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Time-dependent *CP* asymmetries in *D* and *B* decays

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We examine measurements of time-dependent *CP* asymmetries that could be made in new and future flavour facilities. In charm decays, where they can provide a unique insight into the flavor changing structure of the Standard Model, we examine a number of decays to *CP* eigenstates and describe a framework that can be used to interpret the measurements. Such measurements can provide a precise determination of the charm mixing phase, as well as constraints on the Standard Model description of *CP* violation an possible new physics contributions. We make a preliminary assessment, based on statistical considerations, of the relative capabilities of LHCb with data from pp collisions, with Belle II and Super *B* using data from B_d, B_s and charm thresholds. We discuss the measurements required to perform direct and indirect tests of the charm unitarity triangle and its relationship with the usual B_d triangle. We find that, while theoretical and experimental systematic uncertainties may limit their interpretation, useful information on the unknown charm mixing phase, and on the possible existence of new physics can be obtained. We point out that, for B_d decays, current experimental bounds on $\Delta\Gamma_{B_d}$ will translate into a significant systematic uncertainty on future measurements of $\sin 2\beta$ from $b \to c\bar{c}s$ decays. The possibilities for simplified B_s decay asymmetry measurements at Super *B* and Belle II are also reviewed.

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I. INTRODUCTION

The standard model (SM) description of quark mixing and *CP* violation is described by the Cabibbo-Kobayashi-Maskawa (CKM) matrix [1,2]. This matrix can be written as

$$V_{\rm CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}, \quad (1)$$

where the V_{ij} are coupling strengths for up-type to down-type quark transitions. Unitarity of the CKM matrix gives rise to six triangles in a complex plane, one of which, the bd triangle, has been extensively studied by the *B* factories, and has earned the name of "The unitarity triangle." The unitarity triangle is[1]

$$V^*_{ud}V_{ub} + V^*_{cd}V_{cb} + V^*_{td}V_{tb} = 0. \quad (2)$$

The angles of the unitarity triangle are $\alpha = (91.4 \pm 6.1)°$, $\beta = (21.1 \pm 0.9)°$, and $\gamma = (74 \pm 11)°$ [3,4]. Given that, for three generations, these angles must add up to 180°, the measurements of α and β provide a strong constraint on the value of γ in the SM. Precision tests of the CKM mechanism have been made only for transitions of down-type-quarks from the second and third generations, and time-dependent *CP* asymmetries have only been measured in the third generation (*B* decays). In order to study the corresponding phenomena with an up-type quark, one has to study the charm decays as top quarks hadronize before being able to form a quasi-stable meson. In this paper, we concentrate on examining approaches that may be usable, at existing and future experimental facilities, in order to study time-dependent *CP* asymmetries in the charm sector, and how results might be interpreted in the context of the CKM matrix. In particular, we focus on ways in which the SM expectations of the "charm triangle," defined in Eq. (3), might be examined—something that has yet to be done. In addition to this, we make a few observations on measurements of B_d and B_s decays in Secs. VIII and IX.

In addition to the bd triangle, unitarity of the CKM matrix also gives rise to the charm (cu) triangle

$$V^*_{ud}V_{cd} + V^*_{us}V_{cs} + V^*_{ub}V_{cb} = 0, \quad (3)$$

which depends on the weak phase γ by virtue of the presence of the factor V_{ub}. The angles of the charm triangle can be written as α_c, β_c, and γ_c. Some time ago, Bigi and Sanda [5] pointed out that $\gamma_c \simeq \gamma$, and $\alpha_c = 180° - \gamma_c + \mathcal{O}(\lambda^4) = 180° - \gamma + \mathcal{O}(\lambda^4)$.[2] Hence the charm and unitarity triangles are related in a simple way. Using the Buras *et al.* variant of the Wolfenstein

[1] We depart from the usual convention by defining the complex conjugates of the triangle sides.

[2] Our nomenclature differs from that used in Ref. [5]. Our angles can be related to theirs as follows: $\beta_c \equiv \phi_3^{cu}$, $\gamma_c \equiv \phi_2^{cu}$, and $\alpha_c \equiv \phi_1^{cu}$.

parameterization [6,7] of the CKM matrix up to $\mathcal{O}(\lambda^5)$, the weak phase β_c can be estimated to be $\sim 0.035°$. Hence the sum of α_c and γ_c should be essentially 180°. Existing constraints on the Wolfenstein parameters can be used to give a clean prediction of the *cu* triangle parameters. In order to verify if the CKM matrix is the correct description of quark mixing, the angles α_c, β_c, and γ_c need to be measured, as well as the sides of this triangle. The e^+e^- collider experiment Super *B* is the only facility where one can, in principle, perform all of the necessary measurements to perform a complete cross-check of the two triangles. This requires large samples of *B*, *D*, and D_s mesons, which Super *B* will accumulate through runs at charm threshold, and at the $\Upsilon(4S)$. In order to interpret time-dependent measurements in terms of angles of the *cu* triangle, a precise measurement of the charm mixing phase is required. We propose that one studies $D \to K^+K^-$ to measure this mixing phase, and the difference between measurements of $D \to K^+K^-$ and $D \to \pi^+\pi^-$ will then give $-2\beta_{c,\mathrm{eff}}$. A single measurement of, or constraint on, β_c, predicted to be $(0.0350 \pm 0.0001)°$, would clearly be of interest, but this will require a careful study of effects of other amplitudes and possible long-range effects. Any observed deviation from this expectation would then be an indication of new physics (NP). Indeed it is worth noting that the measurement of $\sin 2\beta$ from *B* meson decays to final states including Charmonium and a neutral kaon are inconsistent with the SM at the level of 3.2σ [8]. This result is a strong motivation to perform the corresponding studies of the SM in the charm sector, which is the subject of this paper.

The possibility of large *CP* violation effects in charm decays has been discussed elsewhere [9–12], however, until now these have focused on time-integrated measurements, and ignored possible time-dependent effects. It is clear that we need precision experimental tests of the unitarity triangle, in particular, the angle γ, and the sides of both the charm and unitarity triangles. In addition to this we also need to start measuring the charm triangle angles precisely in order to validate the CKM description of *CP* violation for up-type quarks. Theoretical uncertainties will ultimately limit the constraints that can be placed on the SM, and we discuss some of the issues here. The remainder of this paper outlines the details required to perform time-dependent *CP* measurements in the charm sector, using *CP* eigenstate decays, and constraints on the sides of the charm triangle, before making a few observations on B_d and B_s decays.

II. THE CKM MATRIX

The CKM matrix given in Eq. (1) can be parameterized in a number of different ways. The Wolfenstein parameterization [6] is an expansion in terms of $\lambda = \sin\theta_c$, *A*, ρ, and η, where θ_c is the Cabibbo angle. A variant on this parameterization has been proposed Buras *et al.* [7], and has the advantage of preserving unitarity to all orders in λ in the "*bd*" triangle, though possibly not in the "*cu*" triangle. The Buras *et al.* variant of the CKM matrix up to and including terms $\mathcal{O}(\lambda^5)$ is given by

$$V_{\mathrm{CKM}} = \begin{pmatrix} 1 - \lambda^2/2 - \lambda^4/8 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda + A^2\lambda^5[1 - 2(\rho + i\eta)]/2 & 1 - \lambda^2/2 - \lambda^4(1 + 4A^2)/8 & A\lambda^2 \\ A\lambda^3[1 - (1 - \lambda^2/2)(\rho + i\eta)] & -A\lambda^2 + A\lambda^4[1 - 2(\rho + i\eta)]/2 & 1 - A^2\lambda^4/2 \end{pmatrix} + \mathcal{O}(\lambda^6). \quad (4)$$

The choice of convention used to interpret data in terms of physical observables is irrelevant as long as sufficient terms in the expansion are used. Expansions to $\mathcal{O}(\lambda^3)$, have been sufficient for the *B* factories era, however one should consider additional terms as we move into the era of LHCb and the Super Flavor Factories (SFF's).

The apex of the unitarity triangle, obtained when Eq. (2) is normalized by $V_{cd}V^*_{cb}$, is given by the coordinates $(\bar{\rho}, \bar{\eta})$, where

$$\bar{\rho} = \rho[1 - \lambda^2/2 + \mathcal{O}(\lambda^4)], \quad (5)$$

$$\bar{\eta} = \eta[1 - \lambda^2/2 + \mathcal{O}(\lambda^4)]. \quad (6)$$

The CKM matrix may be written in terms of $\bar{\rho}$ and $\bar{\eta}$ as

$$V_{\mathrm{CKM}} = \begin{pmatrix} 1 - \lambda^2/2 - \lambda^4/8 & \lambda & A\lambda^3(\bar{\rho} - i\bar{\eta})(1 + \lambda^2/2) \\ -\lambda + A^2\lambda^5[1 - 2(\bar{\rho} + i\bar{\eta})]/2 & 1 - \lambda^2/2 - \lambda^4(1 + 4A^2)/8 & A\lambda^2 \\ A\lambda^3[1 - \bar{\rho} - i\bar{\eta}] & -A\lambda^2 + A\lambda^4[1 - 2(\bar{\rho} + i\bar{\eta})]/2 & 1 - A^2\lambda^4/2 \end{pmatrix} + \mathcal{O}(\lambda^6). \quad (7)$$

TABLE I. Constraints on the CKM parameters A, λ, ρ, η, $\bar{\rho}$, and $\bar{\eta}$ obtained by the UTFit and CKM fitter groups.

Parameter	UTFit	CKM fitter	Mean used
λ	$0.22\,545 \pm 0.000\,65$	$0.22\,543 \pm 0.000\,77$	$0.22\,544 \pm 0.007\,05$
A	0.8095 ± 0.0095	$0.812^{+0.013}_{-0.027}$	0.811 ± 0.015
ρ	0.135 ± 0.021	–	–
η	0.367 ± 0.013	–	–
$\bar{\rho}$	0.132 ± 0.020	0.144 ± 0.025	0.138 ± 0.022
$\bar{\eta}$	0.358 ± 0.012	$0.342^{+0.016}_{-0.015}$	0.350 ± 0.014

Current constraints on the CKM parameters A, λ, ρ, η, $\bar{\rho}$, and $\bar{\eta}$ from global fits [3,13] are given in Table I.

The angles of the unitarity triangle given in Eq. (2) are α, β, and γ, where

$$\alpha = \arg[-V_{td}V_{tb}^*/V_{ud}V_{ub}^*] = (91.4 \pm 6.1)^\circ, \tag{8}$$

$$\beta = \arg[-V_{cd}V_{cb}^*/V_{td}V_{tb}^*] = (21.1 \pm 0.9)^\circ, \tag{9}$$

$$\gamma = \arg[-V_{ud}V_{ub}^*/V_{cd}V_{cb}^*] = (74 \pm 11)^\circ. \tag{10}$$

The most precisely measured angles are α and β using B meson decays into $\rho\rho$ [14,15] and charmonium final states [16,17], respectively. Given unitarity, in the SM with just three generations, only two of these angles are independent, hence γ is, in principle, a redundant cross-check of the CKM matrix.

These angles can also be computed from values and uncertainties for A, λ, $\bar{\rho}$, and $\bar{\eta}$. Taking simple averages of CKM Fitter and UTFit values in Table I, the angles, computed to order λ^6, are

$$\alpha = (89.4 \pm 4.3)^\circ, \tag{11}$$

$$\beta = (22.1 \pm 0.6)^\circ, \tag{12}$$

$$\gamma = (68.4 \pm 3.7)^\circ. \tag{13}$$

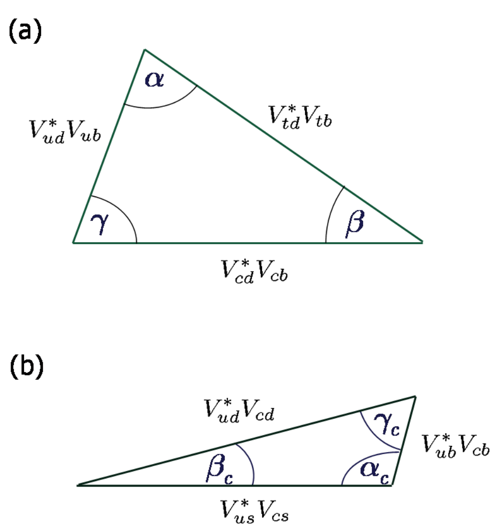

FIG. 1 (color online). (top) The bd unitarity triangle in Eq. (2), and (bottom) the cu unitarity triangle of Eq. (3).

Comparing Eqs. (9) and (10) with Eq. (4), one can see that $V_{td} \simeq |V_{td}|e^{-i\beta}$, and $V_{ub} \simeq |V_{ub}|e^{-i\gamma}$. These relations are exact for low orders of λ, and the equality breaks down as V_{cd} is complex at order λ^5.

The angles of the charm unitarity triangle given in Eq. (3) are

$$\alpha_c = \arg[-V_{ub}^*V_{cb}/V_{us}^*V_{cs}], \tag{14}$$

$$\beta_c = \arg[-V_{ud}^*V_{cd}/V_{us}^*V_{cs}], \tag{15}$$

$$\gamma_c = \arg[-V_{ub}^*V_{cb}/V_{ud}^*V_{cd}], \tag{16}$$

where as already noted $\gamma_c \simeq \gamma$ and $\alpha_c = 180^\circ - \gamma + \mathcal{O}(\lambda^4)$. Again, using the averages of CKM Fitter and UTFit values for A, λ, $\bar{\rho}$, and $\bar{\eta}$ and their errors, we predict that, to order λ^6

$$\alpha_c = (111.5 \pm 4.2)^\circ, \tag{17}$$

$$\beta_c = (0.0350 \pm 0.0001)^\circ, \tag{18}$$

$$\gamma_c = (68.4 \pm 0.1)^\circ. \tag{19}$$

These predictions for the angles of the charm triangle could, and should, be tested experimentally, either directly (through time-dependent *CP* asymmetries) or indirectly (through measurements of the sides of the triangle). On comparing Eq. (15) with Eq. (4), one can see that $V_{cd} = |V_{cd}|e^{i(\beta_c - \pi)}$. Both the bd and cu triangles are shown in Fig. 1.

III. TIME-DEPENDENT EVOLUTION

Neutral meson mixing is a phenomenon that only occurs for K, D, and $B_{d,s}$ mesons (Charge conjugation is implied throughout). Here, in describing the formalism common to these systems, we refer to the mesons as P. The effective Hamiltonian describing neutral meson mixing is given by

$$\mathcal{H}_{\rm eff} = \mathbf{M} - \frac{i\mathbf{\Gamma}}{2}, \tag{20}$$

$$= \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} - \frac{i}{2}\begin{pmatrix} \Gamma_{11} & \Gamma_{12} \\ \Gamma_{21} & \Gamma_{22} \end{pmatrix}. \tag{21}$$

Hence, neutral meson mixing can be described by

$$i\frac{\partial}{\partial t}\begin{pmatrix}|P^0\rangle\\|\bar{P}^0\rangle\end{pmatrix} = \left(M - \frac{i}{2}\Gamma\right)\begin{pmatrix}|P^0\rangle\\|\bar{P}^0\rangle\end{pmatrix}, \quad (22)$$

where $|P^0\rangle$ and $|\bar{P}^0\rangle$ are strong eigenstates of neutral B, D, or K mesons. The matrix elements in Eq. (22) must satisfy $M_{11} = M_{22}$ and $\Gamma_{11} = \Gamma_{22}$ in order to be consistent with CPT symmetry. A further constraint can be obtained in the limit of CP or T invariance, where $\Gamma_{12}/M_{12} = \Gamma_{21}/M_{21}$ must be a real quantity.

One can write the mass eigenstates as an admixture of the strong eigenstates in the following way:

$$|P_{1,2}\rangle = p|P^0\rangle \pm q|\bar{P}^0\rangle, \quad (23)$$

where $q^2 + p^2 = 1$ to normalize the wave function, and

$$\frac{q}{p} = \sqrt{\frac{M_{12}^* - i\Gamma_{12}^*/2}{M_{12} - i\Gamma_{12}/2}}. \quad (24)$$

The magnitude of q/p is very nearly one in the SM. If one considers the mass eigenstates under the CP operator, it follows that $|P_1\rangle$ is CP even, and $|P_2\rangle$ is CP odd. The mass and width differences ΔM and $\Delta\Gamma$ between the mass eigenstates are given by

$$\Delta M = M_2 - M_1, \quad (25)$$

$$\Delta\Gamma = \Gamma_1 - \Gamma_2, \quad (26)$$

where neutral mesons oscillate from particle to antiparticle state with the characteristic mixing frequency ΔM. Detailed discussions of this formalism can be found in a number of text books.

A. Uncorrelated meson production

It can be shown that the general form of the time-evolution of a neutral meson decaying into some final state f is given by

$$\Gamma(P^0 \to f) \propto e^{-\Gamma_1 t}\left[\frac{(1 + e^{\Delta\Gamma t})}{2} + \frac{\mathrm{Re}(\lambda_f)}{1 + |\lambda_f|^2}(1 - e^{\Delta\Gamma t}) + e^{\Delta\Gamma t/2}\left(\frac{1 - |\lambda_f|^2}{1 + |\lambda_f|^2}\cos\Delta M t - \frac{2\mathrm{Im}(\lambda_f)}{1 + |\lambda_f|^2}\sin\Delta M t\right)\right], \quad (27)$$

$$\Gamma(\bar{P}^0 \to f) \propto e^{-\Gamma_1 t}\left[\frac{(1 + e^{\Delta\Gamma t})}{2} + \frac{\mathrm{Re}(\lambda_f)}{1 + |\lambda_f|^2}(1 - e^{\Delta\Gamma t}) + e^{\Delta\Gamma t/2}\left(-\frac{1 - |\lambda_f|^2}{1 + |\lambda_f|^2}\cos\Delta M t + \frac{2\mathrm{Im}(\lambda_f)}{1 + |\lambda_f|^2}\sin\Delta M t\right)\right], \quad (28)$$

where

$$\lambda_f = \frac{q}{p}\frac{\bar{A}}{A}, \quad (29)$$

and A ($\bar{A}$) is the amplitude for the P ($\bar{P}$) decay to a final state f. Note that λ_f is not related to the CKM expansion parameter λ discussed above, but is a complex parameter related to mixing and decay transitions. The time $t = 0$ is defined by the production of a definite meson state (flavor, CP or mixed flavor), that subsequently evolves as a $P - \bar{P}$ admixture until it too decays. The identification of the flavor of a meson state at some fixed point in time is critical for a time-dependent measurement and is discussed in Sec. IV. If $|q/p| \neq 1$, then there is CP violation in mixing, and if $|A|^2 \neq |\bar{A}|^2$, there is direct CP violation, hence a measurement of the real and imaginary parts of λ_f (or equivalently the magnitude and phase) is able to probe the combination of these two effects, i.e. interference between mixing and decay. It should be noted that for all time-dependent CP asymmetry measurements of B_d^0 decays made by experiments until now the assumption that $\Delta\Gamma = 0$ has been used. We discuss the ramifications of this assumption in Sec. VIII.

A time-dependent decay rate asymmetry can be computed from Eqs (27) and (28) as follows:

$$\begin{aligned}\mathcal{A}(t) &= \frac{\bar{\Gamma}(t) - \Gamma(t)}{\bar{\Gamma}(t) + \Gamma(t)},\\ &= 2e^{\Delta\Gamma t/2}\frac{(|\lambda_f|^2 - 1)\cos\Delta M t + 2\mathrm{Im}\lambda_f\sin\Delta M t}{(1 + |\lambda_f|^2)(1 + e^{\Delta\Gamma t}) + 2\mathrm{Re}\lambda_f(1 - e^{\Delta\Gamma t})},\end{aligned} \quad (30)$$

where $\bar{\Gamma}(t)$ and $\Gamma(t)$ are the time-dependent rates, respectively, for $\bar{P}^0 \to \bar{f}$ and $P^0 \to f$ transitions. The asymmetry depends on the real and imaginary parts of λ_f as well as $|\lambda_f|^2$, hence it is possible to extract λ_f from data in terms of only two parameters, the real and imaginary parts of λ_f, as $|\lambda_f|^2$ is completely correlated with those parameters. This formalism is normally written in terms of hyperbolic functions, and one can derive those results by combining the exponential factors in the equations above. In the limit that $\Delta\Gamma = 0$, Eq. (30) reduces to the familiar result

$$\mathcal{A}(t) = -C\cos\Delta M t + S\sin\Delta M t, \quad (31)$$

where

$$S = \frac{2\mathrm{Im}\lambda_f}{1 + |\lambda_f|^2}, \quad \text{and} \quad C = \frac{1 - |\lambda_f|^2}{1 + |\lambda_f|^2}. \quad (32)$$

This approximation has been used in the B-factory measurements of the angles in the bd unitarity triangle. For future measurements, we note, the validity of the assumption that $\Delta\Gamma = 0$ will need further checking.

B. Correlated production of neutral mesons

Neutral K, D, or B mesons are produced in correlated pairs in e^+e^- collections with center of mass energies corresponding to the ϕ, $\psi(3770)$, or $\Upsilon(4S)$ resonances, respectively. The time-dependence of such mesons is complicated by the issue that the pairs of neutral mesons are produced in a coherent wave function consisting of exactly one $|P^0\rangle$ and one $|\bar{P}^0\rangle$ state until one of the mesons decays and the correlated wave function collapses. At that point in time t_1, the second P meson starts to oscillate with mixing frequency ΔM, until eventually this also decays at some later time t_2. The time-difference Δt between these two meson decays replaces the variable t used to describe the evolution of uncorrelated mesons. The sign of Δt is taken to be the difference between the decay time of a meson into a CP eigenstate minus that of the decay into a flavor specific final state (See Sec. IV). Hence, events where the CP eigenstate decay is the second one to occur have positive values of Δt, and those where the CP eigenstate decay occurs first have negative values of Δt.

The corresponding time-dependence is given by

$$\Gamma(P^0 \to f) \propto e^{-\Gamma_1|\Delta t|}\left[\frac{h_+}{2} + \frac{\mathrm{Re}(\lambda_f)}{1+|\lambda_f|^2}h_- + e^{\Delta\Gamma\Delta t/2}\left(\frac{1-|\lambda_f|^2}{1+|\lambda_f|^2}\cos\Delta M\Delta t - \frac{2\mathrm{Im}(\lambda_f)}{1+|\lambda_f|^2}\sin\Delta M\Delta t\right)\right], \tag{33}$$

$$\Gamma(\bar{P}^0 \to f) \propto e^{-\Gamma_1|\Delta t|}\left[\frac{h_+}{2} + \frac{\mathrm{Re}(\lambda_f)}{1+|\lambda_f|^2}h_- + e^{\Delta\Gamma\Delta t/2}\left(-\frac{1-|\lambda_f|^2}{1+|\lambda_f|^2}\cos\Delta M\Delta t + \frac{2\mathrm{Im}(\lambda_f)}{1+|\lambda_f|^2}\sin\Delta M\Delta t\right)\right], \tag{34}$$

where

$$h_\pm = 1 \pm e^{\Delta\Gamma\Delta t}. \tag{35}$$

Hence, the time-dependent CP asymmetry becomes

$$\begin{aligned}\mathcal{A}(\Delta t) &= \frac{\bar{\Gamma}(\Delta t) - \Gamma(\Delta t)}{\bar{\Gamma}(\Delta t) + \Gamma(\Delta t)} \\ &= 2e^{\Delta\Gamma\Delta t/2}\frac{(|\lambda_f|^2-1)\cos\Delta M\Delta t + 2\mathrm{Im}\lambda_f\sin\Delta M\Delta t}{(1+|\lambda_f|^2)h_+ + 2h_-\mathrm{Re}\lambda_f}\end{aligned} \tag{36}$$

and is similar to that for uncorrelated P^0 production. In this case, however, at $\Delta t = 0$, the two P's are completely correlated[3] so that the decay of either one is "filtered" by the decay mode of the other. When $\Delta\Gamma = 0$, $h_+ = 2$, and $h_- = 0$.

For charm decays, the measured parameters normally used are x and y (or a pair of variables related to x and y by a simple rotation), where

$$x = \frac{\Delta M}{\Gamma}, \quad \text{and} \quad y = \frac{\Delta\Gamma}{2\Gamma}. \tag{37}$$

Current experimental constraints [4] give $x \sim 0.005$ and $y \sim 0.01$. In order to illustrate Eq. (36), the distribution $\mathcal{A}(\Delta t)$ for D^0 decays assuming $\mathrm{Re}\lambda_f = \mathrm{Im}\lambda_f = 1/\sqrt{2}$ is shown in Fig. 2 using $x = 0.005$ and $y = 0.01$ [4]. It is clear from this illustration that oscillations in the charm sector are slow compared with those from B_d or B_s decays, and the CP asymmetry varies almost linearly with Δt. While an asymmetry is observable, one will require large statistics to be accumulated in order to make a nontrivial

[3]E.g., if the first decays to a $CP = -1$ eigenstate then, at $\Delta t = 0$, the other has $CP = +1$ and no odd-CP components will appear in its own decay.

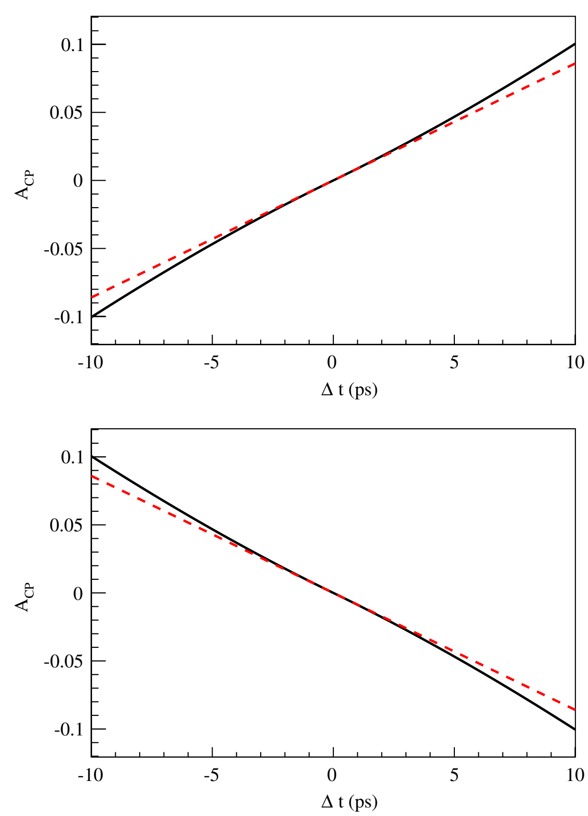

FIG. 2 (color online). Distribution of $\mathcal{A}(\Delta t)$ for D^0 mesons with (top) $\mathrm{Re}\lambda_f = \mathrm{Im}\lambda_f = 1/\sqrt{2}$ and $CP = +1$ and (bottom) the expected asymmetry for $CP = -1$ decay with the same value of λ_f. These distributions assume $q/p = 1$, and the solid (dashed) line corresponds to $y = 0.01$ (0.00). For B_d decays, 1.5 full sinusoidal oscillations are observed in the time interval presented here.

measurement. It should also be noted from Eq. (36) that precise knowledge of both $\Delta\Gamma$ and ΔM will be required in order to translate the slope of the asymmetry into a constraint on λ_f for a given decay channel, as indicated by the two curves shown in Fig. 2.

C. Plausibility of measuring time-dependent *CP* asymmetries

We will be considering three current or planned experimental scenarios where measurements of time-dependence of *CP* asymmetry in charm decays might be possible. These correspond to (a) LHCb, (b) a 2nd generation SFF—either Super *B* [18–20] or Belle II [21,22] running e^+e^- collisions at the $\Upsilon(4S)$, or (c) Charm threshold—Super *B* running at the $\psi(3770)$ ($D^0\bar{D}^0$ threshold where the *D*'s are produced in a coherent state).

Event yields in each scenario will depend on the specific final states considered. Estimates can be made based on the proven performance of *BABAR* and Belle and of the current performance of LHCb [23], assuming that the current trigger efficiencies can be maintained and that the cross section will increase by a factor two for an energy increase to 14 TeV from the current 7 TeV. For the decays to *CP* eigenstates ($D^0 \to K^+K^-$ or $D^0 \to \pi^+\pi^-$, for example) the yield from a 5 fb^{-1} sample at LHCb is likely to be comparable to that expected from each of the SFF's. An upgrade to LHCb could provide a factor 10 more events. Background levels at LHCb are, however, considerably larger than those anticipated at a SFF. For modes with higher multiplicity, lower trigger efficiencies will probably contribute to LHCb sample sizes that are less competitive with the SFF's, again with larger background levels. Event yields at charm threshold will be lower, since the Super *B* luminosity is expected to be smaller at this energy by a factor of 10. Backgrounds, however, will be lower than at the $\Upsilon(4S)$.

To be able to measure t or Δt, *D* mesons must be produced in flight in the laboratory frame of reference. This means that the flight length of the neutral *D* mesons under study needs to exceed the detector resolution associated with reconstructing each final state studied. We see, however, from Fig. 2, that the asymmetry varies almost linearly with decay time so, in principle, asymmetries need only be measured in a few regions of Δt, perhaps even for just the two regions $\Delta t < 0$ and for $\Delta t > 0$.

LHCb should have decay time resolution that is superior to either of the SFF environments. The D^0's at LHCb are produced with momenta of hundreds of GeV/c and the time resolution is generally quite small compared to the D^0 lifetime. The D^0 mesons at LHCb are produced both promptly and in *B* decays, so care is needed to treat each separately. The LHCb trigger has an efficiency that varies with decay length, and a data-driven way to measure this variation is required if a systematic limit in precision is to be avoided. Certainly, a finer granularity in the time-dependence of any observed *CP* asymmetry is surely possible in the LHCb than in either SFF environment.

Time resolution is more of an issue in the SFF environments. However, prompt D^0's from e^+e^- continuum can be cleanly distinguished from those from *B* decay by applying a kinematic cut in momentum. Also, event selection is not based on decay time, so that efficiency does not depend upon decay time.

D^0's produced in $\Upsilon(4S)$ decays have decay lengths dominated by the break-up momentum they acquire. Assuming the performances of Super *B* and Belle II are comparable, respectively, to those of *BABAR* and Belle, the decay time resolution for these D^0's are expected to be of order one-half the D^0 lifetime. This was sufficient for both Belle and *BABAR* to observe mixing in the D^0-$\bar{D}^0$ system. Therefore, with the yields at the Super *B* expected to be at least two orders of magnitude greater, with backgrounds that are similarly small, and with proven data-driven techniques to estimate charge asymmetry effects, observation of *CP* asymmetries and their time-dependence are at least conceivable.

At charm threshold, where coherent D^0 pairs come from decays of the $\psi(3770)$, the break-up momentum is small so that measurements of Δt rely upon the boost from the asymmetric operation of Super *B*. At the $\Upsilon(4S)$, the boost ($\beta\gamma \sim 0.23$) is such that the decay length (one lifetime) for B^0 mesons is approximately 50 μm. Again, this corresponds to a time resolution of about one-half a B^0 lifetime. As estimated by the Super *B* proponents, this is sufficient for measurement of $\sin 2\beta$ to a precision of 0.1° [18]. The D^0 lifetime is, however, 3.8 times shorter than that of the B^0. At the $\psi(3770)$, therefore, a boost ($\beta\gamma$) that is approximately four times larger is required to maintain the same time resolution—providing the detector performance is comparable. Decays of D^0 mesons to *CP* eigenstates have branching fractions about an order of magnitude larger than those of B^0 mesons. If the larger boost is achievable at the $\psi(3770)$, measurements of the angle β_c with a precision similar to that of β are conceivable. A detailed simulation is required to fully understand this, however we discuss results of a simple simulation study in Sec. VII focusing on the potential for measuring time-dependent asymmetries using the $D^0 \to K^+K^-$ and $D^0 \to \pi^+\pi^-$ channels.

IV. FLAVOR TAGGING

Flavor tagging is required to synchronize the time t in the case of uncorrelated decays and Δt for correlated mesons. Flavor tagging works on the principle of identifying flavor-specific final states that can unambiguously be used to determine the flavor of a neutral meson decaying into a *CP* state of interest. A flavor tag has an associated probability that the assignment is incorrect. This so-called mistag probability is denoted by ω, and the figure of merit used to discuss how useful a particular process or set of channels is for flavor tagging is the dilution $D = 1 - 2\omega$. More generally one also considers possible differences between the mistag probability of a particle ω and that of the antiparticle $\bar{\omega}$, where $\Delta\omega = \omega - \bar{\omega}$, and the dilution factor becomes $D + \Delta\omega = 1 - 2\omega + \Delta\omega$.

An important consideration is that the effect of a nonzero value for $\Delta\omega$ is an overall shift in CP asymmetries at all times and is, therefore, functionally similar to the effect of a nonzero value for $|\lambda_f|^2 - 1$. Uncertainty in $\Delta\omega$ is, therefore, strongly correlated with that in $|\lambda_f|$. Any variation in this quantity with decay time must also be well understood if measurements of λ_f are to be meaningful.

A. Flavor tagging of uncorrelated mesons

Flavor tagging of uncorrelated D^0 mesons can be accomplished by identification of ''slow'' (low momentum) pions from the processes $D^{*+} \to D^0\pi^+$ or CP conjugate process is $D^{*-} \to \bar{D}^0\pi^-$. Hence, if one can identify a sample of events where neutral D mesons originate from a $D^{*\pm}$, the charge of the associated pion can be used to infer if the D meson is a D^0 or a $\bar{D}^0$ at the time of decay where $t = 0$. The technical challenge for experiments is in identifying the so-called bachelor $\pi^\pm$ from the $D^{*\pm}$ as this has a low momentum and is therefore more challenging to reconstruct. At the B factories, the D^* from $c\bar{c}$ continuum can be cleanly separated from the $D^\pm$ from B decay by making a momentum cut above the kinematic threshold imposed by the B mass. At LHCb, the majority of the D^* mesons of interest for studying CP asymmetries are secondary particles produced in the primary decay of a B meson. In either case, D^* tagged events will have nontrivial mistag probabilities arising from misreconstruction, wrongly associated slow pions, and from background. A further source of mistagging, though small, could come from mixing of a D^0 used for tagging.

One can account for mistag probabilities by considering the physical decay rates rather than the theoretical ones. These are given by

$$\Gamma^{\rm Phys}(t) = (1 - \omega)\Gamma(t) + \bar{\omega}\,\bar{\Gamma}(t), \qquad (38)$$

$$\bar{\Gamma}^{\rm Phys}(t) = \omega\Gamma(t) + (1 - \bar{\omega})\bar{\Gamma}(t), \qquad (39)$$

where $\Gamma(t)$ and $\bar{\Gamma}(t)$ are from Eqs (27) and (28). It is straightforward to compute the physical CP asymmetry by inserting these results into Eq. (30). Any precision measurement of a CP asymmetry using this method would require detailed control of the systematic uncertainties associated with D^* flavor tagging. This provides a limit on the ultimate precision attainable for a given measurement.

B. Flavor tagging of correlated mesons

The set of flavor specific final states of a D meson can be used to unambiguously identify if a decay into a CP state of interest is that of a D^0 or a $\bar{D}^0$. In analogy with the methods used for B decay tagging (for example, see [17]), one can use a variety of modes for flavor tagging D mesons. The advantage of charm over beauty can be seen, for example, in the use of semileptonic decays for flavor tagging. The decays $D \to K^{(*)-}\ell^+\nu$ account for 11% of all D decays, and unambiguously assign the flavor: a D^0 decay is associated with a ℓ^+ in the final state, and a $\bar{D}^0$ is associated with a ℓ^-. The corresponding situation for tagging D's from B decays is more ambiguous since wrong-sign leptons can arise from decays of B's to $D^*\ell\nu$. In addition, the flavor of each D^0 is unambiguously known at $\Delta t = 0$ in the correlated case. For uncorrelated D^0's, however, the one decaying to a CP eigenstate may have mixed so that its flavor at $t = 0$ is unknown. Thus 11% of all events recorded at the $\psi(3770)$ can be flavor tagged with a mistag probability of essentially zero. Events with kaon or pions in the final state can also be used for flavor tagging, however for these the mistag probability will be nonzero.

From the perspective of performing a precision measurement, which will be an inevitable requirement for testing the SM, minimization of systematic uncertainties will be of paramount importance. Here the benefit of accumulating data at charm threshold is clear as one can choose to restrict the analysis to using only semileptonic tag decays with an 11% efficiency. In doing so, an essentially pure CP sample can be reconstructed with $\omega \simeq \bar{\omega} \simeq 0$.

The viability of including other final states in the tagging algorithm, for example, $D^0 \to K^{*-}(\pi^+, \rho^+)$, etc., introduces experimental issues that may need to be understood. These decays can proceed by a tree-level Cabibbo allowed transition, and the CP conjugate final state can proceed via a doubly Cabibbo suppressed transition. This introduces an ambiguity in the flavor tag assignment (hence dilution), and as D mesons can mix there are several amplitudes from initial to final state. This raises the issue of possible tag-side interference which is a well-known effect for hadronic B tagging [24].

One can account for mistag probabilities by considering the physical decay rates as a function of Δt. These are given by

$$\Gamma^{\rm Phys}(\Delta t) = (1 - \bar{\omega})\Gamma(\Delta t) + \omega\bar{\Gamma}(\Delta t), \qquad (40)$$

$$\bar{\Gamma}^{\rm Phys}(\Delta t) = \bar{\omega}\Gamma(\Delta t) + (1 - \omega)\bar{\Gamma}(\Delta t), \qquad (41)$$

where $\Gamma(\Delta t)$ and $\bar{\Gamma}(\Delta t)$ are from Eqs (33) and (34). Note that the mistag probabilities are interchanged when moving from the uncorrelated (same side tagging) to the correlated (opposite side tagging) case. The CP asymmetry obtained when allowing for tagging dilution is given by

$$\mathcal{A}^{\rm Phys}(\Delta t) = \frac{\bar{\Gamma}^{\rm Phys}(\Delta t) - \Gamma^{\rm Phys}(\Delta t)}{\bar{\Gamma}^{\rm Phys}(\Delta t) + \Gamma^{\rm Phys}(\Delta t)}, \tag{42}$$

$$= -\Delta\omega + \frac{(D+\Delta\omega)e^{\Delta\Gamma\Delta t/2}[(|\lambda_f|^2-1)\cos\Delta M\Delta t + 2{\rm Im}\lambda_f \sin\Delta M\Delta t]}{h_+(1+|\lambda_f|)^2/2 + {\rm Re}(\lambda_f)h_-}. \tag{43}$$

Hence a nonzero mistag probability results in a dilution of the amplitude of oscillation, and any particle-antiparticle mistag probability difference results in an overall offset in the asymmetry.[4] Eq. (43) highlights the attraction of using data from charm threshold to minimize systematic uncertainties associated with tagging. To a good approximation $\Delta\omega = 0$, and $D = 1$ for semileptonic tagged decays, hence the error on λ_f from this source will be relatively small. Furthermore as mentioned above, there is only a single amplitude contributing to the semileptonic tagged side of the event, hence tag-side interference is not an issue. Thus if one observes a nonzero asymmetry, this can readily be identified as a physical effect. For any other tagging category, a significant amount of work would need to be done in order to establish first if the systematic uncertainties were under control in terms of tagging performance, and second, if there is a significant issue related to tag-side interference that could otherwise manifest large fake signals of *CP* violation.

V. ANALYSIS OF *CP* EIGENSTATES

We have considered a number of two- and three-body *CP* eigenstate decays of neutral D mesons in order to determine the CKM element contributions to the decay amplitude, and hence the corresponding weak-phase information that could be extracted from a given decay. The full set of modes is listed in Table II, where we have considered contributions from tree, color-suppressed tree, loop (penguin), and weak-exchange topologies. Possible long-distance contributions have been neglected in this paper. The Feynman diagrams for these topologies, in the case of two body final states, are shown in Fig. 3.

It is clear from Table II that the modes we are considering do not contain contributions from all four topologies, which simplifies the situation somewhat. One should note that the $\pi^0\pi^0$ final state typically consists of four photons, however, it would be possible to reconstruct a vertex and perform a time-dependent analysis for events where photon conversion in detector material had occurred. Also, in about one in 40 instances, one of the π^0's will internally convert in a Dalitz decay $\pi^0 \to e^+e^-\gamma$, in which the e-pair with nonzero opening angle will provide an excellent location of the vertex position.

[4]Note that for uncorrelated decays, one interchanges ω and $\bar{\omega}$, hence the sign of the $\Delta\omega$ terms changes.

In general we are interested in the value of λ_f as given in Eq. (29) when exploring *CP* violation. This can be written as

$$\lambda_f = \left|\frac{q}{p}\right| e^{i\phi_{\rm MIX}} \left|\frac{\bar{A}}{A}\right| e^{i\phi_{CP}}, \tag{44}$$

where $\phi_{\rm MIX}$ is the phase of $D^0\bar{D}^0$ mixing, and ϕ_{CP} is the overall phase of the $D^0 \to f_{CP}$ decay, where f_{CP} is a *CP* eigenstate. The amplitude A in general can have contributions from different topologies, and as a result ϕ_{CP} is not necessarily directly related to an angle of the charm unitarity triangle. This can be seen from the following:

$$A = |T|e^{i\phi_T} + |CS|e^{i\phi_{CS}} + |W|e^{i\phi_W} + \sum_{q=d,s,b} |P_q|e^{i\phi_q}, \tag{45}$$

where the ϕ_j, $j = T, CS, W, q$ are phases of the tree, color-suppressed tree, W exchange, and penguin amplitudes, respectively, and the coefficients of the exponentials are the magnitudes corresponding to those amplitudes. In general one should note that ϕ_j consists of a strong phase (δ_j which is invariant under *CP*) and a weak phase (ϕ_j^W which changes sign under *CP*), thus $\phi_j = \phi_j^W + \delta_j$.

If one considers the tree-dominated decays such as $D \to K^+K^-$, $\pi^+\pi^-$, $K^0K^+K^-$, and $K^0\pi^+\pi^-$, assuming that there is a negligible penguin or color-suppressed tree (and in the case of a $\pi^+\pi^-$ final state, one also neglects W exchange) contribution, then it follows that

$$\lambda_f = \left|\frac{q}{p}\right| e^{i\phi_{\rm MIX}} e^{-2i\phi_T^W}, \tag{46}$$

where $|T|$ and the strong phase $e^{i\delta_T}$ cancel in the ratio of $\bar{A}/A$. While this may be adequate for a rudimentary *CP* asymmetry measurement, eventually it would be necessary to understand the role of the penguin contribution to the two-body final states, and that of the color-suppressed tree for the three-body nonresonant case. It is also clear that in order to interpret any *CP* asymmetry measurement in terms of an angle of the charm triangle, one needs to obtain a precision measurement of q/p in the neutral charm meson system. It should be noted that the same arguments also apply for excited states where, for example, pseudoscalar mesons are replaced by vector or axial-vector particles. For final states with two spin-one particles, one must perform an angular analysis in order to disentangle *CP*-even and *CP*-odd components of the decay.

TABLE II. CP eigenstate modes considered in this paper indicating the topologies contributing to each process in terms of the CKM factors associated with T (tree), CS (color-suppressed tree), P_q (penguin where q is a down-type quark), and W_{EX} (W-exchange) transitions. Blank entries in the table denote that a given topology does not contribute to the total amplitude of the decay, and the relative strengths of these amplitudes decrease from left to right. Nonresonant modes are indicated by NR in order to differentiate from the resonant contributions with the same final state (but different CP eigenvalues and CKM element contributions).

Mode	η_{CP}	T	CS	P_q	W_{EX}
$D^0 \to K^+K^-$	$+1$	$V_{cs}V_{us}^*$		$V_{cq}V_{uq}^*$	$V_{cs}V_{us}^*$
$D^0 \to K_S^0K_S^0$	$+1$				$V_{cs}V_{us}^* + V_{cd}V_{ud}^*$
$D^0 \to \pi^+\pi^-$	$+1$	$V_{cd}V_{ud}^*$		$V_{cq}V_{uq}^*$	$V_{cd}V_{ud}^*$
$D^0 \to \pi^0\pi^0$	$+1$		$V_{cd}V_{ud}^*$	$V_{cq}V_{uq}^*$	$V_{cd}V_{ud}^*$
$D^0 \to \rho^+\rho^-$	±1	$V_{cd}V_{ud}^*$		$V_{cq}V_{uq}^*$	$V_{cd}V_{ud}^*$
$D^0 \to \rho^0\rho^0$	±1		$V_{cd}V_{ud}^*$	$V_{cq}V_{uq}^*$	$V_{cd}V_{ud}^*$
$D^0 \to \phi\pi^0$	$+1$		$V_{cs}V_{us}^*$	$V_{cq}V_{uq}^*$	
$D^0 \to \phi\rho^0$	±1		$V_{cs}V_{us}^*$	$V_{cq}V_{uq}^*$	
$D^0 \to f^0(980)\pi^0$	-1		$V_{cs}V_{us}^* + V_{cd}V_{ud}^*$	$V_{cq}V_{uq}^*$	
$D^0 \to \rho^0\pi^0$	$+1$		$V_{cd}V_{ud}^*$	$V_{cq}V_{uq}^*$	$V_{cd}V_{ud}^*$
$D^0 \to a^0\pi^0$	-1		$V_{cd}V_{ud}^*$	$V_{cq}V_{uq}^*$	$V_{cd}V_{ud}^*$
$D^0 \to K_S^0K_S^0K_S^0$	$+1$				$V_{cs}V_{ud}^* + V_{cd}V_{us}^*$
$D^0 \to K_L^0K_S^0K_S^0$	-1				$V_{cs}V_{ud}^* + V_{cd}V_{us}^*$
$D^0 \to K_L^0K_L^0K_S^0$	$+1$				$V_{cs}V_{ud}^* + V_{cd}V_{us}^*$
$D^0 \to K_L^0K_L^0K_L^0$	-1				$V_{cs}V_{ud}^* + V_{cd}V_{us}^*$
$D^0 \to K_S^0\pi^0$	-1		$V_{cs}V_{ud}^* + V_{cd}V_{us}^*$		$V_{cd}V_{us}^*$
$D^0 \to K_S^0\omega$	-1		$V_{cs}V_{ud}^* + V_{cd}V_{us}^*$		$V_{cd}V_{us}^*$
$D^0 \to K_S^0\eta$	-1		$V_{cs}V_{ud}^* + V_{cd}V_{us}^*$		$V_{cs}V_{ud}^* + V_{cd}V_{us}^*$
$D^0 \to K_S^0\eta'$	-1		$V_{cs}V_{ud}^* + V_{cd}V_{us}^*$		$V_{cs}V_{ud}^* + V_{cd}V_{us}^*$
$D^0 \to K_S^0\pi^+\pi^-$ (NR)	$+1$		$V_{cs}V_{ud}^*$		$V_{cd}V_{us}^* + V_{cs}V_{ud}^*$
$D^0 \to K_S^0\rho^0$	-1		$V_{cs}V_{ud}^* + V_{cd}V_{us}^*$		$V_{cd}V_{us}^*$
$D^0 \to K_S^0K^+K^-$ (NR)	-1	$V_{cd}V_{us}^*$	$V_{cs}V_{ud}^*$		
$D^0 \to K_S^0\phi$	-1				$V_{cs}V_{ud}^* + V_{cd}V_{us}^*$
$D^0 \to K_S^0f^0$	$+1$		$V_{cd}V_{us}^*$		$V_{cd}V_{us}^* + V_{cs}V_{ud}^*$
$D^0 \to K_S^0a^0$	$+1$		$V_{cd}V_{us}^*$		$V_{cd}V_{us}^* + V_{cs}V_{ud}^*$
$D^0 \to K_L^0\pi^0$	-1		$V_{cs}V_{ud}^* + V_{cd}V_{us}^*$		$V_{cd}V_{us}^*$
$D^0 \to K_L^0\omega$	-1		$V_{cs}V_{ud}^* + V_{cd}V_{us}^*$		$V_{cd}V_{us}^*$
$D^0 \to K_L^0\eta$	-1		$V_{cs}V_{ud}^* + V_{cd}V_{us}^*$		$V_{cs}V_{ud}^* + V_{cd}V_{us}^*$
$D^0 \to K_L^0\eta'$	-1		$V_{cs}V_{ud}^* + V_{cd}V_{us}^*$		$V_{cs}V_{ud}^* + V_{cd}V_{us}^*$
$D^0 \to K_L^0\pi^+\pi^-$ (NR)	$+1$		$V_{cs}V_{ud}^*$		$V_{cd}V_{us}^* + V_{cs}V_{ud}^*$
$D^0 \to K_L^0\rho^0$	-1		$V_{cs}V_{ud}^* + V_{cd}V_{us}^*$		$V_{cd}V_{us}^*$
$D^0 \to K_L^0K^+K^-$ (NR)	-1	$V_{cd}V_{us}^*$	$V_{cs}V_{ud}^*$		
$D^0 \to K_L^0\phi$	-1				$V_{cs}V_{ud}^* + V_{cd}V_{us}^*$
$D^0 \to K_L^0f^0$	$+1$		$V_{cd}V_{us}^*$		$V_{cd}V_{us}^* + V_{cs}V_{ud}^*$
$D^0 \to K_L^0a^0$	$+1$		$V_{cd}V_{us}^*$		$V_{cd}V_{us}^* + V_{cs}V_{ud}^*$

In the more general case of two amplitudes contributing to the final state (here we consider the case for a tree and a single penguin contribution P as a simplification), then

$$\lambda_f = \left|\frac{q}{p}\right| e^{i\phi_{\rm MIX}} \frac{e^{-i\phi_T} + re^{-i\phi_P}}{e^{i\phi_T} + re^{i\phi_P}}, \qquad (47)$$

where the penguin to tree ratio $r = |P|/|T|$ is an unknown quantity that needs to be evaluated from data.

If we now return to the amplitudes in Table II, it is possible to determine the relative strengths of the different contributions by considering the number of vertices in the corresponding Feynman diagram and the CKM factors related to these vertices. The distinct products of CKM factors appearing in the table are summarized, up to $\mathcal{O}(\lambda^6)$, in the following:

$$V_{cs}V_{us}^* = \lambda - \frac{\lambda^3}{2} - \left(\frac{1}{8} + \frac{A^2}{2}\right)\lambda^5, \qquad (48)$$

$$V_{cd}V_{ud}^* = -\lambda + \frac{\lambda^3}{2} + \frac{\lambda^5}{8} + \frac{A^2\lambda^5}{2}[1 - 2(\bar{\rho} + i\bar{\eta})], \qquad (49)$$

$$V_{cb}V_{ub}^* = A^2\lambda^5(\bar{\rho} + i\bar{\eta}), \qquad (50)$$

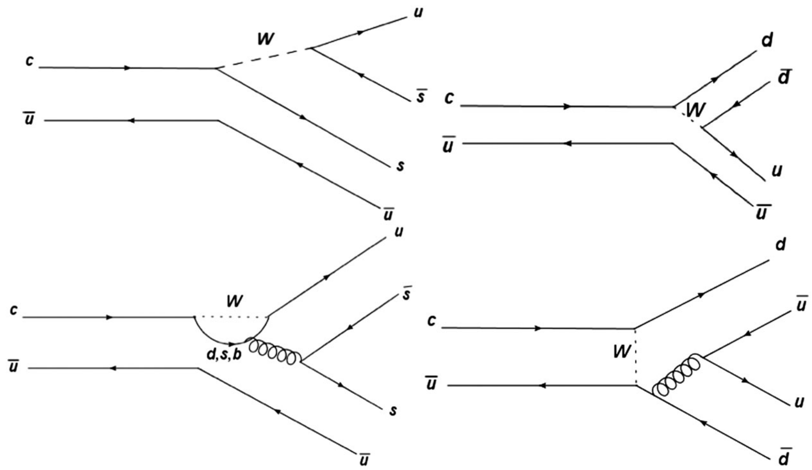

FIG. 3. Feynman diagrams for (top-left to bottom-right) tree, color-suppressed tree, penguin, and W exchange topologies.

$$V_{cs}V_{ud}^{*} = 1 - \lambda^2 - \frac{A^2\lambda^4}{2} + A^2\lambda^6\left[\frac{1}{2} - \bar{\rho} - i\bar{\eta} - \bar{\eta}^2 - \bar{\rho}^2\right], \tag{51}$$

$$V_{cd}V_{us}^{*} = -\lambda^2 + \frac{A^2\lambda^6}{2}[1 - 2(\bar{\rho} + i\bar{\eta})]. \tag{52}$$

Four of the five amplitudes are complex; $V_{cb}V_{ub}^{*}$ has a large phase (γ_c), while $V_{cd}V_{ud}^{*}$ and $V_{cd}V_{us}^{*}$ (both have the phase of V_{cd} which is $\beta_c - \pi$) are related to a small weak phase. The remaining term $V_{cs}V_{ud}^{*}$ also has a small phase, entering at $\mathcal{O}(\lambda^6)$ in the amplitude. It is interesting to note that the amplitude $V_{cb}V_{ub}^{*}$ only proceeds via a penguin process, and is always accompanied by a tree (color allowed or suppressed) and two other penguin amplitudes which will dominate. Hence it is unlikely that one will ever be able to collect data with sufficient statistical precision to measure γ_c from processes involving a $c \to u$ penguin transition.

The next most promising phase to measure is associated with transitions mediated by $V_{cd}V_{ud}^{*}$ where the imaginary component of this amplitude is $\mathcal{O}(\lambda^5)$. Modes involving this transition at leading order include $D \to \pi^+\pi^-$, $\rho^+\rho^-$, h^0h^0, where $h = \pi^0$, ρ^0, a^0. These are discussed in Sec.s V B and V C.

The combination of CKM elements with the smallest phase to this order in λ is $V_{cd}V_{us}^{*}$ which is doubly Cabibbo suppressed. This CKM factor appears in the W exchange amplitudes for $D^0 \to 3K^0$, however, it does so in conjunction with other exchange amplitudes that are Cabibbo allowed. This story is repeated for almost all of the other D^0 modes we consider with a neutral or charged kaon in the final state. The exceptions $D^0 \to K^0 f^0$ and $K^0 a^0$ have a color-suppressed tree proceeding with a CKM factor of $V_{cd}V_{us}^{*}$, and a W exchange amplitude with a factor of both $V_{cd}V_{us}^{*}$ and $V_{cs}V_{ud}^{*}$. Hence while the $\Delta S \neq 0$ modes contain weak phase information, it will be difficult to experimentally distinguish between the amplitudes contributing to the decay and extract a precision measurement of β_c.

A. $D^0 \to K^+K^-$ and related modes

$D^0 \to K^+K^-$ measures the phase of $V_{cd}V_{ud}^{*}$ only in a subdominant penguin transition, and is otherwise dominated by a real tree amplitude with a CKM factor of $V_{cs}V_{us}^{*}$. Hence to first order, one would expect to observe an asymmetry consistent with the mixing phase $\phi_{\rm MIX}$, with no CKM weak phase contribution. This channel provides, therefore, a useful cross-check of detector reconstruction and calibration. It also provides measurements of $|q/p|$ and $\phi_{\rm MIX}$ to complement others that may be available. Given that the SM prediction of the asymmetry in this channel is small, this is also an ideal mode to use when searching for NP. It is interesting to note that V_{cs} is complex at $\mathcal{O}(\lambda^6)$ using the convention of [7]. Ultimately a measurement of β_c could be possible, however, this is not likely to be the most promising mode to measure the angle.

The same is true for the vector-vector final state $K^{*+}K^{*-}$. Using the naive factorization framework the fraction of longitudinally polarized events f_L in the decay of a spin zero meson decaying into two vector mesons can be estimated as [25]

$$f_L = 1 - \frac{m_V^2}{M^2}, \tag{53}$$

where m_V is the vector meson mass, and M is the mass of the decaying parent particle. Using this, we can estimate f_L for $D^0 \to K^{*+}K^{*-}$ to be $\sim$0.77. Hence, one would be required to perform an angular analysis in order to extract CP asymmetry parameters from this decay.

B. $D^0 \to \pi^+\pi^-$ and related modes

$D^0 \to \pi^+\pi^-$ measures the phase of $V_{cd}V_{ud}^{*}$ in the leading-order tree, one of the penguin amplitudes, and the W exchange topologies. Of the remaining two penguin amplitudes that contribute to this decay, one is completely negligible (mediated by a b quark loop) and the other is of the order of λ. The nontrivial penguin topologies are doubly Cabibbo suppressed loops and proceed at order λ^2, where as the tree amplitude is singly Cabibbo suppressed. A rudimentary measurement of this process could in principle ignore the penguin contribution, in which case $\mathrm{Im}\lambda_f \simeq \sin(\phi_{\mathrm{MIX}} - 2\beta_c)$. Thus, there will be a four-fold ambiguity in any measurement of β_c. However, one should note that a more complete analysis would be required in order to extract the weak phase and disentangle the contribution from the $c \to s \to u$ penguin.

Bigi and Sanda have pointed out [5] that there are two isospin amplitude contributions to $D \to \pi^+\pi^-$. Actually, the situation is almost exactly the same as the $B \to \pi\pi$, as we have an isospin $1/2$ meson (a B or a D) decaying into two pions. The only differences are that, in general, we need to assume $\Delta\Gamma \neq 0$, for charm decays, which is a generalization that the existing measurements of $B^0 \to \pi\pi$ have not yet considered, and we neglect the W exchange amplitude (which has the same weak phase as the tree). The ramification of this is straightforward—instead of measuring S and C of Eq. (32) in order to determine the weak phase, one measures the real and imaginary parts of λ as given in Eq. (29). One also measures the amplitudes for the isospin related $\pi^+\pi^-$, $\pi^+\pi^0$, $\pi^0\pi^0$ decays to perform an isospin amplitude decomposition of $\pi\pi$ final states, as described below, in order to disentangle the phase contribution from the tree and penguin amplitudes.

Similar considerations apply to other final states with two-body combinations of $\pi^\pm$, $\rho^\pm$, and $a_1^\pm(1260)$. Such states that include two spin-one particles would require an angular analysis in order to disentangle *CP* even and odd parts and correctly measure the time-dependent *CP* asymmetry parameters. For example, in the $D \to \rho\rho$ case, we expect $f_L \sim 0.83$. As in the B meson system, one can apply the same isospin analysis procedure in order to bound penguins for $D \to \rho\rho$ decays, although one should take care to establish whether there is evidence of any $I = 0$ component arising from the finite width of the ρ [26]. Based on the penguin hierarchy observed in B decays, we expect that, unless long-distance effects play an important role in $c \to u\bar{u}d$ transitions, that $D \to \rho\rho$ might have a smaller penguin contribution than $D \to \pi\pi$. If this turns out to be the case, then $D \to \rho^+\rho^-$ may provide a more precise constraint on β_c than $D \to \pi^+\pi^-$, and should not be overlooked by experimentalists. It should be noted that, while a Quasi-2-Body approach (where the intermediate resonances are treated as particles) may be sufficient for a preliminary study, a full amplitude analysis would eventually be required in order to extract weak phase information from $D \to \rho\rho$ decays.

For decays like, for example, $D \to \rho\pi$, the isospin structure can be more complex, in general [27]. We note, however, that a complete decay amplitude analysis of the $\pi^0\pi^+\pi^-$ Dalitz plot has been performed by both CLEO [28] and by *BABAR* [29] and that, in a subsequent isospin analysis of this three-body final state [30,31], it has been found that the amplitude is dominated by a single ($I = 0$) component. This situation is found to be consistent with a decay model with no penguin contribution [32] but by T, W, and CS amplitudes, all with the same phase. This makes this channel particularly suitable for extraction of β_c. The *BABAR* $\Upsilon(4S)$ sample was very clean and a factor of five larger than for the $\pi^+\pi^-$ channel. A similar statement can be made for CLEOc running at charm threshold. For LHCb, the trigger is known to be less efficient for multibody final states, which in general produce fewer tracks with high transverse momenta to trigger on. As a result, we do not expect LHCb to be able to make a competitive measurement of $D \to \pi^+\pi^-\pi^0$ decays when compared with the potential of future e^+e^- experiments. The analysis of this channel is certainly more complex than that for $\pi^+\pi^-$, but it has been found in both *BABAR* and in Belle experiments that the multibody channels add useful constraints and provide reliable results.

An isospin analysis of $D \to \pi\pi$ *and* $D \to \rho\rho$ *decays*

For these decays, the tree and penguin decay amplitudes are distinguished by their isospin changing structures. The prescription given here parallels the one described in Ref. [33] which outlines how to measure the unitarity triangle angle α from $B \to \pi\pi$ decays and to constrain so-called penguin pollution. Bose symmetry dictates that, for either B^0 or D^0 decays, the two-pion final states can be in either an $I = 0$ or an $I = 2$ final state. In this case, triangular relationships between amplitudes $A^{ij}(\bar{A}^{ij})$ for $D(\bar{D}) \to h^i h^j$ decays ($h = \pi$ or ρ) exist, as follows:

$$\frac{1}{\sqrt{2}} A^{+-} = A^{+0} - A^{00}, \tag{54}$$

$$\frac{1}{\sqrt{2}} \bar{A}^{-+} = \bar{A}^{-0} - \bar{A}^{00}, \tag{55}$$

where the charges are i, $j = +1, -1, 0$. These two triangles can be aligned with a common base given by $A^{+0} = \bar{A}^{-0}$, in which case the angle between A^{+-} and $\bar{A}^{-+}$ is the shift in the measured phase resulting from penguin contributions.

Obviously, one must measure rates for $D^0 \to h^+h^-$, $D^+ \to h^+h^0$, and $D^0 \to h^0h^0$ in order to extract the weak phase of interest: β_c. The amplitude of sinusoidal oscillation given in Eqs. (30) or (36) is related to $\lambda_f = \sin(\phi_{\mathrm{MIX}} - 2\beta_{c,\mathrm{eff}})$. The proposed isospin analysis would enable one to translate a measurement of $\beta_{c,\mathrm{eff}}$ to a constraint on β_c, given a precise determination of the mixing phase and the amplitudes of D decays to hh final states. As final states with more than one neutral particle are required for the isospin analysis, it will only be possible to measure the weak phase using $D^0 \to hh$ decays in an e^+e^- environment. Ultimately, the viability of this method will depend upon theoretical control of any relevant topologies that have been neglected, for instance, long-distance and isospin-breaking effects.

C. $D^0 \to \rho^0\rho^0$ and related modes

$D^0 \to \rho^0\rho^0$ measures the phase of $V_{cd}V_{ud}^*$ via the color-suppressed tree, one penguin, and W exchange amplitudes. Of the remaining two penguin amplitudes that contribute to this decay, one is completely negligible (mediated by a b quark loop) and the other is of the order of λ. Hence, the method to extract the weak phase from this decay is a repeat of the situation for $D^0 \to \pi^+\pi^-$ discussed in Sec. V B. In order to disentangle the penguin contribution to the time-dependent CP asymmetry measurement, one would have to measure $D^0 \to \rho^+\rho^-$, which includes two π^0 mesons in the final state. So once again, this process can only be used to precisely constrain the weak phase in an e^+e^- environment. It should be noted that with $\rho^0\rho^0$, one can easily measure the time-dependent asymmetry, and use the result to reduce the number of ambiguities in the $D \to \rho\rho$ isospin analysis.

D. New physics

The topologies summarized in Table II are conveniently categorized in a way where one can envisage different types of NP affecting the amplitudes contributing to the decay rate. NP can manifest itself in any of the topologies, and while one normally ignores the possibility of NP in tree contributions it is worth noting that the measurement of $\sin 2\beta$ from $B \to J/\psi K^0$ is currently inconsistent with SM expectations at a level of 3.2σ [8]. This highlights the importance of embarking on a quest to measure both the mixing phase and β_c as proposed here. In particular, the penguin amplitudes could be affected by NP in loop transitions mediated via SUSY partners replacing the SM quarks and $W^\pm$. Hence, the modes $D^0 \to h^0h^0$, where $h = \pi^0$, ρ^0, ϕ are particularly good candidates to probe NP manifest through this mechanism. The remaining modes considered here could be used to detect NP contributions from amplitudes that compete with the SM tree or exchange amplitudes. In general, any large observation of CP violation in charm decays is expected to be a sign of NP [34]. If one does observe a signal, then care must be taken in order to disentangle the weak phase of interest from the D^0-$\bar{D}^0$ mixing phase. This, in turn, will require significantly better measurement of mixing parameters than are currently available.

VI. CONSTRAINING THE SIDES OF THE CHARM TRIANGLE

The charm unitarity triangle given in Eq. (3) can also be constrained by measurements of the sides, essentially magnitudes of the elements of the CKM matrix. The difference in the lengths of the two long sides $V_{ud}^*V_{cd}$ and $V_{us}^*V_{cs}$ must be able to accommodate the geometry of the third side indicated in Fig. 1. While the direct measurement of CP violating effects is the focus of this paper, the indirect measurements required to constrain the shape of the triangle independently of CP asymmetry measurements are also important and worthy of a mention. We briefly examine each of these elements in turn in the following to highlight how one can increase current knowledge of the triangle via indirect measurements.

(i) $|V_{ud}|$: This has been precisely measured using nuclear beta decay, and the experimental level of precision reached is at the level of 0.022% [35].

(ii) $|V_{us}|$: This quantity can be measured precisely in kaon decays, however, that has reached a natural conclusion of being dominated by systematic uncertainties. The level of precision reached for this quantity by averaging results from kaon and τ decays is 1% [35]. Future precision measurements of $|V_{us}|$ may be possible via studies of τ decays into final states with charged kaons. Thus the Super B and Belle II experiments will be required to improve our knowledge of this quantity.

(iii) $|V_{ub}|$: The limiting factor for improving constraints on this element comes from a combination of theoretical and experimental issues relating to B decays into semileptonic final states related to $b \to u$ transitions. While there has been a lot of work in this area, there is still a lot of room for improvement both in terms of theoretical and experimental developments. The current level of uncertainty obtained for this quantity is 11% [35]. From the experimental perspective, the inclusive and exclusive results obtained for $|V_{ub}|$ are not in good agreement with each other [36]. Thus the Super B and Belle II experiments will be required to improve our knowledge of this quantity.

(iv) $|V_{cd}|$: Precision measurements of semileptonic D decays can improve our knowledge of $|V_{cd}|$ beyond the current level of precision (4.8% [35]). This measurement can be improved upon by the BES III experiment at IHEP, and also by the Super B and Belle II experiments. Super B will have the advantage of being able to accumulate at data sample 50 times larger than BES III at charm threshold. It is unlikely that Belle II would ultimately be competitive with a measurement of $|V_{cd}|$ as that experiment has no plans to run at charm threshold.

(v) $|V_{cs}|$: The most precise determinations of $|V_{cs}|$ come from measurements of semileptonic D_s decays. The current level of precision obtained for $|V_{cs}|$ is 3.5%. This can be improved by the BES III experiment at IHEP, and also by the Super B experiment, using data collected just above charm threshold.

(vi) $|V_{cb}|$: The limiting factor for improving constraints on this element comes from a combination of theoretical and experimental issues relating to B decays into semileptonic final states related to $b \to c$ transitions. While there has been a lot of work in this area, there is still a lot of room for improvement both in terms of theoretical and experimental developments. The current level of precision achieved by measurements of $|V_{cb}|$ is 3.2% [35]. From the experimental perspective, the inclusive and exclusive results obtained for $|V_{cb}|$ are not in good agreement with each other [36]. Thus Super B and Belle II will be required to improve our knowledge of this quantity.

$|V_{ud}|$ is the most precisely constrained quantity required to reconstruct the triangle using the sides, having been measured to 0.022%. Hence, improved measurements of this quantity will not play an important role in improving our understanding of the charm triangle. All of the other quantities are known to precisions of the order of 1%–10%. Thus in order to improve indirect constraints of the charm triangle, (i) we need to wait for the Super B and Belle II experiments to improve the limiting factors in terms of measuring the above quantities, and (ii) the corresponding theoretical developments should also be pursued in order for experiment to remain a limiting factor. It should also be noted that the BES III experiment will be able to improve the precision of measurements of $|V_{cd}|$ and $|V_{cs}|$ from semileptonic D and D_s decays before the SFF's start collecting data.

Interestingly enough, the quantities $|V_{ub}|$ and $|V_{cb}|$ also currently limit the precision of the sides constraint of the unitarity triangle for B decays, and again the only routes to experimental improvements on that test are via Super B and Belle II.

VII. NUMERICAL ANALYSIS

In this section, we compare the three experimental scenarios, (i) charm threshold (ii) the SFF's at $\Upsilon(4S)$, and (iii) LHCb, relating to the measurement of CP violation in $D^0 \to f_{CP}$ decays, where f_{CP} is a CP eigenstate. We neglect resolution effects related to the reconstruction of vertices in the detector and translation of this spatial distance into values of Δt or t. Finally, based on the expectations from these simulations, we discuss the direct constraint on the apex of the cu triangle in Sec. VII E.

For the numerical analysis and the extrapolation to the expected precision in $\beta_{c,\text{eff}}$, we generate a set of 100 Monte Carlo data samples in each experimental scenario, For Super B running at charm threshold, we do this for both semileptonic and also kaon decays as tags. In each sample, we generate D^0 and $\bar{D}^0$ events with no time-integrated asymmetry, each according to their respective time dependences described in Sec. III. We simulate effects of mistagging either D^0 or $\bar{D}^0$, then perform a binned fit to the resulting asymmetry given in Eq. (43) and the corresponding form in terms of t. In these fits, $\arg(\lambda_f)$ and $|\lambda_f|$ are allowed to vary and the values for ω and $\Delta\omega$ are fixed at those used in the event generation. We repeat this analysis for different possible values of the phase $\arg(\lambda_f)$ from $-10°$ to $+10°$ in 10° steps. As a figure of merit for each experimental scenario, we take the average uncertainty, σ_ϕ, in this phase from the 100 fits, observing that this is consistent with the spread of central values from the individual fits.

A. Charm threshold

D meson pairs produced at the $\psi(3770)$ are quantum-correlated, so that the time evolution is given by Eqs (33) and (34). If one accounts for tagging dilution, then the time-dependent CP asymmetry is given by Eq. (43). On restricting time-dependent analyses to the use only of semileptonic tagged decays, the asymmetry simplifies as there is no dilution, since both ω and $\Delta\omega$ terms can be neglected, and any systematic uncertainty in the asymmetry arising from $D \simeq 1$ becomes small. Furthermore, the $e^+e^- \to \psi(3770)$ environment is extremely clean, so that systematic uncertainties from background contributions are also small and under control. These are important points to stress as we know that the CP phase of interest is expected to be small, hence in order to make a precision measurement, the systematic uncertainties must be minimized.

With 500 fb^{-1} of data at charm threshold, one can expect to accumulate approximately 1.8×10^9 D meson pairs. With a data sample of 281 pb^{-1} CLEO-c obtain 89 $D^0 \to \pi^+\pi^-$ candidates with the other D meson decaying semileptonically into $X^+e\nu_e$. Their efficiency for such events is 50% [37]. Assuming the same efficiency applies,[5] we anticipate that Super B could record 158 000 $Xe\nu_e$ tagged $D^0 \to \pi^+\pi^-$ events, corresponding to 489 500 events when using the full set of $K^{(*)}\ell\nu_\ell$ tagged events, $\ell = e,\ \mu$. We expect about three times the number of events for $D^0 \to K^+K^-$. Figure 4 shows the results obtained for the average uncertainties in the phase $\arg(\lambda_f) \equiv \phi = \phi_{\text{MIX}} + \phi_{CP}$ as a function of that phase.

These results are only for semileptonic tags. We also consider use of hadronically tagged events, for example, $D^0 \to K^-X$ (K^+X), where X is anything, which correspond to 54% (3%) of all neutral D meson decays. From these modes alone, one would expect $\omega \simeq 0.03$, and that the asymmetry in particle identification of K^+ and K^- in the detector will naturally lead to a small, but nonzero value of $\Delta\omega$. We expect that there would be approximately 2.2×10^6 kaon tagged $D^0 \to \pi^+\pi^-$ events in 500 fb^{-1} at charm threshold. Using these data alone, one would be able to measure ϕ to a precision of 4°. Hence, if one combines the results from semileptonic and kaon tagged events, a precision of $\sigma_\phi \sim 3.4°$ is achievable. This represents

[5]Preliminary studies indicate that this is a reasonable assumption.

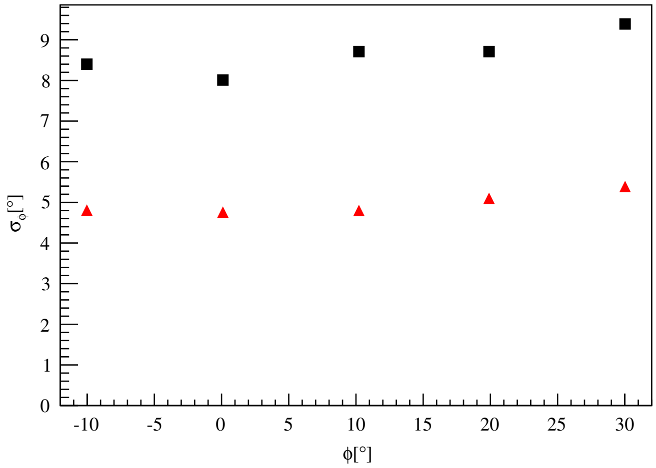

FIG. 4 (color online). The uncertainty in the measured phase $\phi = \phi_{\rm MIX} - 2\beta_c$ as a function of the value of ϕ for (squares) $D^0 \to \pi^+\pi^-$ (triangles) $D^0 \to K^+K^-$ decays at charm threshold with 500 fb^{-1} of data, assuming that the mistag probability is negligible, and only using the full set of semileptonic tagged decays.

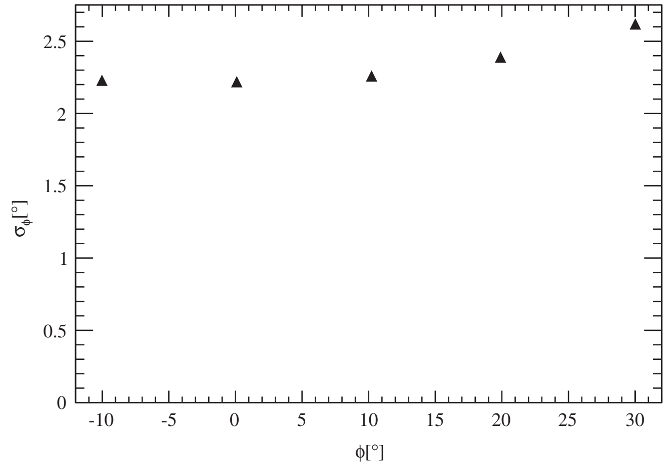

FIG. 5. The uncertainty in the measured phase $\phi = \phi_{\rm MIX} - 2\beta_c$ as a function of the value of ϕ for $D^0 \to \pi^+\pi^-$ decays at the $\Upsilon(4S)$ with 75 ab^{-1} of data assuming $\omega = \bar{\omega} = 0.01$.

a significant improvement in precision over just using semileptonic tagged events.[6]

B. Uncorrelated decays at the $\Upsilon(4S)$

The scenario at the $\Upsilon(4S)$ is somewhat more complicated than the situation encountered at the $\psi(3770)$. First, in order to remove background from D mesons produced in B meson decay, one restricts the analysis to mesons with high momentum. In addition to nontrivial backgrounds, one also has to consider nonzero tagging dilution, where the asymmetry is similar to that given in Eq. (43), but with t substituted for Δt, and ω and $\bar{\omega}$ interchanged (hence a sign flip for the $\Delta\omega$ terms). Thus it is not obvious that $\Delta\omega$ can be neglected, and indeed $D \neq 1$. *BABAR* recorded 30 679 D^* tagged $D^0 \to \pi^+\pi^-$ events at the $\Upsilon(4S)$ in 384 fb^{-1} of data [38], with a purity of 98%, and where the mistag probability for these events is $\sim 1\%$ [39]. From this we estimate that one could reconstruct 6.6×10^6 D^* tagged $D^0 \to \pi^+\pi^-$ events in a data sample of 75 ab^{-1}. We obtain the sensitivities for $\arg(\lambda_f) \equiv \phi$ as a function of the phase shown in Fig. 5 assuming this yield and dilution.

To offset the aforementioned background and dilution issues, the increased boost in this $\Upsilon(4S)$ scenario does slightly reduce the effects of time resolution that are ignored in our analysis here.

C. Uncorrelated decays at LHCb

The final scenario considered is that of measuring time-dependent asymmetries from uncorrelated D mesons in a hadronic environment. Preliminary time-integrated results from CDF [40] and LHCb [23] indicate that such a measurement is possible. Dilution and background effects will, however, be more severe in this hadronic environment than at an e^+e^- machine. The measurement of $|\lambda_f|$ is expected to be dominated by such systematic uncertainties, though $\arg(\lambda_f)$ may be less affected, provided that any variation of ω or $\Delta\omega$ as a function of decay time can be carefully controlled. It is not clear at this point what the ultimate precision obtained from LHCb will be. The best way to ascertain this would be to perform the measurement.

Based on the result in Ref. [23], we estimate that LHCb will collect 7.8×10^6 D^* tagged $D^0 \to \pi^+\pi^-$ decays in 5 fb^{-1} of data, based on an initial 37 pb^{-1} of data. Based on the data shown in the reference, we estimate a purity of $\simeq 90\%$ and $\omega \simeq 6\%$. From these values, we obtain the sensitivities for $\arg(\lambda_f) \equiv \phi$ as a function of the phase shown in Fig. 6 assuming this yield and mistag probability.

D. Summary of sensitivity estimates

Measurements of $\arg(\lambda_f)$ in all scenarios will require good knowledge of the $D^0\bar{D}^0$ mixing parameters. These should be available from SFF's running at $\Upsilon(4S)$ and from LHCb. Super B, for example, expects [18] to measure these with precisions of a few times 10^{-4} for x and y, $\sim 1.5°$ for $\phi_{\rm MIX}$, and a few percent in $|q/p|$. More information comes from the $D^0 \to K^+K^-$ sample, and it is likely that LHCb will further improve these parameters. Hence, in a fit combining these measurements, it will be possible to separate out contributions from the mixing and weak phase in $D^0 \to \pi^+\pi^-$ decays. More accurately, the difference between phases of λ_f measured in $D^0 \to K^+K^-$ and $D^0 \to \pi^+\pi^-$ decays is $\phi_{CP} = -2\beta_{c,\rm eff}$. If loop contributions can be well measured and both long-distance and weak-exchange contributions are negligible, then this constraint can be translated into a measurement of β_c.

[6]Use of the K tag events will introduce tag side interference. For B_d analyses this amounts to a few parts per thousand, but it will need to be evaluated for the specific D modes that are used.

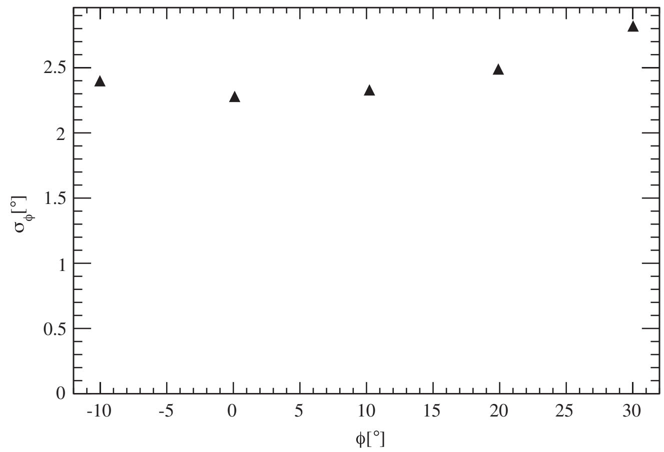

FIG. 6. The uncertainty in the measured phase $\phi = \phi_{\rm MIX} - 2\beta_c$ as a function of the value of ϕ for $D^0 \to \pi^+\pi^-$ decays at LHCb with 5 fb^{-1} of data, assuming $\omega = \bar{\omega} = 0.06$.

In order to relate the measurement of the weak phase, $\beta_{c,\rm eff}$ of λ_f to β_c, one needs to measure a set of isospin related $D \to hh$ decays. This is something that will require the e^+e^- environment, as it will not be possible for LHCb to reconstruct $D^0 \to \pi^0\pi^0$, or $D^0 \to \rho^+\rho^-$. Both $D^+ \to \pi^+\pi^0$ and $D^+ \to \rho^+\rho^0$ would also be challenging measurements for LHCb. Nonetheless, a search for *CP* violation in $D^0 \to \pi^+\pi^-$ and $D^0 \to \rho^0\rho^0$ decays and a measurement of $\phi_{\rm MIX}$ at LHCb would be of considerable interest.

The corresponding sensitivity estimates for arg(λ_f) in the different scenarios considered are summarized in Table III. It should be recalled, however, that effects from time resolution, or from time-dependencies in efficiency or mistag rates are neglected here. We estimate that it should be possible to measure ϕ_{CP} to $\sim$2.6° using this approach. Assuming that penguin contributions can be measured precisely, then the error on β_c that could be obtained by Super *B* would be $\sim$1.3°. LHCb will require input from Super *B* on the decay modes with neutral particles in the final state in order to translate a measurement of $\beta_{c,\rm eff}$ to one on β_c. Further work is required to understand how penguins and other suppressed amplitudes affect the translation of $\beta_{c,\rm eff}$ to β_c, however, it is clear that there will be a significant contribution from penguins given the size of the $D^0 \to \pi^0\pi^0$ branching fraction. It is worth noting that *BABAR* and Belle could be able to make a measurement of $\beta_{c,\rm eff}$ with a precision of $\sim$25°, using the nominal values of *x* and *y* measured for charm mixing available today. We have highlighted several decays that could be used to measure this angle, including $D \to \pi\pi$, $\rho\pi$, $\rho\rho$, $a_1\pi$, $K^0 f^0$, and $K^0 a^0$. Ultimately, it will be important to measure $\beta_{c,\rm eff}$ in each of these modes in order to cross-check the consistency of all of the measurements, constrain NP, and bound possible corrections to the CKM mechanism.

TABLE III. Summary of expected uncertainties from 500 fb^{-1} of data at charm threshold, 75 ab^{-1} of data at the $\Upsilon(4S)$, and 5 fb^{-1} of data from LHCb for $D^0 \to K^+K^-$ and $D^0 \to \pi^+\pi^-$ decays. The column marked SL corresponds to semileptonic tagged events, and the column SL + K corresponds to semileptonic and kaon tagged events at charm threshold.

Parameter	SL	Super *B* SL + K	$\Upsilon(4S)$	LHCb
$\phi(\pi\pi) = \arg(\lambda_{\pi\pi})$	8.0°	3.4°	2.2°	2.3°
$\phi(KK) = \arg(\lambda_{KK})$	4.8°	2.1°	1.3°	1.4°
$\phi_{CP} = \phi_{KK} - \phi_{\pi\pi}$	9.4°	3.9°	2.6°	2.7°
$\beta_{c,\rm eff}$	4.7°	2.0°	1.3°	1.4°

Here, we have concentrated on the determination of the phase arg(λ_f), and one should not neglect the fact that we are also able to constrain $|\lambda_f|$ using these same measurements. An observation of $|\lambda_f| \neq 1$ in data would constitute the measurement of direct *CP* violation in a given decay channel. We estimate that it should be possible to measure $|\lambda_f|$ with a statistical uncertainty of 1%–4% at the future experiments discussed above, though we note that this is limited by any uncertainty in $\Delta\omega$. This is smallest in Super *B* running at charm threshold, but is statistically limited less in other scenarios.

If one compares the relative power of data from charm threshold with that from the $\Upsilon(4S)$, it is clear from Table III that 75(50) ab^{-1} of data at the $\Upsilon(4S)$ is equivalent to approximately 1.2(0.8) ab^{-1} at charm threshold. It is interesting to note that Super *B* proponents expect to accumulate 500 fb^{-1} of data at charm threshold in only three months, whereas 75 ab^{-1} would require five years of running at nominal luminosity. The time-scale involved for the Belle II experimental run at the $\Upsilon(4S)$ is similar to the Super *B* one.

E. Constraint on the *cu* triangle

It is possible to constrain the apex of the *cu* triangle in Fig. 1 by constraining two internal angles, or by measuring the sides. If one considers the representation where the baseline $V^*_{us}V_{cs}$ is normalized to unity, then the angles at vertices corresponding to the coordinates (0, 0) and (1, 0) are β_c and α_c, respectively. The constraint on the apex of the *cu* triangle can be obtained using the CKM prediction of $\gamma_c = (68.4 \pm 0.1)°$ (from the B_d triangle), and any future measurement of β_c. The γ_c constraint is essentially a straight line in the complex plane containing the *cu* triangle. As is the case with the B_d triangle, there are multiple solutions for the apex of the triangle. Even a rudimentary constraint on β_c, made by establishing that $\beta_{c,\rm eff}$ is compatible with zero, would constitute a test of the SM. A precision measurement of $\beta_{c,\rm eff}$ would require a detailed treatment of theoretical uncertainties to determine if any small deviation from the expected value of β_c was due to new physics, or compatible with the SM. This is an area that will require work in the future. We are currently working on determining the effect of penguin pollution in $D \to hh$ decays. In addition to this effect, other potential sources of theoretical uncertainty that may be relevant include isospin-breaking effects, long-distance topologies, or failure of the factorization hypothesis. The coordinates of the apex of the triangle are given by

$$X + iY = 1 + \frac{A^2\lambda^5(\bar{\rho} + i\bar{\eta})}{\lambda - \lambda^3/2 - \lambda^5(1/8 + A^2/2)}, \qquad (56)$$

neglecting contributions from all higher orders in λ. Given that the apex of the bd triangle is $\bar{\rho} + i\bar{\eta}$, one can over-constrain the SM by testing the prediction of $X + iY$ from existing constraints on the apex of the bd triangles. We find that

$$X = 1.00025, \qquad (57)$$

$$Y = 0.00062, \qquad (58)$$

using the existing constraints on the Wolfenstein parameters.

In order to measure $\beta_{c,\text{eff}}$, one needs to precisely constrain ϕ_{MIX}. The current method to measure the mixing phase is via a time-dependent Dalitz plot analysis of D decays to self conjugate final states. Here, we propose to use a time-dependent analysis of decays such as $D \to K^+K^-$, which have an overall phase dominated by the mixing phase in the SM assuming the CKM parameterization, and rate larger than the $\pi\pi$ channel. Having determined ϕ_{MIX}, one can then decouple the mixing phase contribution in $D \to \pi^+\pi^-$ decays, and by performing an isospin analysis one can translate a measurement of λ_f into a constraint on $\beta_{c,\text{eff}}$. Alternatively, one can use a model-independent measurement of the mixing phase, to decouple ϕ_{MIX} and $\beta_{c,\text{eff}}$ from the measurement of λ_f. One would have to control both theoretical and systematic uncertainties to below one per mille in order to be sure of measuring a nonzero value of β_c. At this time, it is unclear if this will be achievable, however, the Super B experiment has the added advantage of being able to study the time-dependence in two ways, and hence may be able to avoid limitations inherent to the D^*-tagged analyses.

VIII. B_d DECAYS

The effect of a nonzero $\Delta\Gamma$ on the time-dependent CP asymmetry distribution is an alteration of the phase of oscillation, and of the amplitude of the oscillation as a function of t or Δt. Until now, all time-dependent CP asymmetry measurements in B_d decays have assumed that $\Delta\Gamma = 0$, which was a reasonable assumption based on theoretical expectations. However, it should be noted that it is possible to bound the systematic uncertainty in the measurement of the unitarity triangle angles α and β by making this assumption using the known experimental constraint on $\text{sign}(\text{Re}\lambda_f)\Delta\Gamma/\Gamma = 0.010 \pm 0.037$ [35]. If one compares the asymmetry obtained assuming $\Delta\Gamma$ corresponding to the experimental bound, then it is possible to estimate the bias and systematic uncertainty time-dependent asymmetry measurements made in B decays arising from the assumption that $\Delta\Gamma = 0$.

We have performed a Monte Carlo-based simulation for the scenario of $S = 0.7$ and $C = 0.0$ taking the uncertainty in $\Delta\Gamma$ to be Gaussian. The ratio of amplitudes for the first maximum/minimum obtained as an estimate of the systematic effect on $S = \sin 2\beta$ is 0.007 ± 0.027, and the corresponding distribution is shown in Fig. 7. This is comparable to the statistical uncertainty in sin2 β measurements [16,17].

Moving on to the measurements related to α, if one considers $B^0 \to \pi^+\pi^-$ decays, where $S = -0.65 \pm 0.07$ and $C = -0.38 \pm 0.06$ [41,42], then the systematic uncertainty in the measurement of S and C is 0.009 ± 0.032. In this case, the systematic effect resulting from the assumption that $\Delta\Gamma = 0$ is also nontrivial, but does not dominate the total uncertainty. The most important channel for the constraint on α is, however, $B^0 \to \rho^+\rho^-$, where $S = -0.05 \pm 0.17$ and $C = -0.06 \pm 0.13$ [14,15]. The corresponding systematic effect on S and C is -0.008 ± 0.038, which is currently small compared to the experimental determination of those quantities. Therefore, while the $\Delta\Gamma = 0$ bound may impact upon the β constraint imposed on the unitarity triangle, it will have little effect on the measurement of α.

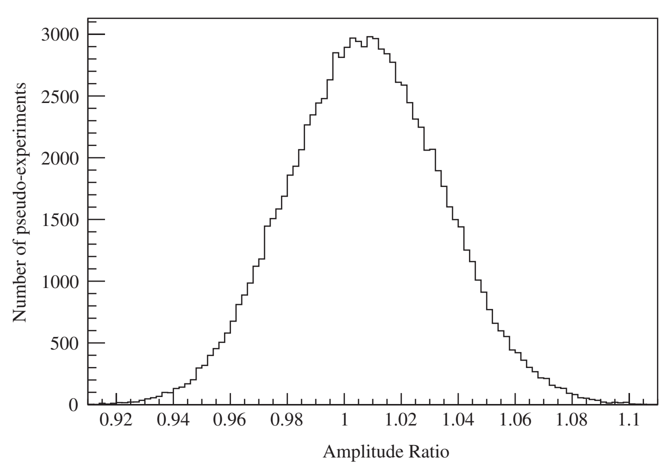

FIG. 7. The bias on $S = \sin 2\beta$ obtained from a Monte Carlo-based simulation resulting from the assumption that $\Delta\Gamma = 0$. The amplitude ratio plotted is that of the maximum time-dependent amplitude accounting for a nonzero $\Delta\Gamma$ to that where $\Delta\Gamma = 0$.

Therefore, current and future experiments aimed at performing a precision measurement of time-dependent *CP* asymmetries should also strive to increase the precision of the bound on $\Delta\Gamma$ to ensure that this systematic effect does not dominate future measurements.

IX. B_s DECAYS

Oscillations in B_s decays are extremely fast relative to B_d and D mesons, and so neither Super B or Belle II are expected to be able to perform time-dependent asymmetry measurements in B_s decays. It should, however, be noted that if these experiments were to accumulate large samples of events at the $\Upsilon(5S)$, then the distribution of events as a function of Δt would contain information on both the real and imaginary parts of λ_f. Hence, some information from *CP* asymmetries related to the time-dependent measurements being done at hadron collider experiments would be measurable in an e^+e^- environment. This was also discussed in [43] in the context of measurements of $B_s \to J/\psi\phi$ at Super B. This issue is particularly relevant for final states including neutral particles such as $B_s \to \eta'\phi$, the B_s equivalent to the most precisely measured golden $b \to s$ penguin mode $B^0 \to \eta' K^0$. It would be extremely challenging to study this mode in a hadronic environment and so the best way to study *CP* violation in this mode would be using data collected at the $\Upsilon(5S)$.

Other interesting decays to study are $B_s \to \rho K_S^0$, $D_s^\pm K^\mp$, and $B_s \to D\phi$ as these measure γ [44–46]. It would be interesting to compare the values obtained from a B_s decay with the result from the $B_d \to DK$ approach currently being used by experiments. It should be noted that LHCb should be able to perform time-dependent measurements of these modes. Finally, as noted in Ref. [35], the channel $B_s \to \pi^0 K_S^0$ is equivalent to the channel $B_d \to \pi^+\pi^-$. Therefore, it would be interesting to attempt to measure λ_f for this decay. Given the π^0 in the final state, and lack of information to constrain a primary vertex, this could be an excellent candidate for Super B or Belle II to study.

X. CONCLUSIONS

We have outlined the formalism required to experimentally measure time-dependent *CP* asymmetries in charm decays using correlated $D^0\bar{D}^0$ decays as well as D^0 mesons tagged from D^* decays, and discussed the benefits of studying a number of different *CP* eigenstates. The important points to note are that one can use K^+K^- decays to measure the mixing phase quite precisely and other decays can be used to constrain the angle $\beta_{c,\mathrm{eff}}$ which is related to the *cu* unitarity triangle. These observables are also sensitive to possible enhancements from new physics. A data sample of 500 fb^{-1} collected at charm threshold would provide a sufficient test to constrain any potential large NP effects. Similar measurements would also be possible using D^*–tagged decays at Super B, Belle II and LHCb. From event yields currently available, we expect the statistical precision in the measured phase at Super B to be slightly better than results from a 5 fb^{-1} LHCb run. As the *cu* and the *bd* unitarity triangles are related, the measurements proposed here provide a new set of consistency checks on the unitarity of the CKM matrix that can be performed using D decays. Measurements of the sides of these triangles would enable a further, indirect cross-check on the validity of this matrix. Only the Super B experiment will be able to make a complete set of the measurements required to perform direct and indirect constraints of both triangles. As β_c is an extremely small angle, its determination will be limited by theoretical and systematic uncertainties. Super B has a potential advantage over other experiments as it will be able to collect data at charm threshold with a boosted center of mass, as well as being able to explore effects using neutral mesons from D^*-tagged events. Data from charm threshold will be almost pure, with a mistag probability of ~ 0 for semileptonic tagged events, which could be advantageous if systematic uncertainties dominate measurements from $\Upsilon(4S)$ data and from LHCb. The ultimate theoretical uncertainty in relating $\beta_{c,\mathrm{eff}}$ to β_c needs to be evaluated. A measurement of $|\lambda_f| \neq 1$ could also signify direct *CPV*.

We also point out that precision measurements of time-dependent asymmetries in B_d decays require improvements in our knowledge of $\Delta\Gamma_{B_d}$. The current experimental constraint on this observable translates into a systematic effect of the order of 0.007 ± 0.027, which is comparable with the current experimental sensitivity on $\sin 2\beta$ from *BABAR* and Belle. We have also computed the systematic effect of assuming $\Delta\Gamma_{B_d} = 0$ for measurements of α from $B^0 \to \pi^+\pi^-$ and $B^0 \to \rho^+\rho^-$ decays, which is negligible for existing measurements.

It may be possible to measure the real and imaginary parts of λ_f from a simplified time-dependent analysis of B_s decays at Super B and Belle II without the need to observe oscillations. While the approach outlined would not be competitive with modes that could be measured in a hadronic environment, it would provide unique access to observable channels that would be inaccessible to the Tevatron and LHCb. The prime example is that of $B_s \to \eta'\phi$, which is the direct analog of the most precisely measured $B_d^0 \to s$ penguin mode $B_d^0 \to \eta' K^0$ from the B factories.

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