of the LRSM4.

Quark	Q_q	g^q_{VZ}	g^q_{AZ}	$g^q_{VZ'}$	$g^q_{AZ'}$
u, c, t, t'	$+\frac{2}{3}$	$\frac{3-8s_W^2}{6}\left(c_\phi - \frac{s_\phi}{\sqrt{1-2s_W^2}}\right)$	$\frac{c_\phi + s_\phi\sqrt{1-2s_W^2}}{2}$	$\frac{3-8s_W^2}{6}\left(c_\phi + \frac{s_\phi}{\sqrt{1-2s_W^2}}\right)$	$-\frac{c_\phi\sqrt{1-2s_W^2}-s_\phi}{2}$
d, s, b, b'	$-\frac{1}{3}$	$-\frac{3-4s_W^2}{6}\left(c_\phi - \frac{s_\phi}{\sqrt{1-2s_W^2}}\right)$	$-\frac{c_\phi + s_\phi\sqrt{1-2s_W^2}}{2}$	$-\frac{3-4s_W^2}{6}\left(c_\phi + \frac{s_\phi}{\sqrt{1-2s_W^2}}\right)$	$\frac{c_\phi\sqrt{1-2s_W^2}-s_\phi}{2}$

leptons in both strongly coupled scenarios [19–23] and weakly coupled ones [24] are possible.

(iii) Yukawa coupling contributions from a fourth generation improve the convergence of the three SM gauge couplings without invoking supersymmetry [25].

(iv) Electroweak baryogenesis through first-order electroweak phase transition can be achieved with four generations in supersymmetric models [26–28] and in models with a strongly coupled fourth family [29].

(v) In the standard model with a fourth generation of quarks, CP violation in processes involving quarks can be enhanced considerably compared to the three-generation case [30,31].

(vi) A fourth generation can also solve the CP asymmetry puzzles of $B \to K\pi$ [32–34] for a range of extra quark masses within the values allowed by high-precision LEP measurements [9–11].

The paper is organized as follows. Section II contains a brief discussion of the LRSM main features, Sec. III is devoted to the calculation of the $Z' \to \bar{q}q, ggg$ decays, and Sec. IV summarizes our results.

II. THE MODEL

The LRSM is based in the manifest left-right symmetric model developed in [35–37], where left and right gauge couplings are equal $g_L = g_R = g$ and Yukawa matrices are Hermitian. In the extended fermion sector, quarks and leptons are placed in doublets with the following assignment of quantum numbers $(d_L, d_R, B-L)$:

$$Q_{iL} = \begin{pmatrix} u_i' \\ d_i' \end{pmatrix}_L : (2, 1, 1/3), \quad Q_{iR} = \begin{pmatrix} u_i' \\ d_i' \end{pmatrix}_R : (1, 2, 1/3),$$

$$L_{iL} = \begin{pmatrix} \nu_i' \\ l_i' \end{pmatrix}_L : (2, 1, -1), \quad L_{iR} = \begin{pmatrix} \nu_i' \\ l_i' \end{pmatrix}_R : (1, 2, -1),$$

with $i = 1, 2, 3$. Here d_L (d_R) denotes the dimension of the $SU(2)_L$ $(SU(2)_R)$ representation, while the $U(1)$ generator corresponds to $B - L$. The electric charge formula is given by [38]

$$Q = T_{3L} + T_{3R} + \frac{B-L}{2}, \tag{1}$$

where T_{3L} and T_{3R} are the $SU(2)_L$ and $SU(2)_R$ generators, respectively. The minimal Higgs sector requires one bidoublet Φ: $(2, 2, 0)$ to generate fermion masses [39] and two additional triplets Δ_L: $(3, 1, 2)$ and Δ_R: $(1, 3, 2)$ to break the $SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$ symmetry down to $U(1)_{\rm em}$, with the further assumption that only Φ and Δ_R have nonvanishing vacuum expectation values (VEVs).

In this model, after spontaneous symmetry breaking (SSB), we have four nonstandard parameters in the gauge sector, i.e. additional gauge boson masses $M_{W'}$, $M_{Z'}$ and mixing angles in both charged (ζ) and neutral (ϕ) sectors. The neutral-current Lagrangian in the physical basis can be written as

$$\mathcal{L}^q_{\rm NC} = e\sum_{q=1}^{6} Q_q(\bar{q}\gamma_\mu q)A^\mu + \frac{g}{2c_W}\sum_{q=1}^{6}[\bar{q}\gamma_\mu(g^q_{VZ} - g^q_{AZ}\gamma_5) \times qZ^\mu + \bar{q}\gamma_\mu(g^q_{VZ'} - g^q_{AZ'}\gamma_5)qZ'^\mu], \tag{2}$$

where $q_1 = u$, $q_2 = c$, $q_3 = t$, $q_4 = d$, $q_5 = s$, $q_6 = b$, with the corresponding weak charges listed in Table I. Here Z is the lightest neutral boson mass eigenstate, while Z' is the heaviest one. The Z boson can be identified with the neutral gauge boson of the SM in the limit of vanishing ϕ. The current bound on the mixing angle of the neutral gauge sector $|\phi| \leq 0.0042$ [37] yields a prediction for $Z \to ggg$ in LRSM quite similar to that of the standard model.

Adding a fourth generation to this model is straightforward and in Table I we also show the weak charges of the additional quarks which we denote by t' and b'. As shown in [5] the individual contributions of quarks heavier than $M_Z/2$ to $Z \to ggg$ are highly suppressed and the sequential fourth generation yields also negligible contributions to this channel, hence we will focus on Z' decays.

III. $Z' \to \bar{q}q, ggg$ DECAYS

In the following we will work in the framework of LRSM4. Results for LRSM can be obtained by removing the contributions of the fourth generation quarks.

Regarding the fourth-family quark masses, the reported lower bounds from the Particle Data Group [40] are

$$m_{t'} > 256\ \text{GeV}, \qquad m_{b'} > 128\ \text{GeV}, \tag{3}$$

but the newest limits on t' and b' masses from direct searches at the Tevatron are $m_{b'} > 338$ GeV [41] and

$m_{t'} > 311$ GeV [42]. A rough upper bound $m_{t'} < (4\pi v^2/3)^{1/2} = 504$ GeV can be obtained from the renormalization group improved analysis of the s-wave $t'\bar{t}'$ tree-level elastic amplitude [43]. This estimate is based on tree-level calculations and one should not disregard higher masses based only on it. Indeed, a calculation of the effective potential at one-loop level for the scalar sector, assuming that the standard model with four generations is an effective theory valid up to a scale Λ and improved with the running of the couplings using the renormalization group equations, yields a perturbative regime for values of the fourth generation slightly above this bound [24]. Nevertheless, a careful analysis of the dynamics of bound states associated with fundamental or composite scalars related to the fixed points in the renormalization group flow equations for the Yukawa couplings [44] is required and the use of moderate mass values in our numerical calculations is necessary.

According to direct experimental searches for the Z' gauge boson in the context of left-right symmetry, the Z' lower mass limit is $m_{Z'} > 630$ GeV at CDF and $m_{Z'} > 804$ GeV at LEP 2. Besides, in Refs. [8,45] electroweak precision data require $m_{Z'} > 998$ GeV.

The width of the decay of the Z' boson into quarks is given by

$$\Gamma(Z' \to \bar{q}q) = \frac{\alpha N_C m_{Z'}}{12 c_W^2 s_W^2} \sum_{i=1}^{8} f(q_i), \qquad (4)$$

where

$$f(q_i) = \sqrt{1 - \frac{4m_{q_i}^2}{m_{Z'}^2}} \left[(g_{VZ'}^{q_i})^2 \left(1 + \frac{2m_{q_i}^2}{m_{Z'}^2}\right) + (g_{AZ'}^{q_i})^2 \left(1 - \frac{4m_{q_i}^2}{m_{Z'}^2}\right)\right] \theta(m_{Z'}^2 - 4m_{q_i}^2), \qquad (5)$$

with $q_1 = u$, $q_2 = c$, $q_3 = t$, $q_4 = d$, $q_5 = s$, $q_6 = b$, $q_7 = t'$, $q_8 = b'$. The decay width $\Gamma(Z' \to q\bar{q})$ depends on the unknown Z' mass and on the masses of the fourth-generation quarks. In Fig. 1 we plot this decay width as a function of the Z' mass in LRSM and using different values of the mass of the fourth-generation quarks in the case of LRSM4. Although the fourth generation contributes to the decay width, due to the reduced phase space and similar couplings to the first three generations this contribution is small. The decay width is essentially dictated by the first three generations and is of the order of 14 GeV for a mass $M_{Z'} = 1$ TeV.

The $Z' \to ggg$ decay is induced at one-loop level by the box and triangle diagrams depicted in Fig. 2. There are six box and six triangle diagrams but one needs to work out only one of each class. Results for the remaining diagrams can be obtained from those of the diagrams in Fig. 2 using Bose symmetry.

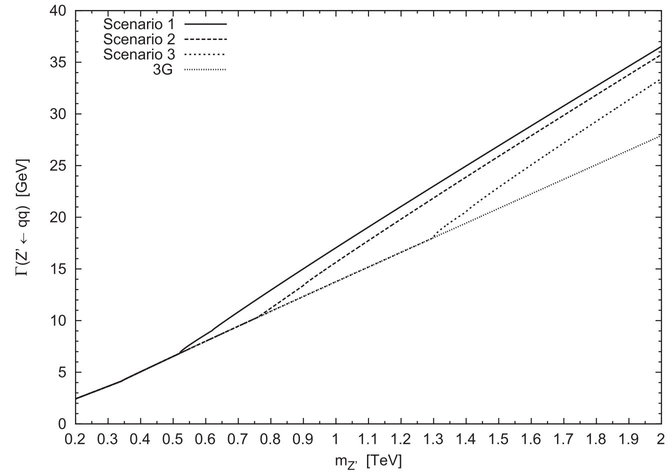

FIG. 1. $Z' \to q\bar{q}$ decay width as a function of the Z' mass in LRSM with three generations (3G) and in LRSM4 with different values of the fourth-generation quark masses: $\{m_{b'} = 260\,\text{GeV}, m_{t'} = 310\,\text{GeV}\}$ (Scenario 1), $\{m_{b'} = 380$ GeV, $m_{t'} = 450$ GeV$\}$ (Scenario 2), and $\{m_{b'} = 650$ GeV, $m_{t'} = 700$ GeV$\}$ (Scenario 3).

As discussed in Ref. [5], the invariant amplitude of the process can be written as

$$\mathcal{M}_{Z' \to ggg} = -\frac{i g_s^3 g_{Z'} N_C}{4\pi^2} (g_{VZ'}^q d_{abc} \mathcal{V}_q + g_{AZ'}^q f_{abc} \mathcal{A}_q), \qquad (6)$$

where

$$\mathcal{V}_q = \sum_{j=1}^{18} f_{V_j}^q T_{V_j}^{\mu_1\mu_2\mu_3\mu_4} \epsilon_{\mu_1}^{*a}(p_1, \lambda_1) \epsilon_{\mu_2}^{*b}(p_2, \lambda_2) \times \epsilon_{\mu_3}^{*c}(p_3, \lambda_3) \epsilon_{\mu_4}(p_4, \lambda_4), \qquad (7)$$

$$\mathcal{A}_q = \sum_{j=1}^{24} f_{A_j}^q T_{A_j}^{\mu_1\mu_2\mu_3\mu_4} \epsilon_{\mu_1}^{*a}(p_1, \lambda_1) \epsilon_{\mu_2}^{*b}(p_2, \lambda_2) \times \epsilon_{\mu_3}^{*c}(p_3, \lambda_3) \epsilon_{\mu_4}(p_4, \lambda_4), \qquad (8)$$

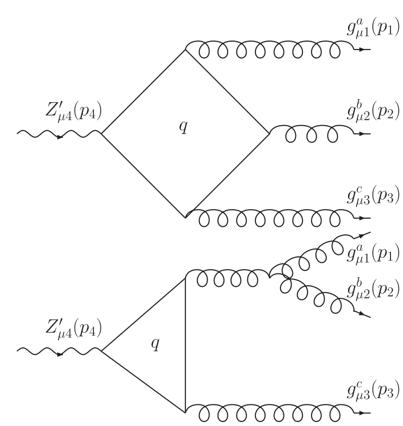

FIG. 2. Examples of box and triangle diagrams contributing to $Z' \to ggg$.

with $f^q_{V,A}$ as finite form factors of the $T^{\mu_1\mu_2\mu_3\mu_4}_{V,A}$ Lorentz structures, which can be expressed in terms of Passarino-Veltman scalar functions. The corresponding $Z' \to ggg$ decay width is given by

$$\begin{aligned}\Gamma(Z'\to ggg) &= \frac{m_{Z'}}{3!256\pi^3}\int_0^1\int_{1-x}^1 |\mathcal{M}|^2 dydx \\ &= \frac{\alpha_s^3(m_{Z'})\alpha N_C^2 m_{Z'}}{384\pi^3 c_W^2 s_W^2}\times\int_0^1\int_{1-x}^1\sum_{k,l=1}^{8} \\ &\times\Bigg[\frac{40}{3}g^{q_k}_{VZ'}g^{q_l}_{VZ'}\bigg(\frac{1}{3}\sum_{\lambda_1,\lambda_2,\lambda_3,\lambda_4}\mathcal{V}_{q_k}\mathcal{V}^*_{q_l}\bigg) \\ &+ 24 g^{q_k}_{AZ'}g^{q_l}_{AZ'}\bigg(\frac{1}{3}\sum_{\lambda_1,\lambda_2,\lambda_3,\lambda_4}\mathcal{A}_{q_k}\mathcal{A}^*_{q_l}\bigg)\Bigg]dydx.\end{aligned} \tag{9}$$

The $Z' \to ggg$ decay width can be written as the sum of three partial widths,

$$\Gamma(Z' \to ggg) = \Gamma_q + \Gamma_{q'} + \Gamma_{qq'}, \tag{10}$$

where Γ_q, $\Gamma_{q'}$, and $\Gamma_{qq'}$ stand for the individual contribution of the SM quarks, the fourth-family quarks, $q_7 = t'$, $q_8 = b'$, and the interference between both classes, respectively. Passarino-Veltman scalar functions are evaluated numerically using FF routines [46]. For the numerical calculations, we use the values from the Particle Data Group [40] for the parameters contained in the amplitude in Eq. (9): $m_u = 0.002\,55$ GeV, $m_d = 0.005\,04$ GeV, $m_s = 0.104$ GeV, $m_c = 1.27$ GeV, $m_b = 4.2$ GeV, $m_t = 171.2$ GeV and $s_W^2 = 0.231\,19$. The mixing angle is set to $\phi = 0$ in the numerical computations.

We will present results for the branching ratio

$$\mathrm{Br}\,(Z' \to ggg) = \frac{\Gamma(Z' \to ggg)}{\Gamma(Z' \to q\bar{q})}. \tag{11}$$

Our results for this observable are shown as a function of $m_{Z'}$ in Figs. 3–6. Results for the first three families are shown in Fig. 3. The first two families yield contributions roughly the same size which are small. In this model, the most important contribution comes from the third family. Furthermore, its interference with the first two families turns out to be constructive, yielding a total branching ratio of the order $1.2 - 2.8 \times 10^{-5}$ for $700\ \mathrm{GeV} \le M_{Z'} \le 1.5$ TeV. Although ruled out by direct searches by CDF and LEP2, it is interesting to note that in the mass region close to the $\bar{t}t$ threshold, the branching ratio abruptly changes due to the opening of this channel in the loops, which produces an imaginary part in the corresponding amplitude. This opening manifests as a dip in the branching ratio at $m_{Z'} = 2m_t$ and a subsequent enhancement above this mass. This enhancement of the branching ratio close to the opening of the $\bar{t}t$ threshold and the fact that the

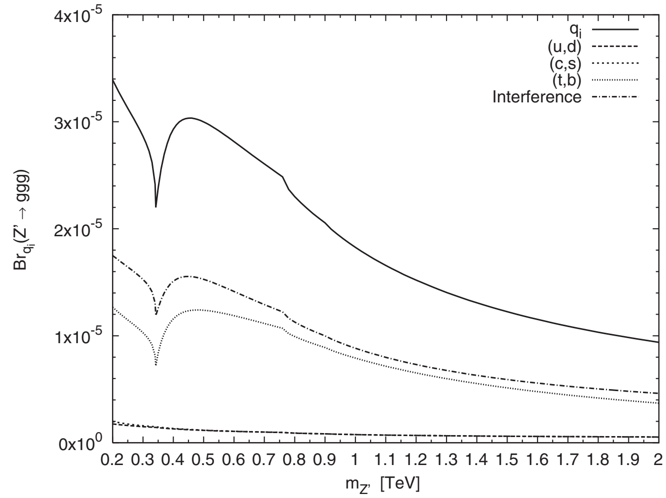

FIG. 3. Separate contribution to $\mathrm{BR}(Z' \to ggg)$ in LRSM for each standard model quark family. The line labeled Interference denotes the interference between the first two families and the third one. The solid line yields the total $\mathrm{BR}(Z' \to ggg)$ in the LRSM.

third generation yields the most important contribution make it appealing to study the effects of a fourth-family of fermions.

We study the effect of a new generation of quarks, considering quark masses barely above the lower bound imposed by direct searches. Explicitly, using as an example $m_{t'} = 450$ GeV, $m_{b'} = 380$ GeV, we obtain the results shown in Fig. 4 from fourth-generation quarks in the loops. Notice that the individual contributions are smaller than those of the standard model quarks and the most important

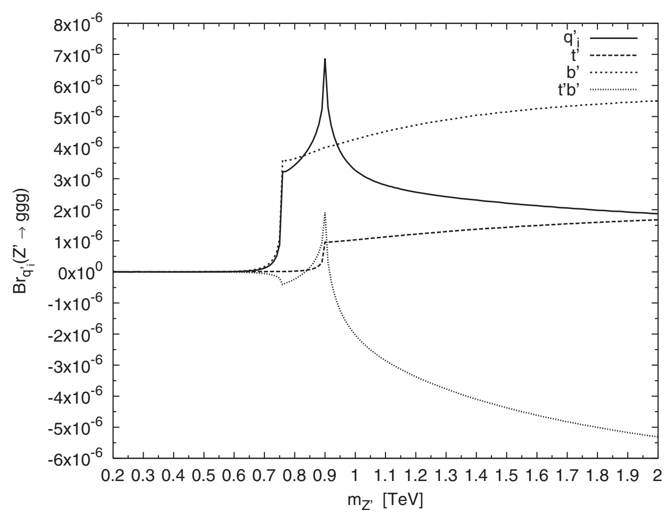

FIG. 4. Contributions from the fourth fermion family to $\mathrm{BR}(Z' \to ggg)$ as a function of $m_{Z'}$ in LRSM4 for $m_{t'} = 450$ GeV, $m_{b'} = 380$ GeV: t' (dashed line), b' (short-dashed line), interference (dotted line), and total (solid line).

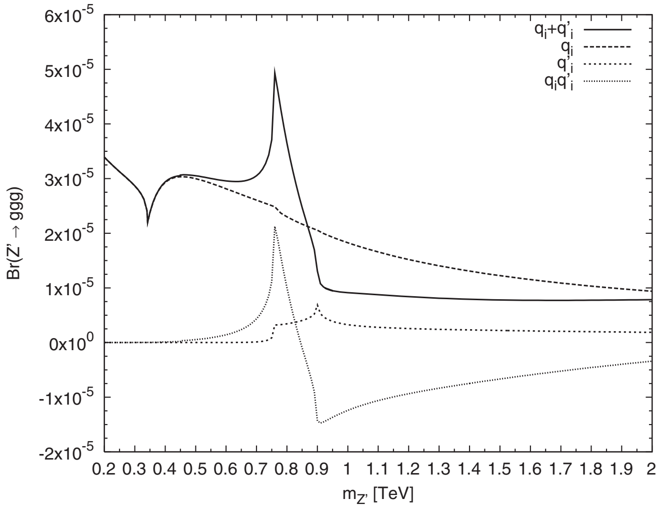

FIG. 5. Contributions to $\mathrm{BR}(Z' \to ggg)$ in LRSM4 for $m_{t'} = 450$ GeV, $m_{b'} = 380$ GeV: standard model quarks (dashed line), fourth-family quarks (short-dashed line), interference (dotted line), and total (solid line).

contributions come from regions in the Z' mass above the $\bar{q}'q'$ threshold, i.e. from the imaginary part in the amplitude associated to on-shell $\bar{q}'q'$ pairs in the loops. Furthermore, in general there is a destructive interference among the t' and b' contributions for the considered masses. This destructive interference can be traced to the opposite sign in the vector and axial couplings, g_V^q and g_A^q, of the Z' shown in Table I and it is complete in the axial case for a degenerate fourth generation. However, in the nondegenerate case there is a small region for $2m_{b'} < m_{Z'} < 2m_{t'}$, where this interference is constructive (the imaginary part of the $\bar{b}'b$ contribution which is dominant has the opposite sign to the real part) and yields contributions of the order of the ones coming from the third generation. This enhancement is quickly surpassed by a similar effect in the t' case rendering a total negative interference for Z' masses above the $\bar{t}'t$ threshold. Nevertheless, an enhancement of the total width in this window for these values of $m_{Z'}$ could be expected similar to the constructive interference between the third family and the first two.

This is indeed the case as can be seen from Fig. 5, where the complete LRSM4 contributions are shown for $m_{t'} = 450$ GeV, $m_{b'} = 380$ GeV. The interference between standard model quarks and fourth-family quarks is constructive for $m_{Z'} \approx 2m_{b'}$ and destructive for $m_{Z'} \geq 2m_{t'}$. This produces an enhancement in the total width for $m_{Z'} \approx 2m_{t'}$ and a strong dip at $m_{Z'} = 2m_{t'}$.

Finally we study the $\mathrm{BR}((Z' \to ggg)$ as a function of the masses of the fourth-generation quarks. Our results are displayed in Fig. 6 and we can see that the general features discussed above do not depend in detail on the specific values of the fourth-generation quark masses, whenever we consider nondegenerate quarks for the fourth generation.

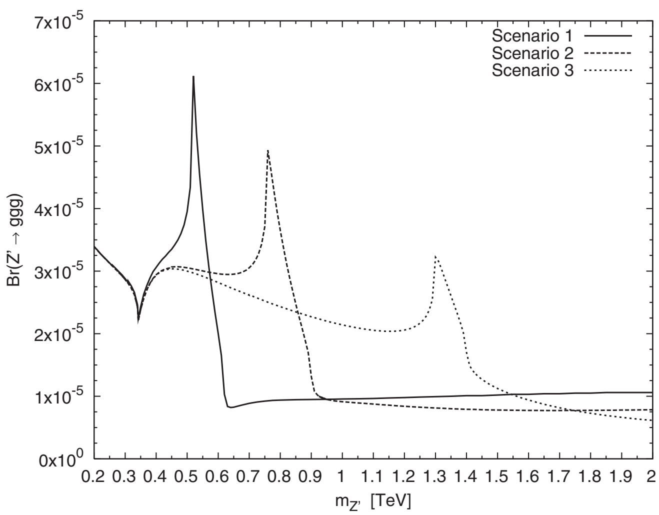

FIG. 6. $\mathrm{BR}((Z' \to ggg)$ for different values of the masses of the fourth-generation quarks: $\{m_{b'} = 260\ \mathrm{GeV}, m_{t'} = 310\ \mathrm{GeV}\}$ (Scenario 1), $\{m_{b'} = 380\ \mathrm{GeV}, m_{t'} = 450\ \mathrm{GeV}\}$ (Scenario 2), and $\{m_{b'} = 650\ \mathrm{GeV}, m_{t'} = 700\ \mathrm{GeV}\}$ (Scenario 3).

IV. SUMMARY

In this work we report a detailed phenomenological analysis of the $Z' \to \bar{q}q, ggg$ decay in a manifest left-right symmetric model with three and four generations. We obtain $Z' \to \bar{q}q \approx 14$ GeV for $m_{Z'} = 1$ TeV which is not strongly influenced by the fourth generation. As to the $Z' \to ggg$ decay, in the conventional (three generations) left-right model, the predicted branching ratio is in the range $\mathrm{BR}(Z' \to ggg) = \frac{\Gamma(Z' \to ggg)}{\Gamma(Z' \to q\bar{q})} = (1.2 - 2.8) \times 10^{-5}$ for $m_{Z'} \in 700$–1500 GeV with the most important contribution coming from the quarks in the third family. For masses of the fourth-family quarks consistent with precision electroweak data we obtain a destructive interference between the up-type (t') and down-type (b') fourth-family quarks due to the opposite sign in the couplings. This interference is complete for the axial part in the case of degenerate quarks. In the case of nondegenerate quarks, the imaginary part of the amplitude coming from the opening of the $\bar{b}'b'$ channel, which has the opposite sign to the real part, produces an enhancement in the window $2m_{b'} < m_{Z'} < 2m_{t'}$ which in turn produces a positive interference with the contributions of the first three families for $m_{Z'} \approx 2m_{b'}$. The branching ratio in the manifest left-right model with four generations lies in the range $(1 - 6) \times 10^{-5}$ for $m_{Z'} \in 700$–1500 GeV.

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[1] J. C. Pati and A. Salam, Phys. Rev. D **10**, 275 (1974); R. N. Mohapatra and J. C. Pati, Phys. Rev. D **11**, 2558 (1975); R. N. Mohapatra and J. C. Pati, Phys. Rev. D **11**, 566 (1975); G. Senjanovic and R. N. Mohapatra, Phys. Rev. D **12**, 1502 (1975); R. N. Mohapatra and P. B. Pal, Phys. Rev. D **38**, 2226 (1988); G. Senjanovic, Nucl. Phys. **B153**, 334 (1979).

[2] R. N. Mohapatra, *Unification and Supersymmetry: The Frontiers of Quark-Lepton Physics* (Springer-Verlag, Berlin, 1986) p. 309.

[3] K. S. Babu and R. N. Mohapatra, Phys. Rev. D **41**, 1286 (1990).

[4] F. Pisano and V. Pleitez, Phys. Rev. D **46**, 410 (1992); P. H. Frampton, Phys. Rev. Lett. **69**, 2889 (1992).

[5] A. Flores-Tlalpa, J. Montaño, F. Ramírez-Zavaleta, and J. J. Toscano, Phys. Rev. D **80**, 033006 (2009).

[6] A. Flores-Tlalpa, J. Montaño, F. Ramírez-Zavaleta, and J. J. Toscano, Phys. Rev. D **80**, 077301 (2009).

[7] J. J. van der Bij and E. W. N. Glover, Nucl. Phys. **B313**, 237 (1989); R. Hopker and J. J. van der Bij, Phys. Rev. D **49**, 3779 (1994); for some reviews on rare Z decays, see E. W. N. Glover and J. J. van der Bij, in "Z *Physics at LEP-1, Proceedings of the Workshop*," CERN Yellow Report No. 89-90, 1989 (unpublished); M. A. Pérez, G. Tavares-Velasco, and J. J. Toscano, Int. J. Mod. Phys. A **19**, 159 (2004).

[8] J. Erler, P. Langacker, S. Munir, and E. R. Pena, J. High Energy Phys. 08 (2009) 017.

[9] M. Maltoni, V. A. Novikov, L. B. Okun, A. N. Rozanov, and M. I. Vysotsky, Phys. Lett. B **476**, 107 (2000).

[10] H. J. He, N. Polonsky, and S. f. Su, Phys. Rev. D **64**, 053004 (2001).

[11] V. A. Novikov, L. B. Okun, A. N. Rozanov, and M. I. Vysotsky, Pis'ma Zh. Eksp. Teor. Fiz. **76**, 158 (2002) [JETP Lett. **76**, 127 (2002)]; Phys. Lett. B **529**, 111 (2002).

[12] G. D. Kribs, T. Plehn, M. Spannowsky, and T. M. P. Tait, Phys. Rev. D **76**, 075016 (2007).

[13] M. Hashimoto, Phys. Rev. D **81**, 075023 (2010).

[14] P. Q. Hung and M. Sher, Phys. Rev. D **77**, 037302 (2008).

[15] J. Erler and P. Langacker, Phys. Rev. Lett. **105**, 031801 (2010).

[16] O. Eberhardt, A. Lenz, and J. Rohrwild, Phys. Rev. D **82**, 095006 (2010).

[17] P. H. Frampton, P. Q. Hung, and M. Sher, Phys. Rep. **330**, 263 (2000).

[18] B. Holdom, W. S. Hou, T. Hurth, M. L. Mangano, S. Sultansoy, and G. Unel, PMC Phys. A **3**, 4 (2009).

[19] B. Holdom, Phys. Rev. Lett. **57**, 2496 (1986); **58**, 177(E) (1987).

[20] C. T. Hill, M. A. Luty, and E. A. Paschos, Phys. Rev. D **43**, 3011 (1991).

[21] J. Carpenter, R. Norton, S. Siegemund-Broka, and A. Soni, Phys. Rev. Lett. **65**, 153 (1990).

[22] P. Q. Hung and C. Xiong, Nucl. Phys. **B847**, 160 (2011).

[23] P. Q. Hung and C. Xiong, Phys. Lett. B **694**, 430 (2011).

[24] D. Delepine, M. Napsuciale, and C. A. Vaquera-Araujo, Phys. Rev. D **84**, 033008 (2011).

[25] P. Q. Hung, Phys. Rev. Lett. **80**, 3000 (1998).

[26] S. W. Ham, S. K. Oh, and D. Son, Phys. Rev. D **71**, 015001 (2005).

[27] M. S. Carena, A. Megevand, M. Quiros, and C. E. M. Wagner, Nucl. Phys. **B716**, 319 (2005).

[28] R. Fok and G. D. Kribs, Phys. Rev. D **78**, 075023 (2008).

[29] Y. Kikukawa, M. Kohda, and J. Yasuda, Prog. Theor. Phys. **122**, 401 (2009).

[30] W. S. Hou, Chin. J. Phys. **47**, 134 (2009).

[31] W. -S. Hou, Y. -Y. Mao, and C. -H. Shen, Phys. Rev. D **82**, 036005 (2010).

[32] A. Soni, A. K. Alok, A. Giri, R. Mohanta, and S. Nandi, Phys. Lett. B **683**, 302 (2010).

[33] W. S. Hou, M. Nagashima, and A. Soddu, Phys. Rev. Lett. **95**, 141601 (2005); W.-S. Hou, M. Nagashima, G. Raz, and A. Soddu, J. High Energy Phys. 09 (2006) 012.

[34] W. S. Hou, H. n. Li, S. Mishima, and M. Nagashima, Phys. Rev. Lett. **98**, 131801 (2007); W.-S. Hou, M. Nagashima, and A. Soddu, Phys. Rev. D **76**, 016004 (2007).

[35] P. Duka, J. Gluza, and M. Zralek, Ann. Phys. (N.Y.) **280**, 336 (2000).

[36] M. Czakon, M. Zralek, and J. Gluza, Nucl. Phys. **B573**, 57 (2000).

[37] M. Czakon, J. Gluza, and M. Zralek, Phys. Lett. B **458**, 355 (1999).

[38] R. N. Mohapatra and R. E. Marshak, Phys. Lett. **91B**, 222 (1980); A. Davidson, Phys. Rev. D **20**, 776 (1979).

[39] R. N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. **44**, 912 (1980).

[40] C. Amsler *et al.* (Particle Data Group) Phys. Lett. B **667**, 1 (2008).

[41] T. Aaltonen *et al.* (CDF Collaboration), Phys. Rev. Lett. **104**, 091801 (2010).

[42] A. Lister (CDF Collaboration), arXiv:0810.3349.

[43] W. J. Marciano, G. Valencia, and S. Willenbrock, Phys. Rev. D **40**, 1725 (1989).

[44] H. Gies and C. Wetterich, Phys. Rev. D **65**, 065001 (2002).

[45] P. Nath *et al.*, Nucl. Phys. B, Proc. Suppl. **200-202**, 185 (2010).

[46] G. J. van Oldenborgh, Comput. Phys. Commun. **66**, 1 (1991).